

## A quantum field theory approach to critical dynamics: the role of multiplicative noise

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Quantum Field Theory has proven to be a very powerful tool for describing continuous phase transitions. The spontaneous symmetry breaking paradigm, complemented with the Renormalization Group (RG), allows us to classify phase transitions into “universality classes”<sup>[J. L. Cardy, *Scaling and renormalization in statistical physics*, Cambridge lecture notes in physics (1996)].</sup> In this way, it is possible to determine different correlation functions and the associated observables in the neighborhood of a critical point. However, the dynamics of a phase transition, when the system is taken out of thermodynamic equilibrium, is still an open research problem. This problem is placed in a much wider context, which is the theoretical formulation of out-of-equilibrium statistical mechanics.

The interest in critical dynamics is rapidly growing up in part due to the wide range of multidisciplinary applications in which criticality is a central issue. For instance, the collective dynamics of biological systems presents critical behavior displaying space-time correlation functions with non-trivial scaling laws<sup>[A. Cavagna, *et al.*, *Phys. Rev. Lett.* **123**, 268001 (2019)].</sup> Other interesting examples come from epidemic spreading models where dynamic percolation is observed near multicritical points<sup>[H. K. Janssen, M. Muller, and O. Stenull, *Phys. Rev.* **E70**, 026114 (2004)].</sup> Moreover, strongly correlated systems, such as antiferromagnets in transition-metal oxides present a very rich phase diagram including ordered as well as topological phases. These compounds are generally described by quantum field theory models<sup>[B. Hsu and E. Fradkin, *Phys. Rev. B* **87**, 085102 (2013)].</sup> (Quantum Lifshitz and related Sine Gordon models) that seem to have anomalous critical dynamics.

The usual approach to critical dynamics is the “Dynamical Renormalization Group (DRG)”, early described in a seminal paper by Hohenberg and Halperin<sup>[P. C. Hohenberg and B. I. Halperin, *Rev. Mod. Phys.* **49**, 435 (1977)].</sup> The simplest starting point is to admit that, very near a critical point, the dynamics of the order parameter is governed by a dissipative process driven by an overdamped additive noise Langevin equation. The critical point is approached by integrating out short distance (high momentum) degrees of freedom in order to obtain the dynamics of an equivalent effective long distance (small momentum) model.

As usual in RG theory, the integration over higher momentum modes generates all kind of couplings, compatible with the symmetry of the problem. For this reason a consistent study of a RG flux should begin, at least formally, with the most general Hamiltonian containing all couplings compatible with symmetry. Interestingly, in a similar way, DRG transformations generate couplings that modify the probability distribution of the original stochastic process. In particular, we will show that one loop perturbative corrections generates couplings compatible with a multiplicative noise stochastic processes, even in the case of assuming an additive processes as a starting point.

In this presentation, we show how to deal with the dynamics of an order parameter near criticality, assuming a dissipative process driven by a general multiplicative noise Langevin equation. We will analyze a simple model of a not conserved real scalar order parameter,  $\phi(\mathbf{x}, t)$  with quartic coupling  $\phi^4(\mathbf{x}, t)$ . We assume, as a starting point, a multiplicative noise Langevin equation, modeled by a general dissipation function  $G(\phi) = G(-\phi)$ , with the same symmetry of the Hamiltonian,  $Z_2$ .

To compute dynamical correlation functions, we use a generalized Martin-Siggia-Rose-Janssen-DeDominicis formalism (MSRJD) that represents the Langevin dynamics by a quantum field theory. We use a recently developed approach to deal with multiplicative noise in a general stochastic prescription<sup>[Z. G. Arenas and D. G. Barci, *Phys. Rev.* **E85**, 041122 (2012); *Journal of Statistical Mechanics: Theory and Experiment* **2012**, P12005 (2012); M. V. Moreno, Z. G. Arenas and D. G. Barci, *Phys. Rev.* **E91**, 042103 (2015)].</sup> In this formalism, the generating functional is written in terms of a functional integral over four fields, two bosonic and two fermionic that, in certain conditions, display Supersymmetry.

As it is well known, the upper critical dimension of this model is  $d_c = 4$ . For  $d > 4$ , the Gaussian fixed point with dynamical critical exponent  $z = 2$  is stable. For,  $d < 4$ , the Gaussian fixed point becomes unstable and the Wilson-Fisher fixed point shows up in a first order expansion around  $\epsilon = 4 - d$ . In this case, the dynamics is governed by  $z = 2 + O(\epsilon^2)$ , and all multiplicative noise coupling constants are irrelevant. In this sense, we recover very well known old results. However, the dynamic dramatically changes at  $d = 2$ . Here, all multiplicative noise couplings are marginally relevant, flowing to a novel stable fixed point dominated by

a multiplicative noise stochastic process\footnote{Nathan Silvano and Daniel G. Barci, {\it Critical Dynamics: multiplicative noise fixed point in 2D systems}, to be published, 2022}.

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