

QCD Workshop | Hervé MOUTARDE

May 4th, 2018

Covariant extension

Phenomenology

- Content of GPDs
- Experimental access
- Tomography
- Dispersion relations

Modeling

- Definition
- Polynomiality
- Radon transform
- Positivity
- Inverse Radon
- Examples

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- Design
- Fits
- Releases

Conclusion

- 1 What do we need for high precision phenomenology?
- 2 How can we implement all theoretical constraints in flexible GPD parameterizations?
- 3 How do we relate all this to actual measurements?

What do we need for high precision phenomenology?

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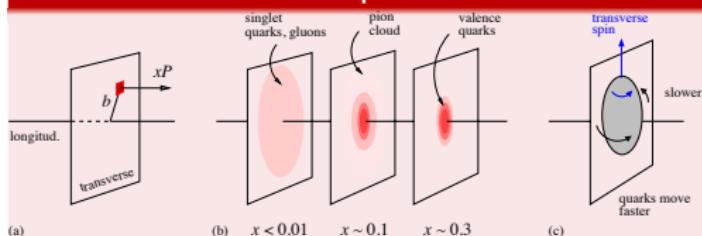
- Probabilistic interpretation of Fourier transform of $GPD(x, \xi = 0, t)$ in transverse plane.

$$\rho(x, b_\perp, \lambda, \lambda_N) = \frac{1}{2} \left[\mathcal{H}(x, 0, b_\perp^2) + \frac{b_\perp^j \epsilon_{ji} S_\perp^i}{M} \frac{\partial \mathcal{E}}{\partial b_\perp^2}(x, 0, b_\perp^2) + \lambda \lambda_N \tilde{\mathcal{H}}(x, 0, b_\perp^2) \right].$$

- Notations : quark helicity λ , nucleon longitudinal polarization λ_N and nucleon transverse spin S_\perp .

Burkardt, Phys. Rev. D62, 071503 (2000)

Can we obtain this picture from exclusive measurements?



Weiss, AIP Conf.
Proc. 1149,
150 (2009)

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- Most general structure of matrix element of energy momentum tensor between nucleon states:

$$\left\langle N, P + \frac{\Delta}{2} \right| T^{\mu\nu} \left| N, P - \frac{\Delta}{2} \right\rangle = \bar{u} \left(P + \frac{\Delta}{2} \right) \left[A(t) \gamma^{(\mu} P^{\nu)} \right. \\ \left. + B(t) P^{(\mu} i\sigma^{\nu)\lambda} \frac{\Delta_{\lambda}}{2M} + \frac{C(t)}{M} (\Delta^{\mu}\Delta^{\nu} - \Delta^2 \eta^{\mu\nu}) \right] u \left(P - \frac{\Delta}{2} \right),$$

with $t = \Delta^2$.

- Key observation: **link between GPDs and gravitational form factors**

$$\int dx x H^q(x, \xi, t) = A^q(t) + 4\xi^2 C^q(t),$$

$$\int dx x E^q(x, \xi, t) = B^q(t) - 4\xi^2 C^q(t).$$

Ji, Phys. Rev. Lett. 78, 610 (1997)

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■ Spin sum rule:

$$\int dx x (H^q(x, \xi, 0) + E^q(x, \xi, 0)) = A^q(0) + B^q(0) = 2J^q.$$

Ji, Phys. Rev. Lett. **78**, 610 (1997)

■ Shear and pressure distributions:

$$\langle N | T^{ij}(\vec{r}) | N \rangle = s(r) \left(\frac{r^i r^j}{\vec{r}^2} - \frac{1}{3} \delta^{ij} \right) + p(r) \delta^{ij}.$$

Polyakov and Shuvaev, hep-ph/0207153

Polyakov, Phys. Lett. **B555**, 57 (2003)■ Energy, radial pressure and transverse pressure distributions (u^μ the 4-velocity at spacetime location χ^ν):

$$\langle N | T^{\mu\nu} | N \rangle = (\epsilon + p_t) u^\mu u^\nu - p_t \eta^{\mu\nu} + (p_r - p_t) \chi^\mu \chi^\nu.$$

Trawinski et al., in preparation



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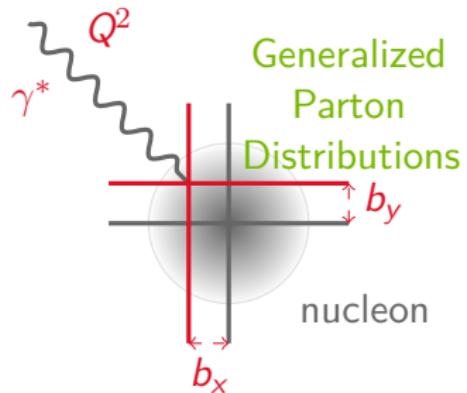
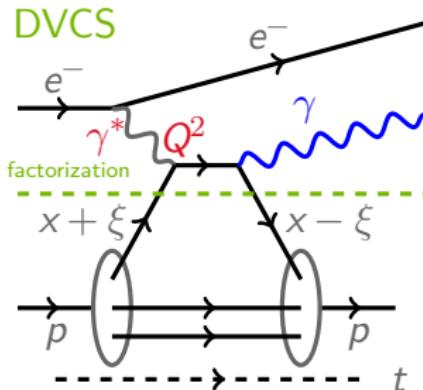
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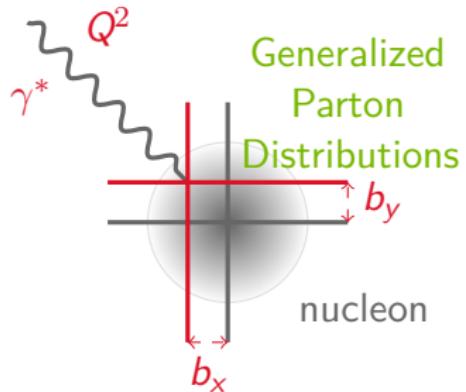
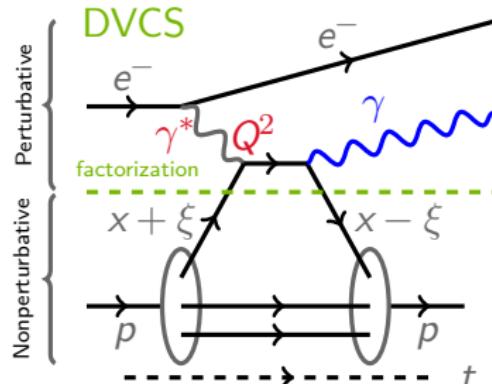
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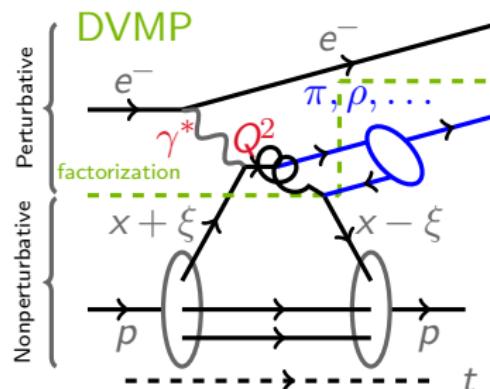
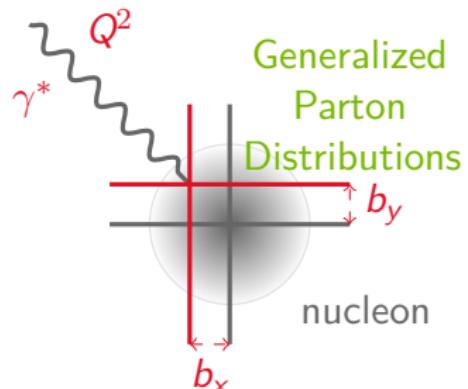
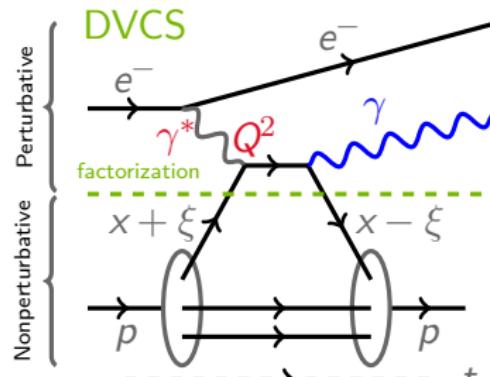
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Exclusive processes of current interest (1/2). Factorization and universality.

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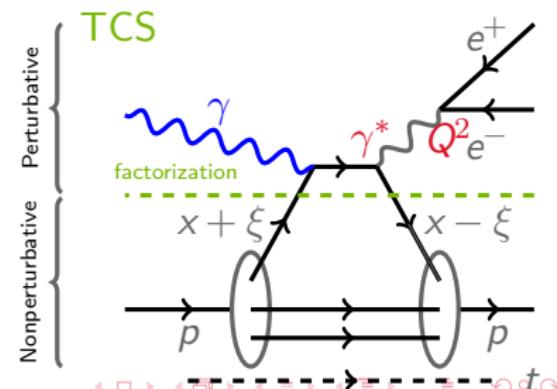
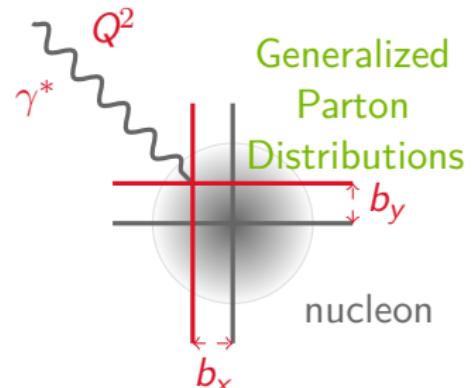
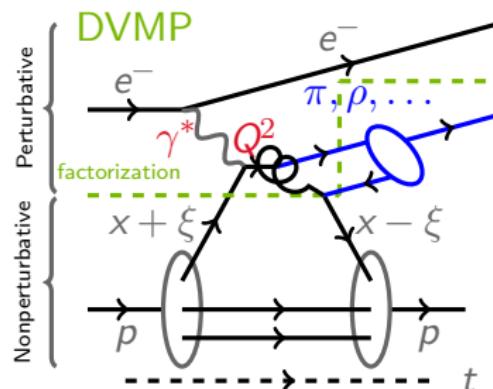
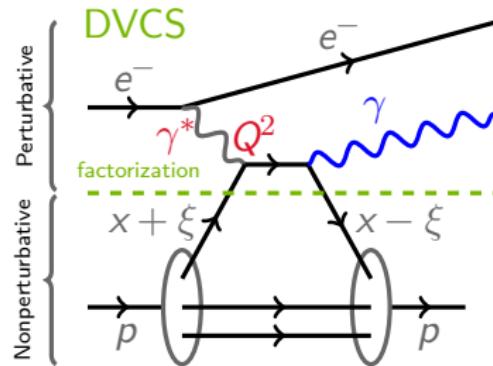
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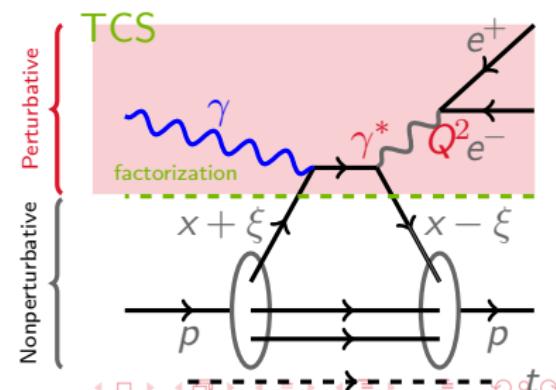
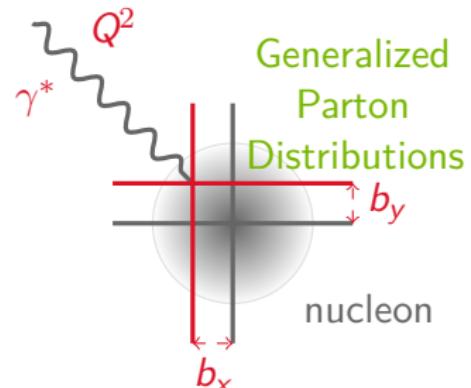
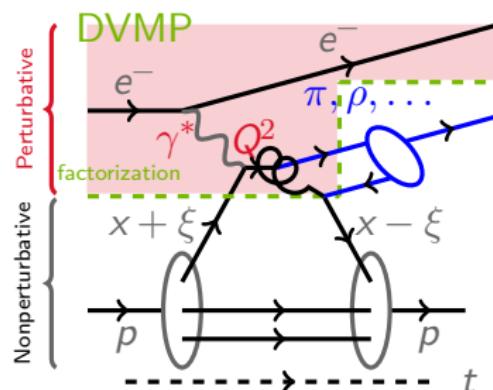
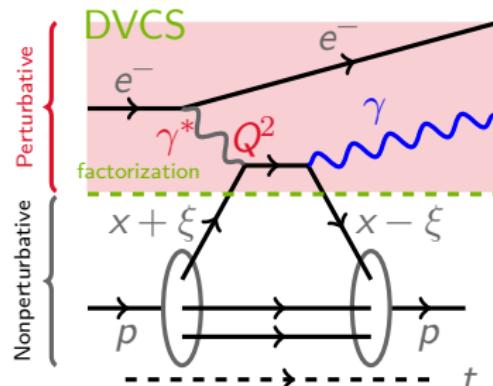
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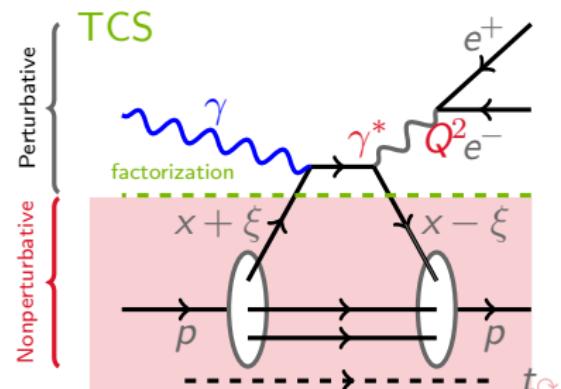
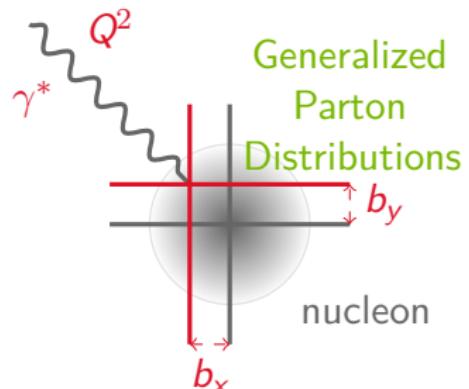
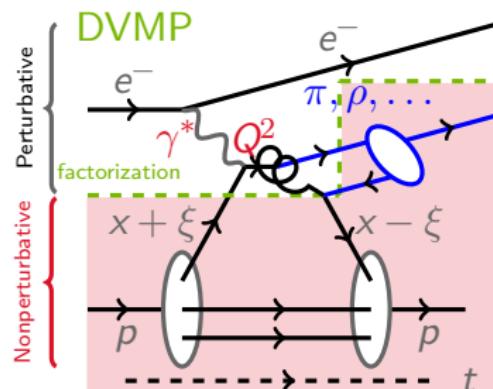
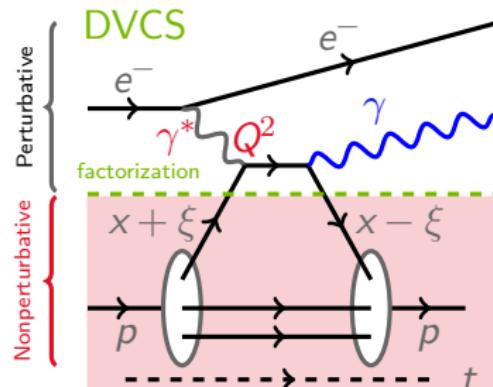
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Bjorken regime : large Q^2 and fixed $xB \simeq 2\xi/(1+\xi)$

- Partonic interpretation relies on **factorization theorems**.
 - All-order proofs for DVCS, TCS and some DVMP.
 - GPDs depend on a (arbitrary) factorization scale μ_F .
 - **Consistency** requires the study of **different channels**.
-
- GPDs enter DVCS through **Compton Form Factors** :

$$\mathcal{F}(\xi, t, Q^2) = \int_{-1}^1 dx C\left(x, \xi, \alpha_S(\mu_F), \frac{Q}{\mu_F}\right) F(x, \xi, t, \mu_F) ,$$

for a given GPD F .

- CFF \mathcal{F} is a **complex function**.

Imaging the nucleon. How?

Extracting GPDs is not enough...Need to extrapolate!

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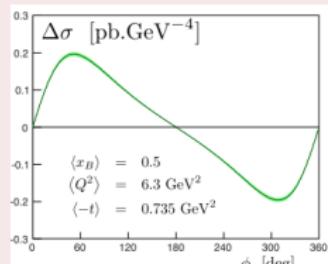
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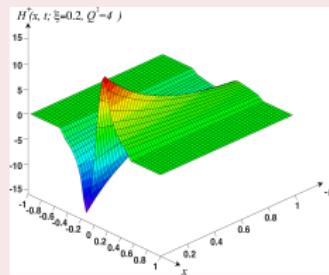
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1. Experimental data fits



2. GPD extraction



3. Nucleon imaging

Images from Guidal et al.,
Rept. Prog. Phys. 76 (2013) 066202

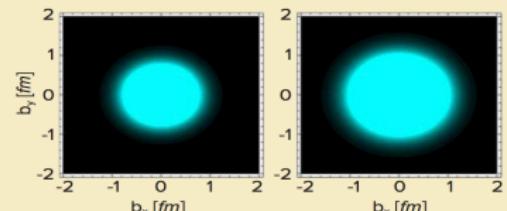
Reaching for the Horizon

The 2015 Long Range Plan for Nuclear Science

Sidebar 2.2: The First 3D Pictures of the Nucleon

A computed tomography (CT) scan can help physicians pinpoint minute cancer tumors, diagnose tiny broken bones, and spot the early signs of osteoporosis. Now physicists are using the principles behind the procedure to peer at the inner workings of the proton. This breakthrough is made possible by a relatively new concept in nuclear physics called generalized parton distributions.

An intense beam of high-energy electrons can be used



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- 1 Extract $H(x, \xi, t, \mu_F^{\text{ref}})$ from experimental data.
- 2 Extrapolate to vanishing skewness $H(x, 0, t, \mu_F^{\text{ref}})$.
- 3 Extrapolate $H(x, 0, t, \mu_F^{\text{ref}})$ up to infinite t and down to vanishing t .
- 4 Compute 2D Fourier transform in transverse plane:
$$H(x, b_\perp) = \int_0^{+\infty} \frac{d|\Delta_\perp|}{2\pi} |\Delta_\perp| J_0(|b_\perp||\Delta_\perp|) H(x, 0, -\Delta_\perp^2).$$
- 5 Propagate uncertainties.
- 6 Control extrapolations with an accuracy matching that of experimental data with **sound** GPD models.

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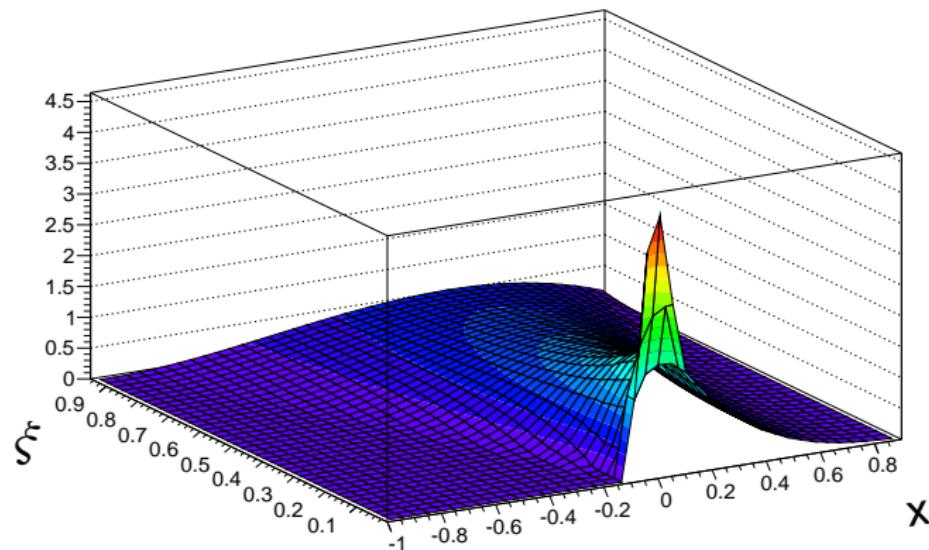
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GPD model: see Kroll et al., Eur. Phys. J. **C73**, 2278 (2013)

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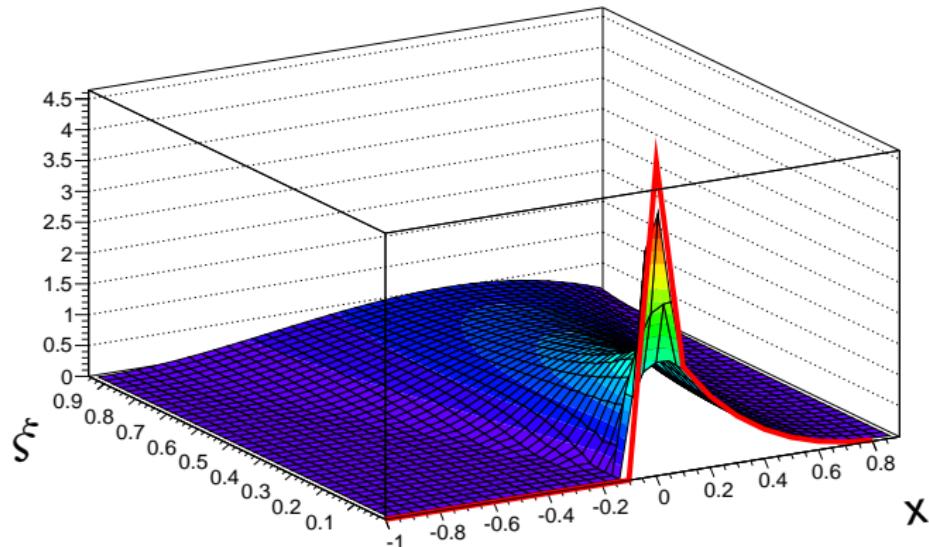
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Need to know $H(x, \xi = 0, t)$ to do transverse plane imaging.GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

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What is the physical region?

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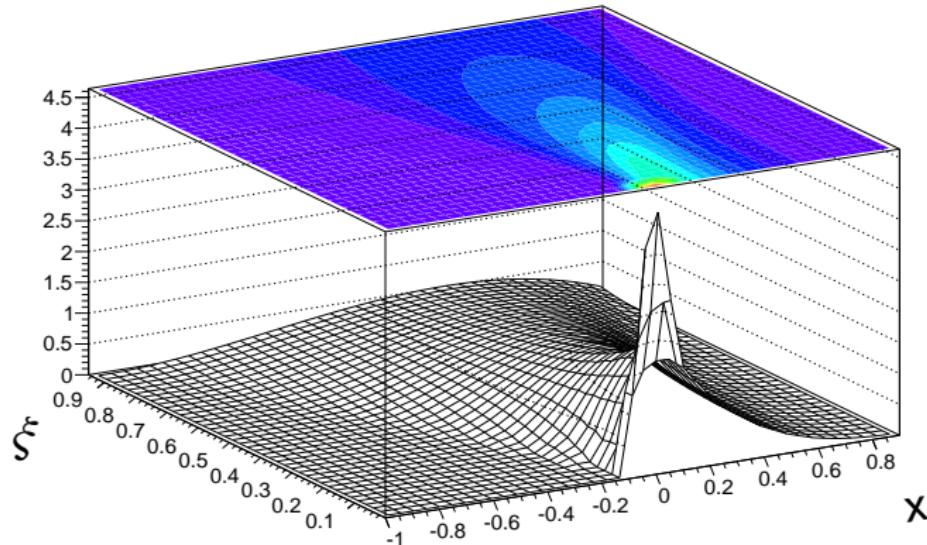
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GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

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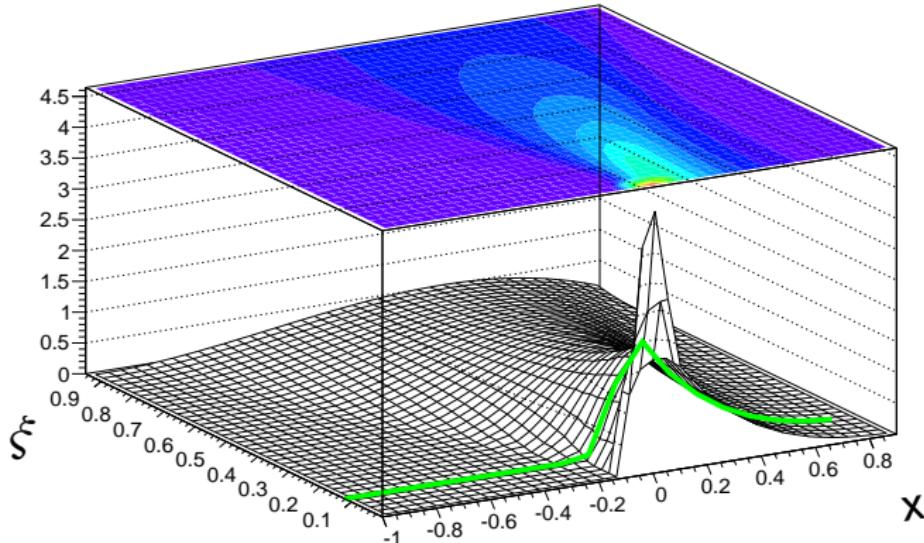
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ξ_{\min} from finite beam energy.



GPD model: see Kroll et al., Eur. Phys. J. **C73**, 2278 (2013)

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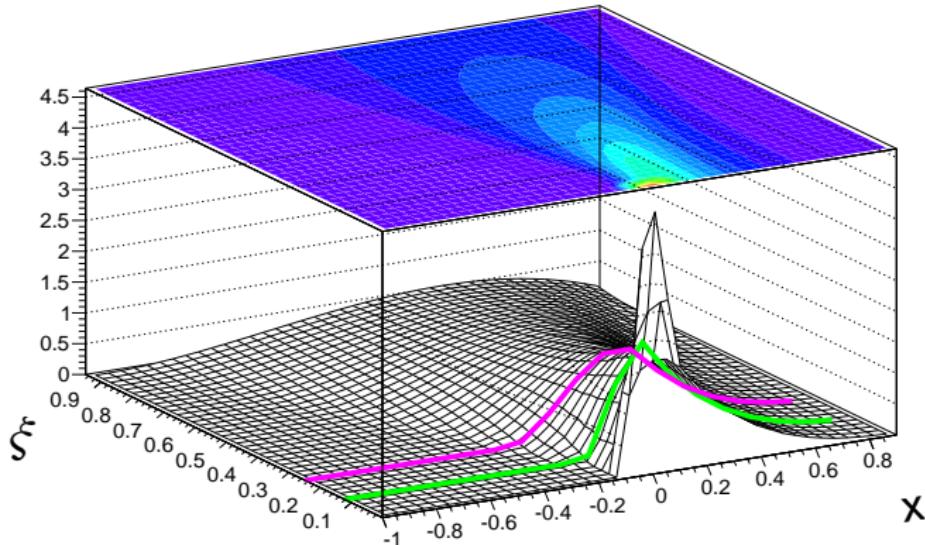
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ξ_{\max} from kinematic constraint on 4-momentum transfer.



GPD model: see Kroll *et al.*, Eur. Phys. J. **C73**, 2278 (2013)

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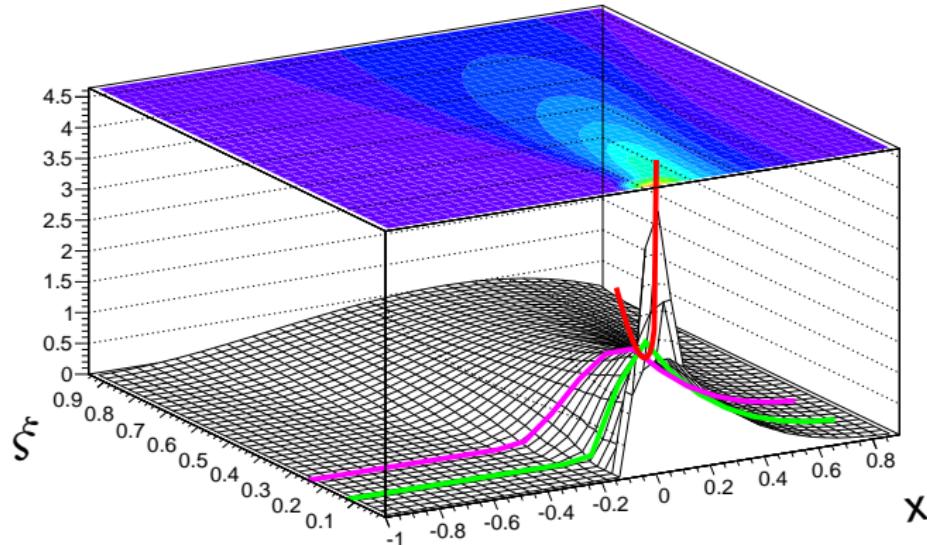
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The cross-over line $x = \xi$.



GPD model: see Kroll et al., Eur. Phys. J. C73, 2278 (2013)

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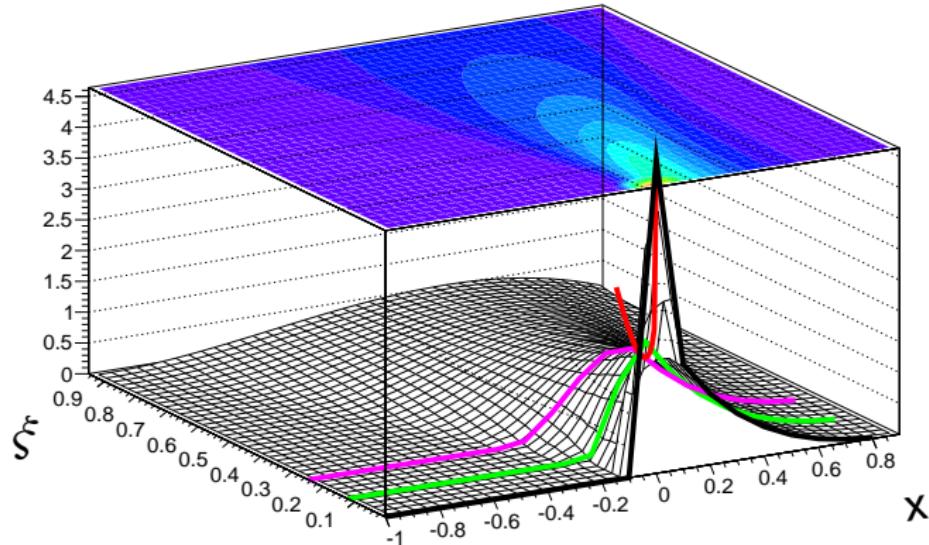
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The black curve is what is needed for transverse plane imaging!



GPD model: see Kroll et al., Eur. Phys. J. **C73**, 2278 (2013)

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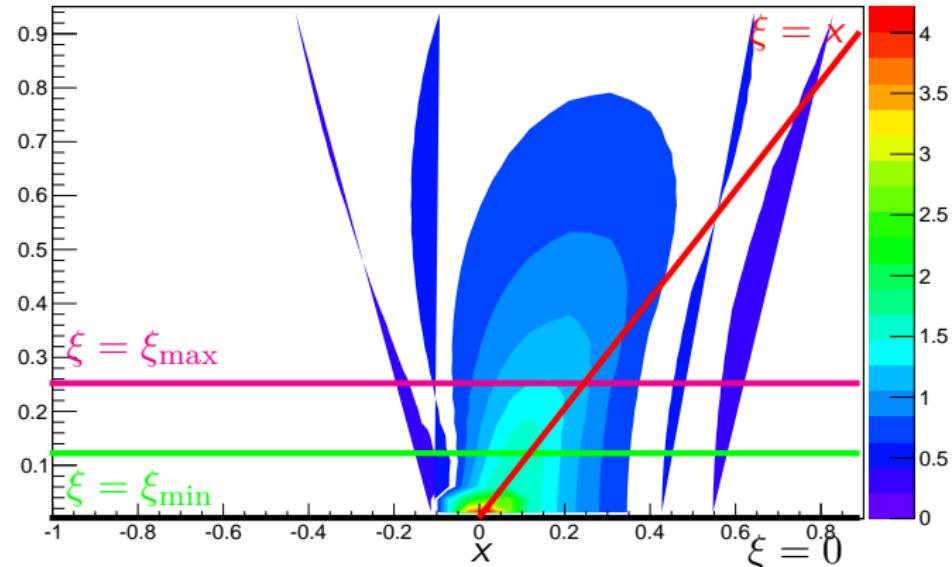
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Density plot of H at $t = -0.23 \text{ GeV}^2$ and $Q^2 = 2.3 \text{ GeV}^2$



GPD model: see Kroll et al., Eur. Phys. J. C73, 2278 (2013)

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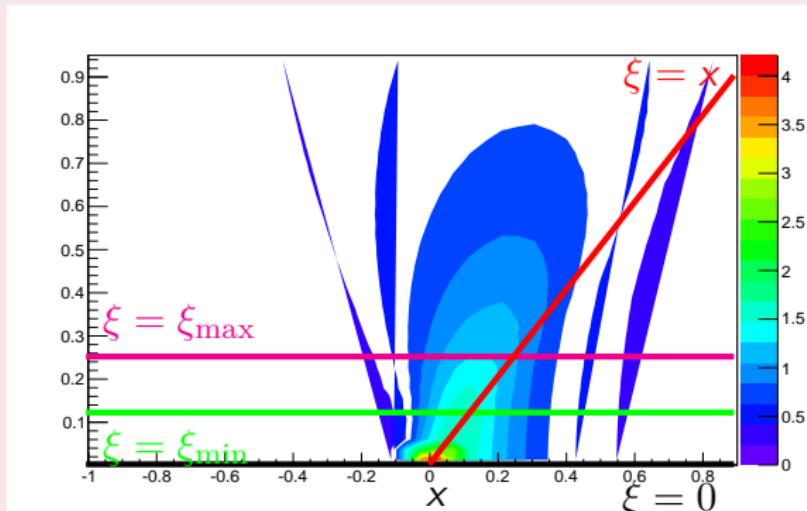
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Density plot of H at $t = -0.23 \text{ GeV}^2$ and $Q^2 = 2.3 \text{ GeV}^2$ 

- Not a hopeless task: x and ξ dependence of GPDs are **strongly tied** (polynomiality)!
- Need for GPD modeling and flexible parameterizations.

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- Write dispersion relation **at fixed t and Q^2** :

$$\text{Re}\mathcal{H}(\xi) = \int_1^\infty \frac{d\omega}{\pi} \text{Im}C(\omega) \left\{ \int_{-1}^{+1} dx \left[\frac{1}{\omega\xi - x} - \frac{1}{\omega\xi + x} \right] H\left(x, \frac{x}{\omega}\right) + \mathcal{I}(\omega) \right\} .$$

Diehl and Ivanov, Eur. Phys. J. **C52**, 919 (2007)

- At **leading order** in α_s (no kinematic corrections):

$$\text{Im}C(\omega) \propto \pi \left[\delta(\omega - 1) - \delta(\omega + 1) \right] .$$

- Dispersion relation simplifies to:

$$\text{Re}\mathcal{H}(\xi) \propto \int_{-1}^{+1} dx \left[\frac{1}{\omega\xi - x} - \frac{1}{\omega\xi + x} \right] H(x, x) + \mathcal{I} ,$$

$$\text{Im}\mathcal{H}(\xi) \propto H(\xi, \xi) - H(-\xi, \xi) .$$

- In principle tomography not possible **at leading order** ... ↗ ↘ ↙ ↛ ↚

DVCS analysis and fits.

No global GPD fit has been obtained so far.

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- GPD fits **only in the small x_B region** with a **flexible** parameterization (kinematic simplifications).
- Global fits of CFFs in the sea and valence regions.
- Some GPD models with non-flexible parameterizations adjusted to experimental DVCS or DVMP data.

Kumerički *et al.*, Eur. Phys. J. **A52**, 157 (2016)

- Unclear model-dependence on tomographic images obtained from CFF fits relying on **leading order** and **leading twist** analysis.

The situation can be improved!

- GPD parameterizations satisfying *a priori* all theoretical constraints on GPDs.
- Computing framework to go beyond leading order and leading twist analysis.

How can we implement all theoretical constraints in flexible GPD parameterizations?

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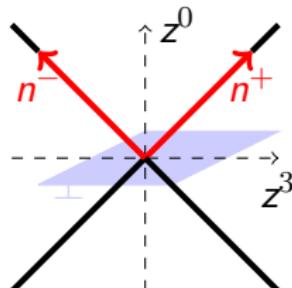
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$$H_\pi^q(x, \xi, t) = \frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P - \frac{\Delta}{2} \right\rangle_{z^+=0, z_\perp=0}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

- Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
- Ji, Phys. Rev. Lett. **78**, 610 (1997)
- Radyushkin, Phys. Lett. **B380**, 417 (1996)

■ PDF forward limit

$$H^q(x, 0, 0) = q(x)$$

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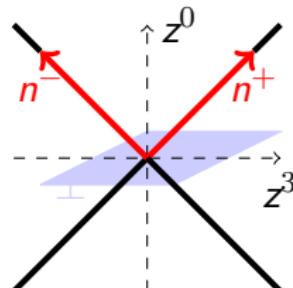
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- Ji, Phys. Rev. Lett. **78**, 610 (1997)
- Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF forward limit
- Form factor sum rule

$$\int_{-1}^{+1} dx H_{\pi}^q(x, \xi, t) = F_1^q(t)$$

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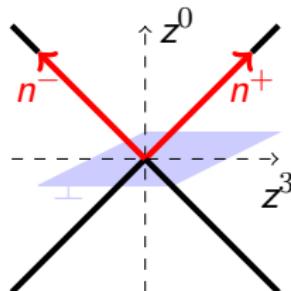
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$$H_\pi^q(x, \xi, t) =$$

$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

- Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
- Ji, Phys. Rev. Lett. **78**, 610 (1997)
- Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF forward limit
- Form factor sum rule
- H^q is an even function of ξ from time-reversal invariance.

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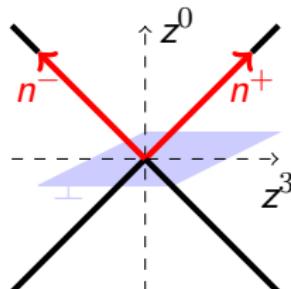
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$$\frac{1}{2} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left\langle \pi, P + \frac{\Delta}{2} \right| \bar{q}\left(-\frac{z}{2}\right) \gamma^+ q\left(\frac{z}{2}\right) \left| \pi, P - \frac{\Delta}{2} \right\rangle_{\substack{z^+=0 \\ z_\perp=0}}$$

with $t = \Delta^2$ and $\xi = -\Delta^+/(2P^+)$.



References

- Müller *et al.*, Fortschr. Phys. **42**, 101 (1994)
- Ji, Phys. Rev. Lett. **78**, 610 (1997)
- Radyushkin, Phys. Lett. **B380**, 417 (1996)

- PDF forward limit
- Form factor sum rule
- H^q is an even function of ξ from time-reversal invariance.
- H^q is real from hermiticity and time-reversal invariance.

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■ Polynomiality

$$\int_{-1}^{+1} dx x^n H^q(x, \xi, t) = \text{polynomial in } \xi$$

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$$H^q(x, \xi, t) \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

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■ H^q has support $x \in [-1, +1]$.

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■ H^q has support $x \in [-1, +1]$.

Relativistic quantum mechanics

■ Soft pion theorem (pion target)

$$H^q(x, \xi = 1, t = 0) = \frac{1}{2} \phi_\pi^q \left(\frac{1+x}{2} \right)$$

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Relativistic quantum mechanics

■ Soft pion theorem (pion target)

Dynamical chiral symmetry breaking

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- H^q has support $x \in [-1, +1]$.

Relativistic quantum mechanics

■ Soft pion theorem (pion target)

Dynamical chiral symmetry breaking

How can we implement *a priori* these theoretical constraints?

- In the following, focus on **polynomiality** and **positivity**.
- Do not discuss the reduction to form factors or PDFs.

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- Express Mellin moments of GPDs as **matrix elements**:

$$\int_{-1}^{+1} dx x^m H^q(x, \xi, t) = \frac{1}{2(P^+)^{m+1}} \left\langle P + \frac{\Delta}{2} \right| \bar{q}(0) \gamma^+ (i \not{D}^+)^m q(0) \left| P - \frac{\Delta}{2} \right\rangle$$

- Identify the **Lorentz structure** of the matrix element:

linear combination of $(P^+)^{m+1-k} (\Delta^+)^k$ for $0 \leq k \leq m+1$

- Remember definition of **skewness** $\Delta^+ = -2\xi P^+$.
- Select **even powers** to implement time reversal.
- Obtain **polynomiality condition**:

$$\int_{-1}^1 dx x^m H^q(x, \xi, t) = \sum_{\substack{i=0 \\ \text{even}}}^m (2\xi)^i C_{mi}^q(t) + (2\xi)^{m+1} C_{mm+1}^q(t).$$

Polynomiality.

Abstract formulation: the range of the Radon transform.

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- Assume the existence of $D^q(z, t)$ such that:

$$\int_{-1}^{+1} dz z^m D(z, t) = C_{mm+1}^q(t) .$$

- $H^q(x, \xi, t) - D(x/\xi, t)$ satisfies polynomiality at order m :

$$\int_{-1}^1 dx x^{\textcolor{red}{m}} \left(H^q(x, \xi, t) - D(x/\xi, t) \right) = \sum_{\substack{i=0 \\ \text{even}}}^{\textcolor{red}{m}} (2\xi)^i C_{mi}^q(t) .$$

- In the Radon transform framework, this is the **Ludwig-Helgason** consistency condition.
- Thus, there exists a function F_D such that:

$$H(x, \xi, t) = D(x/\xi, t) + \int_{\Omega_{DD}} d\beta d\alpha F_D(\beta, \alpha, t) \delta(x - \beta - \alpha \xi) .$$

- The support $\Omega_{DD} = \{|\alpha| + |\beta| \leq 1\}$ is related to the GPD physical domain $|x|, |\xi| \leq 1$.

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- Most general representation of GPD:

$$H^q(x, \xi, t) = \int_{\Omega_{DD}} d\beta d\alpha \delta(x - \beta - \alpha\xi) (F^q(\beta, \alpha, t) + \xi G^q(\beta, \alpha, t))$$

- Support property: $x \in [-1, +1]$.
- Discrete symmetries: F^q is α -even and G^q is α -odd.
- **Gauge**: any representation (F^q, G^q) can be recast in one representation with a single DD f^q :

$$H^q(x, \xi, t) = x \int_{\Omega_{DD}} d\beta d\alpha f_{BMKS}^q(\beta, \alpha, t) \delta(x - \beta - \alpha\xi)$$

Belitsky et al., Phys. Rev. D64, 116002 (2001)

$$H^q(x, \xi, t) = (1 - x) \int_{\Omega_{DD}} d\beta d\alpha f_P^q(\beta, \alpha, t) \delta(x - \beta - \alpha\xi)$$

Pobylitsa, Phys. Rev. D67, 034009 (2003)

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- Choose $F^q(\beta, \alpha) = 3\beta\theta(\beta)$ ad $G^q(\beta, \alpha) = 3\alpha\theta(\beta)$:

$$H^q(x, \xi) = 3x \int_{\Omega} d\beta d\alpha \delta(x - \beta - \alpha\xi)$$

- Simple analytic expressions for the GPD:

$$H(x, \xi) = \frac{6x(1-x)}{1-\xi^2} \text{ if } 0 < |\xi| < x < 1,$$

$$H(x, \xi) = \frac{3x(x+|\xi|)}{|\xi|(1+|\xi|)} \text{ if } -|\xi| < x < |\xi| < 1.$$

- Compute first Mellin moments.

n	$\int_{-\xi}^{+\xi} dx x^n H(x, \xi)$	$\int_{+\xi}^{+1} dx x^n H(x, \xi)$	$\int_{-\xi}^{+1} dx x^n H(x, \xi)$
0	$\frac{1+\xi-2\xi^2}{1+\xi}$	$\frac{2\xi^2}{1+\xi}$	1
1	$\frac{1+\xi+\xi^2-3\xi^3}{2(1+\xi)}$	$\frac{2\xi^3}{1+\xi}$	$\frac{1+\xi^2}{2}$
2	$\frac{3(1-\xi)(1+2\xi+3\xi^2+4\xi^3)}{10(1+\xi)}$	$\frac{6\xi^4}{5(1+\xi)}$	$\frac{3(1+\xi^2)}{10}$
3	$\frac{1+\xi+\xi^2+\xi^3+\xi^4-5\xi^5}{5(1+\xi)}$	$\frac{6\xi^5}{5(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{5}$
4	$\frac{1+\xi+\xi^2+\xi^3+\xi^4+\xi^5-6\xi^6}{7(1+\xi)}$	$\frac{6\xi^6}{7(1+\xi)}$	$\frac{1+\xi^2+\xi^4}{7}$

- Expressions get more complicated as n increases... But they always yield polynomials!

The Radon transform.

Definition and properties.

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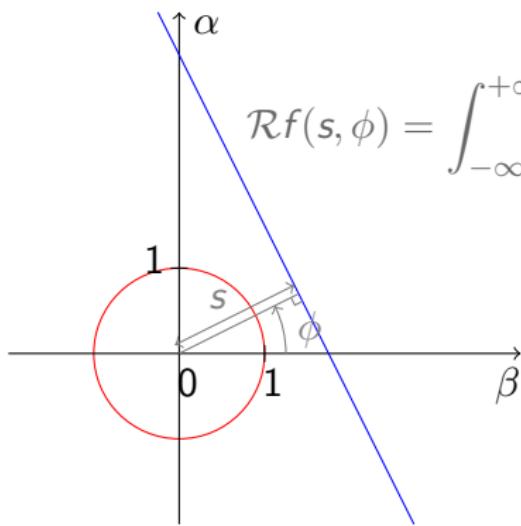
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For $s > 0$ and $\phi \in [0, 2\pi]$:

$$\mathcal{R}f(s, \phi) = \int_{-\infty}^{+\infty} d\beta d\alpha f(\beta, \alpha) \delta(s - \beta \cos \phi - \alpha \sin \phi)$$

and:

$$\mathcal{R}f(-s, \phi) = \mathcal{R}f(s, \phi \pm \pi)$$

Relation to GPDs:

$$x = \frac{s}{\cos \phi} \text{ and } \xi = \tan \phi$$

Relation between GPD and DD in Belitsky *et al.* gauge

$$\frac{\sqrt{1 + \xi^2}}{x} H(x, \xi) = \mathcal{R}f_{\text{BMKS}}(s, \phi),$$

The Radon transform.

Definition and properties.

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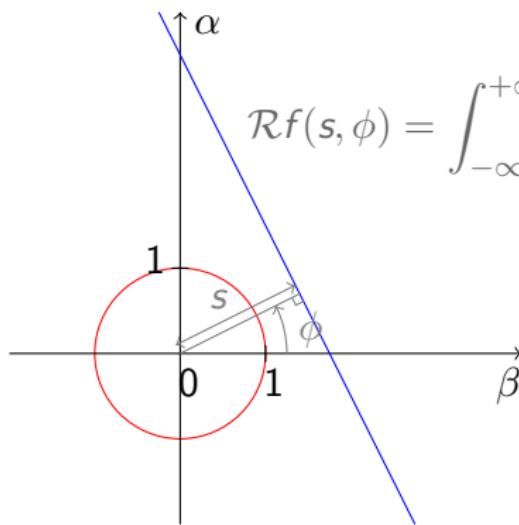
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For $s > 0$ and $\phi \in [0, 2\pi]$:

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$$\mathcal{R}f(-s, \phi) = \mathcal{R}f(s, \phi \pm \pi)$$

Relation to GPDs:

$$x = \frac{s}{\cos \phi} \text{ and } \xi = \tan \phi$$

Relation between GPD and DD in Pobylitsa gauge

$$\frac{\sqrt{1 + \xi^2}}{1 - x} H(x, \xi) = \mathcal{R}f_P(s, \phi),$$

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- The Mellin moments of a Radon transform are **homogeneous polynomials** in $\omega = (\sin \phi, \cos \phi)$.
- The converse is also true:

Theorem (Hertle, 1983)

Let $g(s, \omega)$ an even compactly-supported distribution. Then g is itself the Radon transform of a compactly-supported distribution if and only if the **Ludwig-Helgason consistency condition** hold:

- g is C^∞ in ω ,
- $\int ds s^m g(s, \omega)$ is a homogeneous polynomial of degree m for all integer $m \geq 0$.

- Double Distributions and the Radon transform are the **natural solution** of the polynomiality condition.

Implementing Lorentz covariance.

Extend an overlap in the DGLAP region to the whole GPD domain.

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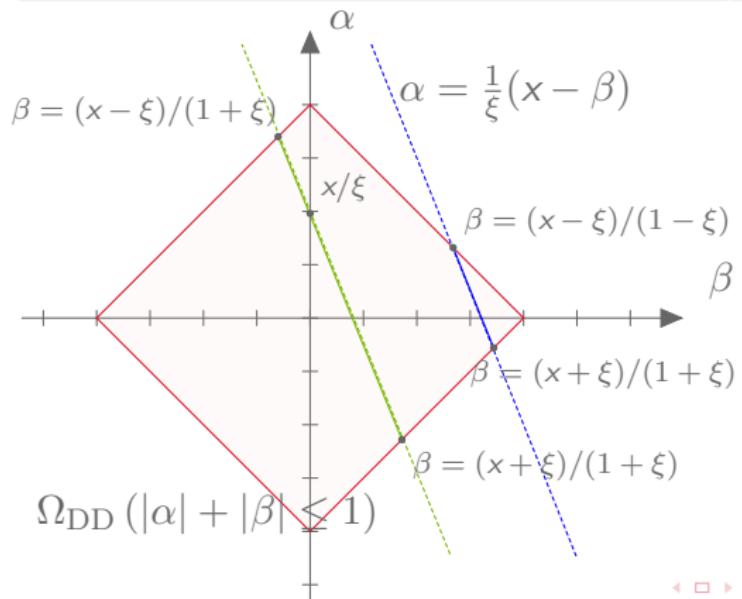
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DGLAP and ERBL regions

$$(x, \xi) \in \text{DGLAP} \Leftrightarrow |s| \geq |\sin \phi| ,$$

$$(x, \xi) \in \text{ERBL} \Leftrightarrow |s| \leq |\sin \phi| .$$



Each point (β, α) with $\beta \neq 0$ contributes to **both** DGLAP and ERBL regions.

Positivity.

A consequence of the positivity of the norm in a Hilbert space.

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- Identify the matrix element defining a GPD as an **inner product** of two different states.
- Apply Cauchy-Schwartz inequality, and identify PDFs at specific kinematic points, e.g.:

$$|H^q(x, \xi, t)| \leq \sqrt{\frac{1}{1-\xi^2} q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right)}$$

- This procedure yields **infinitely many inequalities** stable under LO evolution.

Pobylitsa, Phys. Rev. D66, 094002 (2002)

- The **overlap representation** guarantees *a priori* the fulfillment of positivity constraints.

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- Decompose an hadronic state $|H; P, \lambda\rangle$ in a Fock basis:

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$$|H; P, \lambda\rangle = \sum_{N, \beta} \int [dx d\mathbf{k}_\perp]_N \psi_N^{(\beta, \lambda)}(x_1, \mathbf{k}_{\perp 1}, \dots, x_N, \mathbf{k}_{\perp N}) |\beta, k_1, \dots, k_N\rangle$$

- Derive an expression for the pion GPD in the DGLAP region $\xi \leq x \leq 1$:

$$H^q(x, \xi, t) \propto \sum_{\beta, j} \int [d\bar{x} d\bar{\mathbf{k}}_\perp]_N \delta_{j,q} \delta(x - \bar{x}_j) (\psi_N^{(\beta, \lambda)})^*(\hat{x}', \hat{\mathbf{k}}'_\perp) \psi_N^{(\beta, \lambda)}(\tilde{x}, \tilde{\mathbf{k}}_\perp)$$

with $\tilde{x}, \tilde{\mathbf{k}}_\perp$ (resp. $\hat{x}', \hat{\mathbf{k}}'_\perp$) generically denoting incoming (resp. outgoing) parton kinematics.

Diehl *et al.*, Nucl. Phys. **B596**, 33 (2001)

- Similar expression in the ERBL region $-\xi \leq x \leq \xi$, but with overlap of N - and $(N+2)$ -body LFWFs.

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- Physical picture.
- Positivity relations are fulfilled **by construction**.
- Implementation of **symmetries of N -body problems**.

What is not obvious anymore

What is *not* obvious to see from the wave function representation is however the **continuity of GPDs at $x = \pm \xi$** and the **polynomiality** condition. In these cases both the DGLAP and the ERBL regions must cooperate to lead to the required properties, and this implies **nontrivial relations between the wave functions** for the different Fock states relevant in the two regions. An *ad hoc* Ansatz for the wave functions would **almost certainly lead to GPDs that violate the above requirements**.

Diehl, Phys. Rept. **388**, 41 (2003)

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For any model of LFWF, one has to address the following three questions:

- 1 Does the extension exist?
- 2 If it exists, is it unique?
- 3 How can we compute this extension?

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Consider a GPD H vanishing on the DGLAP region and write it as a Radon transform:

$$H(x, \xi) = \int_{\Omega_{DD}} d\beta d\alpha [F_D(\beta, \alpha) + \delta(\beta)D(\alpha)] \delta(x - \beta - \alpha\xi).$$

- $F_D(\beta, \alpha) = 0$ for all α and $\beta > 0$.
Boman and Todd-Quinto, Duke Math. J. 55, 943 (1987)
- Up to D-term-like contributions, the DGLAP region **completely characterizes** a GPD.
- Modeling strategy:
 - 1 Ensure positivity by modeling the DGLAP region as an overlap of LFWFs.
 - 2 Ensure polynomiality by inverting the Radon transform to identify an underlying DD.

Chouika et al., Eur. Phys. J. C77, 906 (2017)

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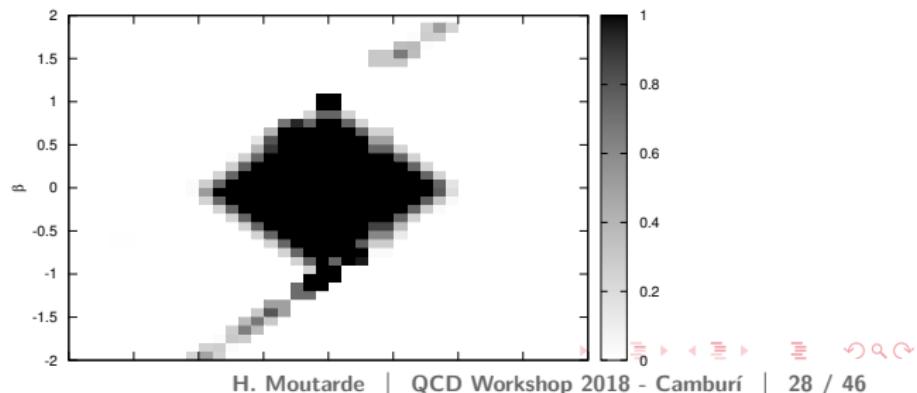
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- Numerical evaluation ***almost unavoidable*** (polar vs cartesian coordinates).
- Ill-posedness by **lack of continuity**.
- The **unlimited** Radon inverse problem is **mildly** ill-posed while the **limited** one is **severely** ill-posed.
- Even if it existed, an analytic expression of the invert Radon transform would be of **limited practical use**.



Computation of the extension. Problem reduction.

Covariant
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How can we get a DD from a GPD in the DGLAP region?

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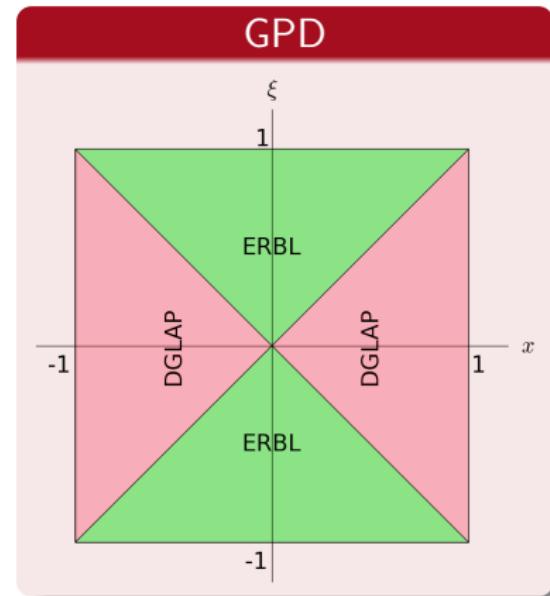
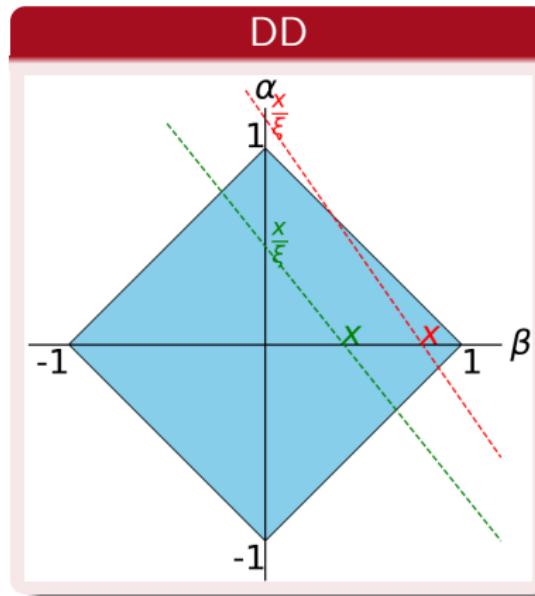
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Covariant extension

How can we get a DD from a GPD in the DGLAP region?

- Restrict to quark GPDs ($\beta > 0$).

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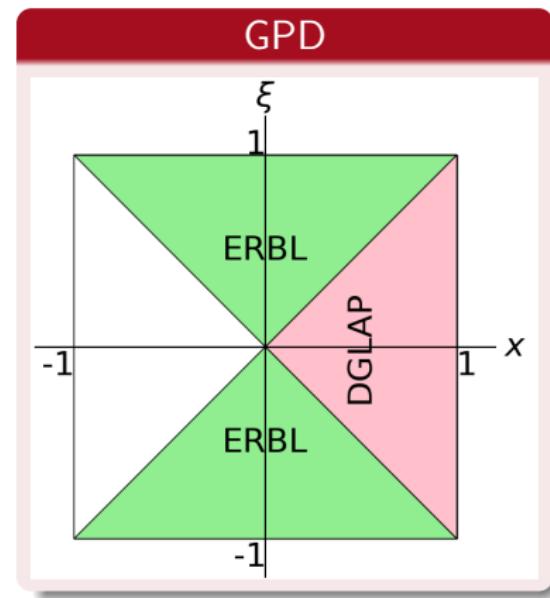
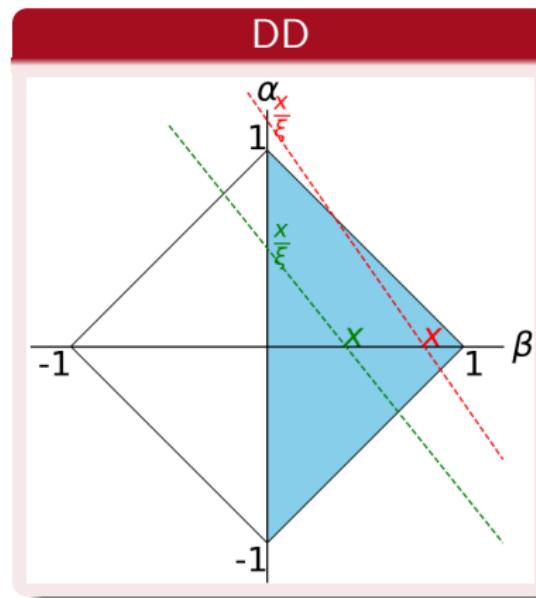
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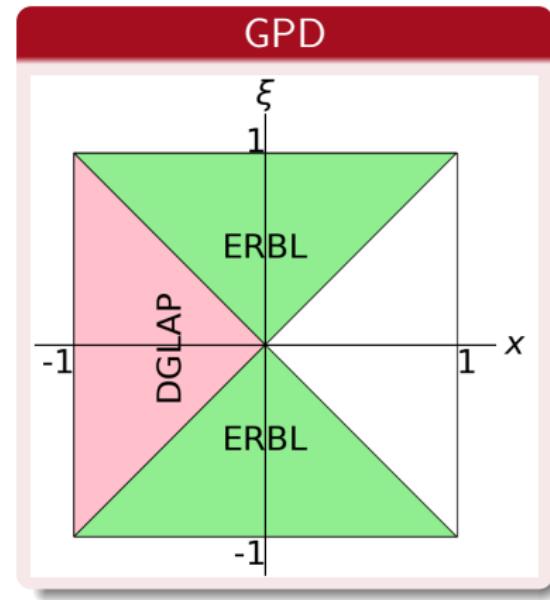
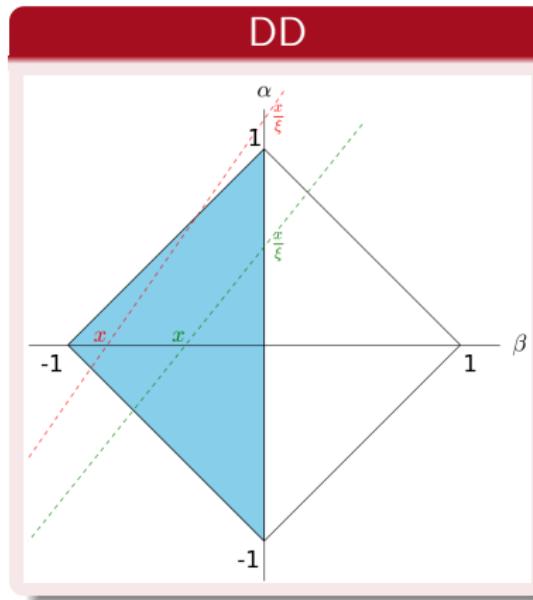
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How can we get a DD from a GPD in the DGLAP region?

- Restrict to quark GPDs ($\beta > 0$).
- Only ERBL region "sees" both $\beta > 0$ and $\beta < 0$.



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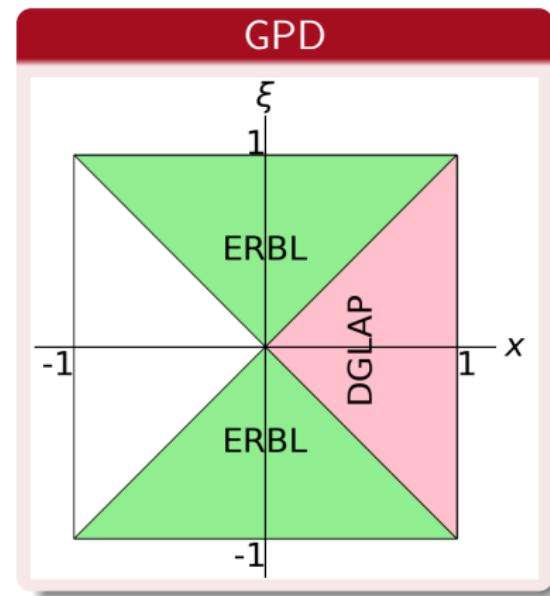
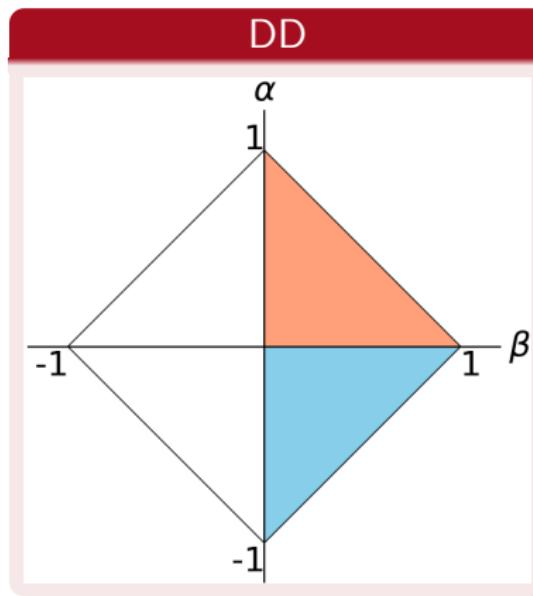
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How can we get a DD from a GPD in the DGLAP region?

- Restrict to quark GPDs ($\beta > 0$).
- Only ERBL region "sees" both $\beta > 0$ and $\beta < 0$.
- Use α -parity of the DD.



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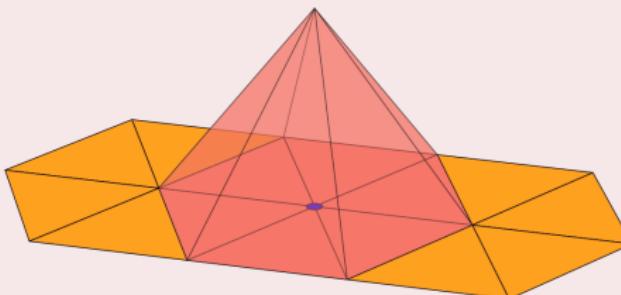
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Example of a P1 basis function



- Discretize the DD on a mesh with $n \simeq 800$ triangular cells.
- Compute the Radon transform of a P1 basis function.
- Sample $m \simeq 4n$ (x, ξ) -lines intersecting the DD support.
- Solve a linear system $AX = B$ with A a sparse $m \times n$ matrix.
- Adopt an iterative regularization method: LSMR.

Fong and Saunders, arXiv:1006.0758

Examples - benchmarks (1/4).

Algebraic Bethe-Salpeter model.

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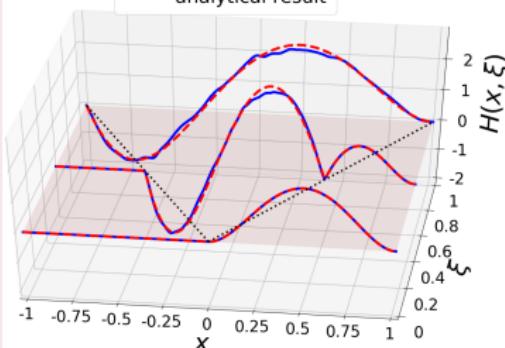
Conclusion

$$\Psi_{I=0}(x, \mathbf{k}_\perp) = 8\sqrt{15}\pi \frac{M^3}{(\mathbf{k}_\perp^2 + M^2)^2} (1-x)x,$$

$$ik_\perp^j \Psi_{I=1}(x, \mathbf{k}_\perp) = 8\sqrt{15}\pi \frac{k_\perp^j M^2}{(\mathbf{k}_\perp^2 + M^2)^2} (1-x)x, \quad j=1,2$$

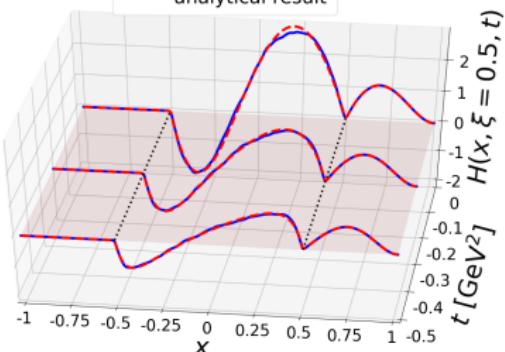
$H(x, \xi, t = 0)$

numerical result
analytical result



$H(x, \xi = 0.5, t)$

numerical result
analytical result



Examples - benchmarks (2/4).

Algebraic spectator model.

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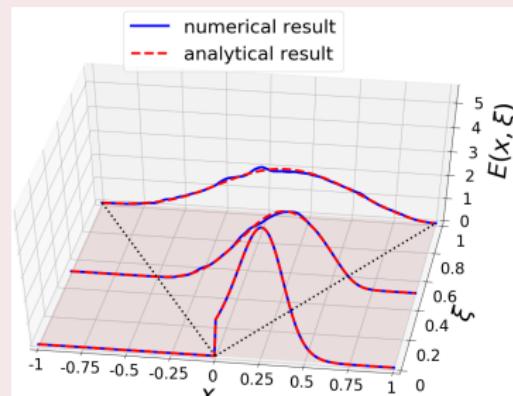
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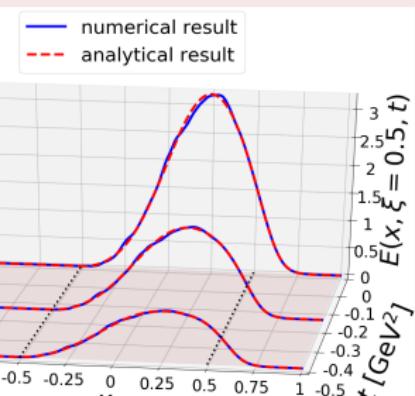
$$\varphi(x, \mathbf{k}_\perp) = \frac{gM^{2p}}{\sqrt{1-x}} x^{-p} \left(M^2 - \frac{\mathbf{k}_\perp^2 + m^2}{x} - \frac{\mathbf{k}_\perp^2 + \lambda^2}{1-x} \right)^{-p-1}$$

Hwang and Müller, Phys. Lett. **B660**, 350 (2008)

$H(x, \xi, t = 0)$



$H(x, \xi = 0.5, t)$



Chouika et al., Eur. Phys. J. **C77**, 906 (2017)

Examples - benchmarks (3/4).

Regge-behaved Radyushkin DD Ansatz model.

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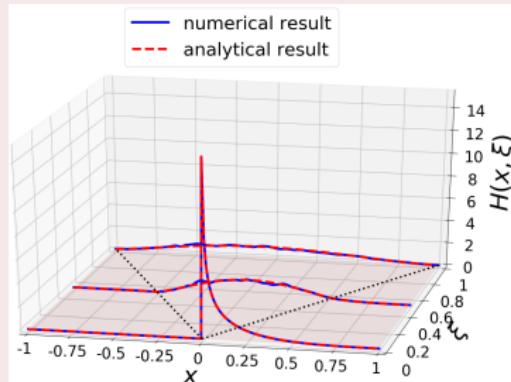
- Design
- Fits
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Conclusion

Radyushkin DD Ansatz with phenomenological PDF:

$$q\text{Regge}(x) = \frac{35}{32} \frac{(1-x)^3}{\sqrt{x}}.$$

$H(x, \xi, t = 0)$



Chouika *et al.*, Eur. Phys. J. C77, 906 (2017)

Examples - benchmarks (4/4).

Gaussian wave function model.

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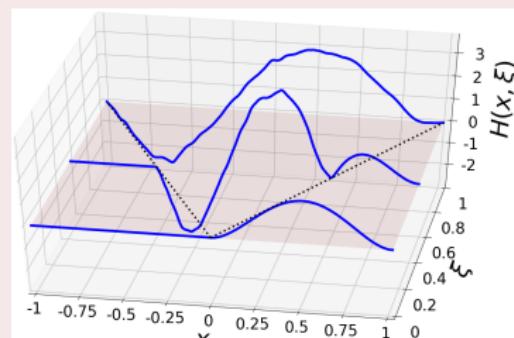
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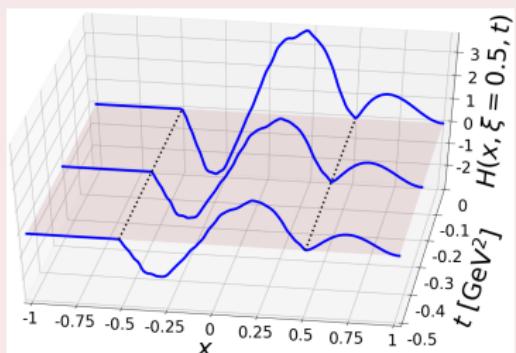
Conclusion

$$\Psi(x, k_{\perp}^2) = \frac{4\sqrt{15}\pi}{M} \sqrt{x(1-x)} e^{-\frac{k_{\perp}^2}{4M^2(1-x)x}}.$$

$H(x, \xi, t = 0)$



$H(x, \xi = 0.5, t)$



Chouika et al., Eur. Phys. J. C77, 906 (2017)

Take home message.

What has been done, what remains to be done.

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- 1 Modeling: GPDs (not CFFs) have to be extracted from measurements to learn about hadron structure.
- 2 Generic procedure to build models satisfying **all theoretical constraints**.
- 3 Remark: soft pion theorem can be fulfilled too!

Chouika *et al.*, Phys. Lett. **B780**, 287 (2018)

- 4 Extension to spin-1/2 hadron in progress.
- 5 Integration in computing chain from GPDs to observables (PARTONS framework) in progress.
- 6 Still have to figure out how to input **phenomenological parameterizations** of PDFs and form factors for **global GPD fits**.

How do we relate all this to actual measurements?



**PARtonic
Tomography
Of
Nucleon
Software**

Covariant extension

Experimental data and phenomenology

Full processes

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Computation of amplitudes

Small distance contributions

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First principles and fundamental parameters

Large distance contributions

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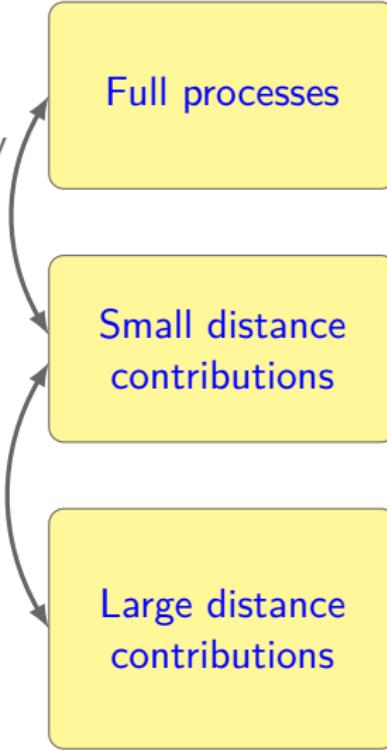
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First principles and fundamental parameters

Full processes

Small distance contributions

Large distance contributions



Computing chain design.

Differential studies: physical models and numerical methods.

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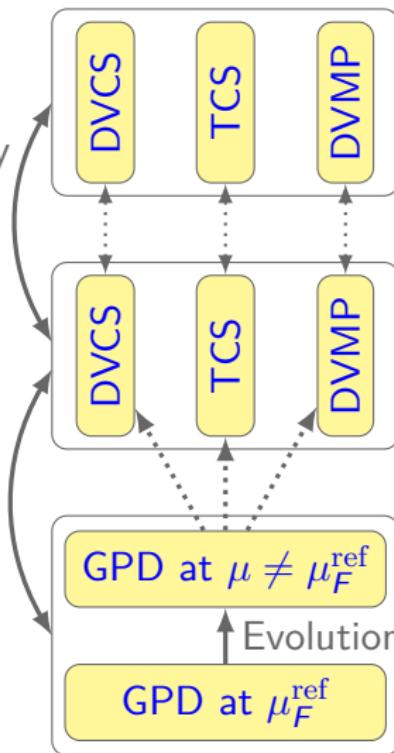
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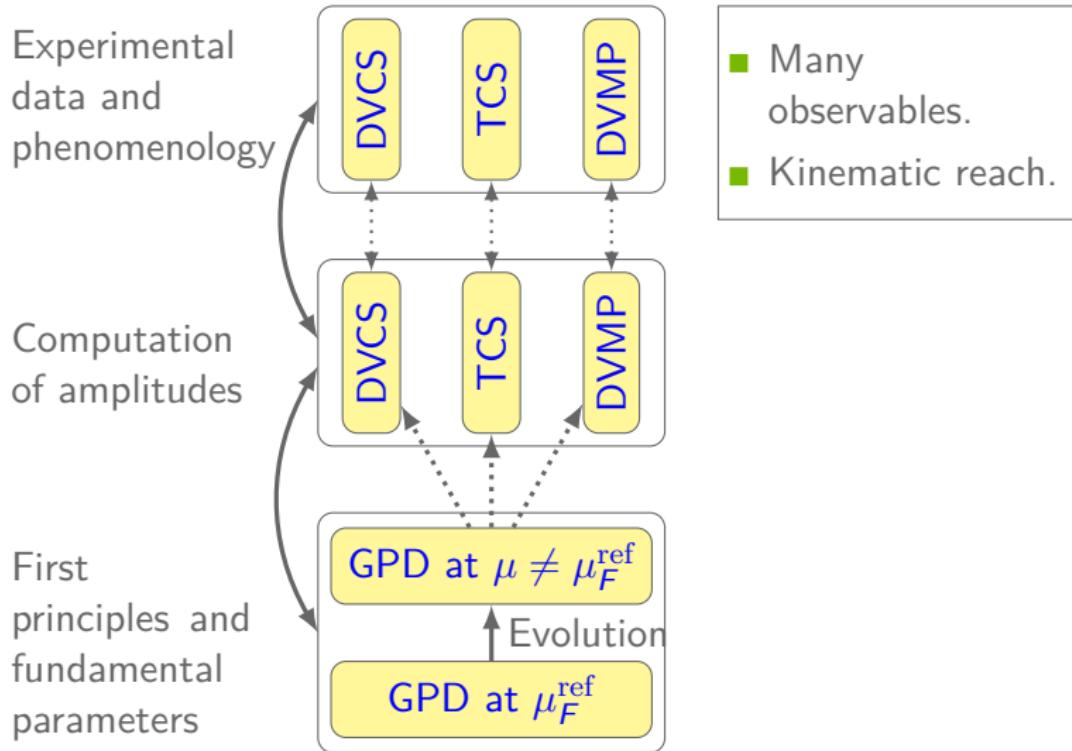
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Computing chain design.

Differential studies: physical models and numerical methods.

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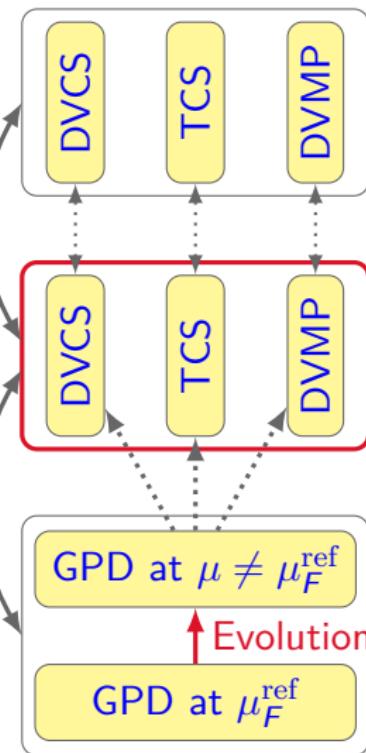
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Experimental data and phenomenology

Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
- Kinematic reach.

- Perturbative approximations.
- Physical models.
- Fits.
- Numerical methods.
- Accuracy and speed.

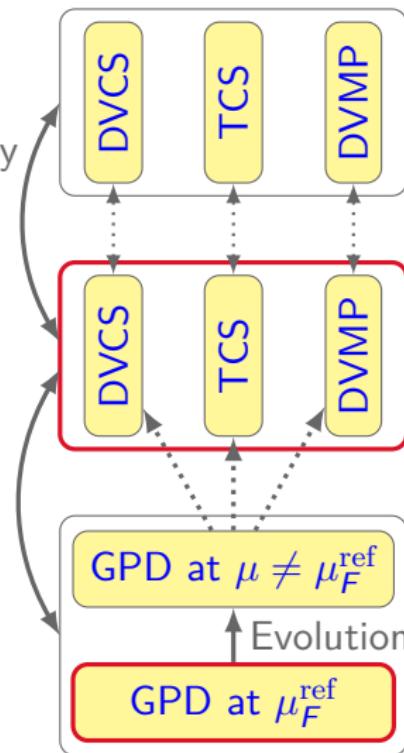
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Computation of amplitudes

First
principles and
fundamental
parameters



- Many observables.
 - Kinematic reach.

- Perturbative approximations.
 - **Physical models.**
 - Fits.
 - Numerical methods.
 - Accuracy and speed.

Computing chain design.

Differential studies: physical models and numerical methods.

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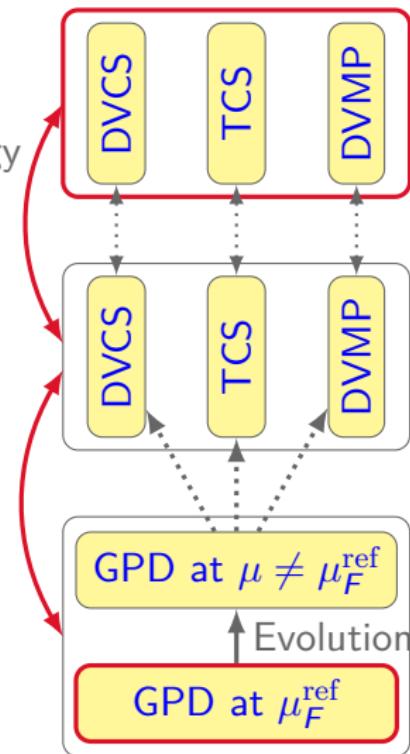
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Need for modularity

Computation of amplitudes

First principles and fundamental parameters



- Many observables.
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Computing chain design.

Differential studies: physical models and numerical methods.

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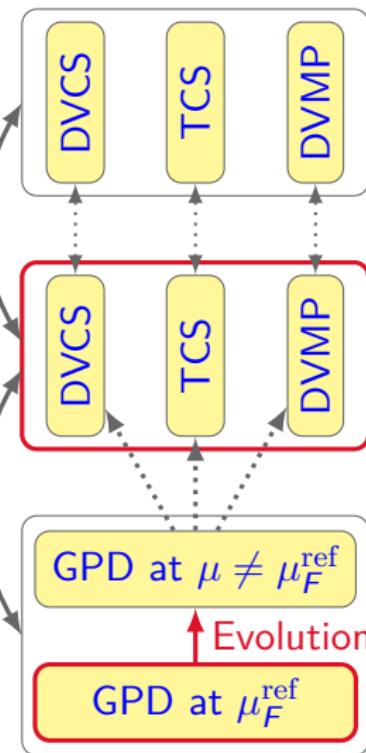
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First principles and fundamental parameters



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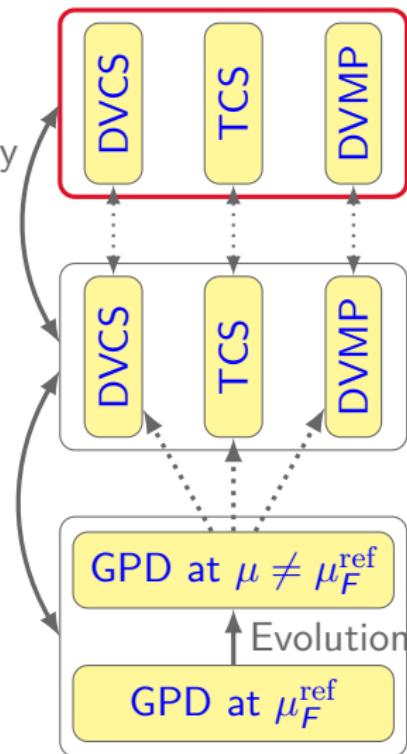
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- Many observables.
 - Kinematic reach.

- Perturbative approximations.
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First local fit of pseudo DVCS data, Sep. 26th, 2016

Mattermost

@herve PARTONS

partons_fits

partons_tests

partons_v0

partons_visualization

radon-inverse

short_distance

Town Square

trello

virtual_machine

More...

PRIVATE GROUPS +

Gitlab

Seville

DIRECT MESSAGES

bryan

cedric

dbinosi

jakub

luca

nchouika

pawel

...

partons_fits v

Mon, Sep 26, 2016

pawel 3:16 PM

```
FCN=1.00128e-11 FROM MIGRAD      STATUS=CONVERGED
TOTAL          EDM=2.00186e-11    STRATEGY= 1    ERROR MATRIX ACCURATE
EXT PARAMETER          VALUE        ERROR        STEP         FIRST
NO. NAME            VALUE        ERROR        SIZE        DERIVATIVE
 1 fit_CFF_H_Re   6.67247e-02  1.34241e+00  2.92531e-05 -7.02262e-07
 2 fit_CFF_H_Im   1.24231e+01  1.07342e+00  1.88608e-05  1.71871e-04
 3 fit_CFF_E_Re  -3.94789e+00   fixed
 4 fit_CFF_E_Im  -1.64116e-01   fixed
 5 fit_CFF_Ht_Re  1.54183e+00   fixed
 6 fit_CFF_Ht_Im  2.59017e+00   fixed
 7 fit_CFF_Et_Re  5.41102e+01   fixed
 8 fit_CFF_Et_Im  3.79052e+01   fixed
EXTERNAL ERROR MATRIX.    NDIM= 25    NPAR= 2    ERR DEF=1
 1.804e+00  7.961e-03
 7.961e-03  1.153e+00
PARAMETER CORRELATION COEFFICIENTS
 NO. GLOBAL      1      2
 1  0.00552  1.000  0.006
 2  0.00552  0.006  1.000
```

The first reasonable fit with PARTONS_Fits! 12 AUL and 12 ALU asymmetries fitted together.

The true values of fit_CFF_H_Re and fit_CFF_H_Im are 0.06672466940113253 and 12.423114181138908

... - - - - -

Write a message...

Help

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Parametric global fit of JLab DVCS data, Apr. 5th, 2017

RESULTS

- Kinematic cuts $Q^2 > 1.5 \text{ GeV}^2$ (where we can rely on LO approximation)
- $-t / Q^2 < 0.25$ (where we can rely on GPD factorization)
- χ^2 / ndf $3272.6 / (3433 - 7) \approx 0.96$
- Free parameters $a_{H\text{sea}}, a_{H\text{val}}, a_{H\tilde{\text{sea}}}, C_{\text{sub}}, a_{\text{sub}}, N_E, N_{\tilde{E}}$
- χ^2 / ndf per data set
 - [1] Phys. Rev. C 92, 055202 (2015)
 - [2] Phys. Rev. Lett. 115, 212003 (2015)
 - [3] Phys. Rev. D 91, 052014 (2015)

Experiment	Reference	Observables	N points all	N points selected	chi2	chi2 / ndf
Hall A	[1] KINX2	σ_{UU}	120	120	135.0	1.19
Hall A	[1] KINX2	$\Delta\sigma_{LU}$	120	120	98.9	0.88
Hall A	[1] KINX3	σ_{UU}	108	108	274.8	2.72
Hall A	[1] KINX3	$\Delta\sigma_{LU}$	108	108	107.3	1.06
CLAS	[2]	σ_{UU}	1933	1333	1089.2	0.82
CLAS	[2]	$\Delta\sigma_{LU}$	1933	1333	1171.9	0.88
CLAS	[3]	AUL, ALU, ALL	498	305	338.1	1.13

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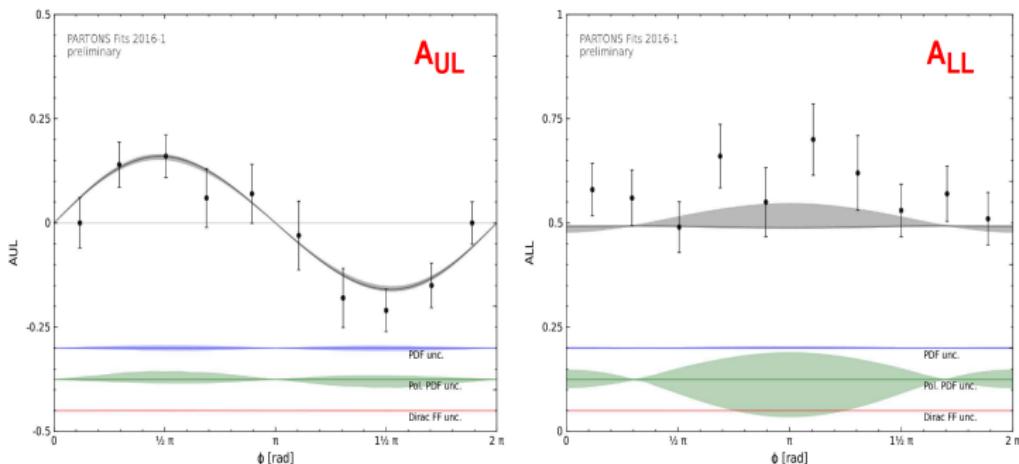
Conclusion

Comparison to CLAS data

RESULTS

CLAS: A_{UL} and A_{LL}
 @ $x_B = 0.26$, $t = -0.23 \text{ GeV}^2$, $Q^2 = 2.0 \text{ GeV}^2$, $E = 5.9 \text{ GeV}$

0.68 c.l.



Good description of experimental data, large systematics coming from Δq

Covariant extension

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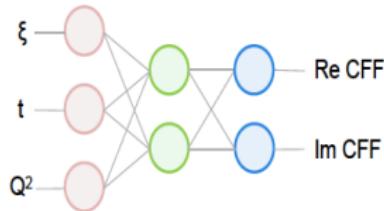
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NEURAL NETWORK



- Our very first attempt to use NN technique → proof of feasibility
- Genetic algorithm (GA) to learn NN
- NN and GA libraries by PARTONS group
- Very simple design of NN
- CLAS asymmetry data only
- $\chi^2 / \text{ndf} = 273.9 / (305 - 68) \approx 1.16$

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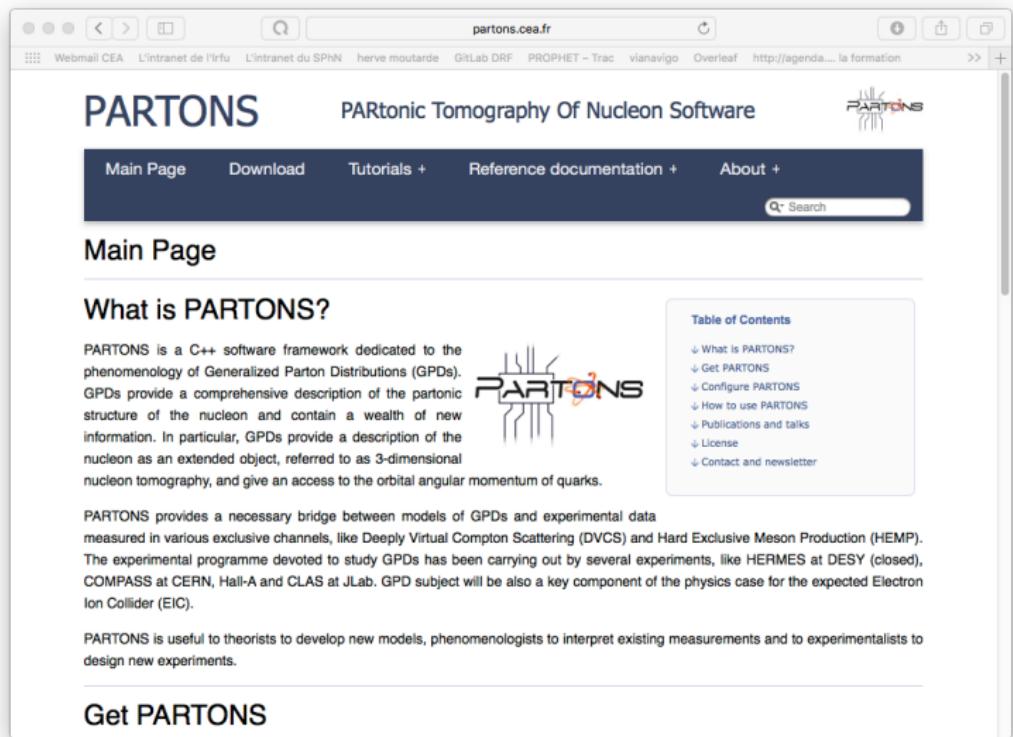
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The screenshot shows the main page of the PARTONS website. The header includes the CEA Paris-Saclay logo, a search bar, and navigation links for Main Page, Download, Tutorials +, Reference documentation +, and About +. A sidebar on the left lists sections for Covariant extension, Phenomenology, Modeling, and PARTONS, with the Releases section highlighted in pink. The main content area features a large "PARTONS" logo, a sub-header "PARtonic Tomography Of Nucleon Software", and a "Main Page" section. Below this, a "What is PARTONS?" section describes the software as a C++ framework for GPD phenomenology, mentioning its role in nucleon tomography and orbital angular momentum of quarks. To the right is a "Table of Contents" sidebar with links to various parts of the documentation. The footer contains a navigation bar with icons for back, forward, search, and other site functions.

PARTONS

PARtonic Tomography Of Nucleon Software

Main Page

What is PARTONS?

PARTONS is a C++ software framework dedicated to the phenomenology of Generalized Parton Distributions (GPDs). GPDs provide a comprehensive description of the partonic structure of the nucleon and contain a wealth of new information. In particular, GPDs provide a description of the nucleon as an extended object, referred to as 3-dimensional nucleon tomography, and give an access to the orbital angular momentum of quarks.

PARTONS provides a necessary bridge between models of GPDs and experimental data measured in various exclusive channels, like Deeply Virtual Compton Scattering (DVCS) and Hard Exclusive Meson Production (HEMP). The experimental programme devoted to study GPDs has been carrying out by several experiments, like HERMES at DESY (closed), COMPASS at CERN, Hall-A and CLAS at JLab. GPD subject will be also a key component of the physics case for the expected Electron Ion Collider (EIC).

PARTONS is useful to theorists to develop new models, phenomenologists to interpret existing measurements and to experimentalists to design new experiments.

Get PARTONS

Covariant extension

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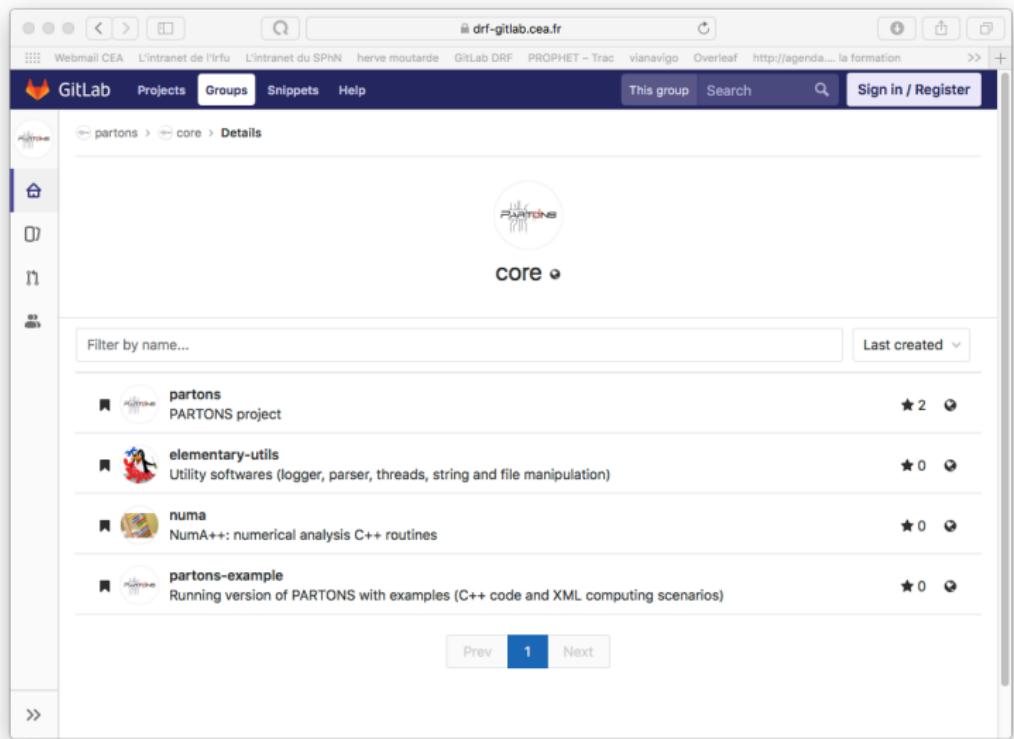
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The screenshot shows a web browser window for the CEA GitLab server at drf-gitlab.cea.fr. The URL bar also lists other projects like Webmail CEA, L'intranet de l'Irfu, etc. The main navigation bar includes GitLab DRF, PROPHET – Trac, vianavigo, Overleaf, and http://agenda... la formation. The current page is 'partons > core > Details' under the 'Groups' tab. The group icon is a circular logo with 'PARTONS' text. The page title is 'core'. A search bar at the top right says 'Last created'. Below it is a table of projects:

Project	Rating	Actions
partons PARTONS project	★ 2	...
elementary-utils Utility softwares (logger, parser, threads, string and file manipulation)	★ 0	...
numa NumA++: numerical analysis C++ routines	★ 0	...
partons-example Running version of PARTONS with examples (C++ code and XML computing scenarios)	★ 0	...

At the bottom, there are navigation buttons for 'Prev', '1', 'Next', and a double arrow '»'.

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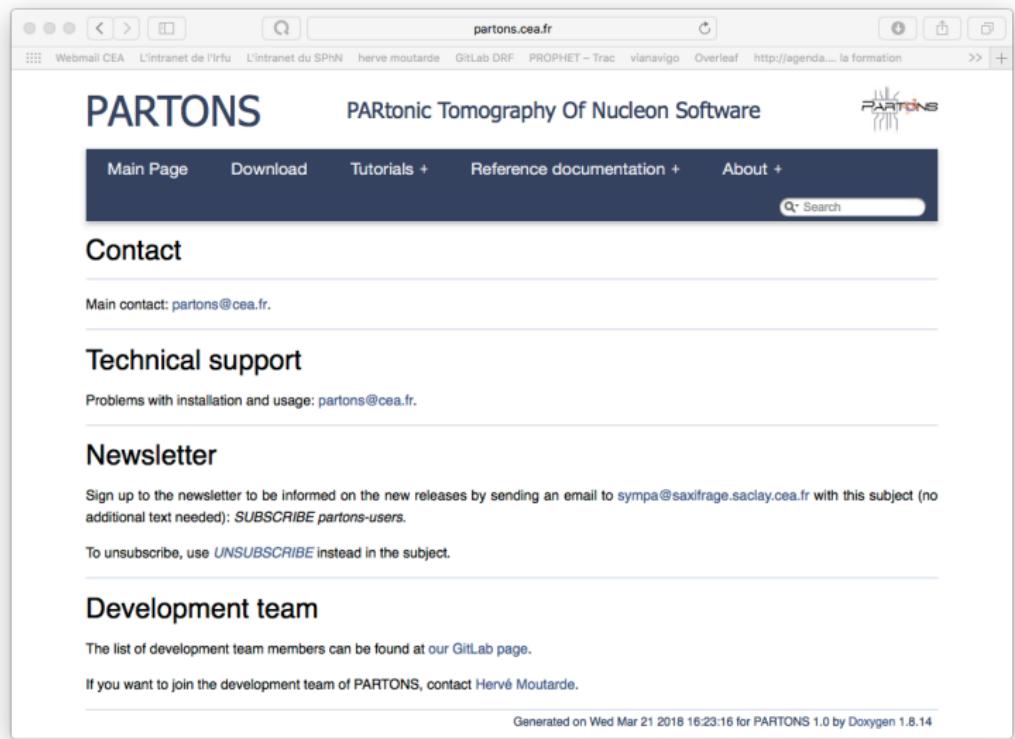
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The screenshot shows a web browser displaying the partons.cea.fr website. The page has a dark blue header with the title "PARTONS" and the subtitle "PARtonic Tomography Of Nucleon Software". Below the header is a navigation bar with links for "Main Page", "Download", "Tutorials +", "Reference documentation +", and "About +". A search bar is also present. The main content area contains sections for "Contact", "Technical support", "Newsletter", and "Development team". The "Newsletter" section includes instructions for signing up via email and unsubscribing. The "Development team" section mentions the availability of a GitLab page for team members. At the bottom, there is a footer note about the generation of the page and a set of navigation icons.

partons.cea.fr

Webmail CEA L'intranet de l'Irfu L'intranet du SPHn herve moutarde GitLab DRF PROPHET – Trac vianavigo Overleaf http://agenda.... la formation >> +

PARTONS

PARtonic Tomography Of Nucleon Software

Main Page Download Tutorials + Reference documentation + About +

Search

Contact

Main contact: partons@cea.fr.

Technical support

Problems with installation and usage: partons@cea.fr.

Newsletter

Sign up to the newsletter to be informed on the new releases by sending an email to sympa@saxfrage.saclay.cea.fr with this subject (no additional text needed): *SUBSCRIBE partons-users*.

To unsubscribe, use *UNSUBSCRIBE* instead in the subject.

Development team

The list of development team members can be found at our GitLab page.

If you want to join the development team of PARTONS, contact Hervé Moutarde.

Generated on Wed Mar 21 2018 16:23:16 for PARTONS 1.0 by Doxygen 1.8.14

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GPD modules

- GK
- VGG
- Vinnikov (evolution)
- MPSSW13 (NLO study)
- MMS13 (DD study)

CFF modules

- LO
- NLO
- NLO Noritzsch

Evolution modules

- Vinnikov (LO)

DVCS modules

- VGG
- GV
- BMJ

α_s modules

- 4-loop perturbation
- constant value

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Channel modules

- DVMP
- TCS
- γM production
- ???

Other modules

- Mellin moments (EM tensor, lattice QCD)
- ???

Hadron structure modules

- DAs
- DDs
- Form factors
- PDFs
- LFWFs
- ???

Nonperturbative QCD modules

- Gap equation solver?
- α_s models?
- ???

Conclusion

Conclusion and prospects.

Putting all the pieces together.

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- We can now build generic GPD model satisfying *a priori* all theoretical constraints.
- We now have tools to **systematically relate** these models to **experimental data**. Open source release under GPLv3.0. of the PARTONS framework.
- We have an **operating fitting engine** for global CFF fits.

New studies become possible!

- Global GPD fits.
- Energy-momentum structure of hadrons.
- Impact of nonperturbative QCD ingredients on 3D hadron structure studies.
- ???

Commissariat à l'énergie atomique et aux énergies alternatives
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