

Relativistic **approaches** for the  
 $\gamma^* N \rightarrow N^*$  transition form factors  
at intermediate and large momentum transfer

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Many Manifestations of Nonperturbative QCD  
Camburi, São Sebastião, SP, Brazil  
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# Motivation - study of $\gamma^*N \rightarrow N^*$ transitions

- **Modern facilities** [Jefferson Lab, MAMI, MIT-Bates, ...] provide very accurate data associated with  $\gamma^*N \rightarrow N^*$  transitions  
 $W = 1.4\text{--}2.0 \text{ GeV}$ ;  $Q^2 = 0\text{--}8 \text{ GeV}^2$
- Intermediate and large (square) momentum transfer  $Q^2 = 2\text{--}12 \text{ GeV}^2$ :  
Calls for **relativistic models** with **relativistic kinematics**  
– **preferentially covariant**
- **Challenges:**  
Interpret the present data  
Make predictions for **higher  $Q^2$**   $\oplus$  **Higher  $W$**  – heavy resonances  
**JLab-12 GeV-upgrade**  
 $N^* = N, \Delta$  – states  $J^P = \frac{1}{2}^\pm, = \frac{3}{2}^\pm$ ,

# Plan of the talk – two different approaches

- **Light-Front holography/Holographic QCD**

Can be used to estimate the **leading order** effects ( $qqq$ -states)  
Nucleon and Roper

- **Covariant Spectator Quark Model**

Constituent Quark Model

Calibrated by  $N$  and lattice data ( $\Delta(1232)$ )

$N(1520)_{\frac{3}{2}}^{-}$ ,  $N(1535)_{\frac{1}{2}}^{-}$ ,  $N(1440)_{\frac{1}{2}}^{+}$

$\Delta(1232)_{\frac{3}{2}}^{+}$  – Quadrupole form factors

# Holographic QCD — Light-Front Holography

- Use connection between LF quantization of QCD and anti-de Sitter conformal field theory (AdS/CFT) — AdS/QCD
- Bottom-up approximation; 5D AdS Lagrangian  $x^\mu \rightarrow (x^\mu, z)$   
5D generalization of 4D couplings:  $\Gamma^A = \frac{R}{z}(\gamma^\mu, -i\gamma^5)$   
 $R = 5\text{D radius (5D space)}$
- Confinement implemented using a **soft-wall** potential  $\Phi = \kappa^2 z^2$   
Define hadron scale  $\kappa \sim m_\rho$
- **Method:**
  - Fix *bare* couplings by large  $Q^2$  data (nucleon)  
Valence quark dominance ( $qqq$  Fock state)  
[GR and D Melnikov, PRD 97, 034037 \(2018\)](#); [GR, PRD 96, 054021 \(2017\)](#)
  - Estimate  $\gamma^* N \rightarrow N(1440)$  form factors  $\oplus$   
nucleon axial form factor  $G_A$  (including meson cloud effects)  
[PRD 97, 073002 \(2018\)](#)

# Light-Front holography – LF – AdS correspondence

Light-Front AdS <sub>5</sub>	$\zeta = \sqrt{\varkappa(1-\varkappa)} \mathbf{b}_\perp $ $z$	$\psi(\varkappa, \mathbf{b}_\perp)$ $\phi(z)$
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$$\psi(\varkappa, \zeta, \varphi) = e^{iL\varphi} \sqrt{\varkappa(1-\varkappa)} \frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

- Correspondence valid for massless quarks ( $m_q = 0$ ) and for Baryon system ( $n = 3$ ) decomposed into a quark  $\oplus (n - 1)$  cluster [GF Teramond and SJ Brodsky, PRL 102, 081601 \(2009\); SJ Brodsky at al, Phys. Rep. 584, 1 \(2015\); T Liu and QM Ma, PRD 92, 096003 \(2015\)](#)
- Parameter impact space – Light-Front equations  $\psi_n(\{\varkappa_j, \mathbf{b}_{\perp j}\}) =$  LF wave func.

$$M_B^2 = \sum_n \prod_{j=1}^{n-1} \int d\varkappa_j d^2\mathbf{b}_{\perp j} \psi_n^*(\{\varkappa_j, \mathbf{b}_{\perp j}\}) \sum_q \left( \frac{-\nabla_{\mathbf{b}_{\perp q}}^2 + m_q^2}{\varkappa_q} \right) \psi_n(\{\varkappa_j, \mathbf{b}_{\perp j}\}) + (\text{interac.})$$

- Quark-spectator decomp: quark( $\varkappa$ )  $\oplus$  spect( $1 - \varkappa$ ):  $\mu_J$  AM dep.;  $\Phi =$  eff. conf.

$$M_B^2 = \int d\varkappa d^2\mathbf{b}_\perp \psi_B^*(\varkappa, \mathbf{b}_\perp) \underbrace{\left( \frac{-\nabla_{\mathbf{b}_\perp}^2}{\varkappa(1-\varkappa)} + \mu_J + \Phi \right)}_{=M_B^2} \psi_B(\varkappa, \mathbf{b}_\perp)$$

- Equivalent to **AdS** equation (variable  $z$ ) with confinement term

# Light-Front holography – AdS<sub>5</sub> formalism

- 5D AdS space defined by the metric  $R=5\text{D}$  radius  
[conformal symmetry: invariance  $(x, z) \rightarrow \lambda(x, z)$ ]

$$ds^2 = \frac{R^2}{z^2} (\eta_{\mu\nu} dx^\mu dx^\nu - dz^2)$$

- Fermion fields  $\Psi - F_L/F_R$  chiral components (Weyl spinors),  $n = 0, 1, \dots$

$$\Psi(x, z) = \frac{1}{\sqrt{2}} \sum_n z^2 \begin{pmatrix} F_{L,n}(z) \\ F_{R,n}(z) \end{pmatrix} \chi_n(x) e^{ix \cdot P}$$

$$S_N = \int d^4x dz \sqrt{-\det g} \bar{\Psi} (\hat{K} - \mu - \Phi) \Psi,$$

$\hat{K}$  = kinetic term,  $\mu$  = 5D mass,  $\Phi$  = effective scalar potential

- Interaction  $V_M$  = electromagnetic current – 3 couplings ( $\eta_p, \eta_m, g_V$ )  $\eta_N = \frac{1}{8} \kappa_N$

$$\hat{V}(x, z) = \hat{Q} \Gamma^M V_M(x, z) + \frac{i}{4} \eta_N [\Gamma^M, \Gamma^N] V_{MN}(x, z) + g_V \Gamma^M \gamma^5 V_M(x, z),$$

$$V_\mu(x, z) = \int \frac{d^4q}{(2\pi)^4} \epsilon_\mu(q) e^{-iq \cdot x} V(-q^2, z), \quad V(Q^2, z) = \kappa^2 z^2 \int_0^1 \frac{d\kappa}{(1-\kappa)^2} \kappa^{\frac{Q^2}{4\kappa^2}} e^{-\frac{\kappa^2 z^2 \kappa}{1-\kappa}}$$

T Gutsche, VE Lyubovitskij, I Schmidt and A Vega PRD 86, 036007 (2012); HR Grigoryan and AV Radyushkin, PRD 76, 095007 (2007)

# Light-Front holography – Nucleon and radial excitations

- Start with AdS equation (equivalent to LF equations) for  $\frac{1}{2}^+$  states

T Gutsche, VE Lyubovitskij, I Schmidt, and A Vega, PRD 86, 036007 (2012); PRD 87, 016017 (2013)

$$\left[ \pm \partial_z + \frac{\mu R + \Phi}{z} \right] F_{L/R}(z) = M_N F_{R/L}(z)$$

Other works: Z Abidin and CE Carlson, PRD 79, 115003 (2009); D Chakrabarti and C Mondal, EPJC 73, 2671 (2013); RS Sufian, GF de Teramond, SJ Brodsky, A Deur, and HG Dosch, PRD 95, 014011 (2017); T Gutsche, VE Lyubovitskij and I Schmidt, PRD 97, 054011 (2018)

- Reduction to second order Schrödinger-like equation  $m = \mu R$

$$\left[ -\frac{d^2}{dz^2} + \frac{m(m-1)}{z^2} + 2\kappa^2 \left( m + \frac{1}{2} \right) F_R + \kappa^4 z^2 \right] F_R = M_{N,n}^2 F_R$$

- Solutions and eigenvalues  $L_n^\alpha =$  generalized Laguerre polynomials;  $n = 0, 1, \dots$

$$F_{R,n} \propto z^m e^{-\kappa^2 z^2/2} L_n^{m-1/2}(\kappa^2 z^2), \quad M_{M,n}^2 = 4\kappa^2 \left( n + m + \frac{1}{2} \right),$$

$m = 3/2$ : pQCD form factors falloff – Z Abidin and CE Carlson, PRD 79, 115003 (2009)

- Transition current  $S_{\text{int}} = \int d^4 dz \sqrt{-\det g} \bar{\Psi}(x, z) \hat{\mathcal{V}}(x, z) \Psi(x, z)$

$$J^\mu = F_1 \left( \gamma^\mu - \frac{q_4^\mu}{q^2} \right) + F_2 \frac{i\sigma^{\mu\nu} q_\nu}{M_N + M_{N1}}$$

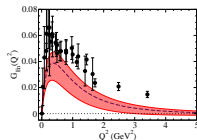
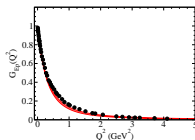
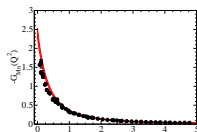
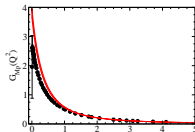
# Light-Front holography – Nucleon e.m. form factors

Analytic expressions in terms of  $a$  ( $\kappa \simeq 0.385$  GeV) and  $g_V$ ,  $\eta_p$ ,  $\eta_n$ ,  $\delta_N = \pm$   
T Gutsche, VE Lyubovitskij, I Schmidt, and A Vega, PRD 87, 016017 (2013)

$$a = \frac{Q^2}{4\kappa^2}$$

$$F_{1N} = e_N \frac{a+6}{(a+1)(a+2)(a+3)} + g_V \delta_N \frac{a}{(a+1)(a+2)(a+3)} + \eta_N \frac{2a(2a-1)}{(a+1)(a+2)(a+3)(a+4)},$$

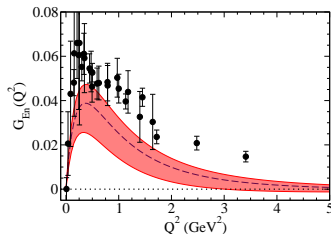
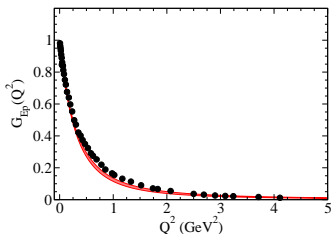
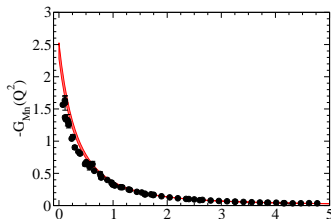
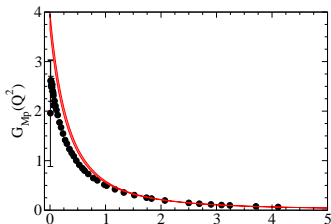
$$F_{2N} = \eta_N \frac{M_N}{2\sqrt{2}\kappa} \frac{48}{(a+1)(a+2)(a+3)},$$



Fit large  $Q^2$  data

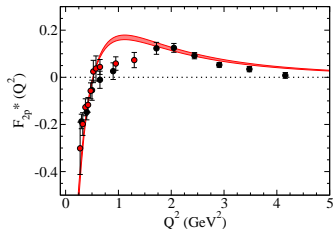
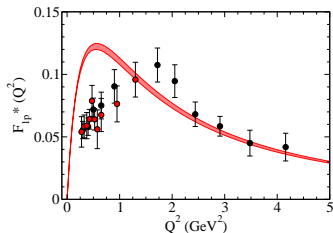


# Light-Front holography – Nucleon e.m. form factors



Red bands  $g_V = 1.28-1.57$ ;  $\eta_p = 0.38-0.42$ ;  $-\eta_n = 0.32-0.36$

# Light-Front holography – $N(1440)$ e.m. form factors



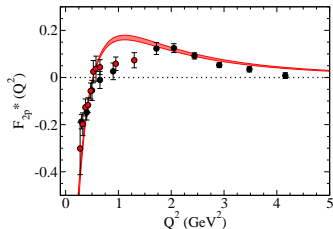
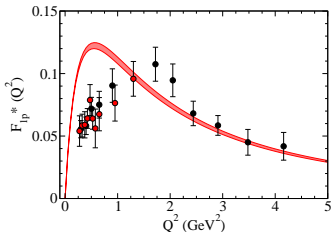
$$F_{1p}^* = \frac{a(\sqrt{2}a + c_1)}{(a+1)(a+2)(a+3)(a+4)} + g_V \frac{a(\sqrt{2}a + c_2)}{(a+1)(a+2)(a+3)(a+4)} + \eta_p \frac{2a(2\sqrt{2}a^2 - c_3a + c_4)}{(a+1)(a+2)(a+3)(a+4)(a+5)}$$

$$F_{2p}^* = \eta_p \left( \frac{M + M_R}{M_R} \right)^2 \frac{M_R}{2\sqrt{3}\kappa} \times \frac{6\sqrt{3}(c_5a - 4)}{(a+1)(a+2)(a+3)(a+4)}$$

$$a = \frac{Q^2}{4\kappa^2}$$

$$c_1 = 4\sqrt{2} + 3\sqrt{3}, \quad c_2 = 4\sqrt{2} - 3\sqrt{3}, \quad c_3 = 9(\sqrt{3} - \sqrt{2}), \quad c_4 = 3\sqrt{3} - 5\sqrt{2}, \quad c_5 = 2 + \sqrt{6}$$

# Light-Front holography – $N(1440)$ e.m. form factors (1')



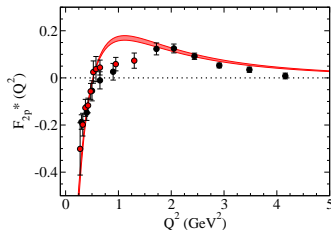
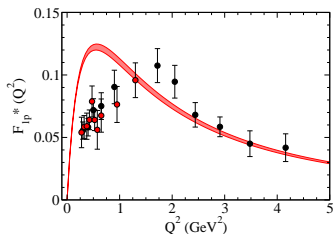
$$F_{1p}^* = \frac{a(\sqrt{2}a + c_1)}{(a+1)(a+2)(a+3)(a+4)} + g_V \frac{a(\sqrt{2}a + c_2)}{(a+1)(a+2)(a+3)(a+4)} + \eta_p \frac{2a(2\sqrt{2}a^2 - c_3a + c_4)}{(a+1)(a+2)(a+3)(a+4)(a+5)}$$

$$F_{2p}^* = \eta_p \left( \frac{M + M_R}{M_R} \right)^2 \frac{M_R}{2\sqrt{3}\kappa} \times \frac{6\sqrt{3}(c_5a - 4)}{(a+1)(a+2)(a+3)(a+4)} \\ a = \frac{Q^2}{4\kappa^2}$$

Accurate description of  $Q^2 > 2 \text{ GeV}^2$  data (narrow window)

based on the parametrization for the nucleon ... good  $F_{2p}^*$  at low- $Q^2$

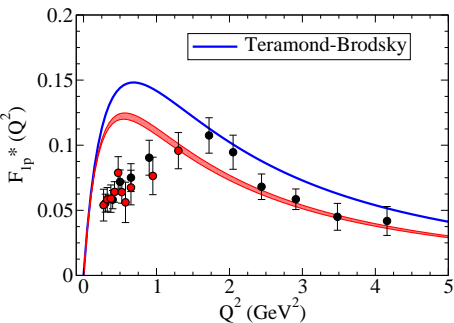
# Light-Front holography – $N(1440)$ e.m. form factors (2)



- Light-Front holographic QCD can be used to estimate elastic and transition form factors in leading twist approximation ( $qqq$  states, first Fock state)
- Expressions depend on 3 bare couplings:  $\kappa_p$ ,  $\kappa_n$  and  $g_V$
- After fixing the couplings (nucleon)  $\Rightarrow$  predictions for large  $Q^2$
- Contrary to previous works, the couplings are fixed by the large  $Q^2$  data  
At low  $Q^2$  we can expect contamination from non valence quark degrees of freedom (mainly  $q\bar{q}$  effects)

# Light-Front holography – Roper -Analytic expressions (1)

Important result from holography – resonances  $N^*$  – Analytic expression  
Roper Dirac form factor – GF Teramond and SJ Brodsky AIP Conf. P 1432, 168 (2012)



$$F_{1p}^* = \frac{\sqrt{2}}{3} \times \frac{\frac{Q^2}{m_\rho^2}}{\left(1 + \frac{Q^2}{m_\rho^2}\right) \left(1 + \frac{Q^2}{m_{\rho 1}^2}\right) \left(1 + \frac{Q^2}{m_{\rho 2}^2}\right)}$$

$$m_\rho = 2\kappa, \quad m_{\rho n} = 2\sqrt{2}\kappa\sqrt{n + \frac{1}{2}} \quad (n = 1, 2, \dots)$$

## Light-Front holography – Roper -Analytic expressions (2)

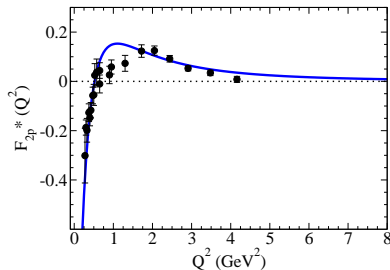
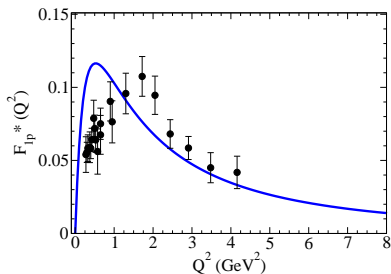
Inspired by the holographic results for  $F_{1p}^*$  and  $F_{2p}^*$  –  $\kappa \simeq 0.385$  GeV  
 $M_N \simeq 2\sqrt{2}\kappa$ ,  $4\kappa \simeq 1.520$  GeV  $\simeq M_{N1}$  (Roper) – 5% deviation

$$G_2 = \frac{1}{\left(1 + \frac{Q^2}{m_\rho^2}\right) \left(1 + \frac{Q^2}{m_{\rho_1}^2}\right) \left(1 + \frac{Q^2}{M_N^2}\right) \left(1 + \frac{Q^2}{M_{N1}^2}\right)}$$

$$G_3 = \frac{1}{\left(1 + \frac{Q^2}{m_\rho^2}\right) \left(1 + \frac{Q^2}{m_{\rho_1}^2}\right) \left(1 + \frac{Q^2}{m_{\rho_2}^2}\right) \left(1 + \frac{Q^2}{M_N^2}\right) \left(1 + \frac{Q^2}{M_{N1}^2}\right)}$$

$\Rightarrow$  Analytic expressions for  $F_{1p}^*$ ,  $F_{2p}^*$ , functions of  $m_{\rho n}$ ,  $M_N$ ,  $M_{N1}$   
— use empirical masses

# Light-Front holography – Roper - Analytic expressions (2')



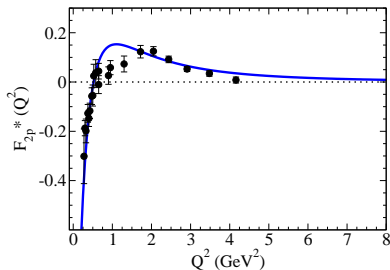
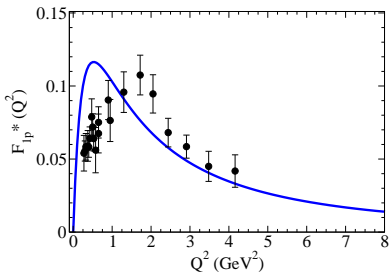
$$\begin{aligned}
 F_{1p}^* &= \frac{1}{12\sqrt{2}}(1 + g_V) \frac{Q^4}{m_\rho^4} G_2 \\
 &+ \frac{1}{24}(c_1 + c_2 g_V) \frac{Q^2}{m_\rho^2} G_2 \\
 &+ \frac{1}{60} \eta_p \left( 2\sqrt{2} \frac{Q^4}{m_\rho^4} - c_3 \frac{Q^2}{m_\rho^2} + c_4 \right) \frac{Q^2}{m_\rho^2} G_3
 \end{aligned}$$

$$\begin{aligned}
 F_{2p}^* &= \frac{\sqrt{3}}{4} \eta_p \left( \frac{M_{N1} + M_N}{M_{N1}} \right)^2 \\
 &\times \left( c_5 \frac{Q^2}{m_\rho^2} - 4 \right) G_2
 \end{aligned}$$

GR, PRD 96, 054021 (2017)

$$c_1 = 4\sqrt{2} + 3\sqrt{3}, \quad c_2 = 4\sqrt{2} - 3\sqrt{3}, \quad c_3 = 9(\sqrt{3} - \sqrt{2}), \quad c_4 = 3\sqrt{3} - 5\sqrt{2}, \quad c_5 = 2 + \sqrt{6}$$

# Light-Front holography – Roper - Analytic expressions (2')



$$\begin{aligned}
 F_{1p}^* &= \frac{1}{12\sqrt{2}}(1 + gv) \frac{Q^4}{m_\rho^4} G_2 \\
 &+ \frac{1}{24}(c_1 + c_2 gv) \frac{Q^2}{m_\rho^2} G_2 \\
 &+ \frac{1}{60} \eta_p \left( 2\sqrt{2} \frac{Q^4}{m_\rho^4} - c_3 \frac{Q^2}{m_\rho^2} + c_4 \right) \frac{Q^2}{m_\rho^2} G_3
 \end{aligned}$$

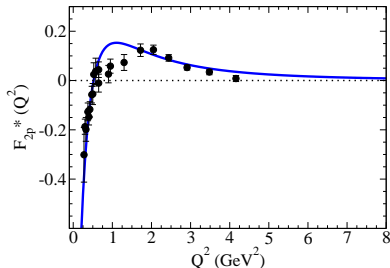
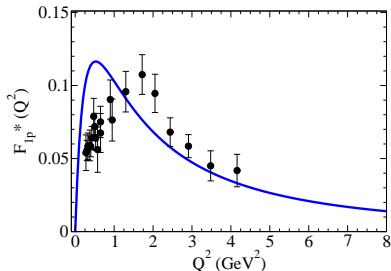
$$\begin{aligned}
 F_{2p}^* &= \frac{\sqrt{3}}{4} \eta_p \left( \frac{M_{N1} + M_N}{M_{N1}} \right)^2 \\
 &\times \left( c_5 \frac{Q^2}{m_\rho^2} - 4 \right) G_2
 \end{aligned}$$

GR, PRD 96, 054021 (2017)

Good analytic representation of the  $F_{1p}^*$  and  $F_{2p}^*$  data for  $Q^2 > 2.5 \text{ GeV}^2$



# Light-Front holography – Roper - Analytic expressions (2')



$$\begin{aligned}
 F_{1p}^* &= \frac{1}{12\sqrt{2}}(1 + g_V) \frac{Q^4}{m_\rho^4} G_2 \\
 &+ \frac{1}{24}(c_1 + c_2 g_V) \frac{Q^2}{m_\rho^2} G_2 \\
 &+ \frac{1}{60} \eta_p \left( 2\sqrt{2} \frac{Q^4}{m_\rho^4} - c_3 \frac{Q^2}{m_\rho^2} + c_4 \right) \frac{Q^2}{m_\rho^2} G_3
 \end{aligned}$$

$$\begin{aligned}
 F_{2p}^* &= \frac{\sqrt{3}}{4} \eta_p \left( \frac{M_{N1} + M_N}{M_{N1}} \right)^2 \\
 &\times \left( c_5 \frac{Q^2}{m_\rho^2} - 4 \right) G_2
 \end{aligned}$$

GR, PRD 96, 054021 (2017)

First analytic representation of  $F_{2p}^*$

# Light-Front holography – Nucleon Axial form factor (1)

Holographic estimate of the meson cloud contribution to nucleon axial form factor

GR PRD 97, 073002 (2018)

- Nucleon wave function: sum of Fock states: (no  $(qqq)g$  states)  
 $qqq$  – leading order  $\oplus$   $qqq(\bar{q}q)$  – 3rd Fock state
- Following T Gutsche, VE Lyubovitskij, I Schmidt and A Vega PRD 86, 036007 (2012)

$$\mathcal{L}_A = [g_A^0 \Gamma^M \gamma_5 \pm \Gamma^M] \mathcal{A}_M(x, z) \frac{\tau_3}{2} + \eta_A [\Gamma^M, \Gamma^N] \gamma_5 \mathcal{A}_{MN}(x, z) \frac{\tau_3}{2}$$

$g_A^0$ ,  $\eta_A$  coupling constants;  $\mathcal{A}_M$  = axial field,  $\mathcal{A}_{MN} = \partial_M A_N - \partial_N A_M$

- Axial form factor:  $c_3$ :  $qqq$  state;  $c_5$ :  $qqq(\bar{q}q)$ ;  $c_3 + c_5 = 1$

$$G_A(Q^2) = c_3 G_A^B(Q^2) + c_5 G_A^{MC}(Q^2)$$

- Compare with Quark Model:  $|N\rangle = Z_N [|qqq\rangle + b_N |MC\rangle]$ ,  $Z_N = 1/(1 + b_N^2)$

$$G_A(Q^2) = Z_N G_A^B(Q^2) + (1 - Z_N) G_A^{MC}(Q^2)$$

Physical  $N$ :  $Z_N$  = probability of  $qqq$  state;  $1 - Z_N$  = probability of MC st.

- $c_3 \equiv Z_N \oplus c_3 \leq 1$  – **Important constraint for holographic models**

# Light-Front holography – Nucleon Axial form factor (2)

Independent parameters:  $c_3$ ,  $g_A^0$  (bare axial coupling),

$\eta_A$  (bare induced pseudoscalar coupling) PRD 86, 036007 (2012)

$a = \frac{Q^2}{4\kappa^2}$ ,  $\kappa = \frac{m_\rho}{2} = 0.385$  GeV – holographic mass scale

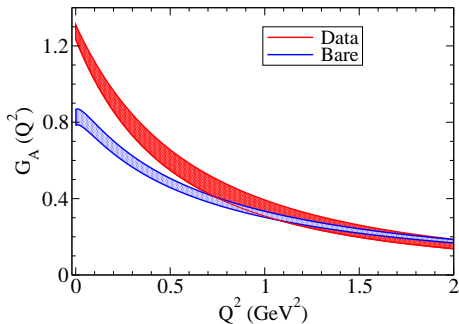
$$G_A^B = \left[ g_A^0 + \frac{a}{6}(g_A^0 - 1) \right] G_1 + \frac{1}{12} \eta_A a (2a + 18) G_2$$

$$G_A^{MC} = \left[ g_A^0 + \frac{a}{10}(g_A^0 - 1) \right] G_3 + \frac{1}{30} \eta_A a (4a + 49) G_4$$

$$G_1 = \frac{1}{(1+a)(1+\frac{a}{2})(1+\frac{a}{3})}, \quad G_2 = \frac{G_1}{1+\frac{a}{4}}, \quad G_3 = \frac{G_2}{1+\frac{a}{5}}, \quad G_4 = \frac{G_3}{1+\frac{a}{6}}$$

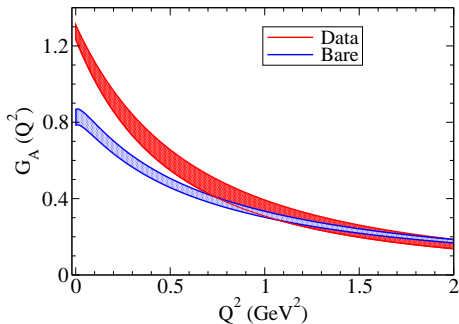
- Bare and MC depend both on  $g_A^0$  and  $\eta_A$
- $G_A^B(0) = g_A^0$ ,  $G_A^{MC}(0) = g_A^0$ ; Thus  $G_A(0) = c_3 g_A^0 + c_5 g_A^0 \equiv g_A^0$   
All holographic models give  $G_A(0) = g_A^0$
- How to fix the parameters  $c_3$  and  $\eta_A$  ?
- A free fit to the  $G_A^{\text{exp}}$  data provides different equivalent solutions  
 $(\eta_A, c_5) = (0.45, -0.41)$  – unphysical solution;  $Z_N = 1.41 > 1$ ;  
 $(\eta_A, c_5) = (0.68, 0.00)$  – no meson cloud
- The data does not fix  $c_3$  unless we provide a estimate for  $G_A^B$   
Use information from lattice QCD

# Light-Front holography – Nucleon Axial form factor (3)



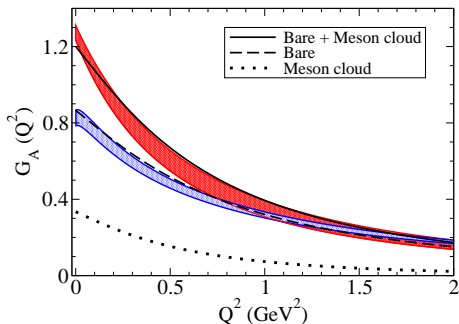
- Red band parametrization of the  $G_A$  data
- Blue band estimate of  $G_A^B$  based on lattice QCD data
- Extrapolation based QM in the range  $m_\pi = 0.35\text{--}0.5 \text{ GeV}^2$   
GR and K Tsumishima,  
PRD 94, 014001 (2016)

# Light-Front holography – Nucleon Axial form factor (3)



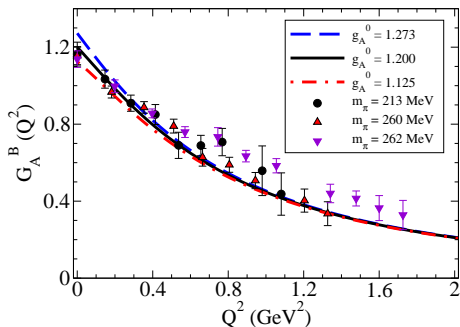
- Red band parametrization of the  $G_A$  data
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GR and K Tsuchima,  
PRD 94, 014001 (2016)
- Best fit:  $g_A^0 = 1.2$ ,  $\eta_A = 1.08$   
 $c_3 = 0.72$  (MC  $\sim 30\%$ )

# Light-Front holography – Nucleon Axial form factor (3')



- **Red band** parametrization of the  $G_A$  data
- **Blue band** estimate of  $G_A^B$  based on lattice QCD data
- Extrapolation based QM in the range  $m_\pi = 0.35\text{--}0.5 \text{ GeV}^2$   
GR and K Tushima,  
PRD 94, 014001 (2016)
- Best fit:  $g_A^0 = 1.2$ ,  $\eta_A = 1.08$   
 $c_3 = 0.72$  (MC  $\sim 30\%$ )
- Very slow falloff for MC ( $\dots$ )  
Slower than in most QM

# Light-Front holography – Nucleon Axial FF – Lattice



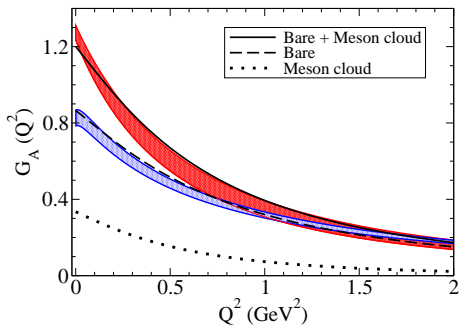
Compare with lattice

$$g_A^0 = 1.12; \mathbf{1.20}, 1.27$$

$$G_A = G_A^B + (1 - c_3)[G_A^{\text{MC}} - G_A^B]$$

- Lattice QCD underestimates  $G_A^{\text{exp}}(0) \simeq 1.27$  in general  
T Yamazaki et al, PRD 79, 114505 (2009)
- Best model (**1.20**) compares well with lattice ( $m_\pi \approx 230$  MeV)
- **Conclusions:**
  - good estimate of bare comp.
  - still small MC effects for  $m_\pi \approx 230$  MeV

# Light-Front holography – Nucleon Axial FF – Summary



## Holographic model

- 3 parameters  
 $G_A = G_A(c_3, g_A^0, \eta_A)$
- Estimate of  $G_A^B$  is necessary to calibrate the model ( $c_3 < 1$ )
- $g_A^0 = 1.2$  – Good estimate of  $G_A(\text{lattice}) \simeq G_A^B$
- **No dependence** on microscopic scales (dimension of bare core); pointlike quarks/mesons
- Very slow falloff of MC components ( $\dots$ )



# Light-Front holography – Summary and outlook

GR and D Melnikov, PRD 97, 034037 (2018); GR, PRD 96, 054021 (2017); PRD 97, 073002 (2018)

- Light-Front holography can be used to estimate elastic and transition form factors in leading twist approximation ( $qqq$  states, first Fock state)
- At small  $Q^2$  we expect contamination of next to leading order corrections (higher Fock states,  $q\bar{q}$ , ...)  
Low  $Q^2$  should not be used to fix the bare couplings
- Formalism can be used to derive analytic expressions for the transition form factors at large  $Q^2$  ( $F_1^*$  and  $F_2^*$ )
- Higher order corrections depend on the mixing coefficients ( $c_3, c_5, \dots$ ) and from the bare couplings  
 $\Rightarrow$  No reference to microscopic baryon-meson couplings & size of baryon bare core — slow falloff of the meson cloud
- Very promising method to estimate  $\gamma^* N \rightarrow N^*$  transition form factors at large  $Q^2$  – leading twist

- **Introduction**

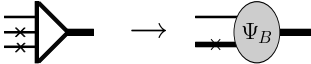
- $N(1520)_{\frac{3}{2}}^{-}$ ,  $N(1535)_{\frac{1}{2}}^{-}$  SR approximation;  $N(1440)_{\frac{1}{2}}^{+}$
- $\Delta(1232)_{\frac{3}{2}}^{+}$  – recent results for the quadrupole form factors

## In collaboration with:

F. Gross (JLab/USA), M.T. Peña (Lisbon/Portugal),  
and K. Tsushima (UCS/Brasil)

# Covariant Spectator Quark Model – Introduction (1)

- Baryons are **three constituent quark** systems
- Wave functions based on the  $SU(6)$  spin-flavor quark states
- Covariant Spectator Theory: quark $\oplus$ (quark-pair) separation  
Integration into the quark-pair degrees of freedom – **eff. diquark mass**  $m_s$

$$\int_{k_1} \int_{k_2} = \int_{4m_q^2}^{+\infty} ds \sqrt{\frac{s - 4m_q^2}{s}} \int \frac{d^3\mathbf{k}}{2\sqrt{s + \mathbf{k}^2}}$$


The diagram shows two quark lines (represented by lines with 'x' marks) entering a triangular vertex from the left. An arrow points from this vertex to a circular loop labeled  $\Psi_B$ , which is connected to a single quark line on the right.

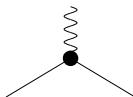
Stadler, Gross and Frank PRC 56, 2396 (1998); Gross and Agbakpe PRC 73, 015203 (2006); Gross, GR and Peña PRC 77, 015202 (2008); PRD **85**, 093005 (2012)

- Radial wave function  $\psi_B(P_B, k)$  determined phenomenologically

$$\psi_B(P_B, k) = \psi_B \left( \frac{(M_B - m_s)^2 - (P_B - k)^2}{m_s M_B} \right)$$

$M_B$  = baryon mass;  $m_s$  = diquark mass

- Quarks with electromagnetic structure  
(**impulse approximation**)



$$j_q^\mu = \left( \frac{1}{6} f_{1+} + \frac{1}{2} f_{1-\tau_3} \right) \gamma^\mu + \left( \frac{1}{6} f_{2+} + \frac{1}{2} f_{2-\tau_3} \right) \frac{i\sigma^{\mu\nu} q_\nu}{2M_N}$$

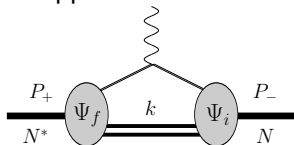
quark form factors  $f_{i\pm}$  parametrize dressing of quarks (gluons and  $q\bar{q}$ )

- **Vector Meson Dominance:**  $f_{i\pm} = f_{i\pm}(Q^2; m_\rho, M_N)$  PRD 80, 033004 (2009)

# Covariant Spectator Quark Model – Introduction (2)

- Transition current –relativistic impulse approximation

$$J^\mu = 3 \sum_\lambda \int_k \bar{\Psi}_f(P_+, k) j_q^\mu \Psi_i(P_-, k)$$



diquark on-shell

- Quark current  $j_q^\mu$  and nucleon radial wave function  $\psi_N(P_N, k)$  calibrated by **nucleon elastic form factor data**

Gross, GR, Peña, PRC 77, 015202 (2008)

- Generalization to lattice QCD:**

- $f_{i\pm}(Q^2; m_\rho, M_N) \rightarrow f_{i\pm}(Q^2; m_\rho^{\text{latt}}, M_N^{\text{latt}})$  – VMD
- $\psi_B(M_B) \rightarrow \psi_B(M_B^{\text{latt}})$

GR, MT Peña, JPG 36, 115011 (2009); PRD 80, 013008 (2009); GR, K Tsushima,

F Gross, PRD 80, 033004 (2009); GR, K Tsushima, AW Thomas, JPG 40, 015102 (2013)

# Semirelativistic approximation - Introduction

$\mathbf{q}$  = photon three-momentum at the  $R$  rest frame

- **Non relativistic regime:**

Othogonality between states is defined when  $|\mathbf{q}| = 0$ .

Both particles at rest

Transition form factors independent of the mass ( $M_N$  or  $M_R$ )

- **Relativistic regime:**

There are ambiguities related with the **relativistic generalization**

Dificulties in defining **othogonality** between states defined in different rest-frames when  $M_R \neq M_N$

- **SemiRelativistic approximation:** GR, PRD 95, 054008 (2017)

- Calculate **elementary** form factors ( $F_1^*, F_2^*/G_1, G_2, G_3$ ) in the limit  $M_R = M_N$

- Use definition of **multipole form factors** and **helicity amplitudes** ( $M_R \neq M_N$ ) to compare with **measured data**.

Cases:  $N(1535)\frac{1}{2}^-$ ,  $N(1520)\frac{3}{2}^-$

## SR approximation – Notation

Use  $R$  rest frame, for simplicity, to define invariant integrals

$$\mathcal{I}_R(Q^2) = \int_k \frac{k_z}{|\mathbf{k}|} \psi_R(P_R, k) \psi_N(P_N, k)$$

We assume that  $\psi_R \equiv \psi_N$ ; In the limit  $M_R \rightarrow M_N$ :

$$\mathcal{I}_R \propto |\mathbf{q}|$$

$\mathcal{I}_R(0) = 0$ : orthogonality between  $N$  (mass  $M_N$ ) and  $R$  (mass  $M_R$ ) states  
**Equal mass limit:**  $M_N, M_R \rightarrow M \equiv \frac{1}{2}(M_N + M_R)$

$$j_i^S \equiv \frac{1}{6} f_{i+} + \frac{1}{2} f_{i-} \tau_3$$
$$j_i^A \equiv \frac{1}{6} f_{i+} - \frac{1}{6} f_{i-} \tau_3$$

## SR approximation – $N(1535)$

$$J^\mu = \bar{u}_R \left[ F_1^* \left( \gamma^\mu - \frac{\not{q} q^\mu}{q^2} \right) + F_2^* \frac{i\sigma^{\mu\nu} q_\nu}{M_N + M_R} \right] \gamma_5 u_N$$

GR, MT Peña, PRD 84, 033007 (2011); GR, PRD 95, 054008 (2017)

$$F_1^* = \frac{1}{2}(3j_1^S + j_1^A)\mathcal{I}_R$$

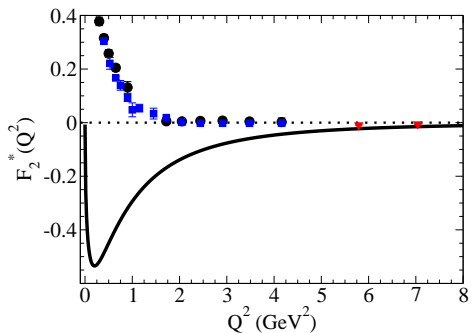
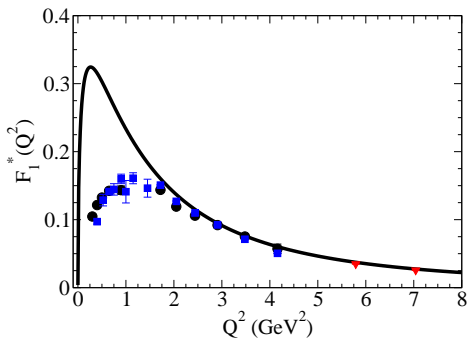
$$F_2^* = -\frac{1}{2}(3j_2^S - j_2^A) \frac{M_N + M_R}{2M_N} \mathcal{I}_R$$

**SR approach: No parameters adjusted**

$$\frac{M_N + M_R}{2M_N} \rightarrow 1, \quad |\mathbf{q}| \rightarrow Q\sqrt{1 + \tau}$$

where  $\tau = \frac{Q^2}{(M_N + M_R)^2}$

# SR approximation – $N(1535)$ – Results



- Data from **CLAS**, **MAID** and **Jlab/Hall C**
- $F_i^*(Q^2) \propto |\mathbf{q}|$ , implying that  $F_i^*(0) = 0$
- Good results for  $F_1^*$  ( $Q^2 > 2 \text{ GeV}^2$ ),  $F_2^*$  wrong sign;  $(F_2^*)_{\text{exp}} \approx 0$  more latter



# SR approximation – $N(1520)$ $P = \frac{1}{2}(P_R + P_N)$

$$J^\mu = \bar{u}_\alpha [G_1 q^\alpha \gamma^\mu + G_2 q^\alpha P^\mu + G_3 q^\alpha q^\mu + \dots] \gamma_5 u_N$$

GR, MT Peña, PRD 89, 094016 (2014); PRD 95, 014003 (2017)  $G_M = -G_E$

$$G_1 = -\frac{3}{2\sqrt{2}} \left[ \left( j_1^A + \frac{1}{3} j_1^S \right) + \frac{M_R + M_N}{2M_N} \left( j_2^A + \frac{1}{3} j_2^S \right) \right] \frac{\mathcal{I}_R}{|\mathbf{q}|}$$

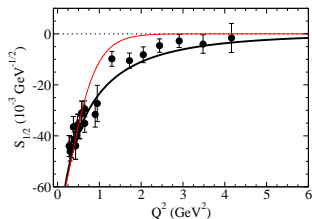
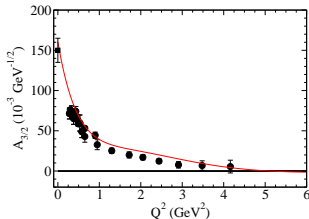
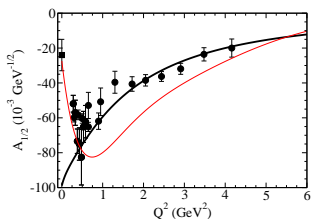
$$G_2 = \frac{3}{2\sqrt{2}M_N} \left[ j_2^A + \frac{1}{3} \frac{1-3\tau}{1+\tau} j_2^S + \frac{4}{3} \frac{2M_N}{M_R + M_N} \frac{1}{1+\tau} j_1^S \right] \frac{\mathcal{I}_R}{|\mathbf{q}|}$$

$$G_3 = \frac{3}{2\sqrt{2}} \frac{M_R - M_N}{Q^2} \left[ j_1^A + \frac{1}{3} \frac{\tau-3}{1+\tau} j_1^S + \frac{4}{3} \frac{M_R + M_N}{2M_N} \frac{\tau}{1+\tau} j_2^S \right] \frac{\mathcal{I}_R}{|\mathbf{q}|}$$

**SR approach:**  $|\mathbf{q}| \rightarrow Q\sqrt{1+\tau}$  **No parameters adjusted**

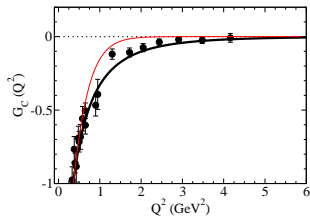
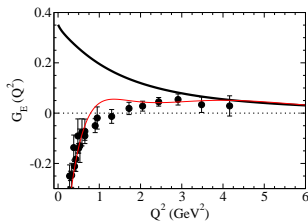
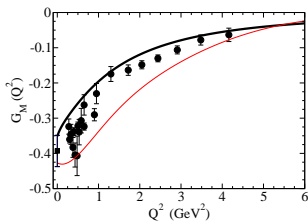
$$\frac{\mathcal{I}_R}{|\mathbf{q}|} \rightarrow \text{const} (M_R \rightarrow M_N), \quad Q^2 G_3 \rightarrow 0$$

# SR approximation – $N(1520)$ – Results (1)



- SemiRelativistic approach (SR); data from CLAS, include MAID fit
- SR very good description of the  $Q^2 > 1.5 \text{ GeV}^2$  data  
Except for  $A_{3/2}$  (CSQM:  $A_{3/2} \equiv 0$ ) –  $A_{3/2} \leftarrow$  dominated by meson cloud ?
- Describe well valence quark degrees of freedom ( $(A_{3/2})_{\text{bare}} \approx 0$ )

# SR approximation – $N(1520)$ – Results (2)



- SemiRelativistic approach (SR); data from CLAS, include MAID fit
- SR very good description of the  $Q^2 > 1.5 \text{ GeV}^2$  data  
Except for  $G_E$  (CSQM:  $G_E \equiv -G_M$ ,  $A_{3/2} \equiv 0$ )
- Describe well valence quark degree of freedom ( $(A_{3/2})_{\text{bare}} \approx 0$ )

# SR approximation – Summary

- In general **SR approach** gives a good description of the **form factors**  
**No parameters adjusted** ( $\psi_R \equiv \psi_N$ )
- Good description of region  $Q^2 > 1.5 \text{ GeV}^2$   
Exceptions:
  - $N(1520)$ :  $A_{3/2}$  and  $G_E$   
Quark models:  $A_{3/2}$  is *usually* very small  
**Interpretation:**  $A_{3/2}$  dominated by meson cloud effects  $A_{3/2} \simeq A_{3/2}^{\text{mc}}$   
[Using  $|A_{3/2}^{\text{mc}}| \gg |A_{1/2}^{\text{mc}}|$ :  $\Rightarrow G_M^{\text{mc}} \simeq \frac{1}{3} G_E^{\text{mc}}$  ]
  - $N(1535)$ :  $F_2^*$  –discussed in next slide

GR, PRD 95, 054008 (2017)

# $\gamma^* N \rightarrow N(1535)$ : Relation between $A_{1/2}$ and $S_{1/2}$

## Implications of $F_2^* = 0$ ?

$$\tau = \frac{Q^2}{(M_R + M_N)^2} \quad Q^2 > 1.5 \text{ GeV}^2$$

$$S_{1/2} \simeq -\frac{\sqrt{1+\tau}}{\sqrt{2}} \frac{M_R^2 - M_N^2}{2M_R Q} A_{1/2}$$

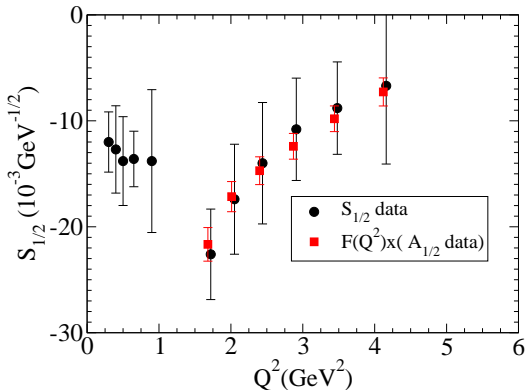
GR, K Tsushima  
PRD 84, 051301 (2011)

GR, D Jido, K Tsushima  
PRD 85, 093014 (2012)

**Cancellation between  
valence and meson cloud**

Consistent with meson cloud  
calculations –  $\chi$  Unitary Model

D Jido, M Doring and E Oset,  
PRC 77, 065207 (2008)



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GR, K Tsushima  
PRD 84, 051301 (2011)

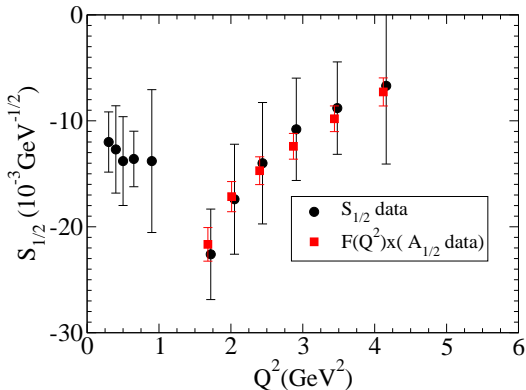
GR, D Jido, K Tsushima  
PRD 85, 093014 (2012)

Cancellation between  
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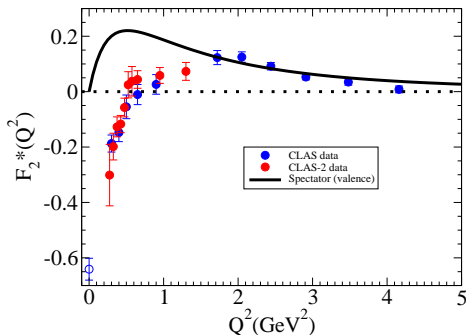
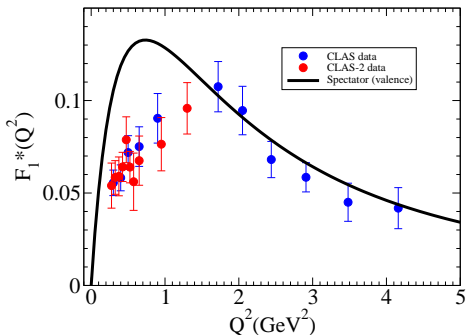
Consistent with meson cloud  
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D Jido, M Doring and E Oset,  
PRC 77, 065207 (2008)

More data are welcome

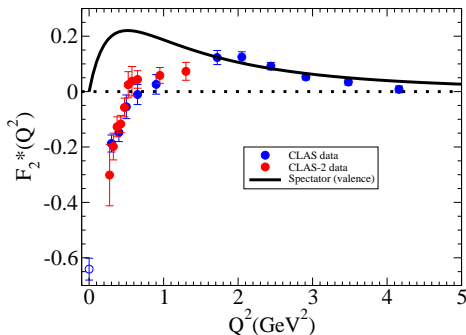
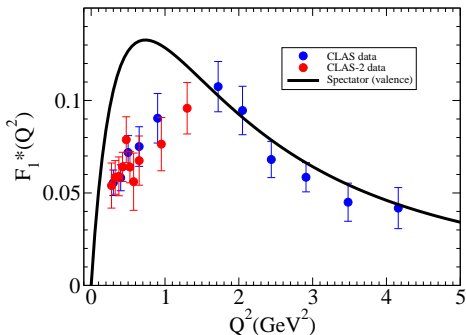


# $\gamma^* N \rightarrow N(1440) - \text{Introduction}$



- **CSQM:** Roper defined as the **1st radial** excitation of the nucleon  
Same **spin/**flavor structure as the nucleon  
Radial wave function defined by the orthogonality with nucleon state  
**GR and K Tsushima, PRD 81, 074020 (2010); PRD 89, 073010 (2014)**
- **No adjustable parameters;** **No meson cloud** components included
- **CLAS data:** **IG Aznauryan et al., PRC 80, 055203 (2009);**  
**VI Mokeev et al., PRC 86, 035203 (2012); PRC 93, 025206 (2016)**

# $\gamma^* N \rightarrow N(1440) - \text{Results}$



- **Good results** for  $Q^2 > 1.5$  GeV<sup>2</sup> – valence quark dominance  
Support **Roper** as **1st radial** excitation of the nucleon – IG Aznauryan PRC 76, 025212 (2007)
- **Failure** for  $Q^2 < 1.5$  GeV<sup>2</sup> – meson cloud ?  
Used to estimate meson cloud from CLAS data  
GR and K Tsushima, AIP Conf.Proc. 1374, 353 (2011)

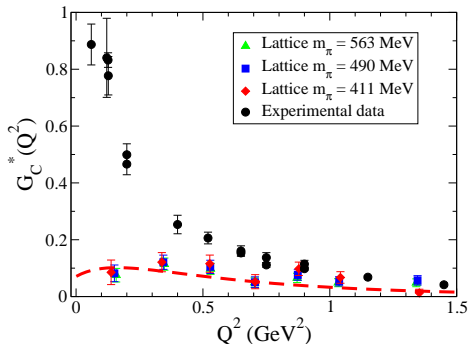
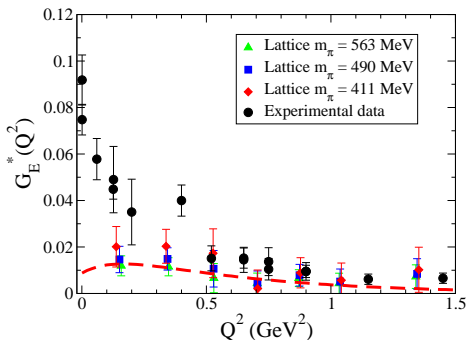


# $\gamma^* N \rightarrow \Delta(1232)$ – Form Factors – Status

- **Magnetic form factors  $G_M^*$** : (not discussed here)
  - Dominated by **valence quark effects** ( $\approx 70\%$ ):  
The gap between QM and the explained as **pion cloud effects**
  - There is today a convergence of results based on different frameworks; Dynamical Models, Dyson-Schwinger, Quark Models; Lattice QCD, ...
- **Quadrupole form factors  $G_E^*$ ,  $G_C^*$  (smaller magnitude)** (next)
  - **Quarks models** (mostly NR) predict only a fraction ( $\approx 10\text{--}20\%$ )
  - **Pion cloud estimates**  
 $SU(6)$  Constituent Quark Models  $\oplus$  Large  $N_c$  models  
 $\Rightarrow G_E^*$ ,  $G_C^*$  are dominated by **pion cloud effects**  
V Pascalutsa and M Vanderhaeghen, PRD 76, 111501 (2007);  
P Grabmayr and AJ Buchmann, PLB 86 (2001)
  - Exceptions: Dyson-Schwinger equations:  
– larger contributions from the bare core  
G Eichmann and D Nicmorus, PRD 85, 093004 (2012);  
J Segovia, and CD Roberts, PRC 94, 042201 (2016)

$\gamma N \rightarrow \Delta$ :  $G_E^*(Q^2)$ ,  $G_C^*(Q^2)$  (bare) (D1  $S_q = 1/2$ ; D3  $S_q = 3/2$ )

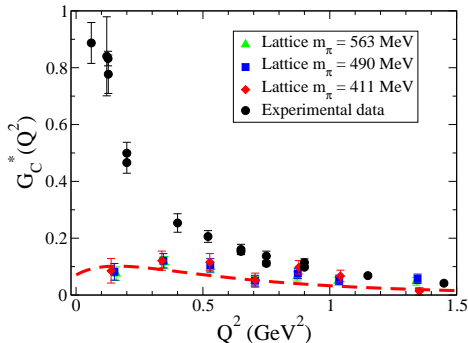
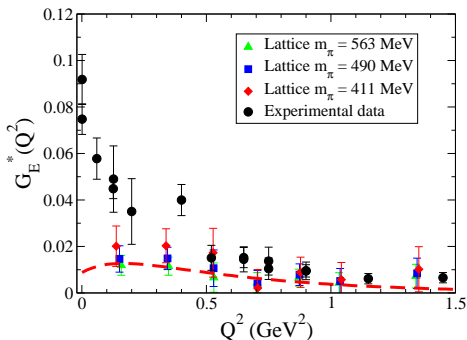
## Small valence quark contributions



- $D$ -state parameters adjusted to lattice QCD data (D1, D3  $\approx 0.7\%$ )
- - - - Result extrapolated to the physical limit

$\gamma N \rightarrow \Delta$ :  $G_E^*(Q^2)$ ,  $G_C^*(Q^2)$  (bare) (D1  $S_q = 1/2$ ; D3  $S_q = 3/2$ )

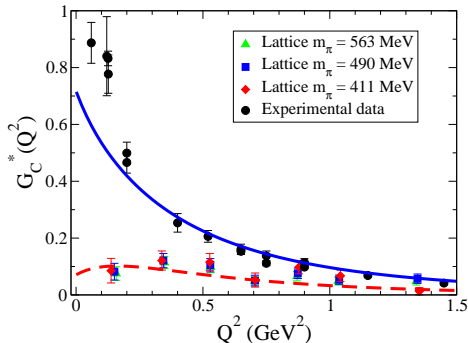
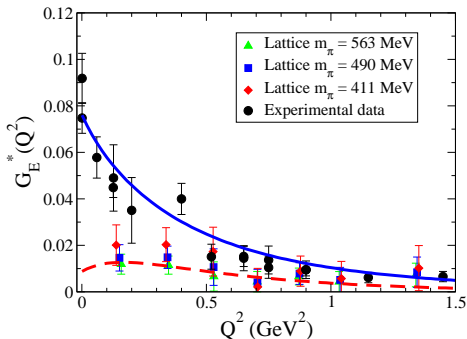
## Small valence quark contributions



- $D$ -state parameters adjusted to lattice QCD data (D1, D3  $\approx 0.7\%$ )
- Lattice data: Alexandrou et al, PRD 77, 085012 (2008)

$\gamma N \rightarrow \Delta: G_E^*(Q^2), G_C^*(Q^2)$  (bare + pion cloud)

## Add pion cloud contribution



- $G_E^*$ : Pascalutsa and Vanderhaeghen, PRD 76 (2007) – Large  $N_c$
- $G_C^*$ : Buchmann PRD 66 (2002) –  $G_{E,C}^* \propto G_{En}/Q^2$

# $\gamma N \rightarrow \Delta$ : $G_E^*(Q^2)$ , $G_C^*(Q^2)$ – Pion cloud effects

- Constituent Quark Models with two-body exchange currents (diagrams with pion and  $q\bar{q}$  states) & Large  $N_c$  limit
    - processes interpreted as pion cloud effects
- V Pascalutsa and M Vanderhaeghen, PRD 76, 111501 (2007);  
P Grabmayr and AJ Buchmann, PLB 86 (2001)

- Exact  $SU(6)$  (symmetric  $N$  and  $\Delta$ )

$$G_E^*(0) = 0, \quad G_C^*(0) = 0, \quad G_{En} \equiv 0,$$

- Broken  $SU(6)$  (deformed  $\Delta$ , asymmetric neutron)

$$G_E^*(0) \propto r_n^2, \quad G_C^*(0) \propto r_n^2, \quad G_{En} \simeq -\frac{1}{6}r_n^2 Q^2,$$

$G_{En} \neq 0$  also interpreted as consequence deformation of  $q\bar{q}$  cloud  
AJ Buchmann and EM Henley, PRC 63, 015202 (2000)

$\gamma N \rightarrow \Delta$ :  $G_E^*(Q^2)$ ,  $G_C^*(Q^2)$  – Siegert's theorem

Siegert's theorem ( $E_{1+}/|\mathbf{q}| \propto S_{1+}/|\mathbf{q}|^2$ )

At the pseudo-threshold (nucleon and  $\Delta$  at rest):  $Q_{pt}^2 = -(M_\Delta - M)^2$

$$G_E^*(Q_{pt}^2) = \overbrace{\frac{M_\Delta - M}{2M_\Delta}}^{=\kappa} G_C^*(Q_{pt}^2)$$

HF Jones and MD Scadron, Ann Phys 81, 1 (1973); AJ Buchmann, E Hernandez, U Meyer, and A Faessler, PRC 58, 2478 (1998); D Drechsel, L Tiator, JPG 18, 449 (1992); GR, PRD 94, 114001 (2016); arXiv:1709.07412 [hep-ph]

$\gamma^* N \rightarrow \Delta$ : quadrupoles – Siegert's theorem  $\kappa = \frac{M_\Delta - M}{2M_\Delta}$  (1)

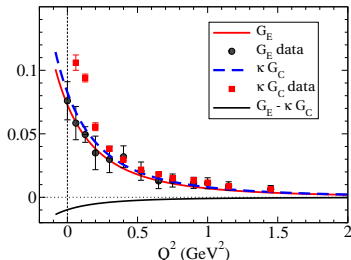
Pascalutsa-Vanderhaeghen-Buchmann parametrization

**violate** Siegert's theorem

Large- $N_c$ :  $\tilde{G}_{En} = G_{En}/Q^2$

$$G_E^\pi = \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M^2}{2\sqrt{2}} \tilde{G}_{En}$$

$$G_C^\pi = \left(\frac{M}{M_\Delta}\right)^{1/2} \sqrt{2} M M_\Delta \tilde{G}_{En},$$



No quark effects

GR, PRD 94, 114001 (2016)

Breaking of Siegert's theorem

$$\begin{aligned} \mathcal{R}_{pt} &= G_E(Q_{pt}^2) - \kappa G_C(Q_{pt}^2) \\ &\simeq - \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{r_n^2}{12\sqrt{2}} Q_{pt}^2 = \mathcal{O}\left(\frac{1}{N_c^2}\right) \end{aligned}$$

$M, M_\Delta = \mathcal{O}(N_c)$ ;  $M_\Delta - M = \mathcal{O}(1/N_c)$

# $\gamma^* N \rightarrow \Delta$ : quadrupoles – Siegert's theorem (2)

## Imposing Siegert's theorem

Corrected Large- $N_c$ :  $\alpha = \frac{Q^2}{2M_\Delta(M_\Delta - M)}$

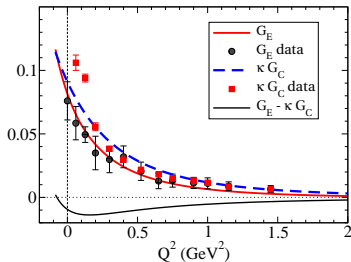
Relative correction  $\mathcal{O}(1/N_c^2)$  in  $G_E^\pi$  at PT

$$G_E^\pi = \left(\frac{M}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M^2}{2\sqrt{2}} \tilde{G}_{En} \frac{\tilde{G}_{En}}{1 + \alpha}$$

$$G_C^\pi = \left(\frac{M}{M_\Delta}\right)^{1/2} \sqrt{2} M M_\Delta \tilde{G}_{En},$$

$\frac{1}{1+\alpha}$  – preserve result at  $Q^2 = 0$

Siegert's theorem:  $\mathcal{R}_{pt} = 0$



Valence quark effects included  
GR arXiv:1709.07412 [hep-ph]/EPJA



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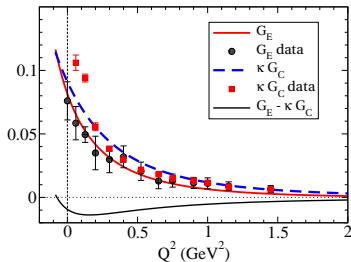
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Valence quark effects included

GR arXiv:1709.07412 [hep-ph]/EPJA

$G_C$  data underestimated

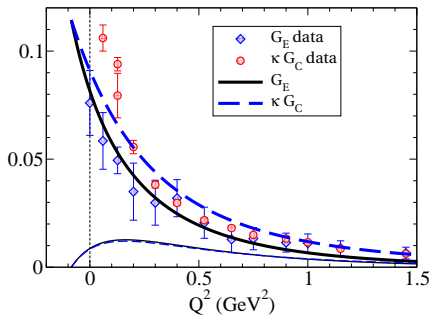
# $\gamma^* N \rightarrow \Delta$ : quadrupoles – Siegert's theorem – new results

Electroexcitation of the  $\Delta^+(1232)$  at low momentum transfer

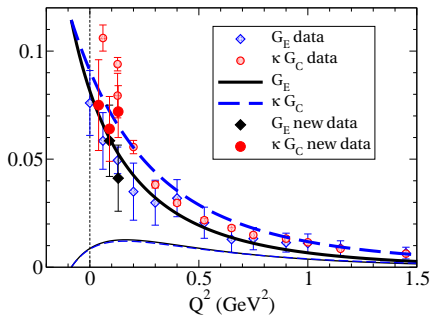
A Blomberg, PLB 760, 267 (2016), JLab/Hall A

- New results at **low momentum transfer region**, where the mesonic cloud dynamics is predicted to be dominant ...
- The new data explore the  $Q^2$  dependence of the resonant quadrupole amplitudes and **for the first time indicate that the Electric and the Coulomb quadrupole amplitudes converge as  $Q^2 \rightarrow 0$**   
[  $\Rightarrow$  **reduction of values for  $G_C$**  ]
- The **source of disagreement** (compared with previous measurements) has been identified in the **extraction procedure of the resonant amplitudes** from the measured MAMI cross sections.

# $\gamma^* N \rightarrow \Delta$ : quadrupoles – Siegert's theorem – new results

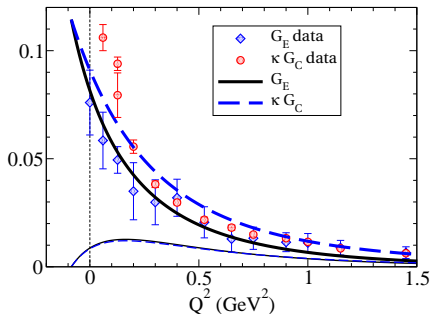


Before 2016

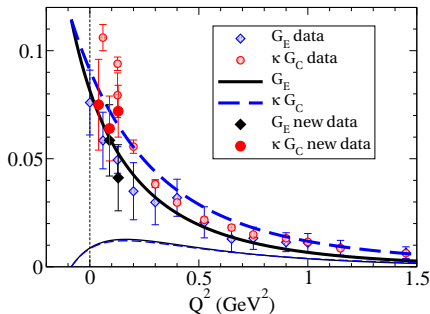


After 2016

# $\gamma^* N \rightarrow \Delta$ : quadrupoles – Siegert's theorem – new results



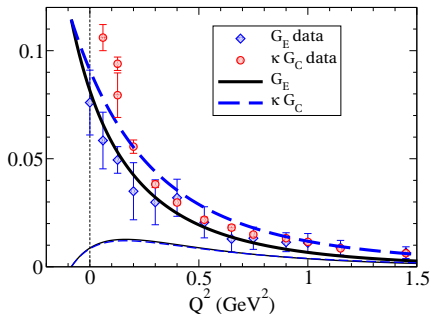
Before 2016



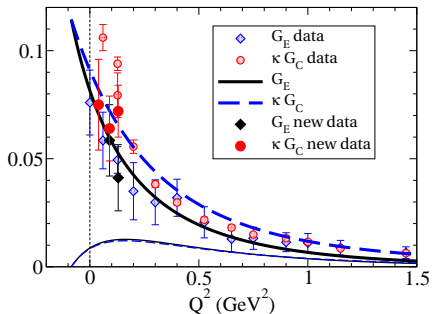
After 2016

- The combination of valence quark component (fixed by lattice QCD)

# $\gamma^* N \rightarrow \Delta$ : quadrupoles – Siegert's theorem – new results



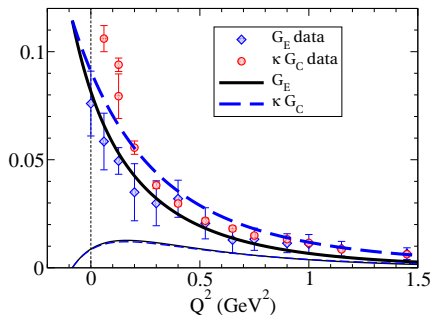
**Before 2016**



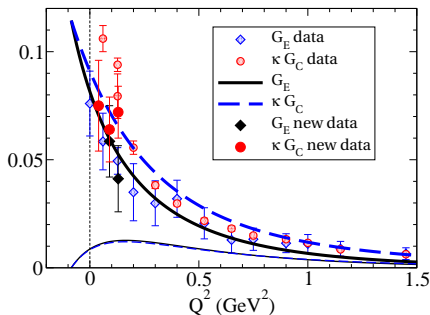
**After 2016**

- The combination of **valence quark component** (fixed by **lattice QCD**)
- ... with well known **pion cloud parametrizations** (parameter-free)

# $\gamma^* N \rightarrow \Delta$ : quadrupoles – Siegert's theorem – new results



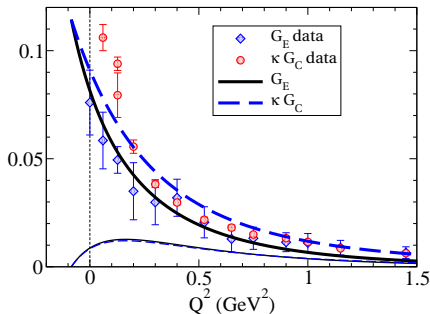
## Before 2016



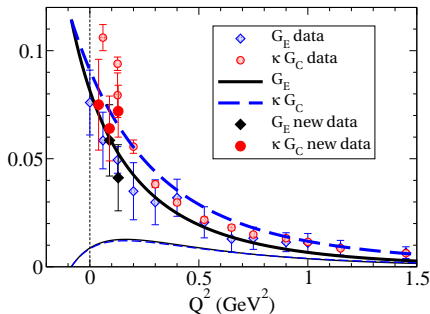
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- $\Rightarrow$  **good description** of the **new data** ...

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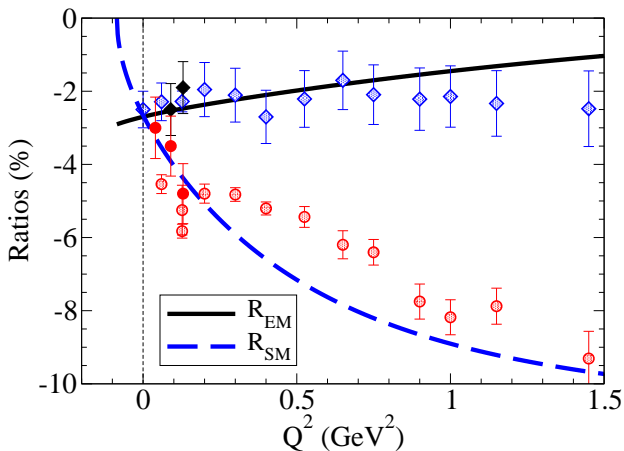
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- The combination of **valence quark component** (fixed by **lattice QCD**)
- ... with well known **pion cloud parametrizations** (parameter-free)
- $\Rightarrow$  **good description** of the **new data** ... and consistence with **ST**

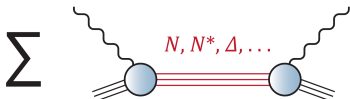
# $\gamma^* N \rightarrow \Delta$ : quadrupoles – new results – EM ratios †





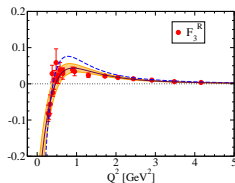
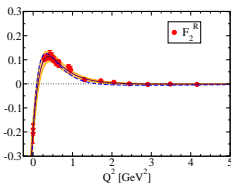
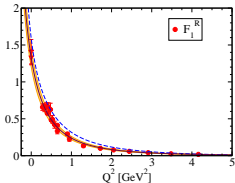
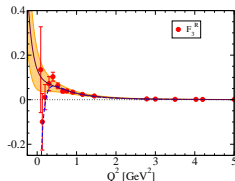
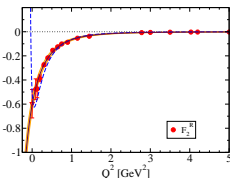
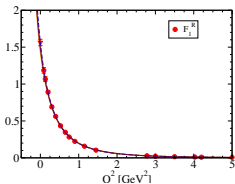
# Other applications

# Compton Scattering – parametrization of $N^*$ form factors

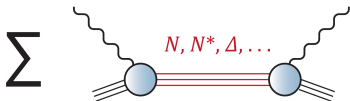


G Eichmann and GR, in preparation

Results dominated by  $\Delta(1232)$  (top) and  $N(1520)$  (bottom)

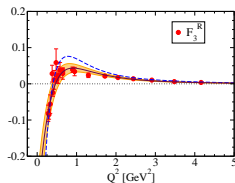
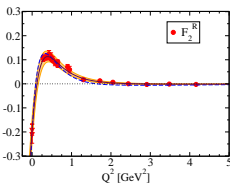
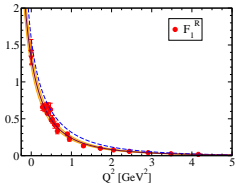
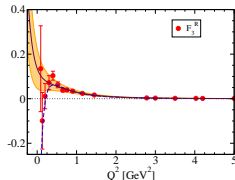
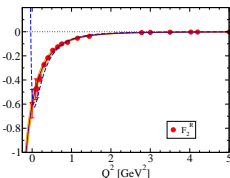
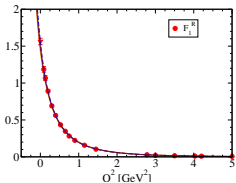


# Compton Scattering – parametrization of $N^*$ form factors



G Eichmann and GR, in preparation

More accurate **low- $Q^2$**  data is necessary to determine the trend of the form factors



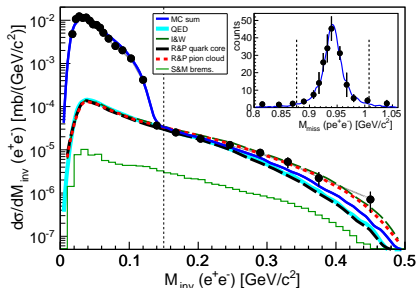
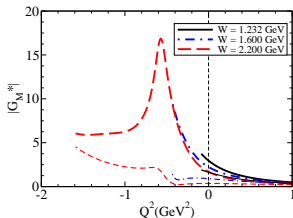
$\gamma^* N \rightarrow \Delta$  timelike FF ( $Q^2 < 0$ ): Dalitz decay ( $\Delta \rightarrow e^+ e^- N$ ) -  $\frac{d\sigma}{dm_{ee}}(pp)$

Form factor  $G_M^*(Q^2, W)$  - timelike regime  $Q^2 \leq -(W - M)^2$  **HADES**

Relation between the  $N - \Delta$  pion cloud and pion form factor  $F_\pi(Q^2) \propto 1/(m_\rho^2 + Q^2)$



$G_M^*(Q^2) \rightarrow G_M^*(Q^2, W)$ :



GR and MT Peña, PRD 85113014 (2012) **Data:** HADES collaboration, PRC 95, 065205 (2017)

GR, MT Peña J Weil, H van Hees and U Mosel, PRD 93, 033004 (2016) -  $W$ -dependence

# Covariant Spectator QM – Summary and conclusions

- Provide covariant estimates for the form factors (helicity amplitudes) for several  $N^*$  states –  $\Delta(1232)$ ,  $N(1440)$ , and  $N(1535)$ ,  $N(1520)$  – SR approach
- In general we have a good agreement with the (large  $Q^2$ ) data ... with a few exceptions

## Role of quark degrees of freedom seems to be under control

- $\Delta(1232)$ – quadrupole transition form factors: large pion cloud  
Pion cloud parametrizations consistent with Siebert's theorem  $\oplus$  valence quark component  
 $\Rightarrow$  very good description of new low- $Q^2$  data ( $G_C$ ,  $G_E$ )
- The model can be extended to higher  $Q^2$  and higher resonances  
Predictions can be tested in the JLab in a near future (JLab-12)  
Also  $N(1650)\frac{1}{2}^-$ ,  $N(1700)\frac{3}{2}^-$ ,  $\Delta(1620)\frac{1}{2}^-$ ,  $\Delta(1700)\frac{3}{2}^-$  – PRD 90, 033010 (2014)

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Thank you

