

## Localized solutions of inhomogeneous nonlinear systems: dimensional reduction and similarity transformation

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Solitary waves or solitons are a special type of solution that emerge due to a perfect balance between the nonlinear and dispersive/diffusive effects of the system. Depending on their form these solutions can be classified as bright, dark, breathers, rogue waves, etc. Various systems are described by the nonlinear Schrödinger (NLS) equation and its generalizations, and the interest includes nonlinear optics [1], Bose-Einstein condensates (BECs) [2], plasmas [3], hydrodynamics [4], photorefractive problems [5], nematic liquid crystals [6], and electrical line transmission [7]. In particular,

new interesting solutions are possible when the NLS equation admits variable coefficients modulated in the spatial and/or temporal coordinates [8-13]. In the case of BECs, inhomogeneity can be easily controlled using Feshbach resonance [14]. Additionally, laser beams can be used to control the pattern of the confinement profile of the system.

BECs can be tightly confined in one or two transverse directions by a strong potential, allowing us to study, respectively, the dynamics of quasi-two-dimensional or quasi-unidimensional of such a system. In this sense, one can apply some approximations to reduce the 3D NLS equation, alias Gross-Pitaevskii equation, to effective 1D [15-18] and 2D [19-21] equations. In particular, the use of the variational approximation for the transverse profile of the wave function, presented in [15, 16], helps to derive an effective 1D nonpolynomial Schrödinger equation, which accurately models the axial dynamics of the cigar-shaped BECs. This method can also be applied for the derivation of effective 2D- NPSEs, when BEC is strongly confined in the axial direction [15, 20]. In addition, by applying the standard adiabatic approximation and using an accurate analytical expression for the corresponding local chemical potential, the authors of Ref. [19] derived an effective 1D equation that governs the axial dynamics of mean-field cigar-shaped condensates with repulsive interatomic interactions, accounting accurately for the contribution from the transverse degrees of freedom. Next, in Ref. [17] was proposed a generalization of Ref. [19] for BECs trapped with anharmonic transverse potentials.

In this presentation, we will briefly review the works on dimensional reduction and on the modulation of localized solutions obtained analytically through the similarity transformation technique.

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