# VII ONTQC

# Recent results on localized structures

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# Contents

# Introduction

- Localized structures arise in Field Theory under the action of scalar and other fields. In particular, one may find kinks, vortices and monopoles;

- Kinks — one spatial dimension, $Z_2$ symmetry;

- Vortices — two spatial dimensions, $U(1)$ symmetry;

- Monopoles — three spatial dimensions, $SU(2)$ symmetry;

- Motivation:
— $U(1) \times U(1)$: superconducting strings [E. Witten, NPB **249**, 557 (1985)];
— Circumventing Derrick's theorem [D. Bazeia, J. Menezes and R. Menezes, PRL **91**, 241601 (2003)];
— Bimagnetic core/shell nanoparticles [M. Estrader et al., Nat. Commun. **4**, 2960 (2013)].

# Vortices

[H.B. Niesen and P. Olesen, NPB **61**, 45 (1973)]

$$S = \int d^2r\,dt \left( -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} + \overline{D_\mu \varphi} D^\mu \varphi - V\left( |\varphi| \right) \right)$$

Ansatz: $A_0 = 0$,

$$\varphi = g(r)\, e^{in\theta} \qquad \text{e} \qquad \overrightarrow{A} = \frac{\hat{\theta}}{r} \left( n - a(r) \right).$$

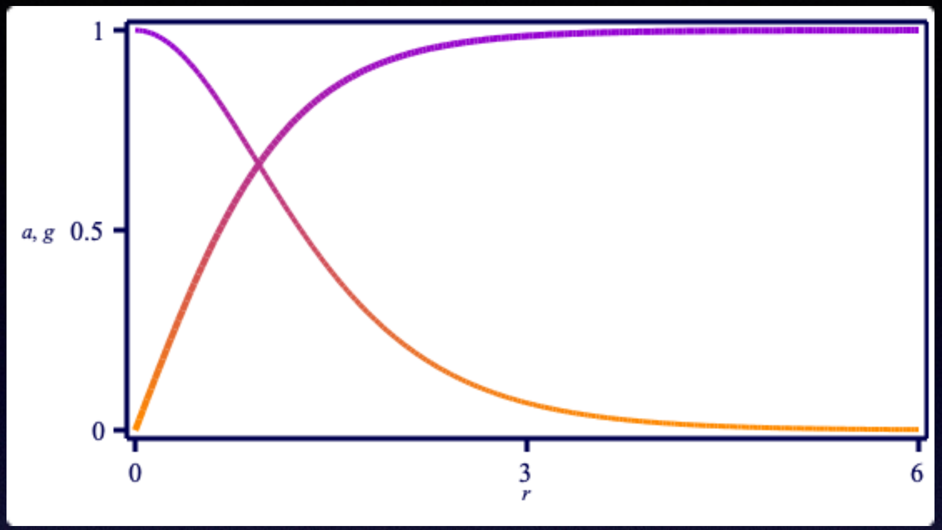
$$a(0) = n,\; g(0) = 0,\; \lim_{r\to\infty} a(r) = 0,\; \lim_{r\to\infty} g(r) = 1.$$

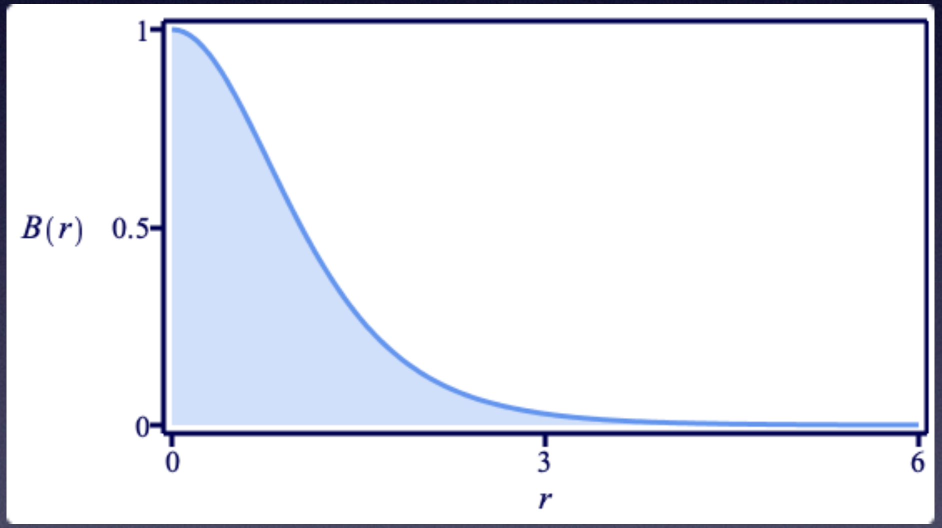
$$B = -\frac{1}{r}\frac{da}{dr} \implies \Phi = 2\pi\, n.$$

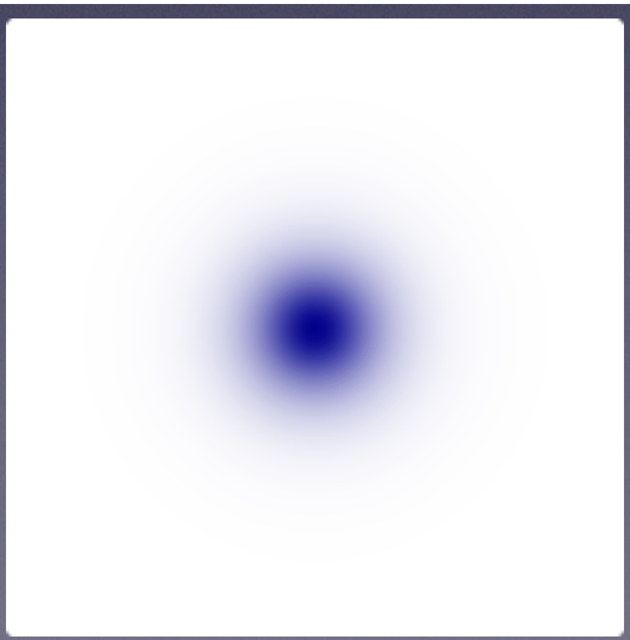
BPS formalism: [E.B. Bogomol'nyi, Sov. J. Nucl. Phys. **24,** 449 (1976)]
[M.K. Prasad and C.M. Sommerfield, PRL **35**, 760 (1975)]

$V(|\varphi|) = \frac{1}{2}\left(1 - |\varphi|^2\right)^2 \implies$ energy is minimized to $E = 2\pi |n|$ for

$$\frac{dg}{dr} = \pm \frac{ag}{r} \qquad \text{e} \qquad -\frac{1}{r}\frac{da}{dr} = \pm \sqrt{2V(g)}\,.$$



# Monopoles

[G. 't Hooft, NPB **79**, 276 (1974); A. M. Polyakov, JETP Lett. **20**, 194 (1974)]

$$S = \int d^3r\, dt \left[ -\frac{1}{4} F^a_{\mu\nu} F^{a\mu\nu} - \frac{1}{2} D_\mu \phi^a D^\mu \phi^a - V(|\phi|) \right].$$

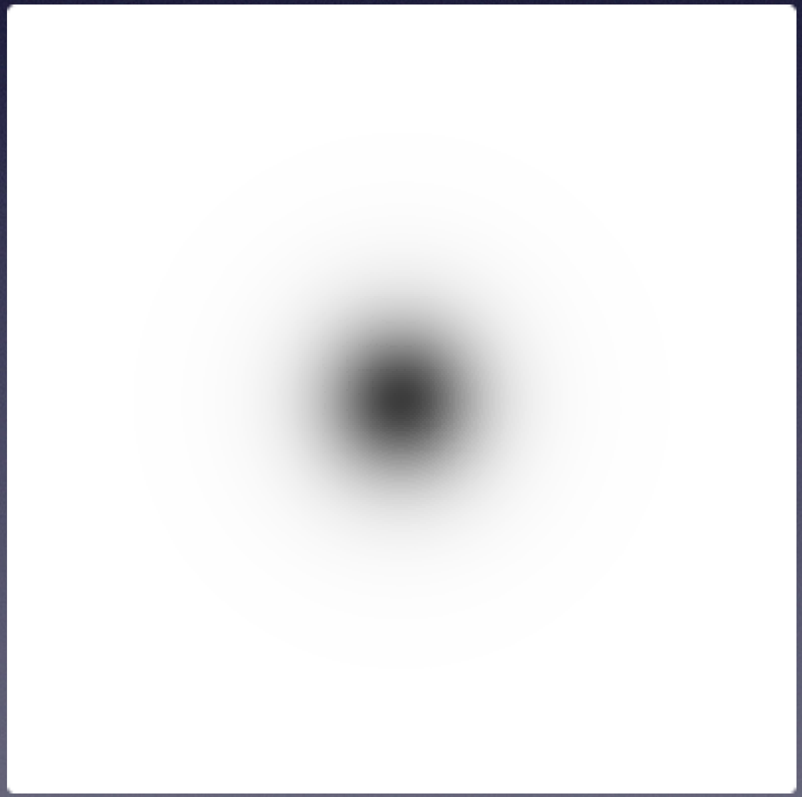
$\eta_{\mu\nu} = \mathrm{diag}\,(-,+,+,+)$

*Ansatz:* $A_0^a = 0$

$$\phi^a = \frac{x_a}{r} H(r) \quad \text{and} \quad A_i^a = \epsilon_{aib} \frac{x_b}{r} \big(1 - K(r)\big).$$

$$H(0) = 0,\ K(0) = 1,\ \lim_{r\to\infty} H(r) = 1,\ \lim_{r\to\infty} K(r) = 0.$$

$$\rho = \frac{K'^2}{r^2} + \frac{(1-K^2)^2}{2r^4} + \frac{1}{2} H'^2 + \frac{H^2 K^2}{r^2} + V.$$



BPS:

$$V \to 0 \implies H(r) = \coth(r) - \frac{1}{r}, \quad K(r) = r\,\mathrm{csch}(r).$$

# Vortices with internal structure

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## Maxwell–Higgs vortices with internal structure

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$$S = \int d^2r\, dt \left( -\frac{P(\chi)}{4} F_{\mu\nu}F^{\mu\nu} + \overline{D_\mu\varphi} D^\mu\varphi + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V\left(|\varphi|,\chi\right) \right)$$

$U(1) \times Z_2$ symmetry

# Vortices with internal structure

*Ansatz:* $A_0 = 0,$

$$\varphi = g(r)\, e^{in\theta}, \qquad \overrightarrow{A} = \frac{\hat{\theta}}{r}\left(n - a(r)\right) \qquad \text{e} \qquad \chi = \chi(r)\,.$$

$$\chi(0) = \chi_0, \qquad \lim_{r\to\infty} \chi(r) = \chi_\infty$$

$$B = -\frac{1}{r}\frac{da}{dr} \implies \Phi = 2\pi\, n.$$

Quantized flux

Equations of motion:

$$\frac{1}{r}\left(r\chi'\right)' = P_\chi \frac{a'^2}{2e^2r^2} + V_\chi,$$

$$\frac{1}{r}\left(rg'\right)' = \frac{a^2 g}{r^2} + \frac{1}{2}V_{|\varphi|},$$

$$r\left(P\frac{a'}{er}\right)' = 2eag^2\,.$$

# Vortices with internal structure

$$\rho = P(\chi)\frac{a'^2}{2r^2} + g'^2 + \frac{a^2 g^2}{r^2} + \frac{1}{2}\chi'^2 + V$$

$$\rho = \frac{P(\chi)}{2}\left(\frac{a'}{r} \pm \frac{(1-g^2)}{P(\chi)}\right)^2 + \left(g' \mp \frac{ag}{r}\right)^2$$

$$+ \frac{1}{2}\left(\chi' \mp \frac{W_\chi}{r}\right)^2 + V - \left(\frac{1}{2}\frac{\left(1-g^2\right)^2}{P(\chi)} + \frac{1}{2}\frac{W_\chi^2}{r^2}\right)$$

$$\pm \frac{1}{r}\left(W - a\left(1-g^2\right)\right)'.$$

Energy requires $\int r\,dr$

# Vortices with internal structure

$$V = \frac{1}{2}\frac{\left(1-g^2\right)^2}{P(\chi)} + \frac{1}{2}\frac{W_\chi^2}{r^2} \quad \Longrightarrow$$

$$\chi' = \pm \frac{W_\chi}{r}$$

Neutral field is independent

$$g' = \pm \frac{ag}{r}$$

$$-\frac{a'}{r} = \pm \frac{(1-g^2)}{P(\chi)}$$

[D. Bazeia, J. Menezes and R. Menezes, PRL **91**, 241601 (2003)]

$\chi$: source for the vortex

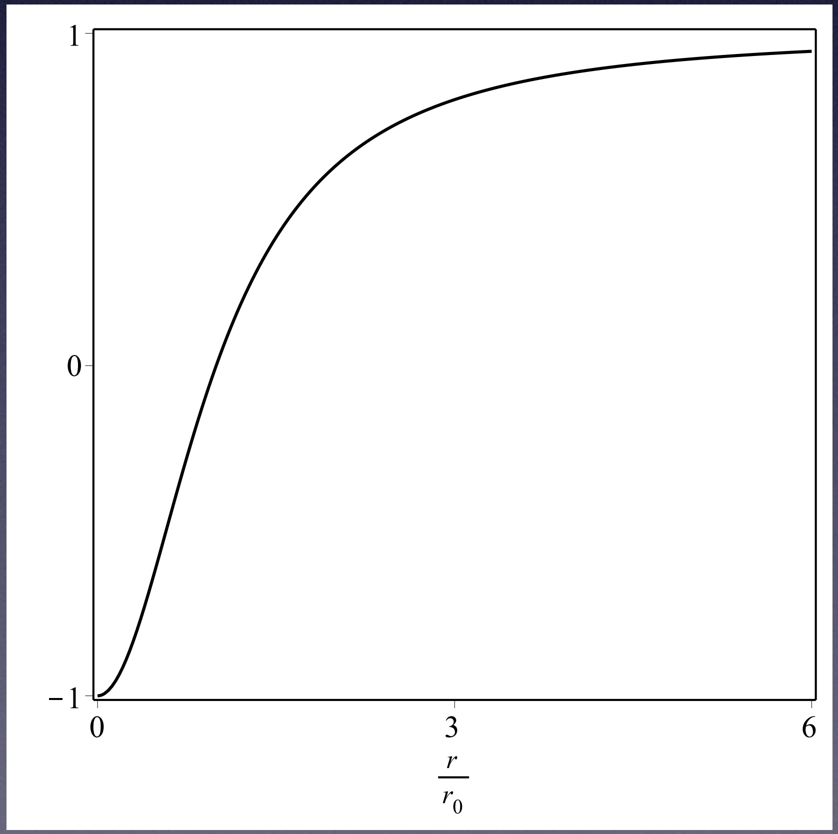
$$E = 2\pi|n| + |W(\chi_\infty) - W(\chi_0)|$$

$$\rho_{vortex} = P(\chi)\frac{a'^2}{r^2} + 2g'^2 \qquad \text{e} \qquad \rho_{source} = \chi'^2\,.$$

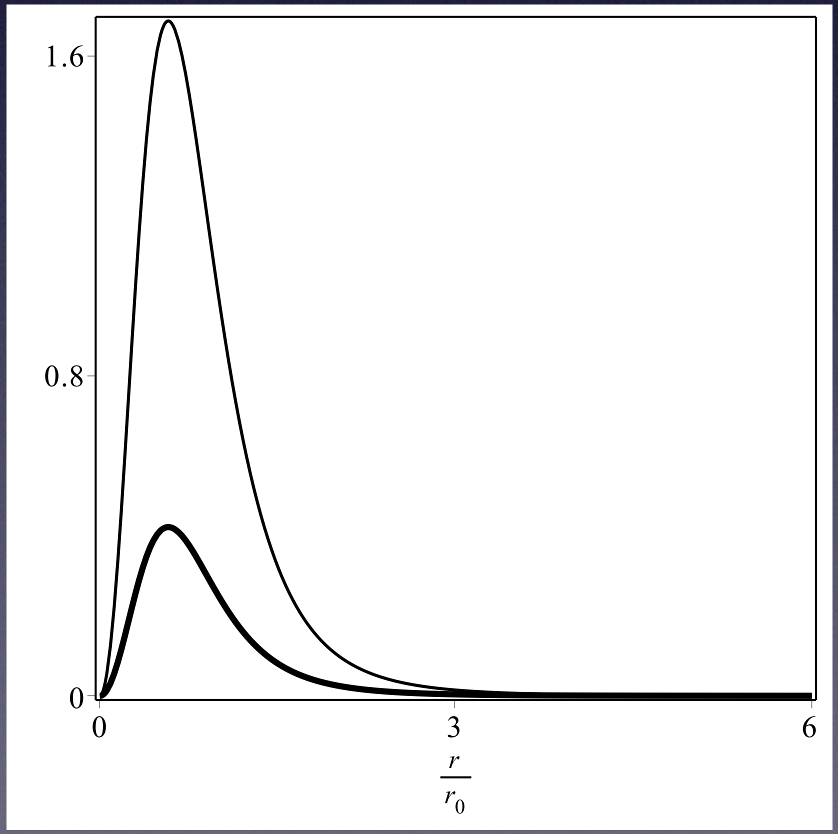
# Vortices with internal structure

Source field:

$$W(\chi) = \chi - \frac{1}{3}\chi^3 \implies \chi(r) = \frac{r^2 - r_0^2}{r^2 + r_0^2} \qquad \text{e} \qquad \rho_{source} = \frac{16\, r_0^4\, r^2}{(r_0^2 + r^2)^4}$$




$\chi$ as a function of $r/r_0$.




$\rho_{source}$

# Vortices with internal structure

Vortex:

$$P(\chi) = \frac{1}{1-\chi^2}$$

Magnetic field for $r_0 = 1$.

$$P(\chi) = \frac{1}{\chi^2}$$

Magnetic field for $r_0 = 1$.

# Multilayered vortices

PHYSICAL REVIEW RESEARCH **1**, 033053 (2019)

## Multilayered vortices

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$$S_1 = \int d^2r\,dt \left( -\frac{1}{4} f(\chi) F_{\mu\nu}F^{\mu\nu} + |D_\mu \varphi|^2 + \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - V(|\varphi|,\chi) \right).$$

$U(1)\times Z_2$ symmetry

$$S_2 = \int d^2r\,dt \left( -\frac{1}{4} f(|\chi|) F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu} + |D_\mu \varphi|^2 + |\mathscr{D}_\mu \chi|^2 - V(|\varphi|,|\chi|) \right).$$

$U(1)\times U(1)$ symmetry

# Multilayered vortices

$$S_2 = \int d^2r\,dt\left(-\frac{1}{4}f(|\chi|)F_{\mu\nu}F^{\mu\nu} - \frac{1}{4}\mathscr{F}_{\mu\nu}\mathscr{F}^{\mu\nu} + |D_\mu\varphi|^2 + |\mathscr{D}_\mu\chi|^2 - V(|\varphi|,|\chi|)\right).$$

$$D_\mu = \partial_\mu + iA_\mu \qquad \mathscr{D}_\mu = \partial_\mu + iq\mathscr{A}_\mu$$

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \qquad \mathscr{F}_{\mu\nu} = \partial_\mu\mathscr{A}_\nu - \partial_\nu\mathscr{A}_\mu$$

*Ansatz:* $A_0 = \mathscr{A}_0 = 0,$

$$\varphi = g(r)\,e^{in\theta}, \qquad \overrightarrow{A} = \frac{\hat{\theta}}{r}\left(n - a(r)\right).$$

$$\chi = h(r)e^{ik\theta}, \qquad \overrightarrow{\mathscr{A}} = \frac{\hat{\theta}}{qr}(k - c(r))$$

$$a(0) = n,\ g(0) = 0,\ \lim_{r\to\infty} a(r) = 0,\ \lim_{r\to\infty} g(r) = 1.$$

$$c(0) = k,\ h(0) = 0,\ \lim_{r\to\infty} c(r) = 0,\ \lim_{r\to\infty} h(r) = w\,.$$

# Multilayered vortices

BPS formalism:

$$V(|\varphi|,|\chi|) = \frac{1}{2}\frac{\left(1-|\varphi|^2\right)^2}{f(|\chi|)} + \frac{q^2}{2}\left(w^2-|\chi|^2\right)^2$$

$$h' = \pm\frac{ch}{r}, \qquad -\frac{c'}{qr} = \pm q\left(w^2-h^2\right)$$ → Hidden sector is independent

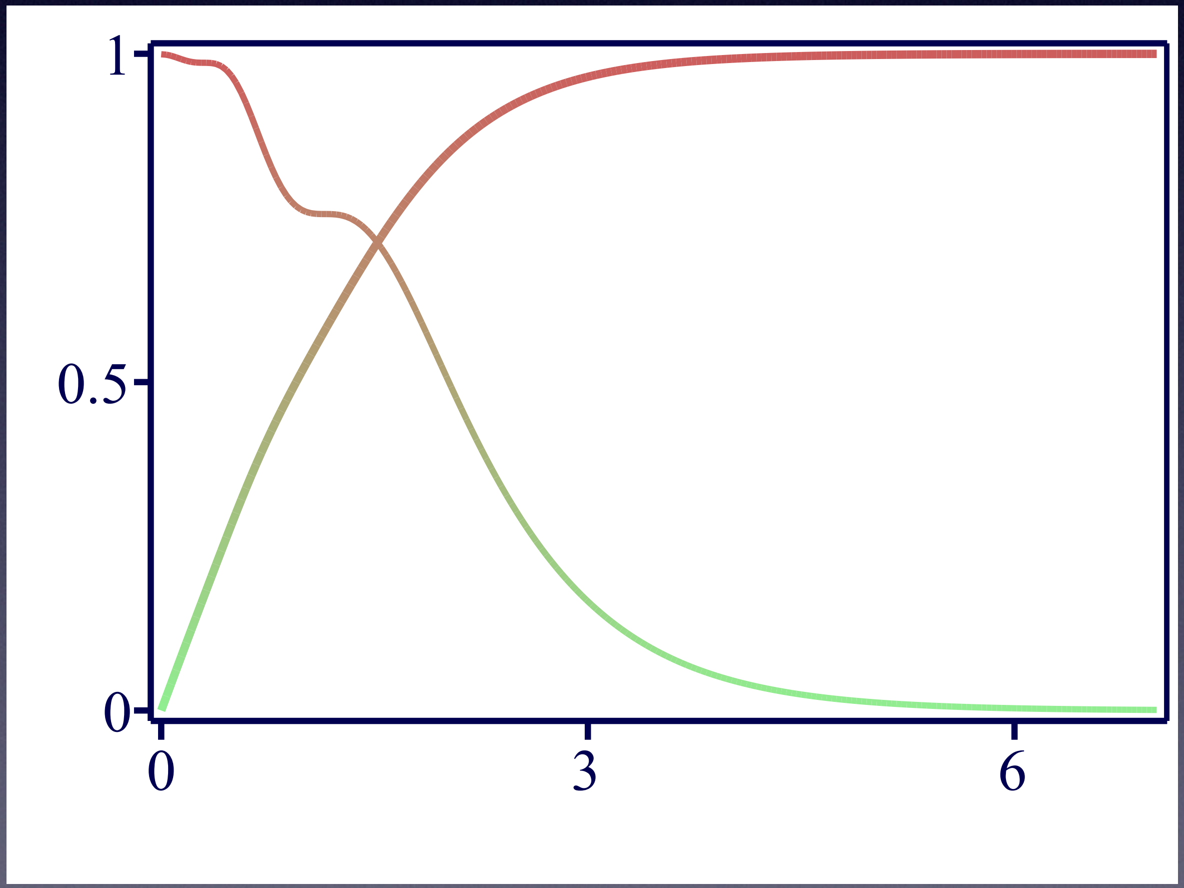
$$g' = \pm\frac{ag}{r}, \qquad -\frac{a'}{r} = \pm\frac{\left(1-g^2\right)}{f(h)}$$ → Hidden sector: source for the visible one

$$E = 2\pi\left(|n|+|k|w^2\right)$$

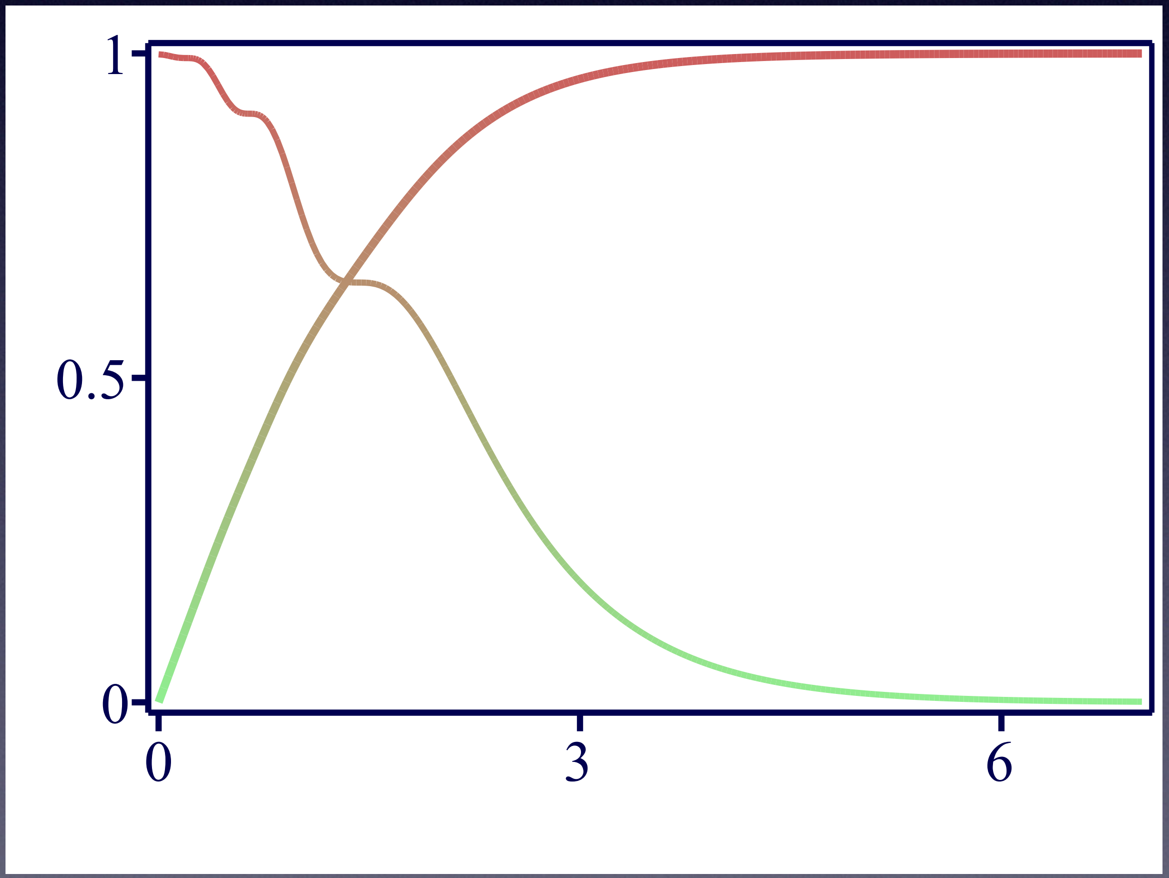
$\rho = \rho_1 + \rho_2,$ onde $\rho_1 = \frac{c'^2}{q^2r^2} + 2h'^2, \qquad \rho_2 = f(h)\frac{a'^2}{r^2} + 2g'^2 .$

# Multilayered vortices

$$f(|\chi|) = \sec^2(2\pi m|\chi|) \quad \Longrightarrow \quad g' = \frac{ag}{r}, \qquad -\frac{a'}{r} = \cos^2\left(2\pi m\, h(r)\right)\left(1 - g^2\right).$$




Solutions; $n = k = 1$, $q = 1$, $m = 2$.




Solutions; $n = k = 1$, $q = 1$, $m = 3$.

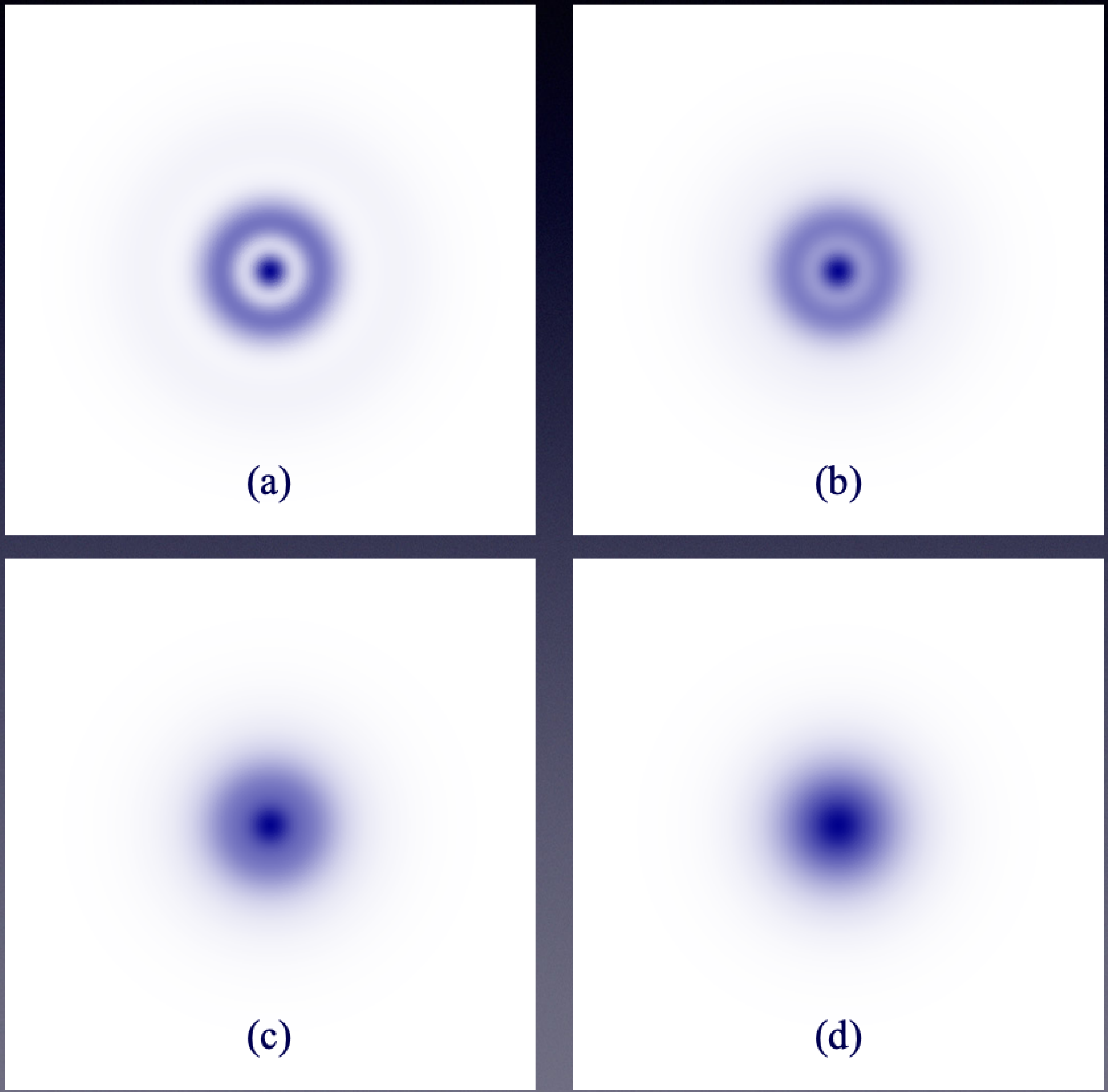
# Multilayered vortices

Magnetic field;
$n = k = 1, q = 1, m = 2.$

Magnetic field;
$n = k = 1, q = 1, m = 3.$

# Multilayered vortices

$$f(|\chi|) = \frac{1+\lambda^2}{\lambda^2 + \cos^2(2\pi m |\chi|)}$$




Magnetic field for
$n = k = 1$, $q = 0{,}5$, $m = 2$, e
$\lambda = 0.5$ (a), $1$ (b), $2$ (c), e $4$ (d).

# Monopoles with internal structure

PHYSICAL REVIEW D **97**, 105024 (2018)

**Magnetic monopoles with internal structure**

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$$S = \int d^3r\,dt \left( -\frac{1}{4}P(\phi)F^a_{\mu\nu}F^{a\mu\nu} - \frac{M(\phi)}{2}D_\mu\chi^a D^\mu\chi^a - \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi,|\chi|) \right)$$

$SU(2) \times Z_2$ symmetry

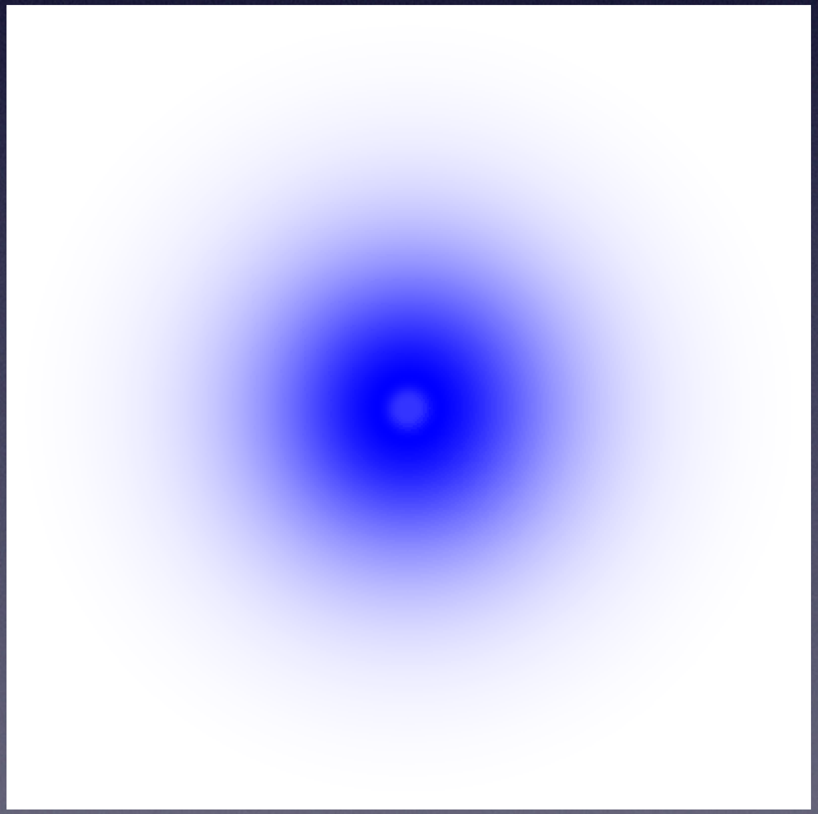
*Ansatz:* $A^a_0 = 0,$

$$\chi^a = \frac{x_a}{r}H(r), \quad A^a_i = \epsilon_{aib}\frac{x_b}{gr^2}(1-K(r)), \quad \text{and} \quad \phi = \phi(r)\,.$$

# Monopoles with internal structure

Compact source: $\phi(r) = \begin{cases} \tanh^3\left(\frac{1}{3r} - \frac{1}{3r_0}\right), & r \leq r_0, \\ 0, & r > r_0\,. \end{cases}$

$P(\phi) = 1 + \phi^2$



Energy density for $r_0 = 1$.

$P(\phi) = \phi^2$

Compact monopole;
energy density for $r_0 = 1$.

# Small and hollow monopoles

PHYSICAL REVIEW D **98,** 025017 (2018)

**Small and hollow magnetic monopoles**

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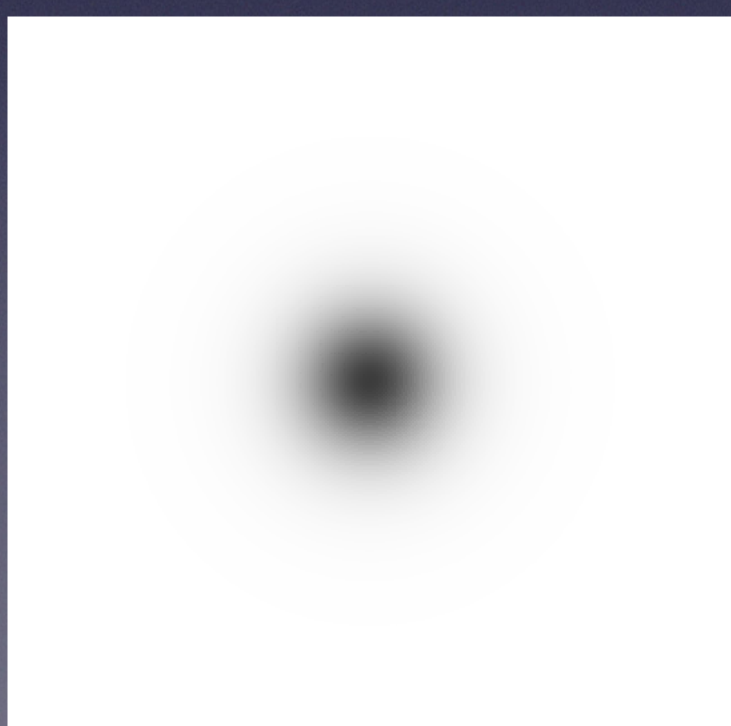
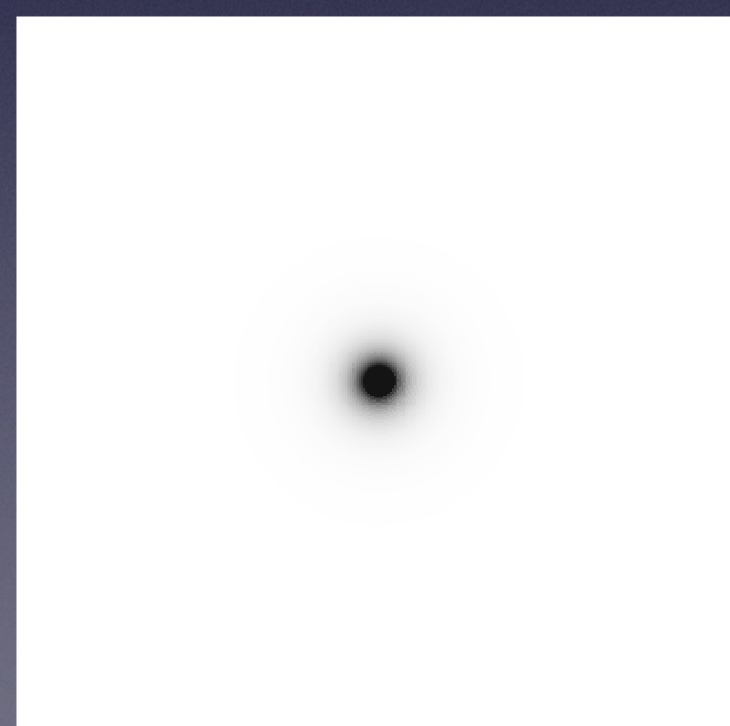
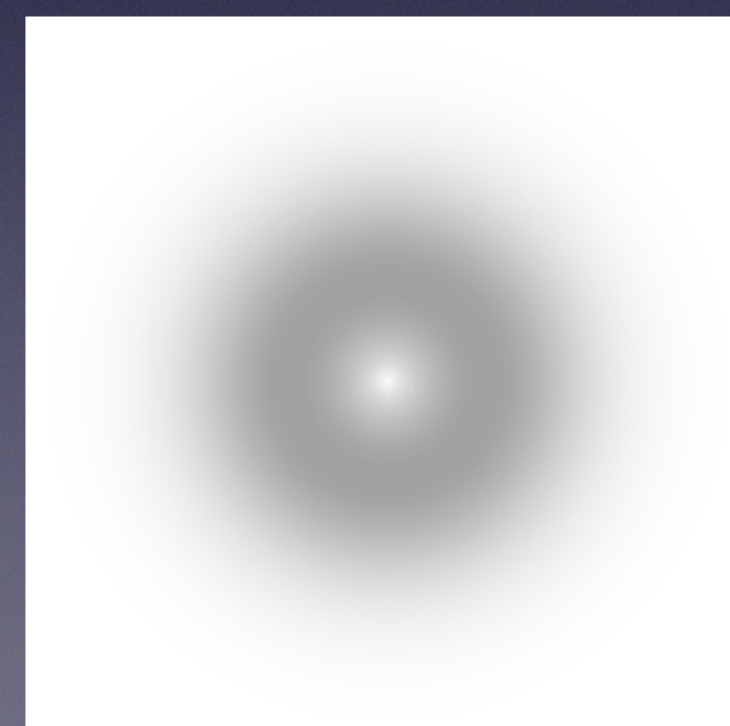
$$S=\int d^3r\,dt\left(-\frac{P(|\phi|)}{4}F^a_{\mu\nu}F^{a\mu\nu}-\frac{M(|\phi|)}{2}D_\mu\phi^a D^\mu\phi^a-V(|\phi|)\right).$$







# Bigmanetic monopoles

PHYSICAL REVIEW D **98**, 065003 (2018)

**Bimagnetic monopoles**

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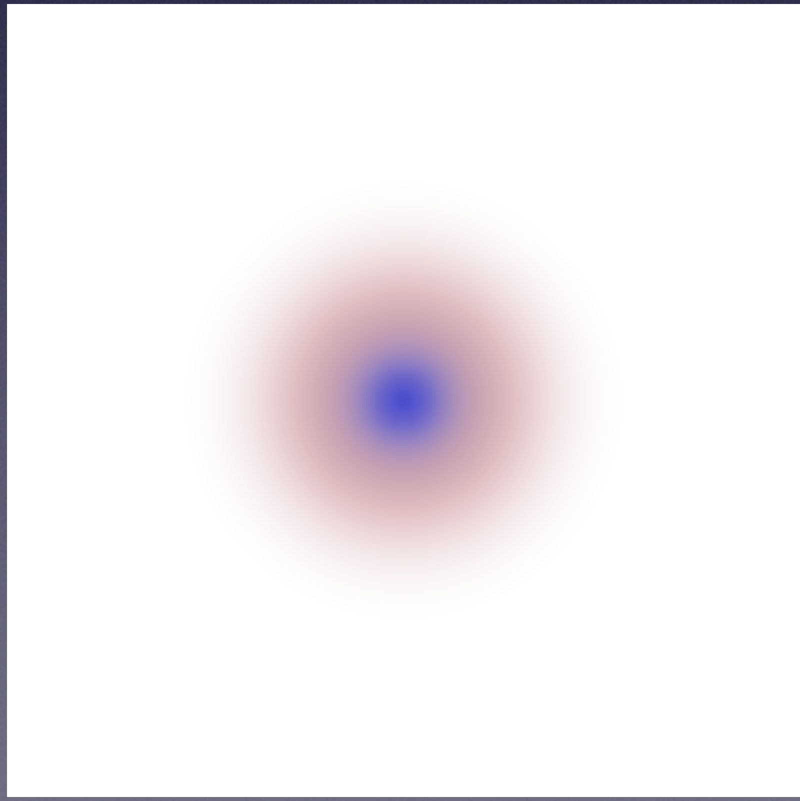
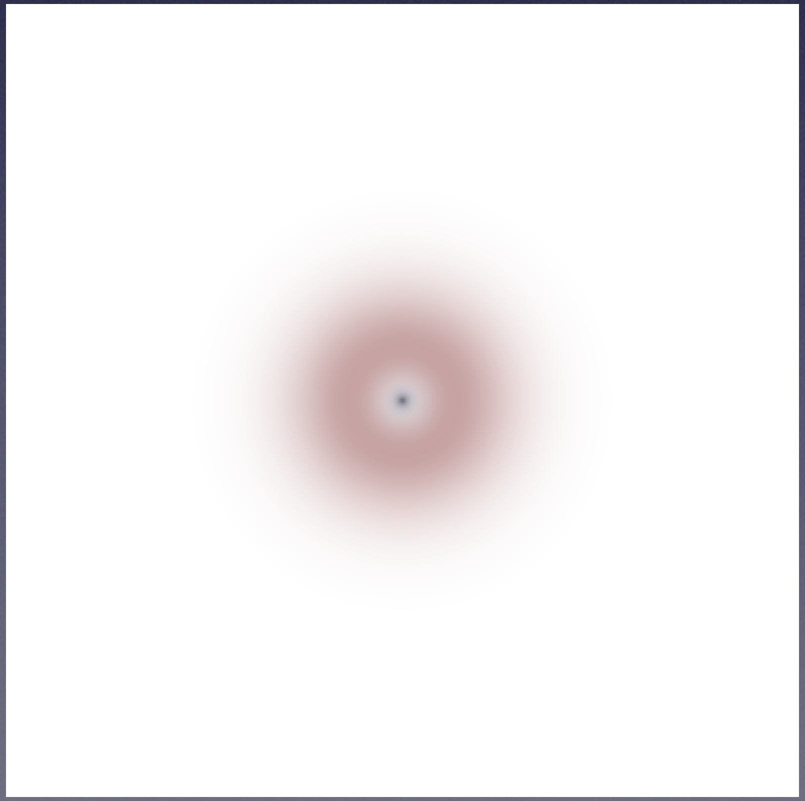
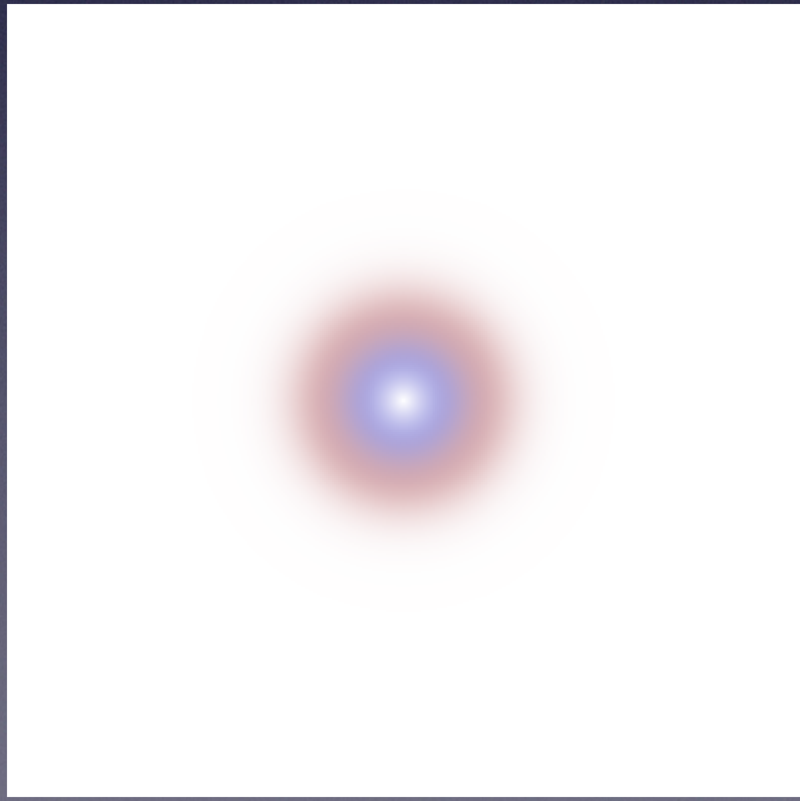
$$S=\int d^3r\,dt\left(-\frac{P(|\chi|)}{4}F^a_{\mu\nu}F^{a\mu\nu}-\frac{M(|\chi|)}{2}D_\mu\phi^a D^\mu\phi^a-\frac{\mathscr{P}(|\chi|)}{4}\mathscr{F}^a_{\mu\nu}\mathscr{F}^{a\mu\nu}-\frac{\mathscr{M}(|\chi|)}{2}\mathscr{D}_\mu\chi^a\mathscr{D}^\mu\chi^a-V(|\phi|,|\chi|)\right).$$

$\hookrightarrow \{SU(2)\times SU(2)\}$







# Multimagnetic Monopoles

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THE EUROPEAN PHYSICAL JOURNAL C

Regular Article - Theoretical Physics

## Multimagnetic monopoles

$$SU(2) \times SU(2) \times \cdots \times SU(2)$$

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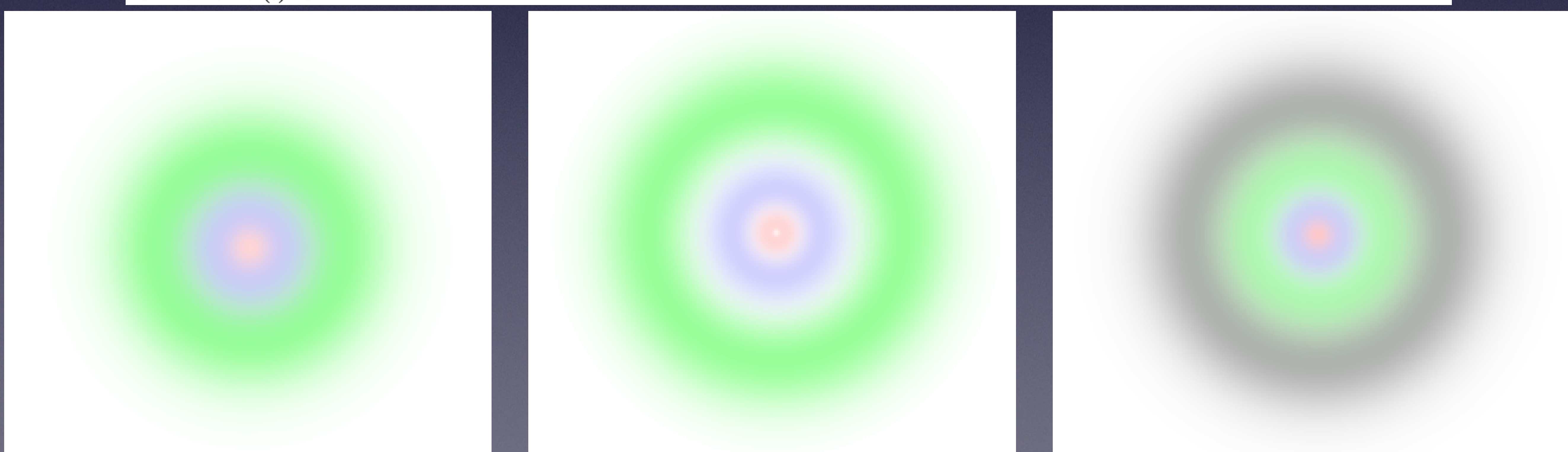

# Electrically charged localized structures

Eur. Phys. J. C (2021) 81:94
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THE EUROPEAN PHYSICAL JOURNAL C

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Regular Article - Theoretical Physics

## Electrically charged localized structures

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$$S_1 = \int d^D r\, dt \left( -\frac{\varepsilon(\phi)}{4} F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - A_\mu j^\mu \right).$$

# Electrically charged localized structures

Single point charge; three spatial dimensions; static

$$\mathbf{E} = \frac{1}{\varepsilon(\phi)} \frac{\hat{r}}{r^2} \qquad \text{and} \qquad r^2 \frac{d}{dr}\left(r^2 \frac{d\phi}{dr}\right) = \frac{d}{d\phi}\left(\frac{1}{2\varepsilon}\right).$$

Energy density: $\rho_f = \frac{1}{2}\left(\frac{d\phi}{dr}\right)^2 + \frac{1}{2r^4\varepsilon(\phi)}$

First order formalism:

$$\varepsilon(\phi) = \frac{1}{W_\phi^2} \qquad \Longrightarrow \qquad \frac{d\phi}{dr} = \pm \frac{W_\phi}{r^2}$$

Energy:

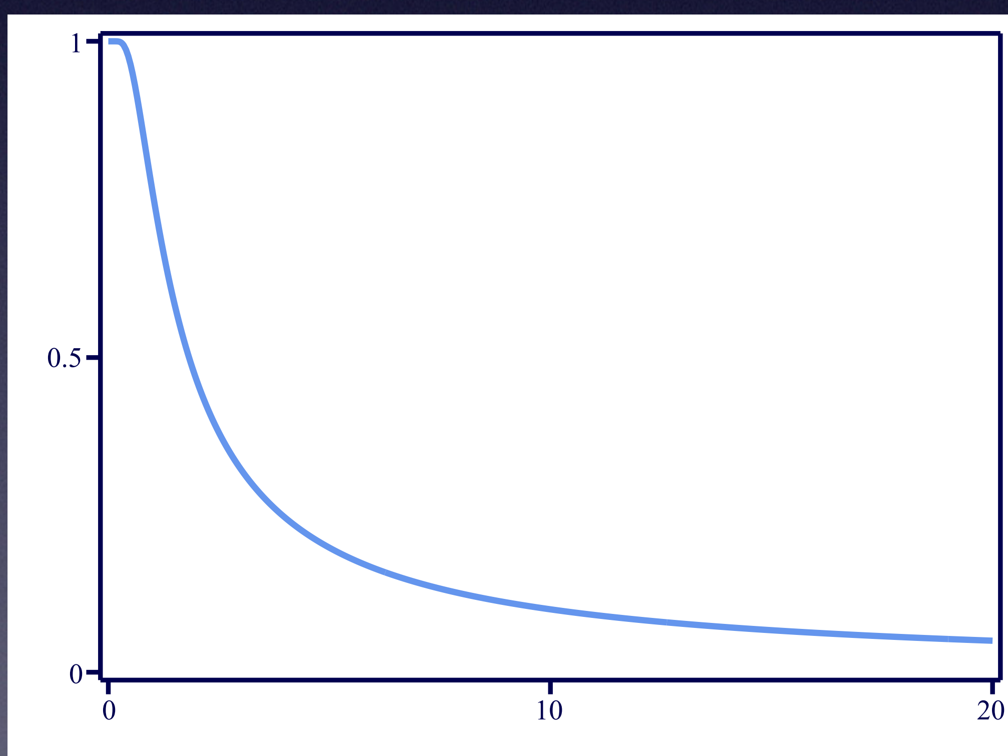
$$E = 4\pi \,|\, W(\phi(r \to \infty)) - W(\phi(r = 0)) \,|\,.$$

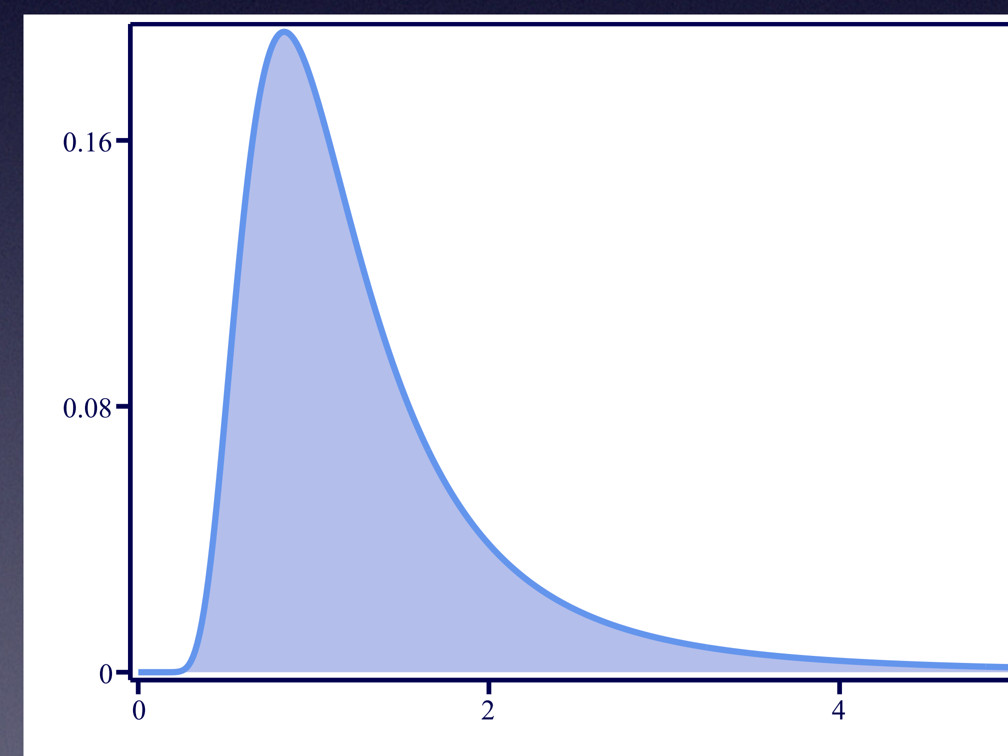
# Electrically charged localized structures

$$W(\phi) = \phi - \frac{1}{3}\phi^3 \implies \varepsilon(\phi) = \left(1 - \phi^2\right)^{-2}.$$

$$\phi(r) = \tanh\left(\frac{1}{r}\right).$$

$$\rho_f(r) = \frac{1}{r^4}\,\mathrm{sech}^4\left(\frac{1}{r}\right).$$




Energy: $E = 8\pi/3$

# Final comments

- Symmetry enhancement allows for modifications in physical properties of localized structures;
- Presence of first order equations;
- Perspectives: curved spacetime, impurities and systems with electric charges.

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CAPES

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OBRIGADO!