The suppression of the decay constants of the pion's excited states in Holographic QCD

Carlisson Miller

Many manifestations of nonperturbative QCD
Camburi- 2018

Institute for Theoretical Physics - Brazil

03/05/2018

Collaboration

Phys.Rev. D91 (2015) 065024

Prof. Dr. Gastão Krein (IFT-UNESP-São Paulo-Brazil)

Dr. Alfonso Bayona (IFT-ICTP-São Paulo-Brazil)

Outline

- **Motivation**
- **Holographic QCD**
- **Pion mass spectrum**
- **Holographic decay constants**
- **The vanishing of the decay constants**
- **Comparison with the Dyson-Schwinger formalism**
- **Conclusions**

QCD Motivation

- The vanishing of the leptonic decay constants of pion's excited states in the chiral limit

PHYSICAL REVIEW C **70**, 042203(R) (2004)

Pseudoscalar meson radial excitations

A. Höll,[1] A. Krassnigg,[1] and C. D. Roberts[1,2]
[1]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439-4843, USA*
[2]*Fachbereich Physik, Universität Rostock, D-18051 Rostock, Germany*
(Received 9 June 2004; published 26 October 2004)

Goldstone modes are the only pseudoscalar mesons to possess a nonzero leptonic decay constant in the chiral limit when chiral symmetry is dynamically broken. The decay constants of their radial excitations vanish. These features and aspects of their impact on the meson spectrum are illustrated using a manifestly covariant and symmetry-preserving model of the kernels in the gap and Bethe-Salpeter equations.

DOI: 10.1103/PhysRevC.70.042203 PACS number(s): 14.40.Cs, 11.10.St, 11.15.Tk, 21.45.+v

- The authors have found a generalized GOR relation

$$f_{\pi_n} M_{\pi_n}^2 = 2m_q(\xi)\rho_{\pi_n}(\xi)$$

QCD Motivation

- Its worth to remind that this relation reproduce the GOR relation, $n = 0$.

$$\rho^0_{\pi_0}(\xi) := \lim_{m\to 0} \rho_{\pi_0}(\xi) = -\frac{1}{f^0_{\pi_0}} \langle \bar{q}q \rangle_\xi \qquad (f^0_{\pi_0})^2 M^2_{\pi_0} = 2m_q(\xi) \langle \bar{q}q \rangle_\xi$$

- When chiral symmetry is dynamically broken, then

$$\rho^0_{\pi_n}(\xi) := \lim_{m\to 0} \rho_{\pi_n}(\xi) < \infty, \quad \forall n.$$

- The authors considered, by assumption

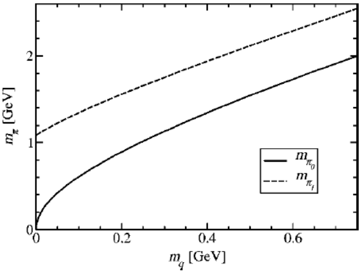
$$M_{\pi_{n>0}} > M_{\pi_0} \qquad \text{hence} \qquad M_{\pi_{n>0}} \neq 0 \text{ (chiral limit)}$$

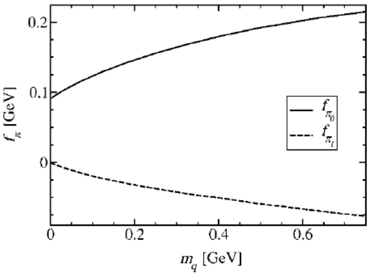
- This leads to vanishing of the decay constant for excited states

$$f^0_{\pi_n} \equiv 0, \quad \forall n \geq 1.$$

QCD Motivation

- The ρ_n function is non-trivial as quark mass. The authors found just for the first excited state.





- In the real world, the light quarks has small mass. So, we expected a suppression for excited states

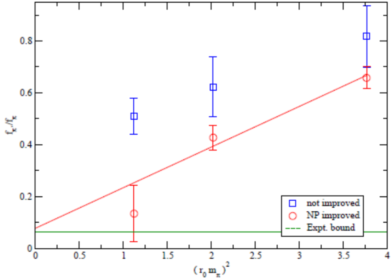
$$\frac{f^0_{\pi_n}}{f^0_{\pi_0}} < 1, \quad \forall n \geq 1.$$

QCD Motivation

- A first QCD lattice result from [UKQCD Collaboration, Phys. Lett. B 642, 244 (2006)].

QCD Motivation

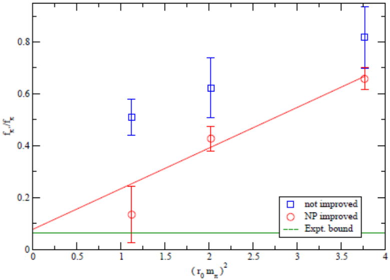
- A first QCD lattice result from [UKQCD Collaboration, Phys. Lett. B 642, 244 (2006)].



$$f_{\pi 1}/f_{\pi} = 0.078$$

QCD Motivation

- A first QCD lattice result from [UKQCD Collaboration, Phys. Lett. B 642, 244 (2006)].



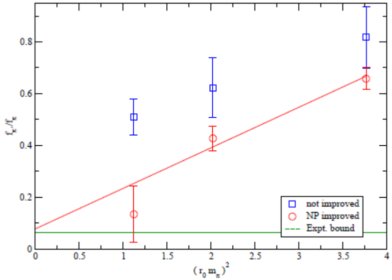
$f_{\pi 1}/f_{\pi} = 0.078$

- In 2001, experimental results showed this suppression [JHEP 0106, 067 (2001)].

$$f_{\pi_1}/f_{\pi_0} < 0.064 \text{ MeV}.$$

QCD Motivation

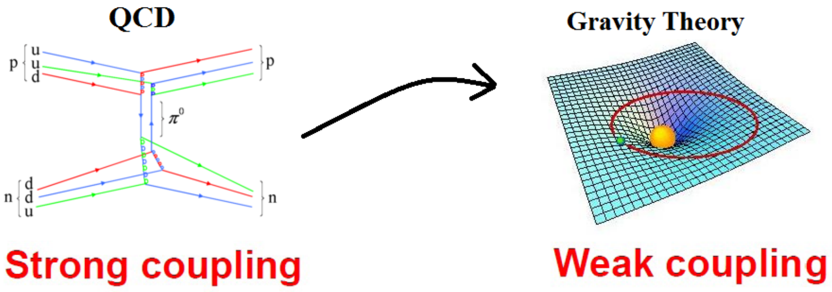
- A first QCD lattice result from [UKQCD Collaboration, Phys. Lett. B 642, 244 (2006)].



$f_{\pi 1}/f_\pi = 0.078$

- Sum rules [Phys. Rev. D 65, 074013 (2002)], effective chiral Lagrangians [Phys. Rev. D 56, 221 (1997)] also find strongly suppressed values for f_{π_1}.

Holographic QCD:
A new approach for Hadron Physics



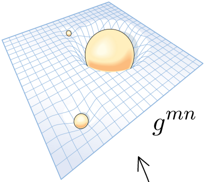
Holographic Dictionary

- Holographic QCD aims to study the strongly coupled regime of QCD using gauge/gravity duality

Gluon Sector

$$-\frac{1}{4}F^{\mu\nu,a}F^{a}_{\mu\nu} \longrightarrow$$

Space-time dynamics



g^{mn}

Flavor Sector

$$\sum_{flavors}\bar{\psi}^{f}_{i}(i\gamma^{\mu}\partial_{\mu}-M)_{ij}\psi^{f}_{j} \longrightarrow L_m \ , X$$

Fields

Holographic Gluon Sector

- Gauge invariant Operators in 4D are mapped into fields in 5D

$$4D : \mathcal{O}(x^\mu) \qquad 5D : \phi(x^\mu, z)$$

$$T^{\mu\nu} \quad \rightarrow \quad g^{mn}(x,z) \qquad \langle T^\mu_\mu \rangle = \frac{\beta(g_s)}{2g_s}\mathrm{Tr}F^2$$

$$\mathrm{Tr}F^2 \quad \rightarrow \quad \Phi(x,z)$$

Phys.Rev. D86 (2012) 034033

- Action for pure gluon sector: Graviton-dilaton action

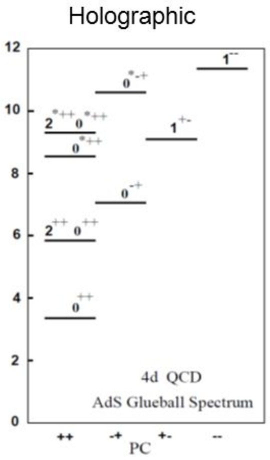
$$S_g = \frac{1}{16\pi G_5}\int d^5x\sqrt{|g|}e^{-2\Phi}(R + 4\partial_M\Phi\partial^M\Phi - V(\Phi))$$

- Glueballs takes place:

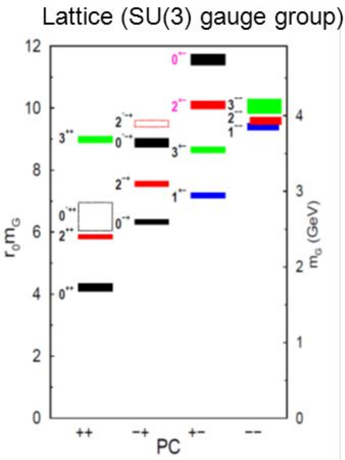
F.Brunner, A. Rebhan – Phys.Lett.B770(2017)124 – 130

K. Hashimoto, C. Tan, S. Terashima – Phys.Rev.D77(2008)086001

Glueballs spectrum



from Brower-Mathur-Tan (2000)



from Morningstar-Peardon

Holographic Flavour sector

- The holographic dictionary for the flavour sector:
  $m^2 = (\Delta - p)(\Delta + p - 4)$

4D: $\mathcal{O}(x^\mu)$	5D: $\phi(x^\mu, z)$	p	Δ	m^2
$\bar{q}_L \gamma_\mu t^a q_L$	$L^a_m(x,z)$	1	3	0
$\bar{q}_R \gamma_\mu t^a q_R$	$R^a_m(x,z)$	1	3	0
$\bar{q}_R q_L$	$\frac{2}{z} X(x,z)$	0	3	-3

- Action for flavour sector: Karch-Katz-Son-Stephanov (KKSS) model

$$S_f = \int d^5x \sqrt{|g|} e^{-\Phi} \mathrm{Tr} \left\{ |DX|^2 - 3|X|^2 - \frac{1}{4g_5^2} \left(L^2_{\mu\nu} + R^2_{\mu\nu} \right) \right\}$$

Holographic QCD

- The full system can be described by the action

$$S_{grav} = S_g + \frac{N_f}{N_c} S_f$$

- Holographic duality conjectures

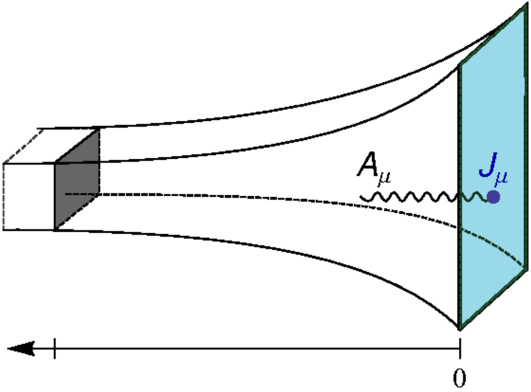
$$Z_{hadron}[J(x)] = Z_{grav}[\phi_0(x)] \sim e^{-S_{grav}[\phi(x,\epsilon)]}|_{\phi \to \phi_0}.$$

- No backreaction: $N_f/N_c \ll 1$
- **QCD** Trace anomaly $\Rightarrow \Phi(z) \neq 0 \Rightarrow$ Deformed AdS_5

$$ds^2 = a(z)\frac{L^2}{z^2}(dt^2 - d\vec{x}^2 - dz^2)$$

- $a(z)$ is determined by the dilation-gravity equation

- The simplest background that simulates confinement is the hard wall background: $a(z) = \delta(z - z_0)$.



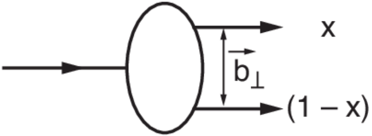
$\text{J. Sonnenschein} - \text{hep} - \text{th}/0009146$

Light-Front Holographic QCD

$$LF(3+1) \longleftrightarrow AdS_5$$

$$\psi(x,\vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}} \longleftrightarrow z$$



$$\psi(x,\zeta,\varphi) = e^{iL\varphi}\sqrt{x(1-x)}\frac{\phi(\zeta)}{\sqrt{2\pi\zeta}}$$

arXiv:1301.2733 [hep-ph]

S. Brodsky, G. Teramond

DCSB is implemented in a nonstandard way

The decay constants of the excited states do not vanishes in the chiral limit

Holographic QCD

- We consider the flavor action in the hard wall background

$$S_f = \int d^5x \sqrt{|g|} \mathrm{Tr} \left\{ |DX|^2 - 3|X|^2 - \frac{1}{4g_5^2} \left(L_{\mu\nu}^2 + R_{\mu\nu}^2 \right) \right\}$$

$$D_m X = \partial_m X - i L_m X + i X R_m,$$
$$L_{mn} = \partial_m L_n - \partial_n L_m - i \left[L_m, L_n \right] \quad L \Leftrightarrow R.$$

- Vacuum structure: $L_{mn} = R_{mn} = 0$

$$\partial_z \left(\frac{1}{z^3} \partial_z X_0 \right) + \frac{3}{z^5} X_0 = 0 \;\rightarrow\; v(z) = 2X_0 = Mz + \Sigma z^3$$

Da Rold and Pomarol - Nucl. Phys. B721 (2005), pp. 79-97

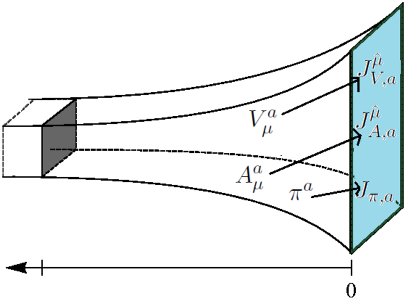
Holographic QCD

- The fluctuations around the vacuum give the mesons

$$X\left(x,z\right)=e^{i\pi^{a}(x,z)t^{a}}X_{0}\left(x,z\right)e^{i\pi^{a}(x,z)t^{a}}$$

- Similarly to QCD, we construct the 5D vector and axial vector fields.

$$V_{\mu}^{a}(x,z)=R_{\mu}^{a}(x,z)+L_{\mu}^{a}(x,z),\quad A_{\mu}^{a}(x,z)=R_{\mu}^{a}(x,z)-L_{\mu}^{a}(x,z)$$



Holographic QCD

- Using the fluctuations and expanding the flavour action, we have

$$S = S_0 + S_2 + S_3 + \dots\,,$$

- We focus in S_2 for the $SU(N_f)$

$$S_2 = \int d^5x\sqrt{|g|}\Big\{ -\frac{1}{4g_5^2}a_a^{mn}a^a_{mn} + \frac{T_A^{ab}}{2}(\partial^m\pi^a - A^{m,a})(\partial_m\pi_b - A_{m,b})\Big\}.$$

$$a_{mn} = \partial_m A_n - \partial_n A_m, \quad \mathrm{Tr}(\{T^a, X_0\}\{T^b, X_0\}) = \frac{T_A^{ab}}{2}\,.$$

- Let us separate the action S_2 in the 4d and z parts, using

$$A^a_{\hat{\mu}} = A^{\perp,a}_{\hat{\mu}} + \partial_{\hat{\mu}}\phi^a.$$

Holographic QCD

- The gauge symmetry of the action allow us to write

$$A^a_{\hat{\mu}} \;\to\; A^{\perp,a}_{\hat{\mu}}, \quad A^a_z \to -\partial_z \phi^a, \quad \pi^a \to \pi^a - \phi^a \,.$$

- The Kaluza-Klein expansion

$$A^{\perp,a}_{\hat{\mu}}(x,z) = g_5 \sum_{n=0}^{\infty} a^{a,n}(z) \hat{A}^{a,n}_{\hat{\mu}}(x),$$

$$\pi^a(x,z) = g_5 \sum_{n=0}^{\infty} \pi^{a,n}(z) \hat{\pi}^{a,n}(x), \quad \phi^a(x,z) = g_5 \sum_{n=0}^{\infty} \phi^{a,n}(z) \hat{\pi}^{a,n}(x).$$

- The gauge-invariant excitations, like mesons, are performed holographically as 5D normalizable modes of bulk fields.

Holographic QCD

- This way, the pseudoscalar part of the action S_2 becomes

$$S_2^\pi = \sum_{n,m=0}^{\infty} \int d^4x \left\{ \frac{1}{2} \Delta_\pi^{a,nm} (\partial_{\hat{\mu}} \hat{\pi}^{a,n})(\partial^{\hat{\mu}} \hat{\pi}^{a,m}) - \frac{1}{2} M_\pi^{a,nm} \hat{\pi}^{a,n} \hat{\pi}^{a,m} \right\}$$

- where the coefficients are

$$\begin{aligned} \Delta_\pi^{a,nm} &= \int \frac{dz}{z} \Big\{ [\partial_z \phi^{a,n}(z)][\partial_z \phi^{a,m}(z)] \\ &+ \beta_A^{aa}(z)[\pi^{a,n}(z) - \phi^{a,n}(z)][\pi^{a,m}(z) - \phi^{a,m}(z)] \Big\}, \\ M_\pi^{a,nm} &= \int \frac{dz}{z} \beta_A^{aa}(z) [\partial_z \pi^{a,n}][\partial_z \pi^{a,m}]. \end{aligned}$$

- where we have defined

$$\beta_A^{ab} = \frac{g_5^2}{z^2} T_A^{ab} .$$

Holographic QCD

- In order to obtain standard kinetic terms we impose

$$\Delta_\pi^{a,nm} = \delta^{nm}, \quad M_\pi^{a,nm} = m^2_{\pi^{a,n}} \delta^{nm}$$

- The above conditions lead to equations

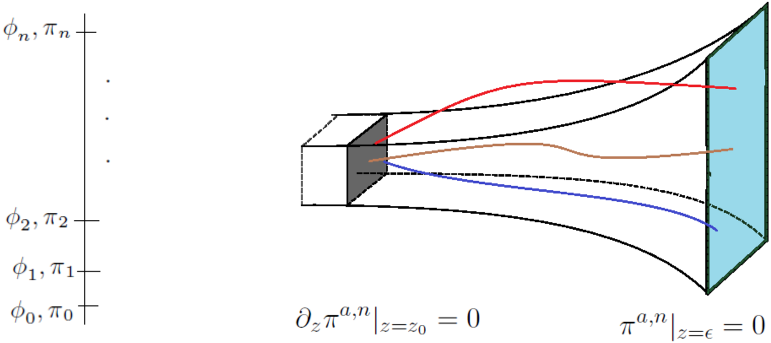
$$\frac{\beta_A^{aa}(z)}{z}\left[\pi^{a,n}(z) - \phi^{a,n}(z)\right] = -\partial_z \left[\frac{1}{z}\partial_z \phi^{a,n}(z)\right],$$
$$\beta_A^{aa}(z)\,\partial_z \pi^{a,n}(z) = m^2_{\pi^{a,n}} \partial_z \phi^{a,n}(z).$$

- The mass spectrum of the axial and pseudoscalar mesons can be obtained using

$$\phi^{a,n}|_{z=\epsilon} = \pi^{a,n}|_{z=\epsilon} = 0,$$
$$\partial_z \phi^{a,n}|_{z=z_0} = \partial_z \pi^{a,n}|_{z=z_0} = 0.$$

- To solve it, we use the following conditions for normalized modes

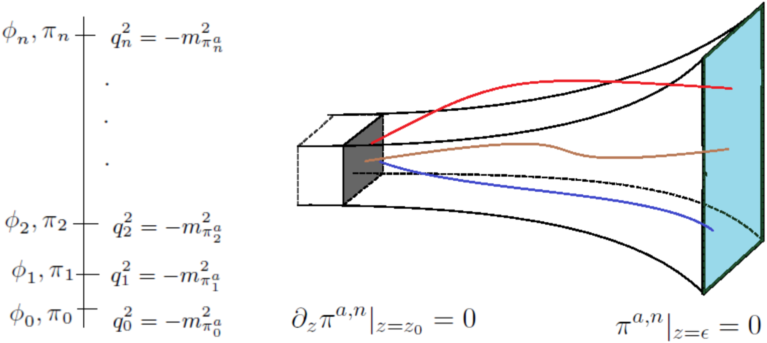
$$\pi^{a,n}|_{z=\epsilon} = 0, \quad \partial_z \pi^{a,n}|_{z=z_0} = 0$$



Holographic QCD

- To solve it, we use the following conditions for normalized modes

$$\pi^{a,n}|_{z=\epsilon} = 0, \quad \partial_z \pi^{a,n}|_{z=z_0} = 0$$



- The eigenvalues are identified as the hadronic masses

Pion spectrum

- The pions are identified by

$$P = \pi^i t^i = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} & \pi^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} \end{pmatrix},$$

- The equations for normalized modes takes the form

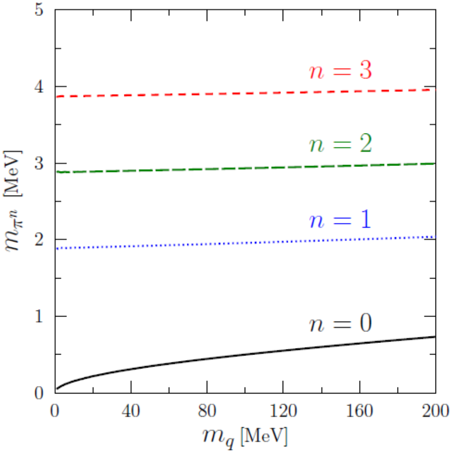
$$\frac{\beta(z)}{z} \left[\pi^n(z) - \phi^n(z)\right] = -\partial_z \left[\frac{1}{z}\partial_z \phi^n(z)\right] ,$$
$$\beta(z)\, \partial_z \pi^n(z) = m^2_{\pi^n} \partial_z \phi^n(z).$$

- where

$$\beta(z) = \frac{g_5^2}{z^2} v(z)^2 = g_5^2 \left(\zeta m_q + \frac{\sigma}{\zeta} z^2\right)^2.$$

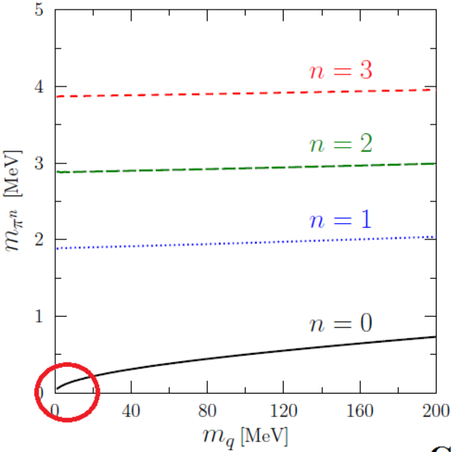
Pion spectrum

- Solving numerically these equations we obtain



Pion spectrum

- Solving numerically these equations we obtain

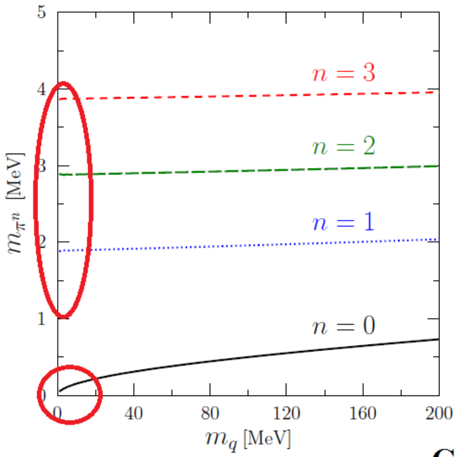


$$m_\pi^2 \propto m_q$$

Gell-Mann-Oakes Renner

Pion spectrum

- Solving numerically these equations we obtain



$$m_{\pi^n} = m^0_{\pi^n} + a_n m_q$$

$$m_\pi^2 \propto m_q$$

Gell-Mann-Oakes Renner

Holographic decay constants

- Variating the action δS_2, we obtain the surface term

$$\delta S_2^{\mathrm{Bdy}} = -\int d^4x \left[\langle J^{\hat{\mu}}_{V,a} \rangle \left(\delta V^a_{\hat{\mu}} \right)_{z=\epsilon} + \langle J^{\hat{\mu}}_{A,a} \rangle \left(\delta A^a_{\hat{\mu}} \right)_{z=\epsilon} + \langle J_{\pi,a} \rangle \left(\delta \pi^a \right)_{z=\epsilon} \right]$$

- where the vev's (holographic currents) are

$$\langle J^{\hat{\mu}}_{V,a}(x) \rangle = P^{z\mu}_{V,a}|_{z=\epsilon} = -\frac{1}{g_5^2} \left(\sqrt{|g|} v^{z\mu}_a \right)_{z=\epsilon},$$

$$\langle J^{\hat{\mu}}_{A,a}(x) \rangle = P^{z\mu}_{A,a}|_{z=\epsilon} = -\frac{1}{g_5^2} \left(\sqrt{|g|} a^{z\mu}_a \right)_{z=\epsilon},$$

$$\langle J_{\pi,a}(x) \rangle = P^{z}_{\pi,a}|_{z=\epsilon} = \left[\sqrt{|g|} T^{ab}_A \left(\partial^z \pi_b - A^z_b \right) \right]_{z=\epsilon}.$$

Holographic decay constants

- Using the Kaluza-Klein expansion in the axial and vector holographic currents

$$\langle J_{\pi,a}(x)\rangle = -\sum_{n=0}^{\infty}\left[\frac{\beta_A^{aa}(z)}{g_5 z}\partial_z \pi^{a,n}(z)\right]_{z=\epsilon}\hat{\pi}^{a,n}(x).$$

$$\langle J^{\hat{\mu}}_{A,a}(x)\rangle = \sum_{n=0}^{\infty} g_{A^{a,n}}\hat{A}^{\hat{\mu}}_{a,n}(x) - \sum_{n=0}^{\infty} f_{\pi^{a,n}}\partial^{\hat{\mu}}\hat{\pi}^{a,n}(x),$$

- The constants are defined as

$$g_{A^{a,n}} = \left[\frac{1}{g_5 z}\partial_z a^{a,n}(z)\right]_{z=\epsilon}, \quad f_{\pi^{a,n}} = -\left[\frac{1}{g_5 z}\partial_z \phi^{a,n}(z)\right]_{z=\epsilon}.$$

Holographic decay constants

- These constants are precisely the decay constants

$$\langle 0|J^{\hat{\mu}}_{A,a}(x)|A^{b,m}(p,\lambda)\rangle = g_{A^{a,m}} e^{-ip\cdot x}\epsilon^{\hat{\mu}}(p,\lambda)\delta^{ab},$$

$$\langle 0|J^{\hat{\mu}}_{A,a}(x)|\pi^{b,m}(p)\rangle = f_{\pi^{a,m}} e^{-ip\cdot x}\delta^{ab}.$$

- After some manipulations, we obtain the PCAC relation

$$\langle J_{\pi,a}(x)\rangle = \partial_{\hat{\mu}}\langle J^{\hat{\mu}}_{A,a}(x)\rangle$$

- This way, we find the following relation

$$f_{\pi^{a,n}} m^2_{\pi^{a,n}} = -\left[\frac{\beta(z)}{g_5 z}\Pi^{a,n}(z)\right]_{z=\epsilon}, \quad \Pi^n(z) = \partial_z \pi^n(z)$$

Holographic decay constants

- By comparing with the relation found by C. Roberts and et.

$$f_{\pi^{a,n}} m^2_{\pi^{a,n}} = 2 m_q \rho_{\pi^{a,n}}$$

- We find a holographic expression for the ρ-function

$$\rho_{\pi^{a,n}} = -\frac{1}{2m_q}\left[\frac{\beta(z)}{g_5 z}\Pi^{a,n}(z)\right]_{z=\epsilon}, \quad \Pi^n(z) = \partial_z \pi^n(z)$$

- Remember that, authors showed that

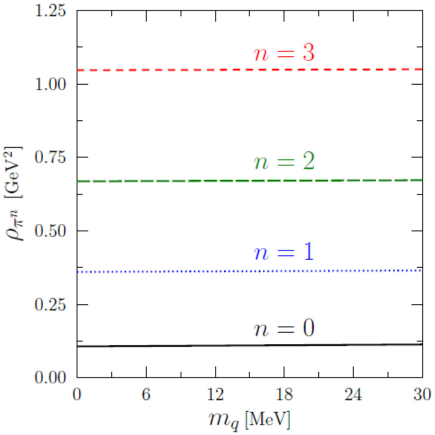
$$\rho_{\pi^{a,n}} < \infty \quad \text{chiral limit}$$

- And, as consequence,

$$f_{\pi^{a,n}} = 0, \quad n > 1 \quad \text{chiral limit}$$

Holographic ρ-function

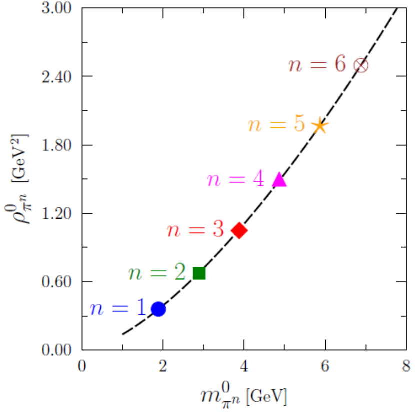
- Numerical results for the ρ-function in Holographic model



Quark mass dependence of ρ_{π^n}.

Holographic ρ-function

- We also found the ρ-function, for the excited states, as function of mass in the chiral limit.



The function $\rho^0_{\pi^n}$, defined in Eq. (1.5), for the first six excited states. The dashed line is a fit to the discrete eigenvalues.

The $\rho^0_{\pi^n}$ function can be fitted as

$$\rho^0_{\pi^n} = \gamma \left(m^0_{\pi^n}\right)^{3/2}, \quad n \geq 1$$

Holographic decay constants

- Finally, the behavior of the decay constant

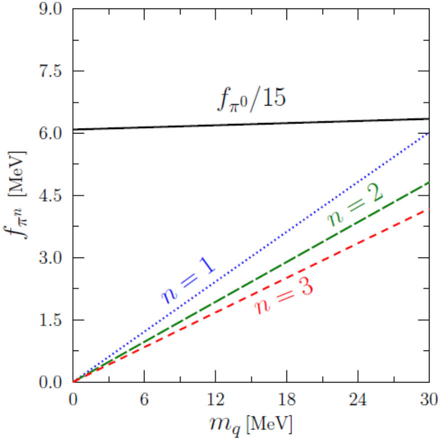


FIG. 4. Quark mass dependence of f_{π^n}.

The decay constant scale as

$$f_{\pi^n} \sim m_q, \quad n \geq 1.$$

Dyson-Shwinger Equations

$$f_{\pi^n} m_{\pi^n}^2 = 2m_q \rho_{\pi^n}$$

Holographic QCD

Dyson-Shwinger Equations

$$f_{\pi^n} m_{\pi^n}^2 = 2 m_q \, \rho_{\pi^n}$$

$$\downarrow$$

ρ_{π^n} is finite by dynamical chiral SB

Dyson-Shwinger Equations

$$f_{\pi^n} m_{\pi^n}^2 = 2m_q \, \rho_{\pi^n}$$

$$\downarrow$$

ρ_{π^n} is finite by dynamical chiral SB

$$\downarrow$$

$$0 \neq M_{\pi_{n>0}} > M_{\pi_0} \quad \text{(assumption)}$$

Holographic QCD

Dyson-Shwinger Equations

$$f_{\pi^n} m^2_{\pi^n} = 2m_q \, \rho_{\pi^n}$$

$$\downarrow$$

ρ_{π^n} is finite by dynamical chiral SB

$$\downarrow$$

$$0 \neq M_{\pi_{n>0}} > M_{\pi_0} \qquad \text{(assumption)}$$

$$\downarrow$$

$$f^0_{\pi_n} \equiv 0, \quad \forall n \geq 1$$

Dyson-Shwinger Equations

$$f_{\pi^n} m_{\pi^n}^2 = 2m_q \, \rho_{\pi^n}$$

$$\downarrow$$

ρ_{π^n} is finite by dynamical chiral SB

$$\downarrow$$

$0 \neq M_{\pi_{n>0}} > M_{\pi_0}$ (assumption)

$$\downarrow$$

$$f^0_{\pi_n} \equiv 0, \quad \forall n \geq 1$$

Holographic QCD

$$f_{\pi^{a,n}} m_{\pi^{a,n}}^2 = -\left[\frac{1}{g_5}\left(\frac{1}{z}\right)\beta(z)\Pi^{a,n}(z)\right]_{z=\epsilon}$$

Dyson-Shwinger Equations

$$f_{\pi^n} m_{\pi^n}^2 = 2m_q \, \rho_{\pi^n}$$

$\downarrow$

ρ_{π^n} is finite by dynamical chiral SB

$\downarrow$

$0 \neq M_{\pi_{n>0}} > M_{\pi_0}$ (assumption)

$\downarrow$

$$f^0_{\pi_n} \equiv 0, \quad \forall n \geq 1$$

Holographic QCD

$$f_{\pi^{a,n}} m_{\pi^{a,n}}^2 = -\left[\frac{1}{g_5}\left(\frac{1}{z}\right)\beta(z)\Pi^{a,n}(z)\right]_{z=\epsilon}$$

$\downarrow$

$\rho^0_{\pi^n}$ has been finite in the chiral limit

$$\rho^0_{\pi^n} = \gamma \left(m^0_{\pi^n}\right)^{3/2}$$

Dyson-Shwinger Equations

$$f_{\pi^n} m_{\pi^n}^2 = 2m_q \rho_{\pi^n}$$

$\downarrow$

ρ_{π^n} is finite by dynamical chiral SB

$\downarrow$

$0 \neq M_{\pi_{n>0}} > M_{\pi_0}$ (assumption)

$\downarrow$

$$f_{\pi_n}^0 \equiv 0, \quad \forall n \geq 1$$

Holographic QCD

$$f_{\pi^{a,n}} m_{\pi^{a,n}}^2 = -\left[\frac{1}{g_5}\left(\frac{1}{z}\right)\beta(z)\Pi^{a,n}(z)\right]_{z=\epsilon}$$

$\downarrow$

$\rho_{\pi^n}^0$ has been finite in the chiral limit

$$\rho_{\pi^n}^0 = \gamma\left(m_{\pi^n}^0\right)^{3/2}$$

$\downarrow$

Numerically, we have shown

$$m_{\pi^n} = m_{\pi^n}^0 + a_n m_q$$

Holographic QCD

Dyson-Shwinger Equations

$$f_{\pi^n} m^2_{\pi^n} = 2m_q \, \rho_{\pi^n}$$

$$\downarrow$$

ρ_{π^n} is finite by dynamical chiral SB

$$\downarrow$$

$$0 \neq M_{\pi_{n>0}} > M_{\pi_0} \quad \text{(assumption)}$$

$$\downarrow$$

$$f^0_{\pi_n} \equiv 0, \quad \forall n \geq 1$$

Holographic QCD

$$f_{\pi^{a,n}} m^2_{\pi^{a,n}} = - \left[\frac{1}{g_5} \left(\frac{1}{z} \right) \beta(z) \Pi^{a,n}(z) \right]_{z=\epsilon}$$

$$\downarrow$$

$\rho^0_{\pi^n}$ has been finite in the chiral limit

$$\rho^0_{\pi^n} = \gamma \left(m^0_{\pi^n} \right)^{3/2}$$

$$\downarrow$$

Numerically, we have shown

$$m_{\pi^n} = m^0_{\pi^n} + a_n m_q$$

$$\downarrow$$

$$f_{\pi^n} = 0 \text{ for } n \geq 1$$

Holographic Conclusions

- Holographic provides a new way to study the hadron physics.
- We have found a generalized GOR relation from holography, where the ρ_n function has a simple form.
- Our results show that the following relation is valid, in the chiral limit

$$m_{\pi_0} < m_{\pi_{n>0}} \neq 0$$

- We have shown, from holography, that the ρ_n function is finite in the chiral limit and scale as $\left(m^0_{\pi^n}\right)^{3/2}$
- This leads us to conclude that all the decay constant vanish in the chiral limit, for excited states.

Collaborators

Prof. Dr. Gastão Krein (IFT-UNESP-São Paulo-Brazil)

Dr. Alfonso Bayona (IFT-ICTP-São Paulo-Brazil)

Funding

CAPES