RELATIVISTIC BEC-BCS CROSSOVER IN A COLD/MAGNETIZED NJL MODEL

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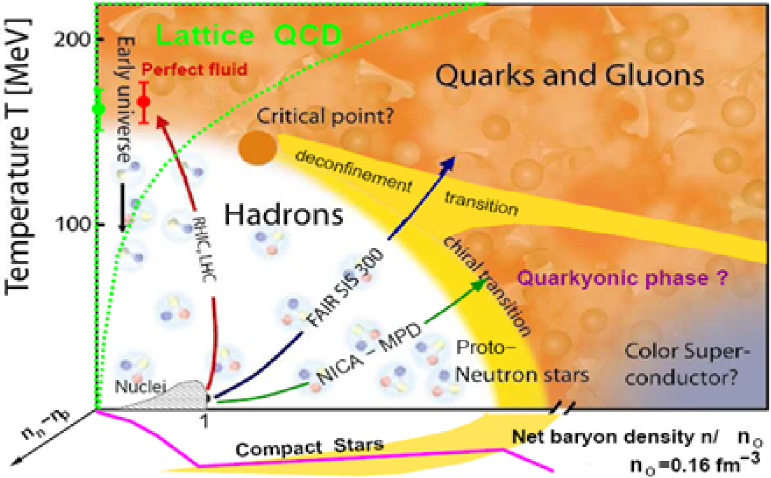
Many Manifestations of Nonperturbative QCD
May 01, 2018

Outline

- Motivation
- The BEC-BCS crossover
- $N_c = 2$ NJL model in the presence of an external magnetic field
- Beyond mean field effecs in the crossover

QCD Phase Diagram

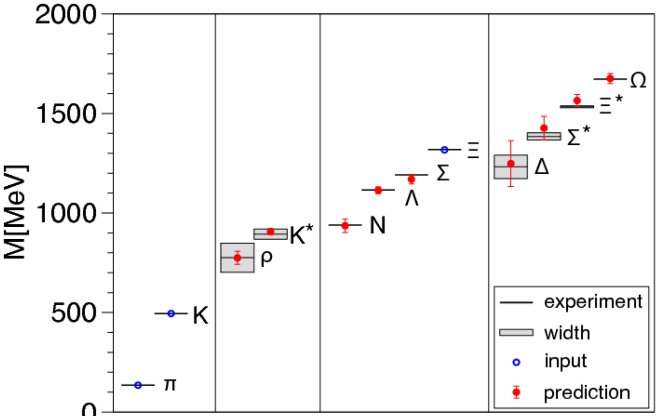


L. McLerran, Nucl.Phys.Proc.Suppl. 195 (2009) 275-280

QCD Phase diagrams - Approaches

- Perturbative: Based on asymptotic freedom $\rightarrow$ high energy experiments, at weak coupling.
- $1/N_c$ Expansion: Initially, $N_c \rightarrow \infty$, and set equal 3 after all calculations. Usually provides qualitative results.
- Dyson-Schwinger equations;
- Holographic QCD, based in the correspondence AdS/CFT
- Effective models/theories: Valid in specific regimes. Ex: ChPT, QMM, NJL/PNJL,...

→ Lattice QCD simmulations: Large computacional resources, but describes situations inaccessible with other methods.



Fodor and Hoelbling, Rev.Mod.Phys.84, 449 (2012)

→ First principles calculations and numerical simulations make use of Monte Carlo method, but the Fermion determinant becomes complex when the chemical potential is finite

Sign problem!

$\Downarrow$

Importance of developing effective models to describe the $T \times \mu$ phase diagram. Ex.: NJL model

$$\mathcal{L}_{NJL} = \bar{\psi}\left(i\partial\!\!\!/ - m_c\right)\psi + G\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma^5\tau_a\psi\right)^2\right];$$

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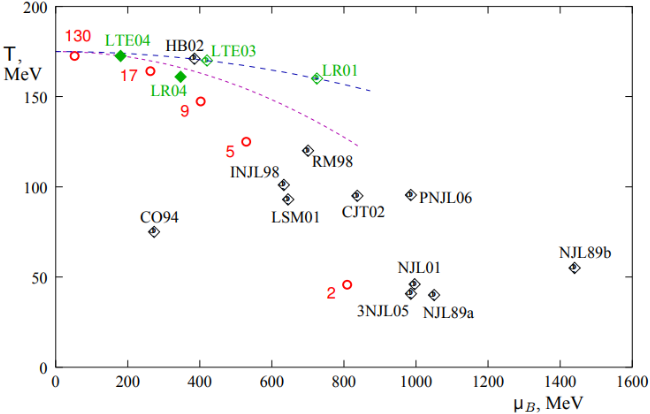
→ Critical point location on the QCD phase diagram, in different approaches:



M. Stephanov, Proc. of Science (2006)

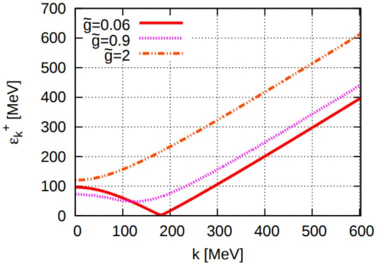
BEC-BCS Crossover

→ Condensed matter: Ultracold Fermi gases can realize a smooth crossover from a BCS superfluid to a Bose-Einstein condensation when the attraction between the difermion molecules increase*.

→ Quark matter: if the coupling constant become sufficiently high at moderate densities:

CSC-BCS → CSC-BEC.

$$\varepsilon_k^+ = \sqrt{\left(\sqrt{k^2+m^2}-\mu\right)^2+\Delta^2}$$

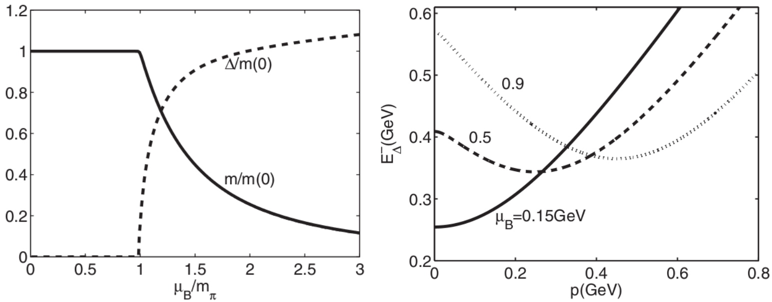


E. Ferrer, Proc. of Compact Stars in the QCD Phase Diagram III (2012)

*D. M. Eagles, Phys. Rev. 186, 456 (1969).

BEC-BCS Crossover

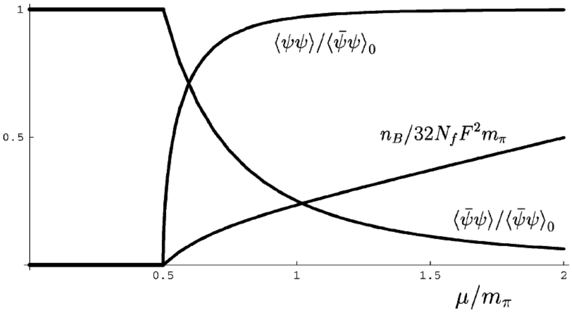
➙ At low temperatures, the phase transition between chirally broken phase at low densities and the color superconducting phase at large densities proceeds in a smooth way instead of being a strong first-order transition: Possibility of a BEC-BCS crossover with the increasing of the density!



Sun, He and Zhuang, Phys. Rev. D 75, 096004 (2007).

BEC-BCS crossover for a $N_c = 2$ model:

→ Comparison with chiral perturbation theory:

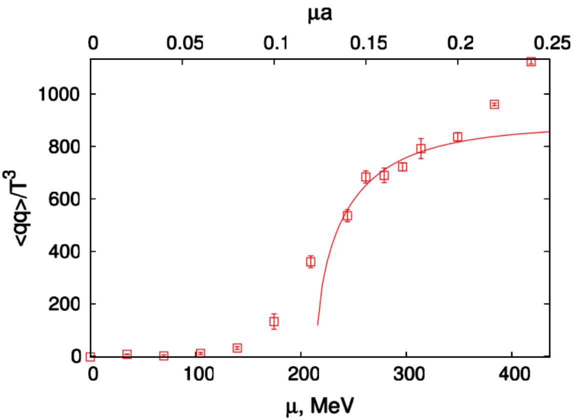


Kogut et. al., Nucl. Phys. B 582 477 (2000).

→ No sign problem!

BEC-BCS crossover for a $N_c = 2$ model:

→ Comparison with lattice simulations.



Braguta, et. al., Phys. Rev.D **94**, *114510 (2016)*

NJL model + Diquarks + Finite eB

➙ Standard Lagrangian Density:

$$\begin{aligned} \mathcal{L} &= \bar{\psi}\left(i\not{D} - m_c\right)\psi + G_S\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\tau\psi)^2\right] \\ &+ G_D\left(\bar{\psi}i\gamma_5\tau_2 t_2 C\bar{\psi}^T\right)\left(\psi^T C i\gamma_5\tau_2 t_2\psi\right) . \end{aligned}$$

$D_\mu = \partial_\mu - iQ\mathcal{A}_\mu$ (Q is the charge matrix $Q = \mathrm{diag}(q_u, q_d)$, and $\mathcal{A}_\mu = \delta_{\mu 2} x_1 B$.)

Fierz transformation on color space:† $G_S = G_D = G$

$$\Omega_0 = \frac{(m - m_c)^2 + \Delta^2}{4G} - 4\sum_{s=\pm1}\int\frac{d^3k}{(2\pi)^3}\sqrt{\left(\sqrt{k^2 + m^2} + s\mu\right)^2 + \Delta^2}\,,$$

➙ Changing in the dispersion comparing to nonrelativistic case
$\Longrightarrow \mu_N = \mu - m$ controls the BEC-BCS crossover, instead of μ only.

†C. Ratti and W. Weise, Phys. Rev. D **70**, 054013 (2004)

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Thermodynamic Potential

➙ Inclusion of finite magnetic field effects:

$$2\int \frac{d^3k}{(2\pi)^3} \to \sum_{f=u}^{d} \frac{|q_f|B}{4\pi} \sum_{l=0}^{\infty} \alpha_l \int_{-\infty}^{+\infty} \frac{dk_3}{2\pi},$$

$$E_k \to E_{k_3,l} = \sqrt{k_3^2 + 2l|q_f|B + m^2}\,,$$

$\alpha_l = 2 - \delta_{l,0} \to$ takes into account the degeneracy of Landau levels.
The thermodynamic potential at $T = 0$ becomes:

$$\Omega_0(m, \Delta, B, \mu) = \frac{(m - m_c)^2 + \Delta^2}{4G} - 2\sum_{f=u}^{d} \frac{|q_f|B}{4\pi} \sum_{s=\pm 1} \sum_{l=0}^{\infty} \alpha_l \int_{-\infty}^{+\infty} \frac{dk_3}{2\pi} \sqrt{(E_{k_3,l} + s\,\mu)^2 + \Delta^2}\,.$$

Regularization: Form factors

➙ Smooth functions multiplying the integrand:

- Wood-Saxon: $U_\Lambda^{WS\alpha}(x) = \left[1 + \exp\left(\frac{x/\Lambda - 1}{\alpha}\right)\right]^{-1}$
- Lorentziano: $U_\Lambda^{\mathrm{Lor}N}(x) = \left[1 + \left(\frac{x^2}{\Lambda^2}\right)^N\right]^{-1}$

Prescription:

$$\sum_{l=0}^{\infty} \int_{-\infty}^{+\infty} \frac{dk_3}{2\pi} \to \sum_{l=0}^{\infty} \int_{-\infty}^{+\infty} \frac{dk_3}{2\pi} U_\Lambda\left(\sqrt{k_3^2 + 2l\left|q_f\right| B}\right)$$

Non-physical oscillations in some physical quantities

Magnetic Field Independent Regularization - MFIR

→ Complete separation of magnetic field dependent contributions and divergent terms: MFIR scheme![‡]!

$$I_f = \frac{|q_f|B}{2\pi} \sum_{s=\pm 1} \sum_{l=0}^{\infty} \alpha_l \int_{-\infty}^{+\infty} \frac{dk_3}{2\pi} \sqrt{(E_{k_3,l} + s\mu)^2 + \Delta^2}$$

$$\pm \frac{|q_f|B}{\pi} \sum_{l=0}^{\infty} \alpha_l \int_{-\infty}^{+\infty} \frac{dk_3}{(2\pi)} \sqrt{E_{k_3,l}^2 + \Delta^2}$$

[‡]P. G. Allen *et al.*, Phys. Rev. D **92**, 074041 (2015).

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Magnetic Field Independent Regularization - MFIR

Final expression for the thermodynamic potential[§]:

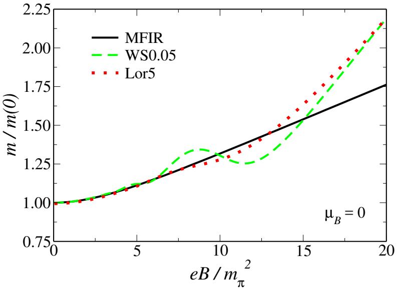
$$
\begin{aligned}
\Omega_0(m,\Delta,B,\mu) &= \Omega_0 \\
&- \frac{N_c}{4\pi^2}\sum_{f=u}^{d}\left(|q_f|B\right)\int_0^\infty dk_3\left\{\sum_{l=0}^{\infty}\alpha_l F\left(k_3^2+2l|q_f|B\right)\right. \\
&- \left. 2\int_0^\infty dy\, F\left(k_3^2+2y|q_f|B\right)\right\} \\
&- \frac{N_c}{2\pi^2}\sum_{f=u}^{d}\left(|q_f|B\right)^2\left[\zeta'\left(-1,x_f\right)-\frac{1}{2}\left(x_f^2-x_f\right)\ln\left(x_f\right)+\frac{x_f^2}{4}\right]
\end{aligned}
$$

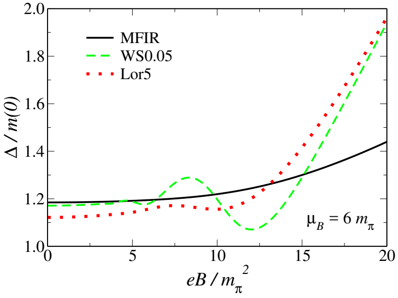
with $x_f = (m^2+\Delta^2)/(2|q_f|B)$

$$F\left(z^2\right)=\sum_{s=\pm1}\left[\sqrt{\left(\sqrt{z^2+m^2}+s\mu\right)^2+\Delta^2}-\sqrt{z^2+m^2+\Delta^2}\right]$$

[§]Duarte et. al., Phys. Rev, D 93, 025017 (2016)

Order parameters m and Δ as functions of eB





Magnetic Field Independent Regularization - MFIR

➙ Effects of temperature and/or external magnetic fields does not generate new divergencies to the theory, and can not modify the behavior of the physical quantities.

➙ Many works had been associated the oscillations that arises due to the wrong regularization to the well known “De Haas–van Alphen” effect[¶].

➙ These oscillations are present in situations where the diquark coupling is small, or when only two quarks participate in the pairing, but in this work it is not the case.

[¶] T. Holstein, *et al.* Phys. Rev. B **8**, 2649, 1973

Model Parametrization

➙ Parameters to be fixed: Λ, G and m_c, that reproduces the empirical values of m_π, f_π and $\langle\bar{\psi}\psi\rangle_0$.

But the experimental values are valid for $N_c = 3$. How to proceed in two color case?

➙ Purpose: Rescaling of the physical quantities by N_c[‖]:

$$f_\pi \propto \sqrt{N_c}\ , \quad \langle\bar{\psi}\psi\rangle_0 \propto N_c\ , \quad \text{and} \quad m_\pi \text{ does not depends on } N_c$$

$$\begin{aligned} f_\pi &\sim 92.4\text{ MeV} \rightarrow 75.45\text{ MeV} \\ \langle\bar{\psi}\psi\rangle_0^{1/3} &\sim -250\text{ MeV} \rightarrow -218\text{ MeV} \\ m_\pi &\sim 140\text{ MeV} \rightarrow 140\text{ MeV} \end{aligned}$$

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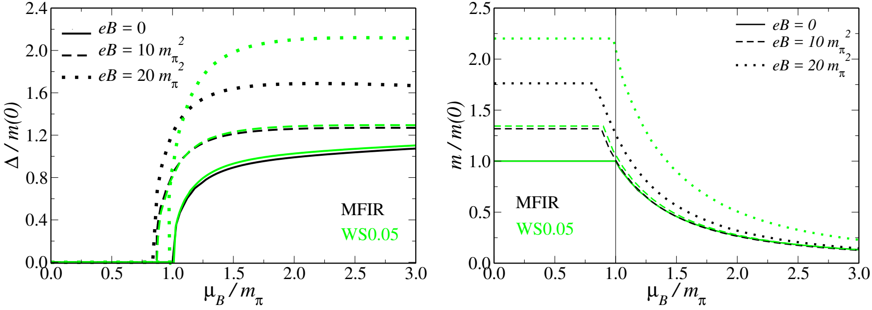
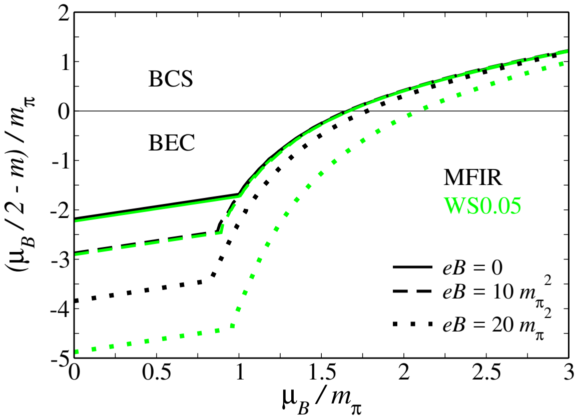
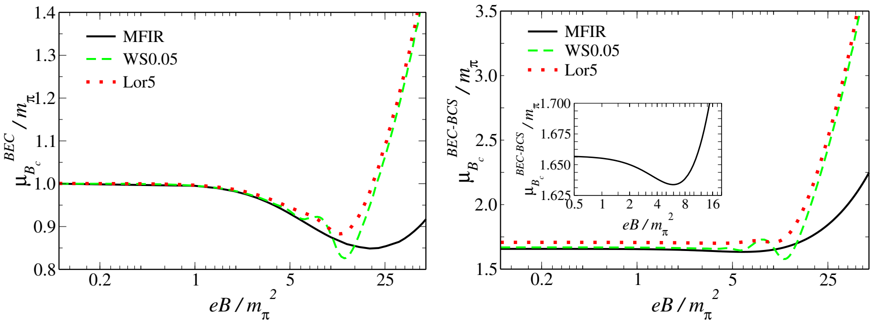
Order parameters, MFIR × Wood-Saxon



μ_N, MFIR $\times$ Wood-Saxon



Critical chemical potentials, MFIR × Form factors



Comparison with Chiral Perturbation Theory

If $eB = 0$, the BEC transition occurs in $\mu_{B_c}^{BEC} = m_\pi$**.

$$\Downarrow$$

If we can write the expression of the pion mass as a function of the magnetic field, we expect that the condition $\mu_{B_c}^{BEC}(eB) = m_\pi(eB)$ still remains valid!

→ Diquarks in BEC phase are neutral, so we express the transition point in terms of effective mass of neutral pion.

Nishida, Y., Abuki, H., Phys. Rev. D, **72 096004, (2005).

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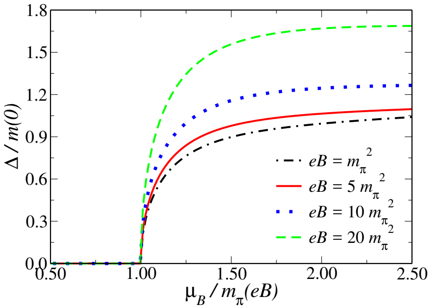
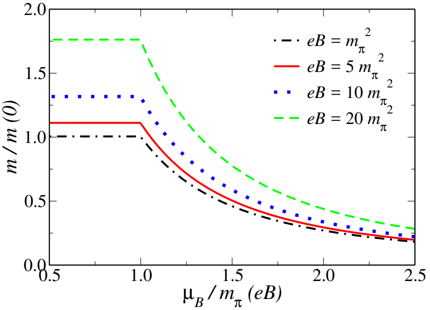
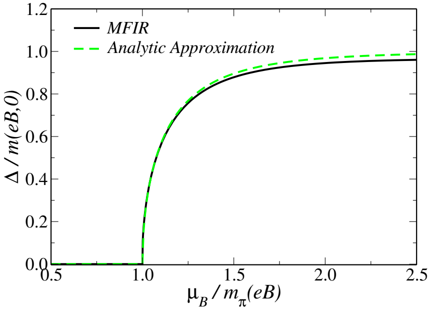
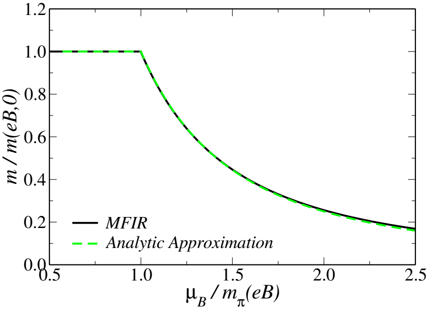




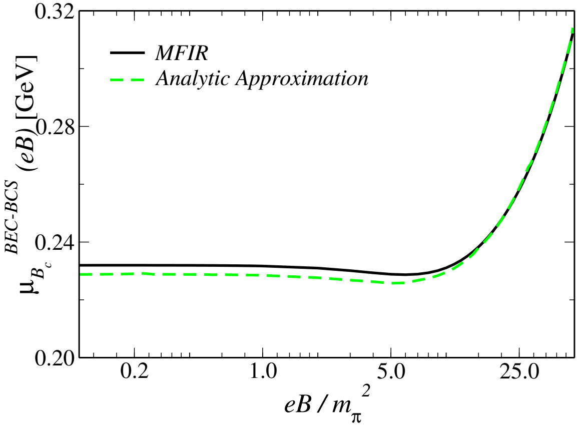
Comparison with Chiral Perturbation Theory

phase	m	Δ
$\mu_B < m_\pi(eB)$	$m(eB,0)$	0
$\mu_B \geq m_\pi(eB)$	$m(eB,0)\left[\frac{m_\pi(eB)}{\mu_B}\right]^2$	$m(eB,0)\sqrt{1-\left[\frac{m_\pi(eB)}{\mu_B}\right]^4}$





$$\mu_B = m_\pi(eB) \Longrightarrow \mu_{B,c}^{BEC-BCS} \simeq \left[2m(eB)m_\pi^2(eB)\right]^{1/3}$$



Final Remarks: $N_c = 2$ model

➙ Separation of medium contributions from divergent integral is crucial to correctly describe the behavior of the system.

➙ With MFIR it is possible:

- To reproduce the usual NJL when $eB = 0$;
- Prevent the non-physical oscillations in order parameters and critical chemical potentials.
- Show that, if the pion mass is a function of the magnetic field, the BEC phase transition will occur in $\mu_B = m_\pi(B)$.
- Obtain expressions equivalent to the well established ChPT in the presence of the external magnetic field.

BEC-BCS Crossover + OPT

→ More realistic problem, considering now three color degrees of freedom, where quarks forms barions that must be neutral in relation to color charge.

$$
\begin{aligned}
\mathcal{L} &= \bar{\psi}\left(i\partial\!\!\!/_\mu - m_c\right)\psi + G_s\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right] \\
&+ \sum_{a=2,5,7} G_d\left[\left(\bar{\psi}i\gamma^5\tau_2\lambda_a C\bar{\psi}^T\right)\left(\psi^T i\gamma^5\tau_2\lambda_a C\psi\right)\right]
\end{aligned}
$$

Fierz transformation: $G_d = 0.75Gs$, but with this value we can not observe the BEC-BCS crossover $\Longrightarrow$ free parameters!

$$
\begin{aligned}
\Omega_0 &= \frac{(m-m_0)^2}{4Gs} + \frac{\Delta}{4G_d} - 4\sum_{s=\pm1}\int\frac{d^3k}{(2\pi)^3}E_\Delta^s \\
&- 4\int\frac{d^3k}{(2\pi)^3}\left[E_k + (\mu_b - E_k)\theta(\mu_b - E_k)\right]
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$E_k = \sqrt{k^2+m^2}; E^{\pm}_\Delta = \sqrt{(E_k-\mu_r)^2+\Delta^2}$

Optimized perturbation Theory application

→ Results beyond mean field: Optimized Perturbation Theory.

$$\begin{aligned}\mathcal{L}_\delta &= \delta\mathcal{L} + (1-\delta)\mathcal{L}_0(\eta_i) \\ &= \mathcal{L}_0(\eta_i) + \delta\left[\mathcal{L} - \mathcal{L}_0(\eta_i)\right]\end{aligned}$$

- $\delta = 0$: Solvable Lagrangean $\mathcal{L}_0$
- $\delta = 1$: Original Lagrangean
- δ : *"bookkeeping"*

→ All physical quantities becomes functions of the parameters η_i. How to determine it?

Principle of minimum sensitivity - PMS (variational):

$$\left.\frac{\partial P}{\partial \eta_i}\right|_{\eta_i=\bar{\eta}_i} = 0$$

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OPT and Parametrization

Since every physical quantity in OPT is dependent on η_i, we have to be careful with the model parametrization, once m_π, f_π and $\langle\bar{\psi}\psi\rangle_0$ also depends on η_i[††].

➙ Equations to solve:

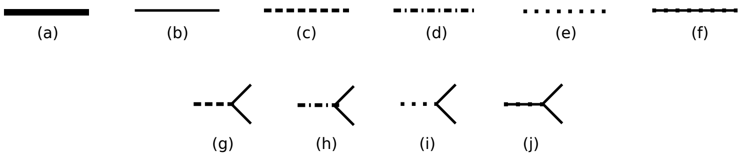
$$
\begin{aligned}
\frac{m_c}{M} &= 4GN_cN_f m_\pi^2 I_1(m_\pi^2) \\
f_\pi^2 &= 2N_cN_f M^2 I_1(0) \\
\langle\bar{\psi}\psi\rangle &= -\frac{M - m_c}{4G} \\
\bar{\eta} &= \sigma_c\left[1 + \frac{1}{2N_cN_f} + \frac{G_d}{G_s}\frac{(N_c-1)}{2N_c^2N_f}\right]
\end{aligned}
$$

with $I_1(q^2) = \int \frac{d^4k}{(2\pi)^4}\frac{1}{(p^2-M^2)[(p+q)^2-M^2]}$.

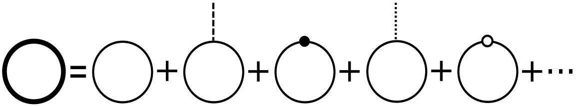
[††] J.-L. Kneur et. al., Phys.Rev. C **81** 065205 (2010).

Thermodynamic potential

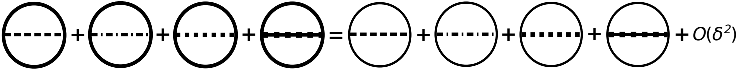
→ At $\mathcal{O}(\delta^1)$ there are contributions of 1 and 2 loops:



Propagators and vertices



One Loop

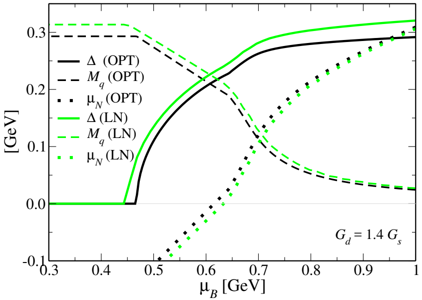


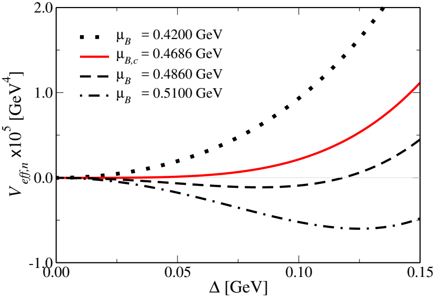
Two Loops

Numerical Results - No color neutrality

$$\frac{\partial V_{eff}}{\partial \sigma} = \frac{\partial V_{eff}}{\partial \Delta} = \frac{\partial V_{eff}}{\partial \eta} = \frac{\partial V_{eff}}{\partial \alpha} = 0$$

$$\mu_r = \mu_b = \frac{\mu_B}{3}$$



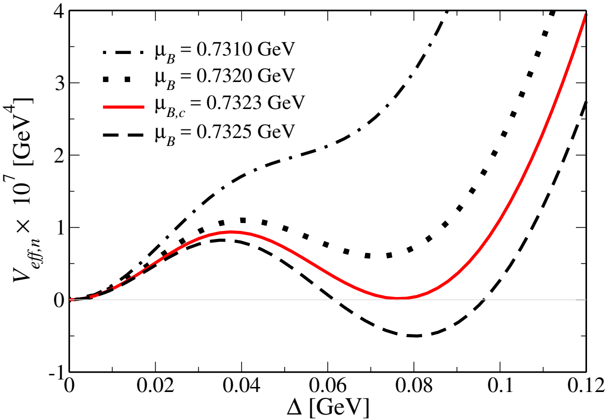


Duarte, Farias, Manso and Ramos, Phys. Rev. D 96, 056009 (2017)

$$G_d = 1.4 G s$$

Numerical Results

➜Including color neutrality.

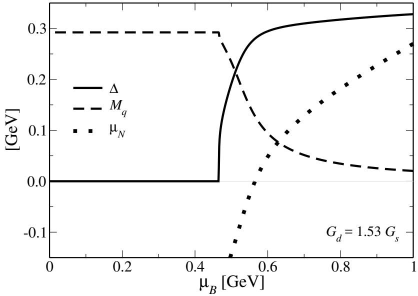


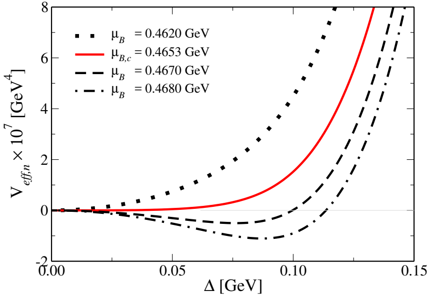
Duarte, Farias, Manso and Ramos, Phys. Rev. D 96, 056009 (2017)

$$G_d = 1.4 G s$$

$$\frac{\partial V_{eff}}{\partial \sigma} = \frac{\partial V_{eff}}{\partial \Delta} = \frac{\partial V_{eff}}{\partial \eta} = \frac{\partial V_{eff}}{\partial \alpha} = -\frac{\partial V_{eff}}{\partial \mu_8} = 0$$

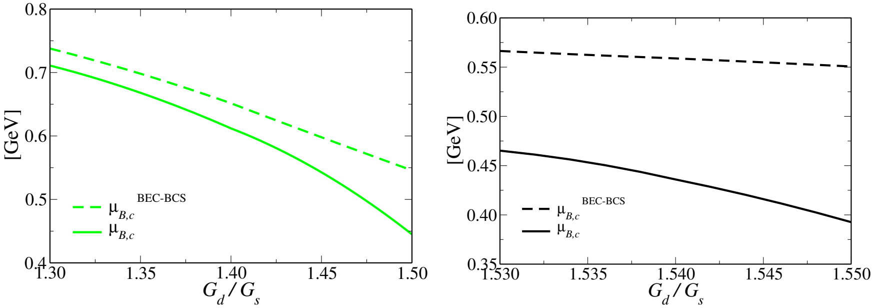
$$\mu_r = \frac{\mu_B}{3} + \frac{\mu_8}{3} \quad \text{and} \quad \mu_b = \frac{\mu_B}{3} - \frac{2\mu_8}{3}$$





Duarte, Farias, Manso and Ramos, Phys. Rev. D 96, 056009 (2017)

Numerical Results - μ_{B_c}



Duarte, Farias, Manso and Ramos, Phys. Rev. D 96, 056009 (2017)

Final Remarks - NJL + OPT

➙ We study the effects of the application of OPT, taking into account contributions beyond mean field approximation, on the BEC-BCS crossover with three color degrees of freedom.

➙ In the case without color neutrality the physical quantities in the OPT has the same behavior that ones calculated in the LN approximation.

➙ Including the color neutrality condition, it is necessary to increase the ratio G_d/G_s to observe the BEC phase, and consequently the BEC-BCS crossover.

Future Perspectives

❶ PNJL with chiral imbalance;

❷ Quark matter in $\beta-$equilibrium;

❸ Meson fluctuation effects, in the presence of an external magnetic field;

❹ Solution of Bethe-Salpeter equation in Minkowski space, including more realistic ingredients to QCD (T. Frederico talk yesterday).

Thanks for your attention!

Natural units

- $1\ \text{GeV}^2 \simeq 5.13 \times 10^{19}$ Gauss. Range $0 \leq eB \leq 3.02 \times 10^{19}$ Gauss in $N_c = 2$ problem.
- Mass: $1\ \text{GeV} \simeq 1.78 \times 10^{-24}$ g.
- Temperature: $1\ \text{GeV} \simeq 1.16 \times 10^{13}$ K.
- Density: $1\ \text{GeV}^3 \simeq 130.149\ \text{fm}^{-3}$ ($\rho_0 \sim 0.16\ \text{fm}^{-3}$).

Approximate values of the densities for LN and OPT

LN			
Ratio	$\mu_{B_c}^{\text{BEC}}$ (GeV)	$\mu_{B_c}^{\text{BEC-BCS}}$ (GeV)	$\mu_B(n_B/n_0 > 1)$ (GeV)
1.3	0.7137	0.7370	0.7380
1.4	0.6144	0.6603	0.6578
1.5	0.4474	0.5767	0.5706

OPT			
Ratio	$\mu_{B_c}^{\text{BEC}}$ (GeV)	$\mu_{B_c}^{\text{BEC-BCS}}$ (GeV)	$\mu_B(n_B/n_0 > 1)$ (GeV)
1.53	0.4653	0.5651	0.5641
1.54	0.4366	0.5573	0.5565
1.55	0.3939	0.5496	0.5496

Critical Points calculation methods (Slide 5)

- CO: Composite Operators
- RM: Random matrix
- HB: Hypotesis Bootstrap
- CJT: Cornwall-Jackiw-Tomboulis
- LR: Lattice Results
- LTE: Lattice Taylor Expansion