Spontaneous chiral symmetry breaking in holographic QCD

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Reference: 2107.10983 [hep-ph] (accepted in Phys.Rev.D)

VII ONTQC

December 8-10 2021

Summary

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2. The AdS/CFT correspondence and holographic QCD
3. Chiral symmetry breaking in holographic QCD
4. Spontaneous chiral symmetry breaking in soft wall models
5. Conclusions

1. Introduction

Quantum Chromodynamics (QCD): The theory of strong interactions

$$L_{QCD} = \bar{\psi}_f \left[\, i\,\gamma^\mu D_\mu - m_f \,\right]\psi_f - \frac{1}{2}\mathrm{Tr}\left[\mathrm{F}_{\mu\nu}\mathrm{F}^{\mu\nu}\right]$$

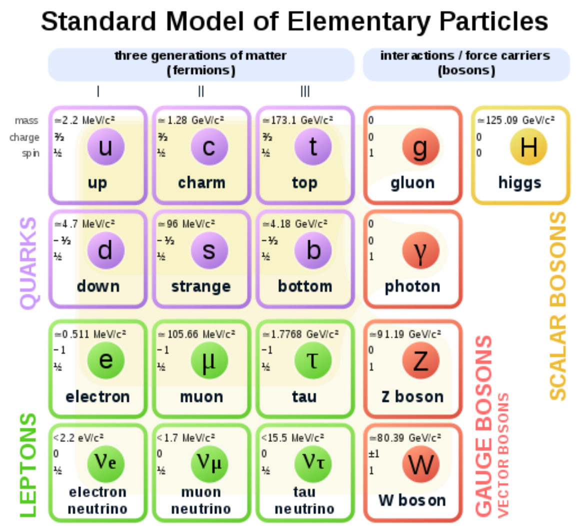
$$D_\mu = \partial_\mu - igA_\mu \;\; , \;\; F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]$$

Quarks are Dirac spinors ψ_f

Gluons are non-Abelian gauge fields $A_\mu = A_\mu^a T^a$

QCD is invariant under the local $SU(3)$ colour symmetry

Quarks and gluons are the elementary particles of the **standard model**



Asymptotic freedom and confinement

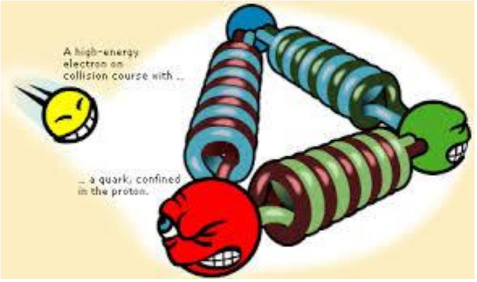
The QCD beta function

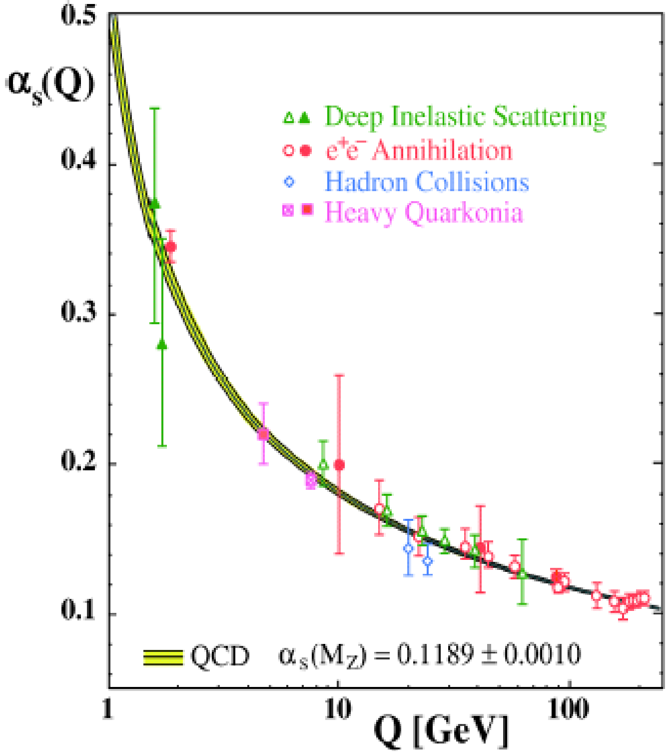
1- loop result

$$\beta_g \equiv \frac{dg}{d\,\mathrm{Log}\,\mu} = -\frac{g^3}{(4\pi)^2}\left[\frac{11}{3}N_c - \frac{2}{3}N_f\right]$$

$N_f = 6$ e $N_C = 3 \rightarrow \beta_g < 0$

Asymptotic freedom in the UV and confinement in the IR

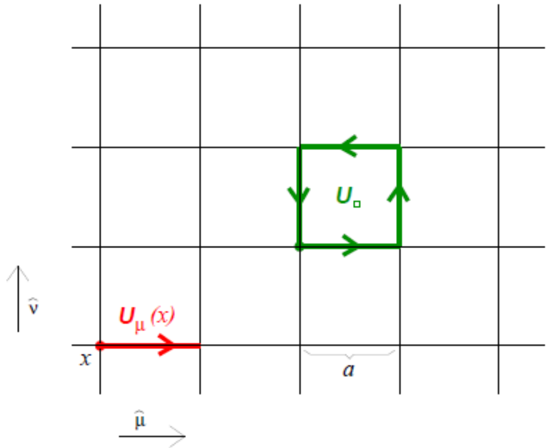




Bethke hep-ex/0606035

Non-perturbative approaches to QCD

- Lattice QCD:



Wilson 1974

- Dyson-Schwinger equations:

$$\left(\frac{\delta S[\phi]}{\delta \phi}\Big|_{\phi=\frac{\delta}{\delta J}} - \mathbf{J}\right)\mathbf{Z}[\mathbf{J}] = \mathbf{0}$$

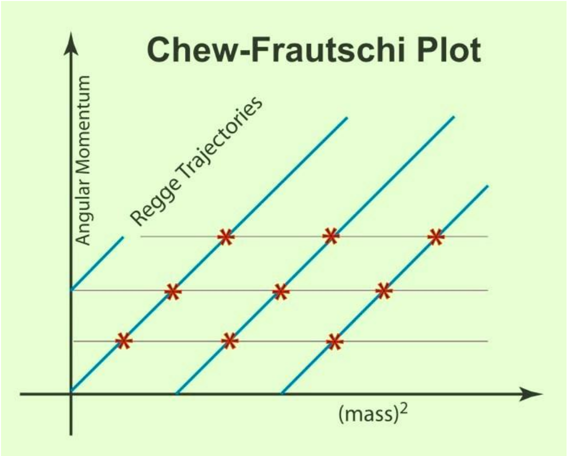
Dyson 1949,

Schwinger 1951

- Other approaches: *Chiral lagrangians, Nambu-Jona-Lasinio model, QCD sum rules, RG flow, …*

Hadrons and string theory

The spectrum of **baryons and mesons** organize approximately into **Regge trajectories**



$$J = J_0 + \alpha' M^2$$

Chew & Frautschi 1962

J : spin , M : mass α' : Regge slope

This can be obtained from **1d** objects (**strings**)

Nambu, Nielsen & Susskind 1969-1970

Problem: **massless spin 2 particle** → **string theory is a theory of gravity**

Veneziano scattering amplitude

$$A(s,t) = \frac{\Gamma(-\alpha(s))\Gamma(-\alpha(t))}{\Gamma(-\alpha(s)-\alpha(t))}$$

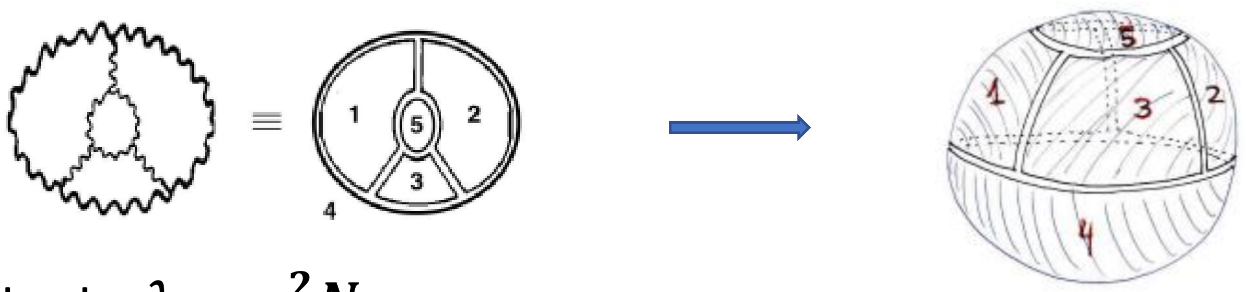
This simple amplitude satisfies the **s—t** duality and has the asymptotic behaviour $s^{J(t)}$ in the Regge limit , expected for hadronic scattering.

This amplitude is obtained from **scattering of strings** *Veneziano & many others 1968-1970*

Yang-Mills/string duality

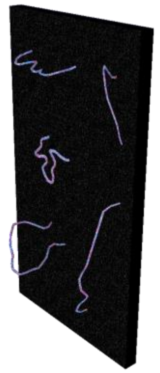
Feynman diagrams of $SU(N_c)$ gauge theories in the large N_c limit can be thought in terms of string theory

't Hooft 1974



't Hooft constant: $\lambda = g^2 N_C$

2. The AdS/CFT correspondence and holographic QCD



D-branes relate the physics of open strings with the physics of closed strings

Polchinski 1995

AdS/CFT is a concrete realization of the **Yang-Mills/string duality**

Maldacena 1997

$SU(N_c)$ theory in the **large** N_c limit with **conformal symmetry** in d dimensions ⟷ string theory in **Anti-de-Sitter** space in d+1 dimensions

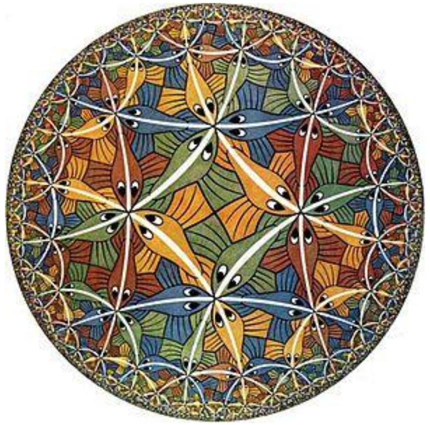
Gauge/gravity duality

In the regime $\lambda \gg 1$ string theory becomes **a classical gravitational theory**

E.g: **4-d** $N = 4$ super Yang-Mills ⟷ **supergravity IIB** in $AdS_5 \times S^5$

The AdS/CFT dictionary

Conformal symmetry group $\boldsymbol{SO(2,4)}$ becomes the $\boldsymbol{AdS_5}$ isometry group



$\boldsymbol{AdS_5}$ in **Poincaré** coordinates

$$ds^2 = \frac{R^2}{z^2}\left[dz^2 - dt^2 + d\,\bar{x}^2\right]$$

Scale transformation $\quad x^\mu \rightarrow \lambda\, x^\mu\,,\ z \rightarrow z\,\lambda$

Fields $\boldsymbol{\phi_{\dots}}$ in $\boldsymbol{AdS_5}$ couple on the **boundary** with operators $\boldsymbol{O_{\dots}}$ of the $\boldsymbol{CFT_4}$

$\boldsymbol{CFT_4}$ **partition function** ↔ gravitational **path integral** in $\boldsymbol{AdS_5}$

$$Z_{CFT}\left[\phi^0_{\dots}, g^0_{\mu\nu}\right] = Z_{AdS}\left[\phi_{\dots}, g_{\mu\nu}\right]$$

Gubser-Klebanov-Polyakov 1998,
Witten 1998

Holographic QCD

Top-down approach

$\boldsymbol{N=4}$ super Yang-Mills in 4d	⟷	IIB string theory in $\boldsymbol{AdS_5 \times S^5}$
↕		↕
QCD-like theories	⟷	deformed string theories

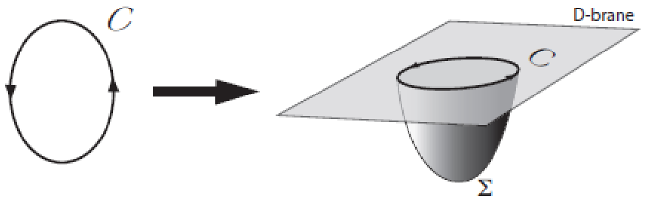
E.g. Klebanov-Witten (1998), Klebanov-Strassler (2000), Maldacena-Nunez (2000), Sakai-Sugimoto (2004), Kuperstein-Sonnenschein (2004)

Bottom-up approach

CFT in 4d	⟷	Einstein gravity in $\boldsymbol{AdS_5}$
↕		↕
QCD as a **deformed CFT**	⟷	**Gravity** in a **deformed** $\boldsymbol{AdS_5}$ space

E.g. Polchinski-Strassler (2000), Erlich-Katz-Son-Stephanov (2005), Karch-Katz-Son-Stephanov (2006), Gursoy-Kiritsis-Nitti (2007), Gubser-Nellore (2008)

Some progress in holographic QCD



Maldacena 1998

Confinement: Wilson loop described by a string world-sheet.

$$\langle W(C)\rangle = Z_{string}(\partial\Sigma = \mathrm{C})$$

$ds^2 = g_{tt}(z)dt^2 + g_{xx}(z)dx_i^2 + g_{zz}(z)dz^2$ satisfies the confinement criterion

$V_{\bar{Q}Q}(L \gg 1\,) = \sigma L$ when $\mathrm{f} = \sqrt{g_{tt}g_{xx}}$ has a minimum $\neq 0$

Kinar, Schreiber and Sonnenschein 1998

Chiral symmetry breaking

$J^{\mu,a}_{L/R} = \bar{q}_{L/R}\,\gamma^\mu T^a q_{L/R}$	⟷	$A^{m,a}_{L/R}$, where $m = (z, \mu)$.
$\bar{q}q$	⟷	X
Global symmetry	⟷	Gauge symmetry

Erlich, Katz, Son and Stephanov & Da Rold and Pomarol 2005

Finite temperature AdS/CFT and holographic QCD

4d **conformal fluid** dual to a **5d AdS black hole**

$$T^{\mu\nu} = \alpha\, T^4[\eta^{\mu\nu} + 4u^\mu u^\nu] - 2\eta\, \sigma^{\mu\nu} + \cdots$$

Universal prediction: η: shear viscosity, **s**: entropy density.

$$\eta/s = 1/(4\pi)$$

Policastro-Son-Starinets 2001, Kovtun-Son-Starinets 2004

Non-linear fluid/gravity correspondence: *Bhattacharyya-Hubeny-Minwalla-Rangamani 2007*

Deconfinement transition in 4d	⟷	Gravitational Hawking-Page transition in 5d

Witten 1998, Herzog 2006, B.B-Boschi-Filho-Braga-Pando Zayas 2007, Gursoy-Kiritsis-Mazzanti-Nitti 2008

Non-conformal fluids

$$T^{\mu\nu} = \rho u^\mu u^\nu + (P - \zeta\theta)[\eta^{\mu\nu} + u^\mu u^\nu] - 2\eta\, \sigma^{\mu\nu} + \cdots$$

Gubser-Nellore-Pufu-Rocha 2008, Gursoy-Kiritsis-Michalogiorgakis-Nitti 2008

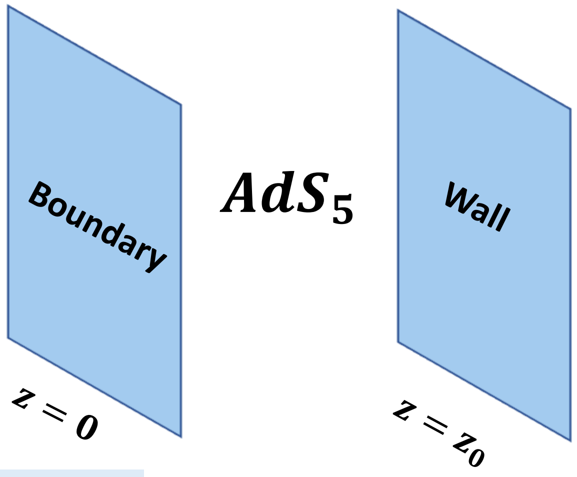
3. Chiral symmetry breaking in holographic QCD

Chiral symmetry breaking in the hard wall model

Erlich, Katz, Son and Stephanov & Da Rold and Pomarol 2005

5d **background**: **AdS** ending in an **IR hard wall**

$$ds^2 = \frac{1}{z^2}\left[-dt^2 + dx_i^2 + dz^2\right] \quad , \quad 0 < z \le z_0$$



This background satisfies the **confinement** criterion.

Adding flavour

$$S = -\int d^5x \sqrt{-g}\, \mathrm{Tr}\left\{ |D_m X|^2 + m_X^2 |X|^2 + \frac{1}{4g_5^2}\left[F_{mn}^{(L)\,2} + F_{mn}^{(R)\,2}\right]\right\}$$

X: tachyon

$A_m^{L/R}$: non-Abelian gauge fields

$$F_{mn}^{(L/R)} = \partial_m A_n^{(L/R)} - \partial_n A_m^{(L/R)} - i\left[A_m^{(L/R)}, A_n^{(L/R)}\right],$$
$$D_m X = \partial_m X - i A_m^{(L)} X + i\, X\, A_m^{(R)}$$

Map between **5d fields** and **4d operators**

$$A_m^{(L/R)} \leftrightarrow J_\mu^{(L/R)}$$
$$X \leftrightarrow \bar{q}_R q_L$$

$m_X^2 = \Delta(\Delta - 4) = -3$ **(tachyonic field)**

Focus on light quarks: $N_f = 2$

Classical background

$$2X_0(z) = v(z)\, 1_{2\times 2} \quad , \qquad A_m^{(L/R)} = 0$$

Tachyon field equation

$$[z^2\partial_z^2 - 3\, z\partial_z + 3\,]v(z) = 0$$

Exact solution

$$v(z) = c_1 z + c_3 z^3$$

Coefficients c_1 and c_3 related to the **quark mass** m_q and **chiral condensate** $\Sigma =< \bar{q}q >$.

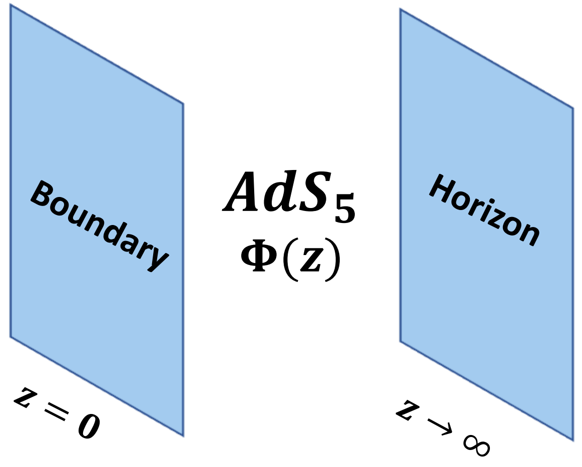
c_3 fixed by **boundary conditions**. This is **different from QCD** w/ Σ is generated dynamically

Chiral symmetry breaking in the soft wall model

Karch, Katz, Son and Stephanov 2006

5d **background**: **AdS** and a scalar field **Φ (the dilaton)**

$$ds^2 = \frac{1}{z^2}\,[-dt^2 + dx_i^2 + dz^2] \quad , \quad \Phi(z) = \phi_\infty z^2$$



$\Phi(z)$ responsible for **conformal symmetry breaking**

Adding flavour

$$S = -\int d^5x\,\sqrt{-g}\; e^{-\Phi}\mathrm{Tr}\,\{\,|D_m X|^2 + m_X^2\,|X|^2 + \frac{1}{4g_5^2}[F_{mn}^{(L)\,2} + F_{mn}^{(R)\,2}]\}$$

Tachyon field equation

$$[z^2\partial_z^2 - (3 + 2\phi_\infty z^2)\,z\partial_z + 3\,]v(z) = 0$$

Exact analytic solution

$$v(z) = \frac{\sqrt{\pi}}{2}c_1 z\,U(\frac{1}{2}, 0; \phi_\infty z^2)$$

Near the boundary,

$$v_{UV}(z) = c_1 z + d_3(c_1)\ln z + c_3(c_1) z^3 + \cdots$$

w/ d_3 and c_3 are **proportional** to c_1 ⟹ $\sigma_q \propto m_q$

The **soft wall model** leads to **explicit chiral symmetry breaking** !

Non-linear extension of the soft wall model

B-B and Mamani 2020

$$S = -\int d^5x \sqrt{-g}\, e^{-\Phi} \mathrm{Tr}\left\{ |D_m X|^2 + m_X^2 |X|^2 + \lambda |X|^4 + \frac{1}{4g_5^2}\left[F_{mn}^{(L)\,2} + F_{mn}^{(R)\,2}\right]\right\}$$

The tachyon field equation becomes

$$[z^2\partial_z^2 - (3 + 2\phi_\infty z^2)\, z\partial_z + 3]v(z) - \frac{\lambda}{2} v^3(z) = 0$$

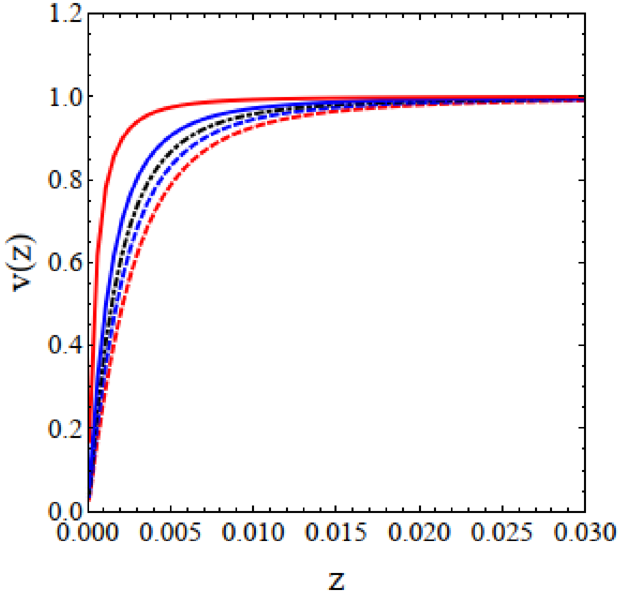
Far from the boundary , the regular solution takes the form

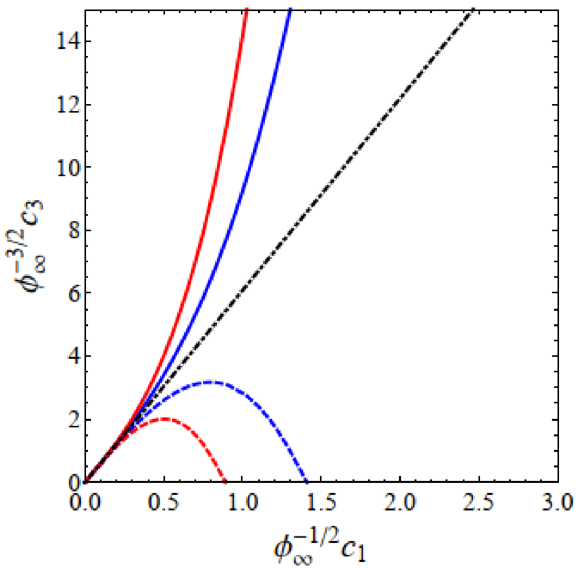
$$v_{IR}(z) = c_0 + c_2(c_0)z^{-2} + \cdots$$

Solutions for $v(z)$ found **numerically**. The UV coefficients c_1 and c_3 depend on a **single parameter**!

The tachyon profile goes from zero at small z (UV) to a constant at large z (IR)

The chiral condensate is a nonlinear function of the quark mass





Solid (dashed) lines correspond to $\lambda > 0$ ($\lambda < 0$)

However, chiral symmetry breaking **remains explicit in the chiral limit**

4. Spontaneous chiral symmetry breaking in soft wall models

B-B, Mamani and Rodrigues 2021

Non-minimal dilaton couplings

$$S = -\int d^5x \sqrt{-g}\,\mathrm{Tr}\{e^{-\mathrm{a}(\Phi)}\,[\,|D_m X|^2 + V(|X|)]\; + \frac{e^{-\mathrm{b}(\Phi)}}{4g_5^2}[F_{mn}^{(L)\,2} + F_{mn}^{(R)\,2}]\}$$

with $\Phi = \phi_\infty z^2$ and $V(|X|) = m_X^2\, X^2 + \lambda\, X^4$

Tachyon field equation becomes

$$\left[z^2\partial_z^2 - (3 + z\, a')z\partial_z - m_X^2\right]v - \frac{\lambda}{2}v^3 = 0$$

Two possibilities for the gauge coupling:

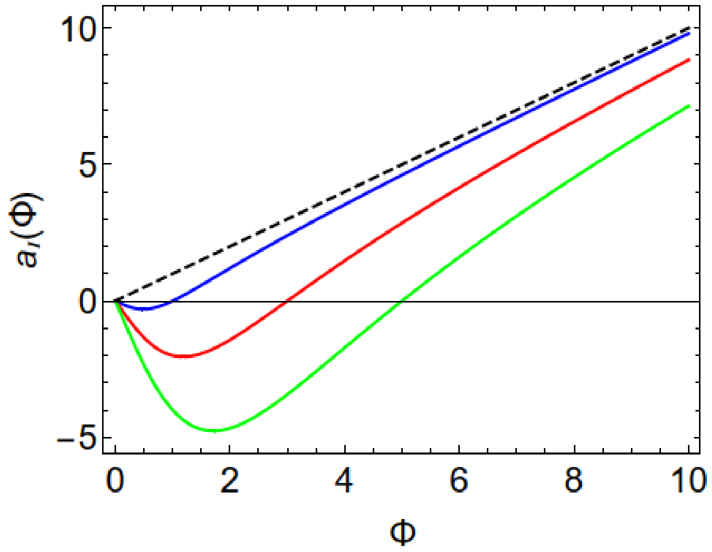
$b(\Phi) = a(\Phi)$ Models of **type A**

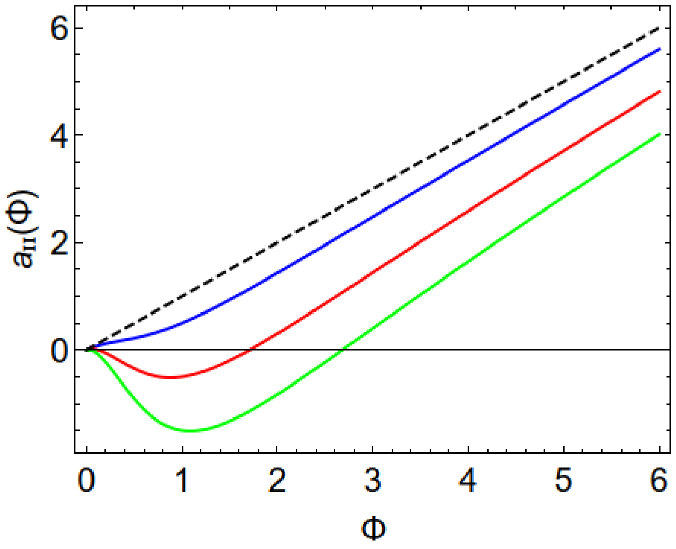
$b(\Phi) = \Phi$ Models of **type B**

Non-minimal dilaton couplings that lead to **spontaneous chiral symmetry breaking**

$$a_I(\Phi) = \Phi \left[\frac{-a_0^2 + \Phi^2}{a_0 + \Phi^2} \right]$$

$$a_{II}(\Phi) = \Phi - \frac{a_0 \Phi^{3/2}}{1 + \Phi^2}$$





Blue, red and green correspond to $a_0 = 1, a_0 = 3$ and $a_0 = 5$

Common property: the **coupling becomes negative** near the boundary

Compatible with previous proposals

Gherghetta, Kapusta and Kelley 2009

Teramond and Brodsky 2009, Zuo 2009, Chelabi et al 2015

Violation of the Breitenlohner-Freedman (BF) bound

Consider a scalar perturbation $\boldsymbol{S(x,z)}$ around the trivial vacuum $\boldsymbol{X = 0}$

It satisfies the linear differential equation

$$[\partial_z + 3A' - a']\partial_z S + \Box S - e^{2A} m_X^2 S = 0$$

where $A = -\ln z$

Redefining the field as $S = e^{a/2}\bar{S}$, the differential equation takes the AdS form

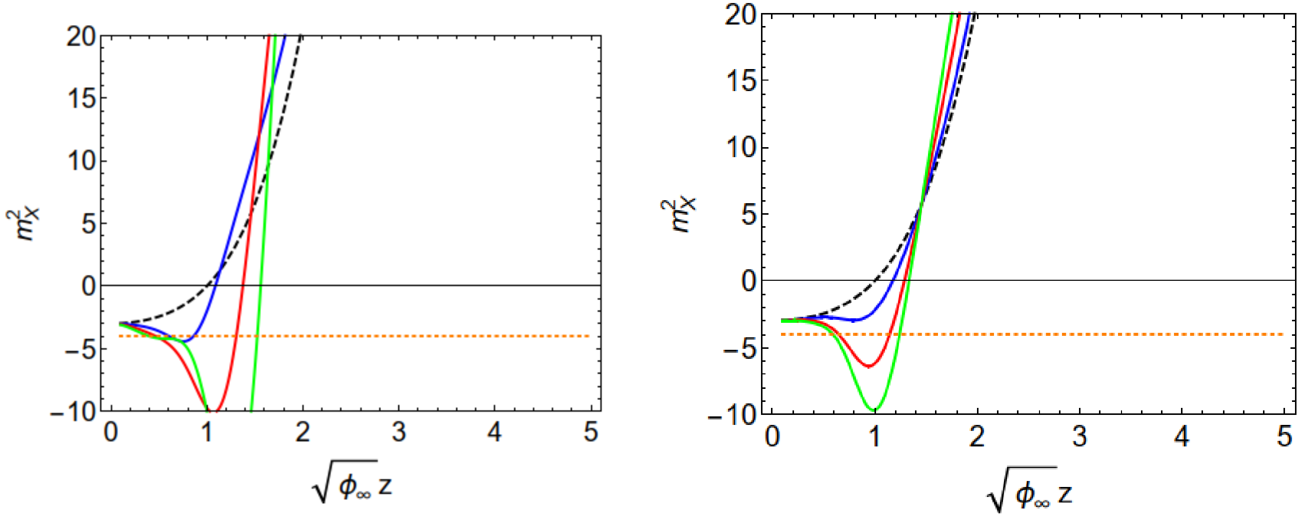
$$[\partial_z + 3A']\partial_z \bar{S} + \Box \bar{S} - e^{2A} \bar{m}_X^2(z) \bar{S} = 0$$

with a **5d running mass**

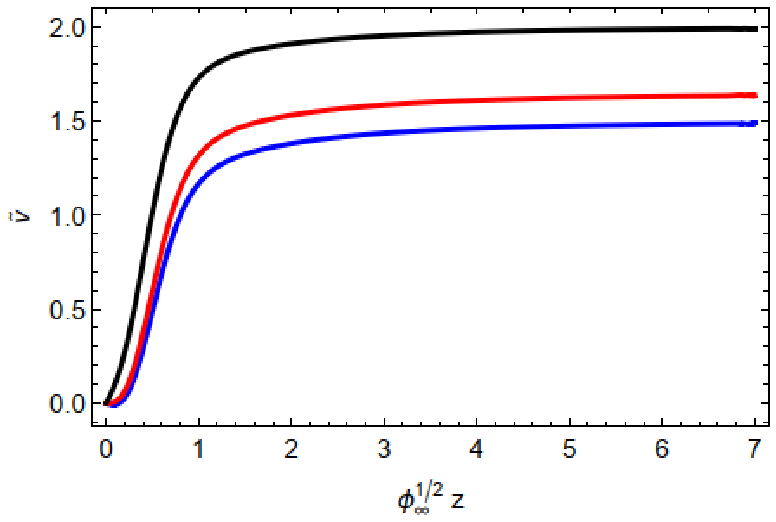
$$\bar{m}_X^2(z) = m_X^2 - e^{2A}[\frac{a''}{2} + \frac{a'}{2}(3A' - \frac{a'}{2})]$$

The non-minimal dilaton couplings induce the **violation of the BF bound**

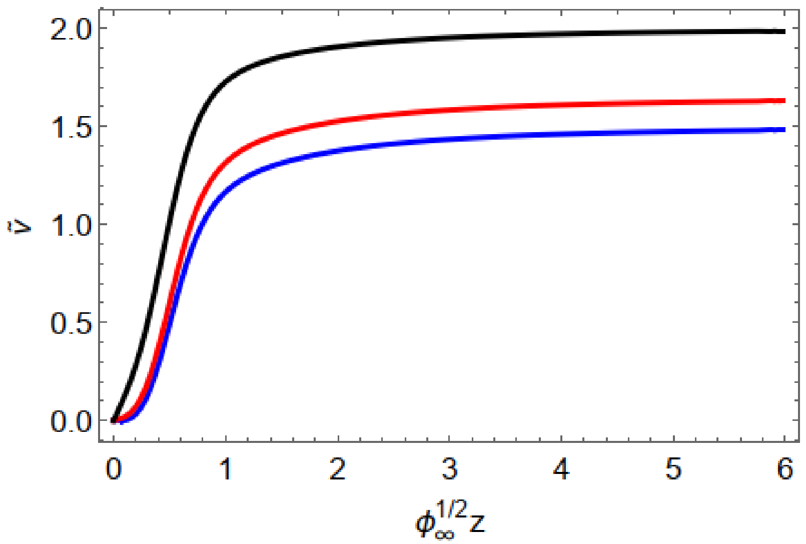
→ the trivial vacuum $\boldsymbol{X = 0}$ is **unstable!**



The **stable vacuum** $\boldsymbol{X(z)}$ found solving (numerically) the tachyon differential equation



Type I

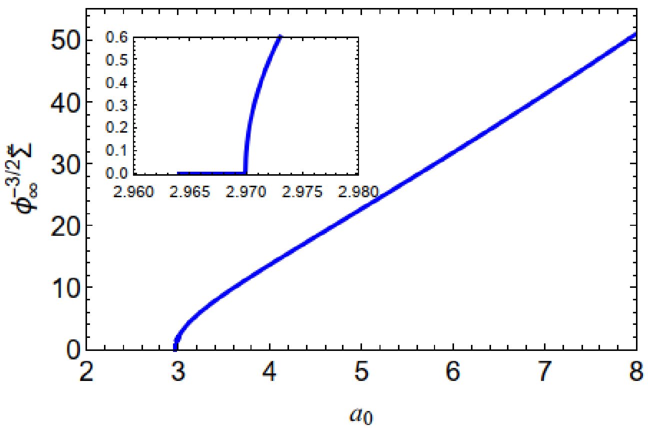


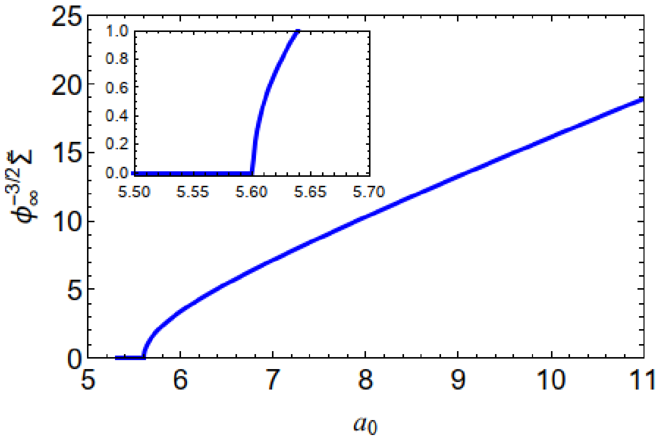
Type II

The chiral condensate in the chiral limit

A non-zero chiral condensate emerges above some critical value a_0^c

⟶ Spontaneous breaking of chiral symmetry!

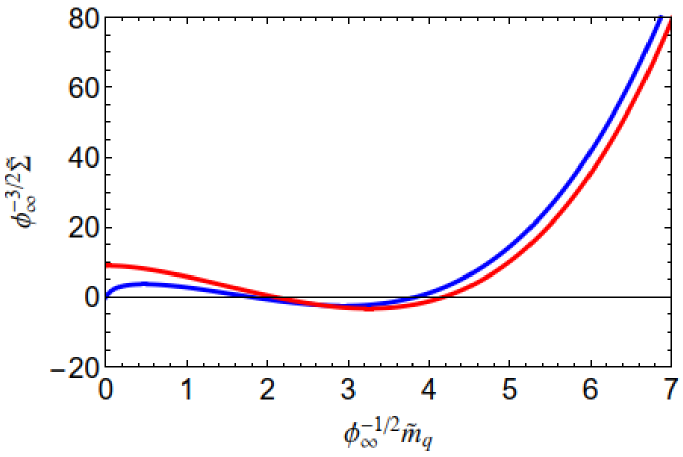




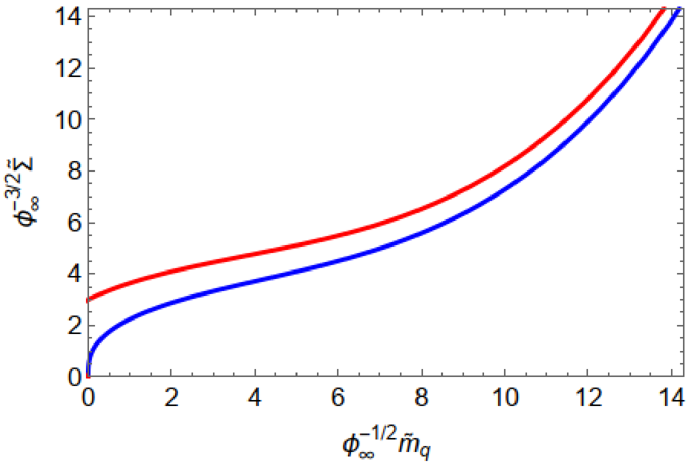
The chiral condensate as a function of the quark mass (fixed a_0)

Blue curve ($a_0 < a_0^c$)

Red curve ($a_0 > a_0^c$)



Type I



Type II

Meson spectrum

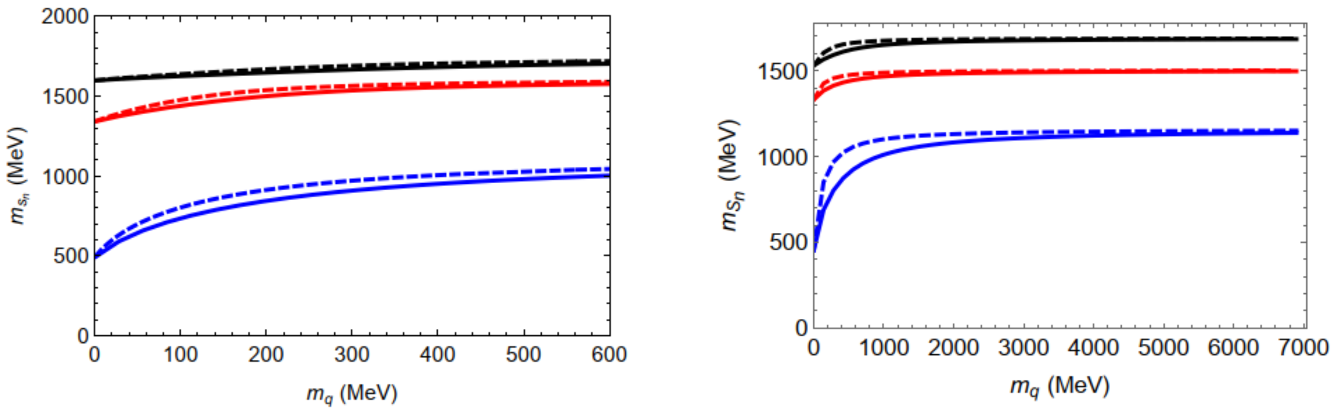
Fixing the parameters:

Parameter	Model IA	Model IB	Model IIA	Model IIB
a_0	3.5	3.5	6.5	6.5
ϕ_∞	$(0.388\,\mathrm{GeV})^2$	$(0.388\,\mathrm{GeV})^2$	$(0.388\,\mathrm{GeV})^2$	$(0.388\,\mathrm{GeV})^2$
λ	160	380	60	413

Masses of vector mesons
(compared with experimental data)

n	Model IA ($a = b$)	Model IB ($a \neq b$)	Model IIA ($a = b$)	Model IIB ($a \neq b$)	GKK [13]	ρ experimental [63]
0	327	776	344	776	475	776 ± 1
1	1280	1097	1208	1097	1129	1282 ± 37
2	1486	1344	1439	1344	1429	1465 ± 25
3	1662	1552	1632	1552	1674	1720 ± 20
4	1823	1735	1802	1735	1884	1909 ± 30
5	2116	1901	1958	1901	2072	2149 ± 17
6	2250	2053	2104	2053	2243	2265 ± 40

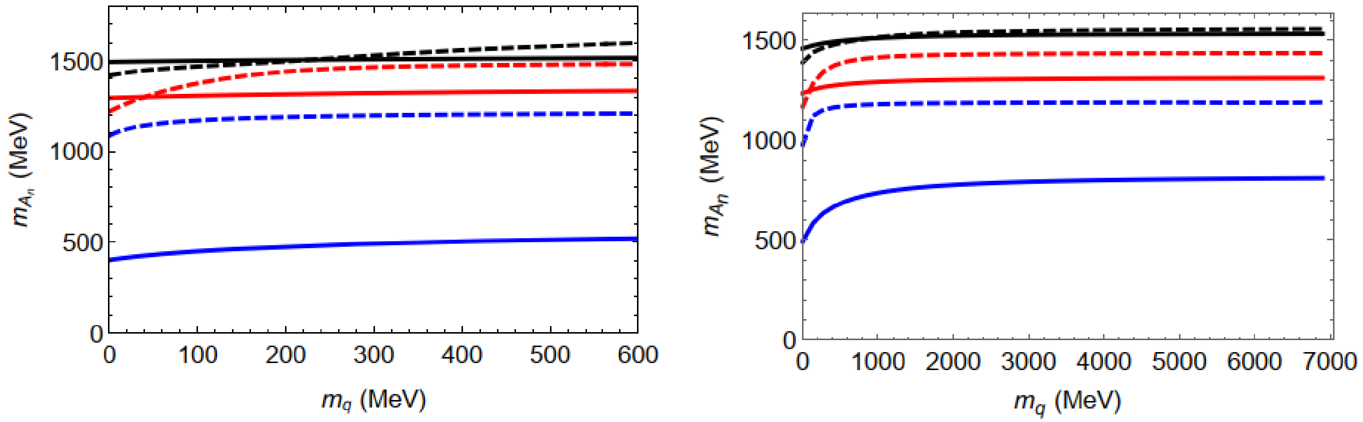
Evolution of the **scalar meson masses** with the quark mass



Masses of scalar mesons for fixed quark mass (compared with experimental data)

n	Model IA ($a=b$)	Model IB ($a \neq b$)	Model IIA ($a=b$)	Model IIB ($a \neq b$)	BM [51]	f_0 experimental [63]
0	526	519	539	546	980	990 ± 20
1	1351	1349	1348	1350	1246	1350 ± 150
2	1600	1599	1540	1541	1466	1505 ± 6
3	1755	1755	1718	1719	1657	1724 ± 7
4	1904	1904	1881	1881	1829	1992 ± 16
5	2048	2048	2032	2032	1986	2103 ± 8
6	2185	2185	2174	2174	2132	2314 ± 25
7	2315	2315	2313	2313	2268	

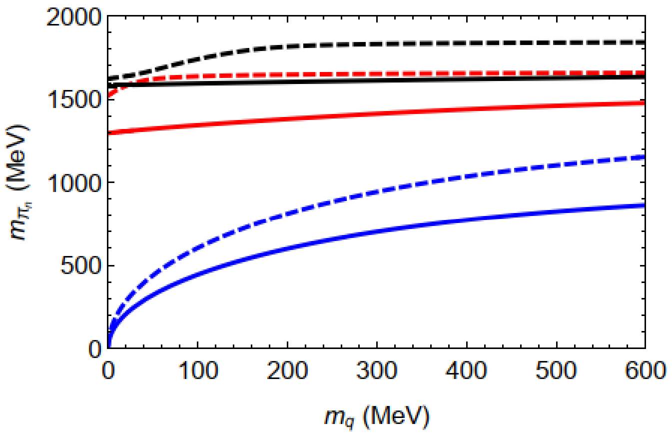
Evolution of the **axial-vector meson masses** with the quark mass

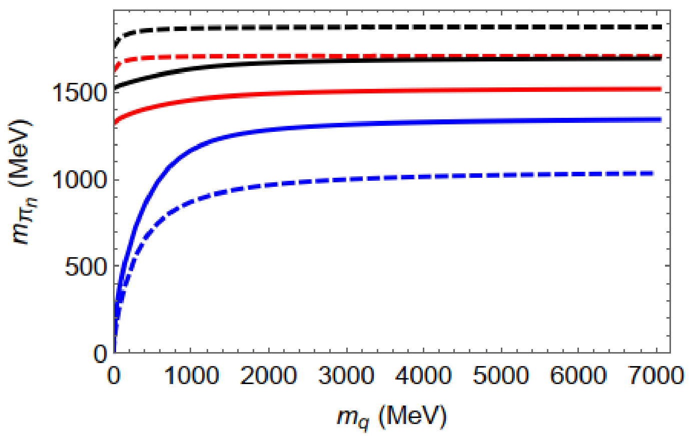


Masses of axial-vector mesons for fixed quark mass (compared with experimental data)

n	Model IA ($a=b$)	Model IB ($a \neq b$)	Model IIA ($a=b$)	Model IIB ($a \neq b$)	GKK [13]	a_1 experimental [63]
0	409	1098	525	1105	1185	1230 ± 40
1	1296	1231	1242	1261	1591	1647 ± 22
2	1494	1423	1463	1431	1900	1930^{+30}_{-70}
3	1669	1625	1651	1625	2101	2096 ± 122
4	1828	1797	1817	1798	2279	2270^{+55}_{-40}
5	1978	1950	1970	1954		
6	2316	2092	2114	2100		

Evolution of the **pseudoscalar meson masses** with the quark mass





The mass of the fundamental states go to zero in the chiral limit (**Nambu-Goldstone bosons**)

Masses of pseudoscalar mesons for fixed quark mass (compared with experimental data)

n	Model IA ($a = b$)	Model IB ($a \neq b$)	Model IIA ($a = b$)	Model IIB ($a \neq b$)	KBK [64]	π experimental [63]
0	140	140	140	140	144	140
1	1301	1539	1338	1675	1557	1300 ± 100
2	1582	1626	1533	1819	1887	1816 ± 14
3	1739	1794	1713	1945	2090	2070
4	1890	1945	1877	2183	2270	2360
5	2036	2083	2028	2301	2434	
6	2175	2212	2170	2422	2586	

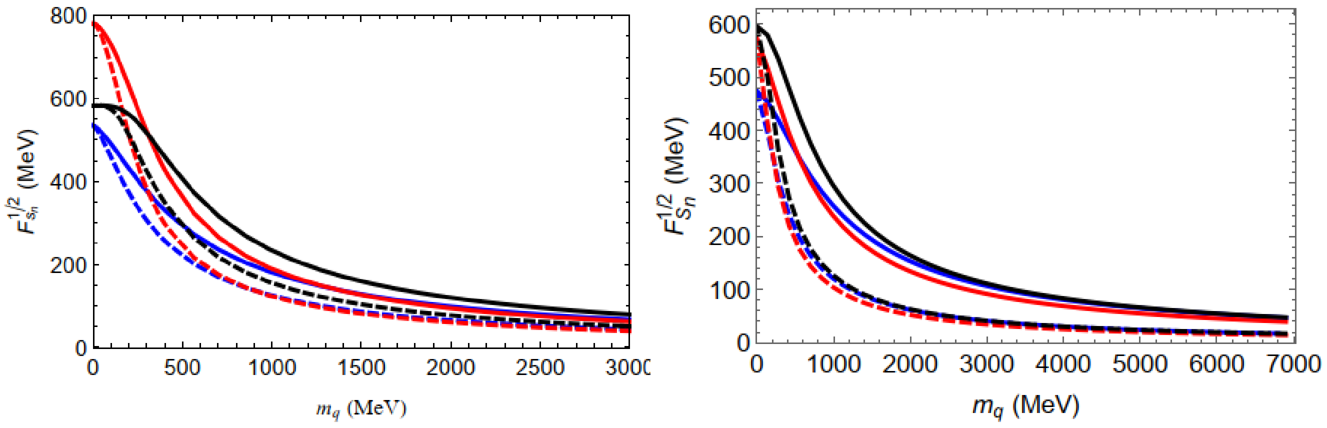
The quark masses were fixed as $m_q = 9$ **MeV** (model **IA**), $m_q = 4.7$ **MeV** (model **IB**), $m_q = 9.8$**MeV** (model **IIA**) and $m_q = 26.8$ **MeV** (model **IIB**)

Decay constants

Decay constants of vector mesons
(compared with experimental data)

	Model IA $(a=b)$	Model IB $(a \neq b)$	Model IIA $(a=b)$	Model IIB $(a \neq b)$	SW [1]	Experimental [65] $(F_{V^a} = g_\rho)$
$F_{V_0}^{1/2}$	235	260	226	260	261	346.2 ± 1.4
$F_{V_1}^{1/2}$	357	310	265	310		433 ± 13
$F_{V_2}^{1/2}$	337	343	314	343		

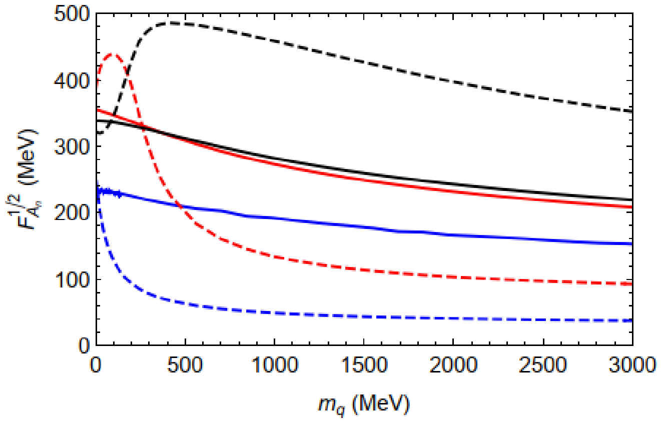
Evolution of the **scalar meson decay constants** with the quark mass

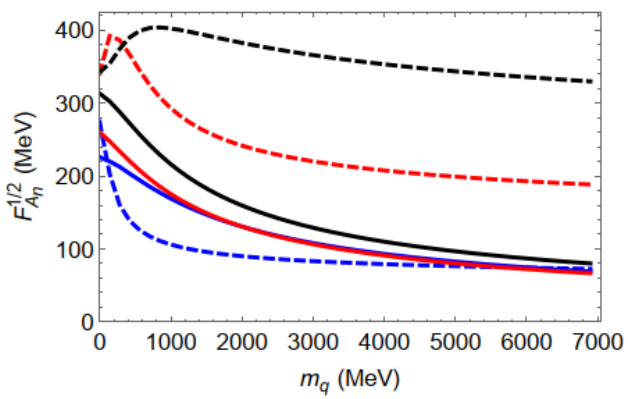


Decay constants of scalar mesons for fixed quark mass (compared with experimental data)

	Model IA $(a=b)$	Model IB $(a \neq b)$	Model IIA $(a=b)$	Model IIB $(a \neq b)$	SW [1]	QCD Results [66]
$F_{s_0}^{1/2}$	532.7	533.2	470.4	426.6	420	425.3
$F_{s_1}^{1/2}$	779.7	780.1	561.1	465.7	499	
$F_{s_2}^{1/2}$	582.5	582.5	595.8	544.5	552	

Evolution of the **axial-vector meson decay constants** with the quark mass

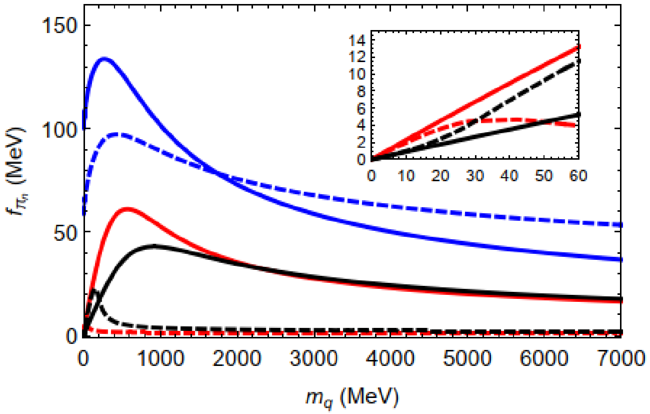


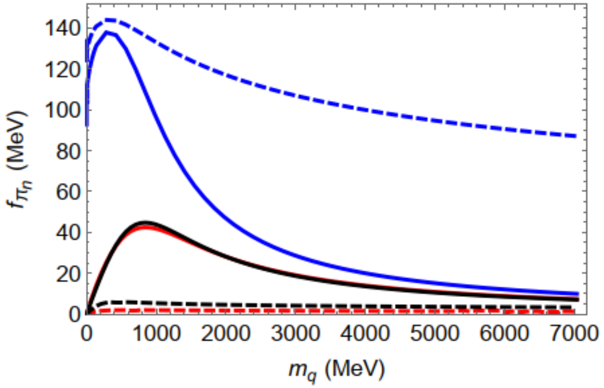


Decay constants of axial-vector mesons for fixed quark mass
(compared with experimental data)

	Model IA ($a = b$)	Model IB ($a \neq b$)	Model IIA ($a = b$)	Model IIB ($a \neq b$)	SW [1]	Experimental ($F_{A^c} = f_{a_1}/\sqrt{2}$) [67]
$F^{1/2}_{A_0}$	141.82	241	226.07	224.24	261	433 ± 13
$F^{1/2}_{A_1}$	324.10	395	257.12	386.50		
$F^{1/2}_{A_2}$	295.17	322	310.91	348.54		

Evolution of the **pseudoscalar meson decay constants** with the quark mass

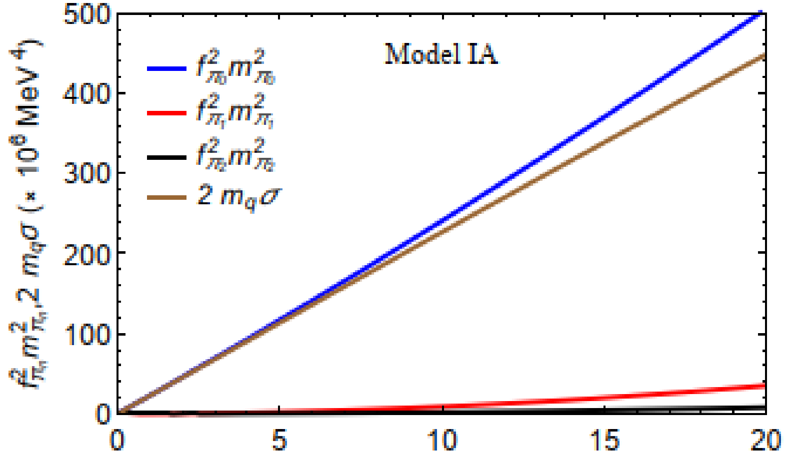


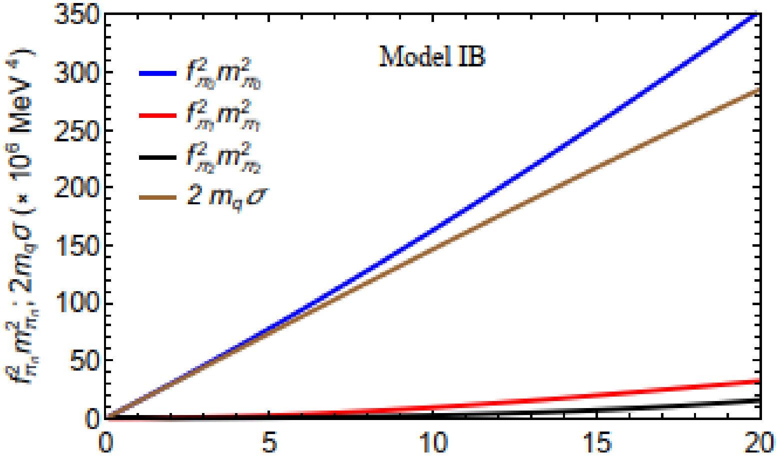


Decay constants of pseudoscalar mesons for fixed quark mass
(compared with experimental data)

	Model IA $(a=b)$	Model IB $(a \neq b)$	Model IIA $(a=b)$	Model IIB $(a \neq b)$	Experimental $(f_{\pi^+}/\sqrt{2})$ [63]
f_{π_0}	104.3	60.9	118.3	138.68	92.1 ± 0.8
f_{π_1}	2.05	0.95	3.94	1.04	
f_{π_2}	0.79	0.42	3.37	2.97	

The Gell-Mann-Oakes-Renner (GOR) relation





The fundamental states in the pseudoscalar sector satisfy the **GOR** relation $\boldsymbol{f_\pi^2 m_\pi^2 = 2 m_q \sigma}$

Conclusions

- A non-linear extension of the soft wall model with non-minimal dilaton couplings allows for the description of spontaneous chiral symmetry breaking in the chiral limit.
- Pions behave as Nambu-Goldstone bosons in the chiral limit, as expected in QCD
- Meson spectrum at physical values of the quark mass seems compatible with experimental data
- Transition to the regime of heavy quarks looks promising but needs more ingredients
- We have followed a bottom-up approach. More sophisticated top-down models built from string theory provide confinement and spontaneous chiral symmetry breaking . There are, however, plenty of d.o.f not present in QCD

Next steps

- Describe not only chiral symmetry breaking but also confinement in a consistent way (Einstein-dilaton-tachyon holographic QCD).
- Turn on the temperature, quark density, magnetic field, etc.