Quark matter within the NJL model: An alternative regularization scheme

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Many Manifestations of nonperturbative QCD

Camburi, São Paulo
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Universidade Federal de Santa Maria 1960

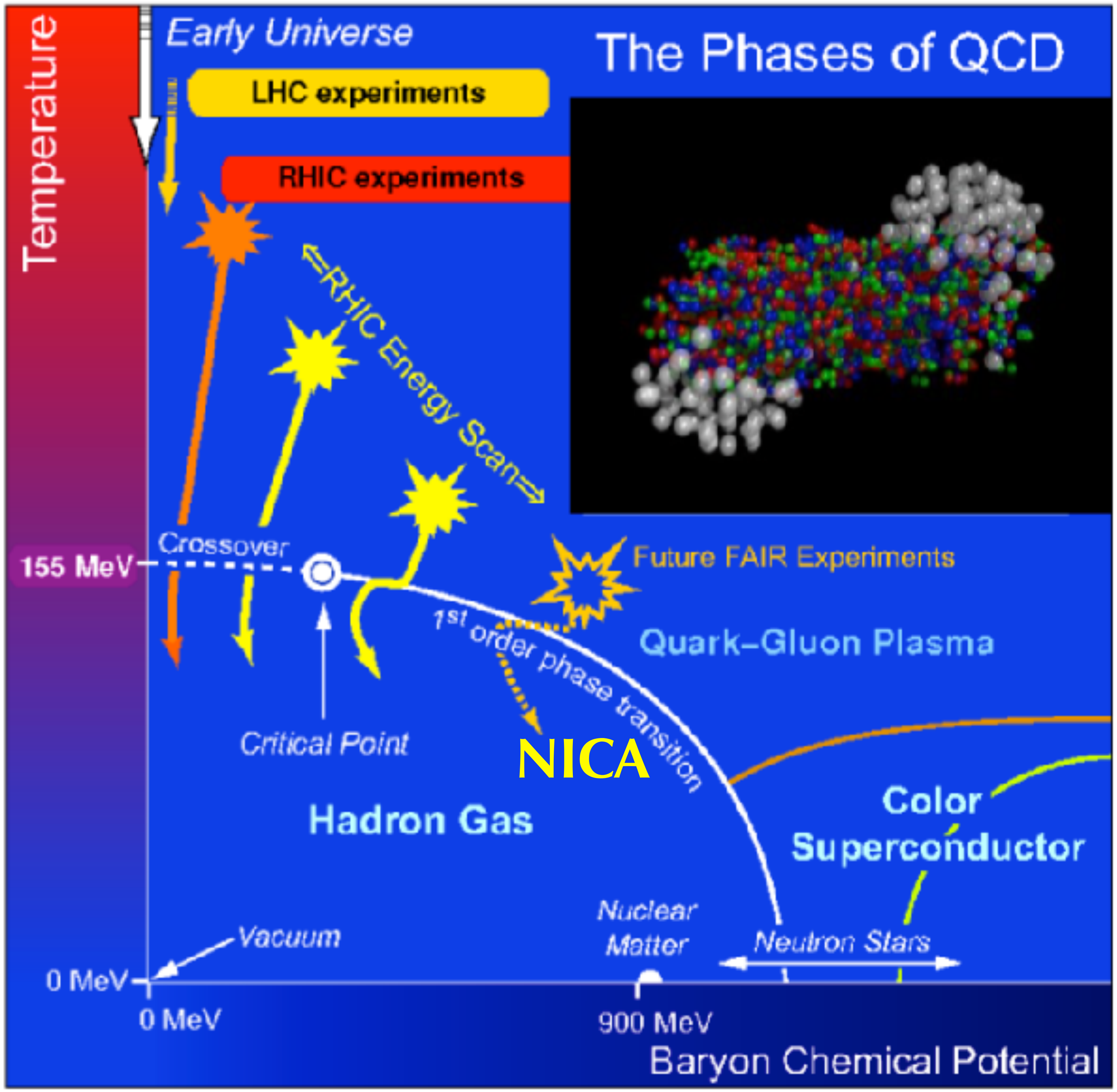
Outline

- Motivation
- QCD Phase diagram
- How a chiral imbalance of right-handed and left-handed quarks can influence the phase diagram of QCD?
- Color superconducting phases
- NJL model - Proper separation of medium effects from divergent integrals - MS Scheme!
- Results and perspectives

In Collaboration with:

- G. Krein - IFT - Unesp
- D.C. Duarte - UFSM / ITA - Brazil
- R.O. Ramos - UERJ - Brazil
- V. Dexheimer - KSU - USA

QCD Phase Diagram



From a BNL webpage

- - Early universe
- - CEP
- Heavy-ion collisions
- - Neutron stars
- Color superconductivity

NICA - Nuclotron-based Ion Collider fAcility

FAIR - Facility for Antiproton and Ion Research


NICA'S
RESTAURANTE

Lattice Results: Sign Problem

- fermion determinant is complex

$$[\det\ M(\mu)]^* = \det\ M(-\mu^*) \in \mathbb{C}$$

- no positive weight in path integral

$$Z = \int \mathcal{D}U\, e^{-S_{YM}}\ \det\ M(\mu)$$

- standard lattice methods base on importance sampling cannot be used!

Sign Problem

- Taylor Series
- Phase structure at imaginary chemical potential
- Complex Langevin dynamics
- ...
- **It is a open problem!**

To make progress

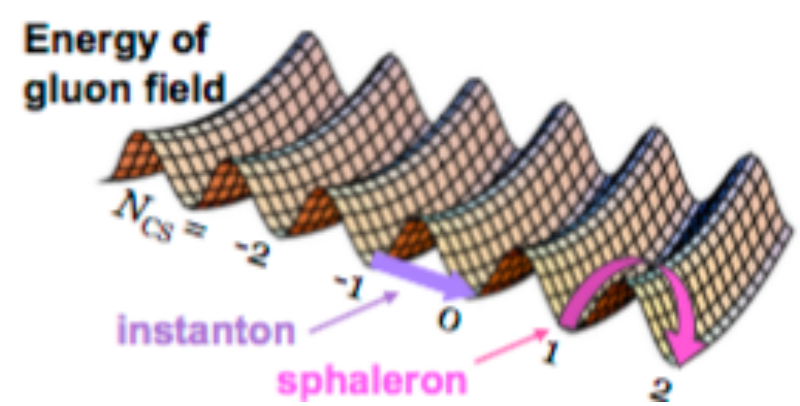
- We use Quantum Field Theory (in medium)
- DSE (beyond RL truncation…)
- Lattice (limitations…)
- Effective models (just a few degrees of freedom): NJL/PNJL, Linear σ model, MIT,…

Motivation

- The nontrivial nature of the vacuum of non-Abelian gauge theories in general, and of QCD in particular, allows for the existence of topological solutions like **instantons** and **sphalerons**.

Topology-induced change of chirality

Right ⟷ Left



- from the Adler-Bell-Jackiw anomaly[&] in the context of QCD, they can generate an asymmetry between the number of left- and right-handed quarks.

R. W. Jackiw, Int.J.Mod.Phys. A25 (2010) 659-667

G. 't Hooft, PRD 14, 3432 (1976); F. R. Klinkhamer and N. S. Manton, PRD 30, 2212 (1984).

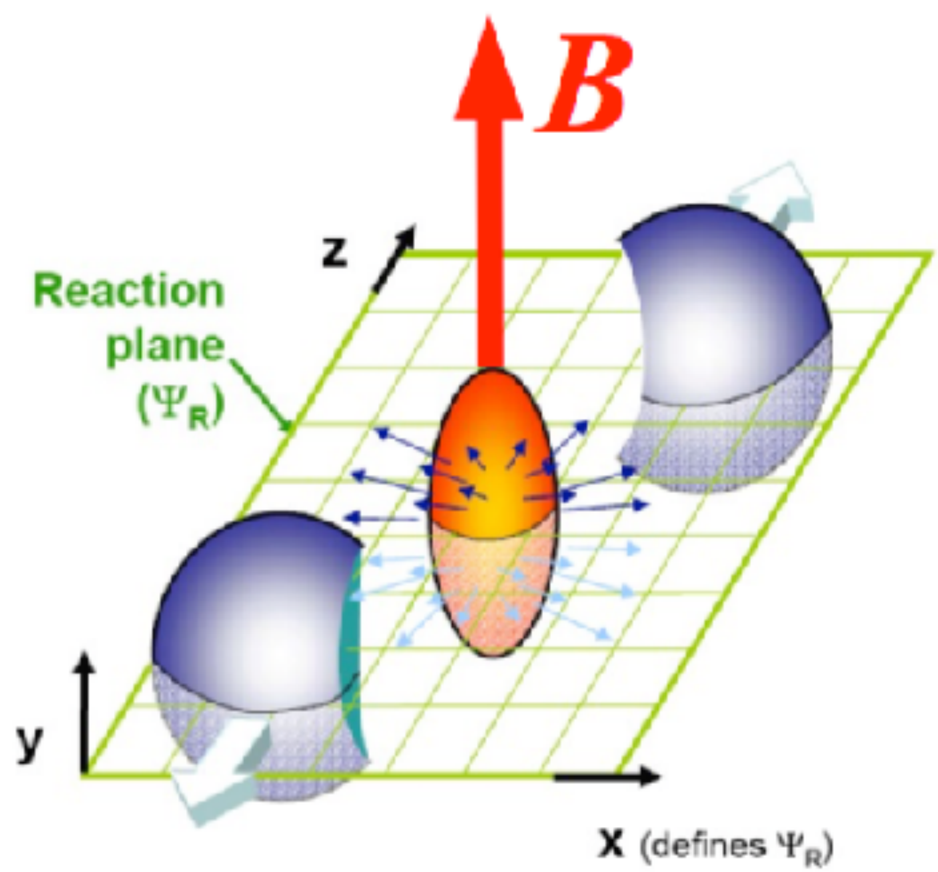
Motivation

- Such a chirality imbalance is expected to occur in event-by-event C and CP violating processes in heavy-ion collisions

Selected for a Viewpoint in *Physics*
PRL 103, 251601 (2009) PHYSICAL REVIEW LETTERS week ending 18 DECEMBER 2009

Azimuthal Charged-Particle Correlations and Possible Local Strong Parity Violation

- In off-central collisions a magnetic field is created and the presence of a chiral imbalance **gives rise to an electric current along the magnetic field**, whose effect is to produce charge separation, an effect dubbed chiral magnetic effect (CME)



Motivation

CME effect is not restricted to QCD, it extends over a wide range of systems, e.g., in

- hydrodynamics and condensed matter systems#
- has been actually observed in recent condensed matter experiments
- which makes it of much wider interest in physics.

#More references in R.L.S.Farias at all, Phys. Rev. D 94, 074011 (2016)

Motivation

NATURE PHYSICS | LETTER

Chiral magnetic effect in $ZrTe_5$

Qiang Li, Dmitri E. Kharzeev, Cheng Zhang, Yuan Huang, I. Pletikosić, A. V. Fedorov, R. D. Zhong, J. A. Schneeloch, G. D. Gu & T. Valla

Affiliations | Contributions | Corresponding authors

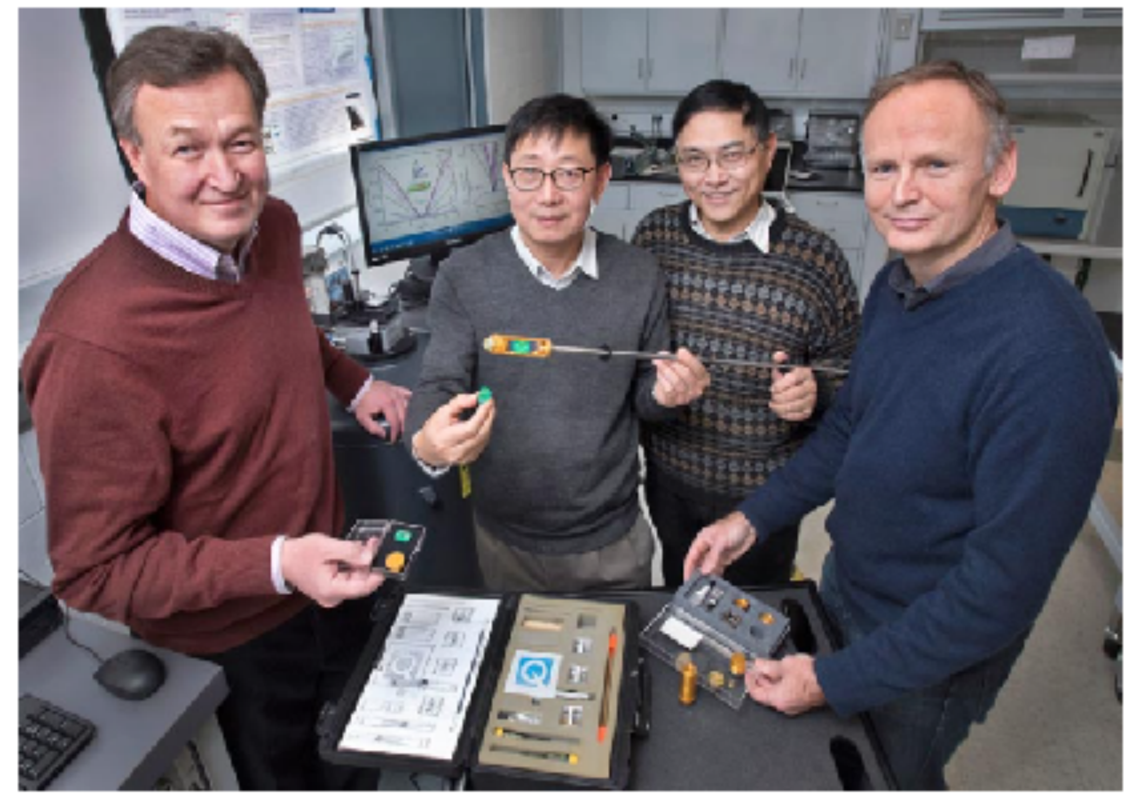
Nature Physics (2016) | doi:10.1038/nphys3648

Received 19 December 2014 Accepted 04 January 2016 | Published online 08 February 2016

Chiral Magnetic Effect Generates Quantum Current

Separating left- and right-handed particles in a semi-metallic material produces anomalously high conductivity

February 8, 2016



Nuclear theorist Dmitri Kharzeev of Stony Brook University and Brookhaven Lab with Brookhaven Lab materials scientists Qiang Li, Genda Gu, and Tonica Valla in a lab where the team measured the unusual high conductivity of zirconium pentatelluride.

- CME recently has been observed in zirconium pentatelluride

From a BNL webpage

Motivation

- The effects of a chiral imbalance in the phase diagram of QCD can be studied in the grand canonical ensemble by the introduction of a chiral chemical potential μ_5

$$\mu_5 \bar{\psi} \gamma_0 \gamma_5 \psi \longrightarrow$$ in the QCD Lagrangian density

PHYSICAL REVIEW D **78,** 074033 (2008)

Chiral magnetic effect

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[2]*Department of Physics, Brookhaven National Laboratory, Upton New York 11973, USA*
(Received 2 September 2008; published 31 October 2008)

PHYSICAL REVIEW D 81, 114031 (2010)

Chiral magnetic effect in the Polyakov–Nambu–Jona-Lasinio model

Kenji Fukushima* and Marco Ruggieri†
Yukawa Institute for Theoretical Physics, Kyoto University, Kyoto 606-8502, Japan

Raoul Gatto‡
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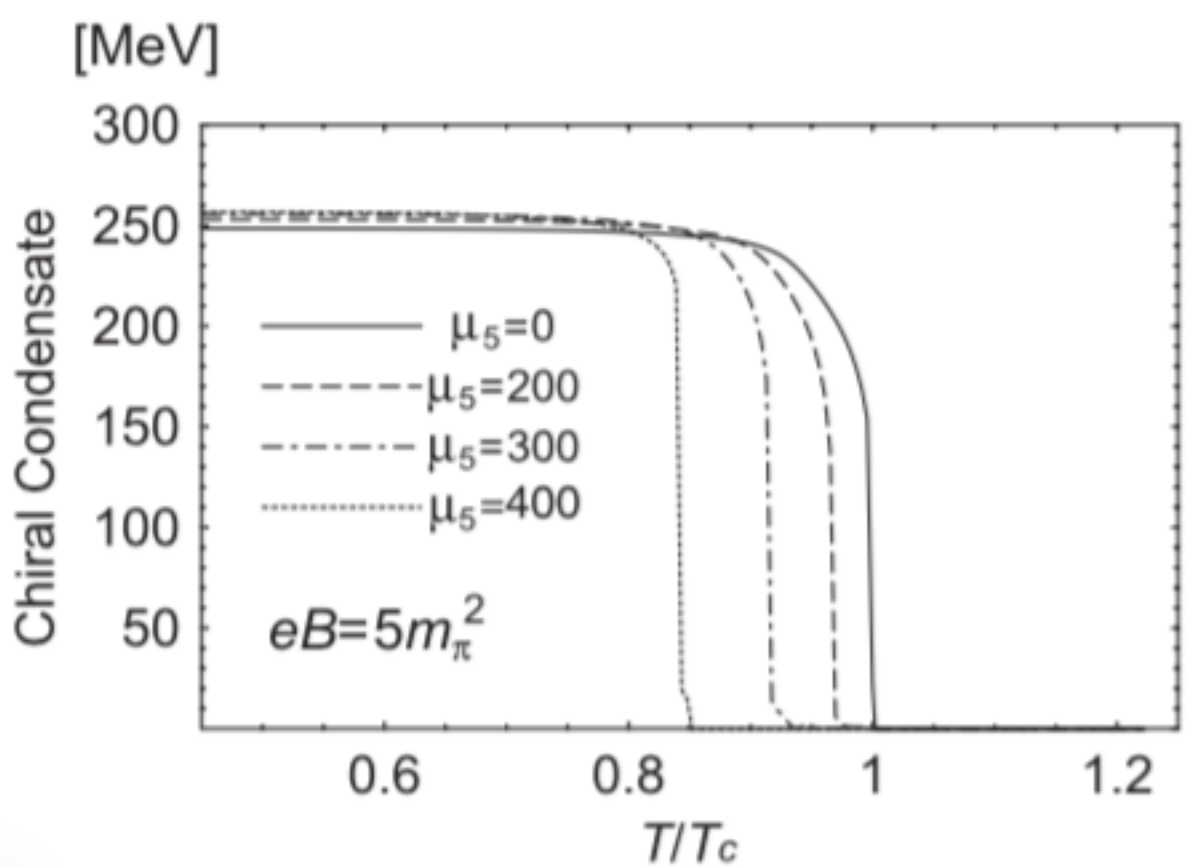
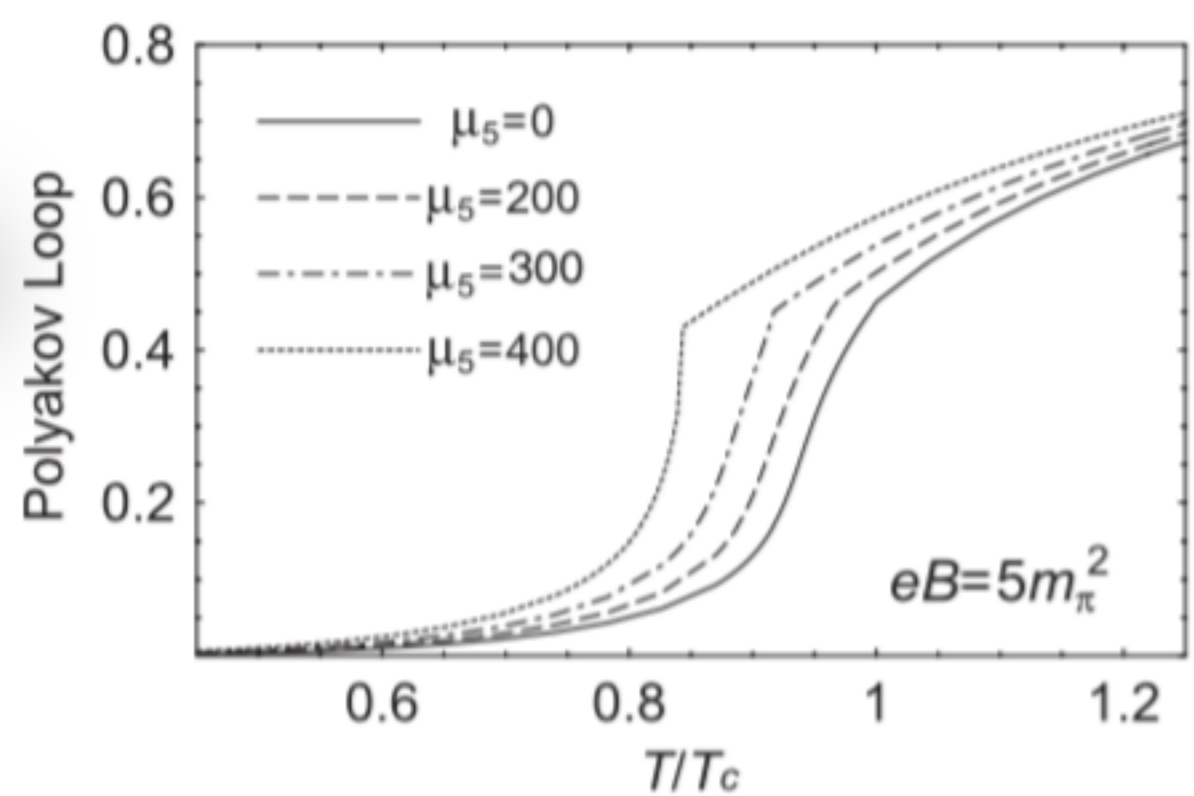
(Received 4 March 2010; published 21 June 2010)

We study the two-flavor Nambu–Jona-Lasinio model with the Polyakov loop in the presence of a strong magnetic field and a chiral chemical potential μ_5, which mimics the effect of imbalanced chirality due to QCD instanton and/or sphaleron transitions. First, we focus on the properties of chiral symmetry breaking and deconfinement crossover under the strong magnetic field. Then we discuss the role of μ_5 on the phase structure. Finally, the chirality charge, electric current, and their susceptibility, which are relevant to the chiral magnetic effect, are computed in the model.

DOI: 10.1103/PhysRevD.81.114031 PACS numbers: 12.38.Aw, 12.38.Mh





PHYSICAL REVIEW D **84**, 014011 (2011)

Critical end point of quantum chromodynamics detected by chirally imbalanced quark matter

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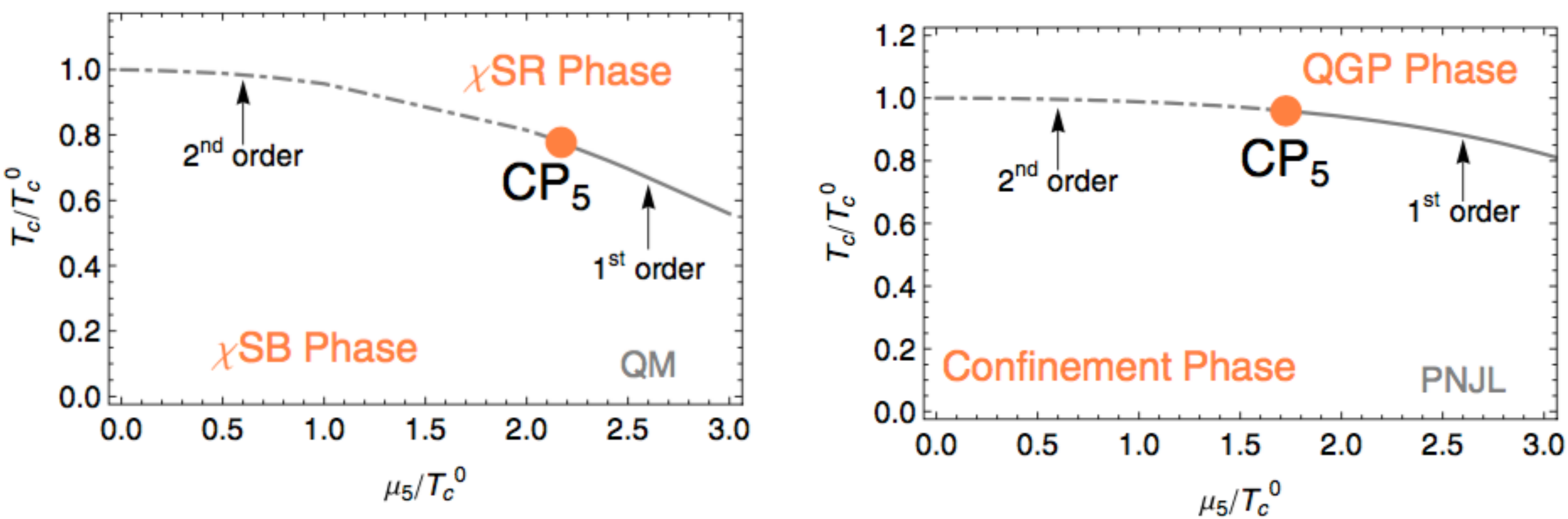
(Received 5 April 2011; published 7 July 2011)

We suggest the idea, supported by concrete calculations within chiral models, that the critical end point of the phase diagram of quantum chromodynamics with three colors can be detected, by means of lattice simulations of grand-canonical ensembles with a chiral chemical potential, μ_5, conjugated to chiral charge density. In fact, we show that a continuation of the critical end point of the phase diagram of quantum chromodynamics at finite chemical potential, μ, to a critical end point in the temperature-chiral chemical potential plane, is possible. This study paves the way for the mapping of the phases of quantum chromodynamics at finite μ, by means of the phases of a fictitious theory in which μ is replaced by μ_5.

DOI: 10.1103/PhysRevD.84.014011 PACS numbers: 12.38.Aw, 12.38.Mh



PHYSICAL REVIEW D 81, 114031 (2010)

Chiral magnetic effect in the Polyakov–Nambu–Jona-Lasinio model

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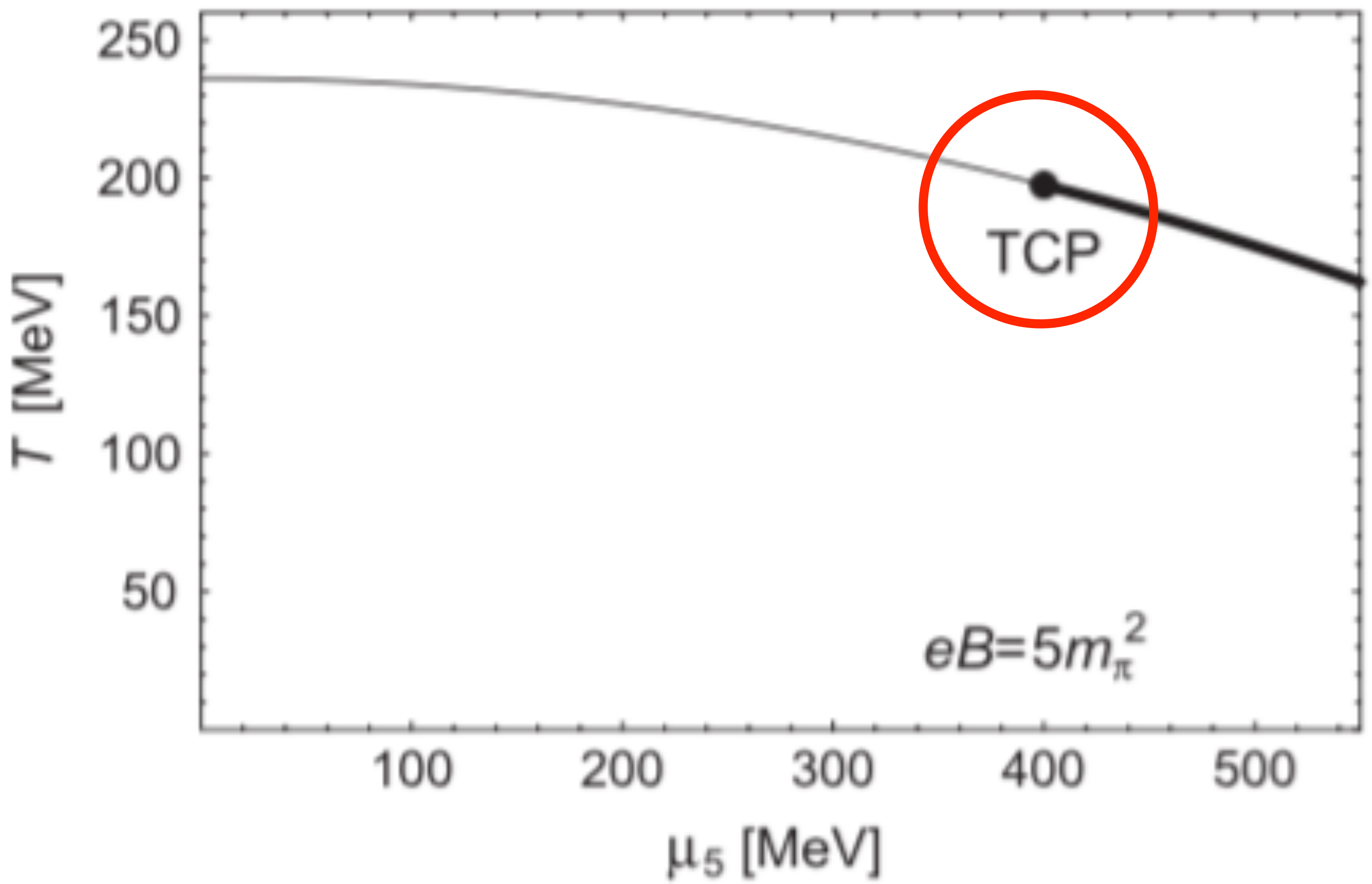
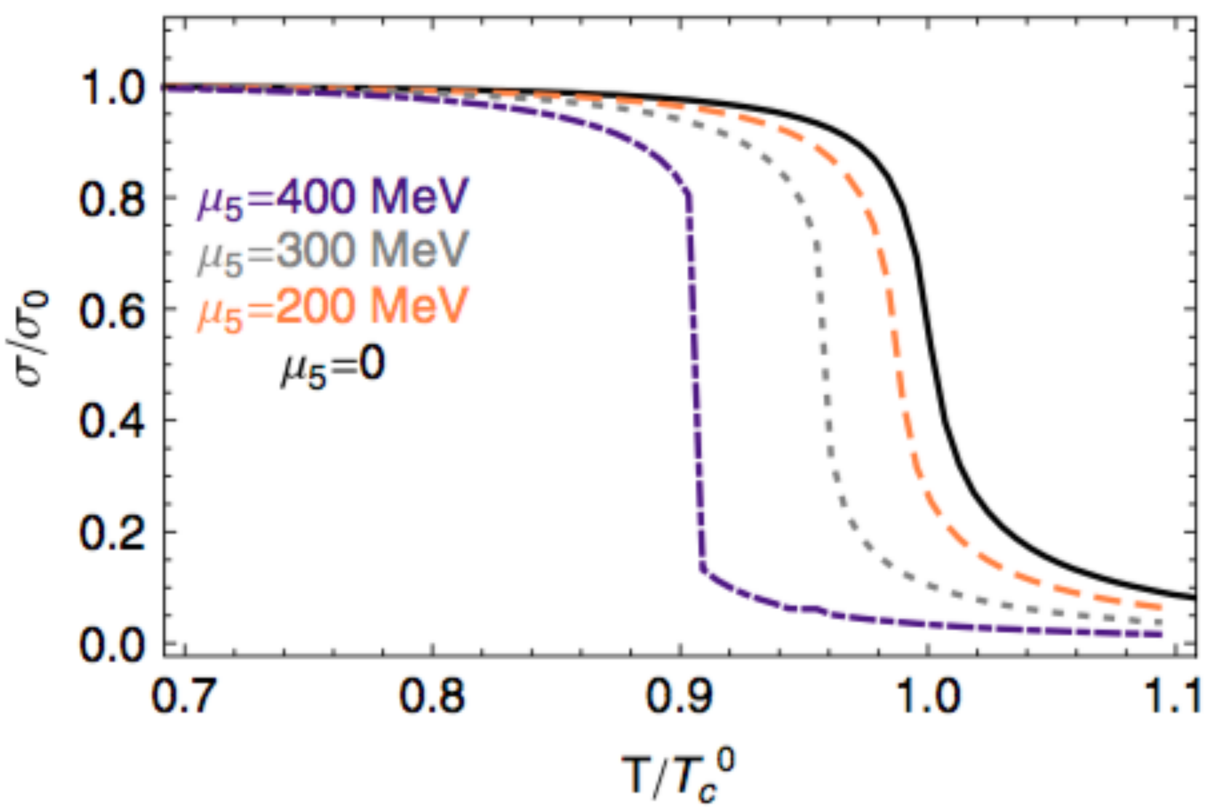
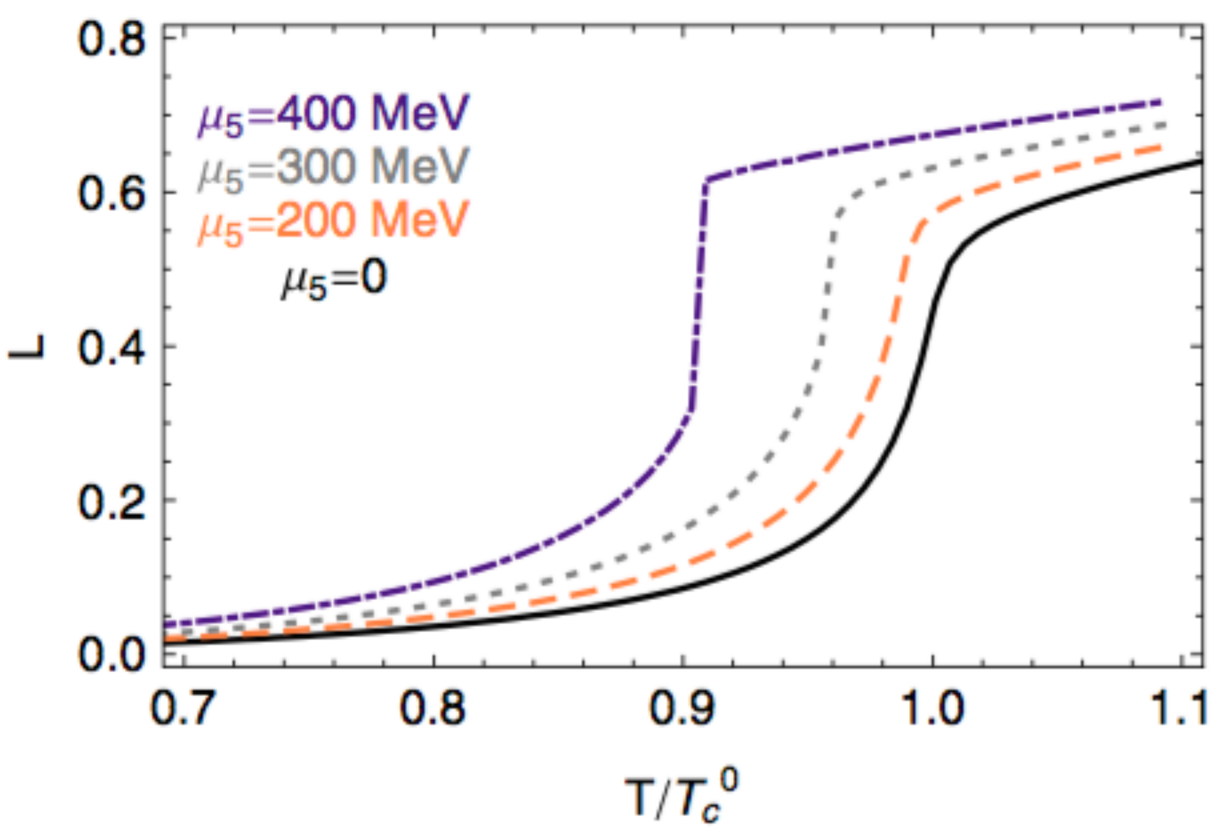


- Critical temperature decrease with μ_5
- Appear a Tricritical point TCP





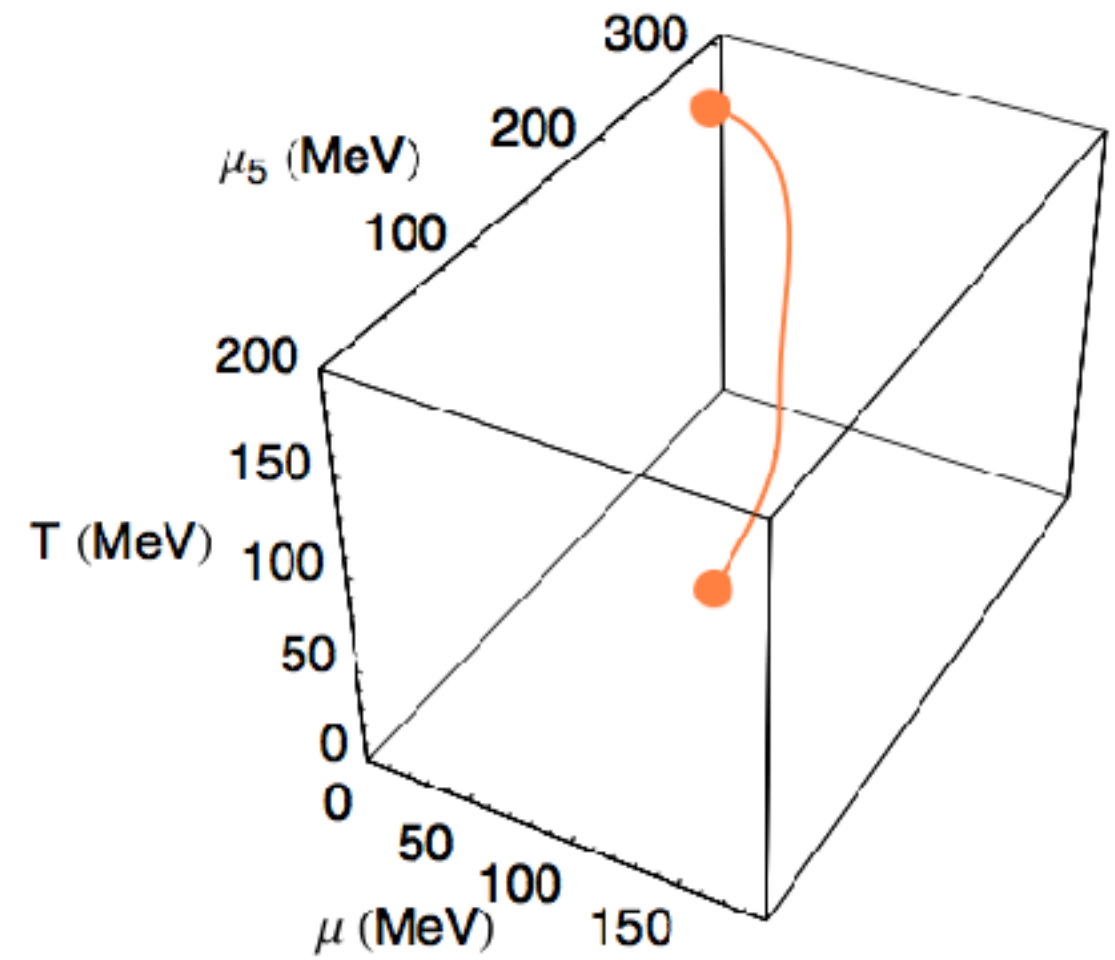


FIG. 3 (color online). Evolution of the critical end point in the $\mu - \mu_5 - T$ space, for the PNJL model.

- Tc decrease with μ_5
- mapping of CP with CP_5

PHYSICAL REVIEW D **91**, 034017 (2015)

Effect of the chiral chemical potential on the position of the critical endpoint

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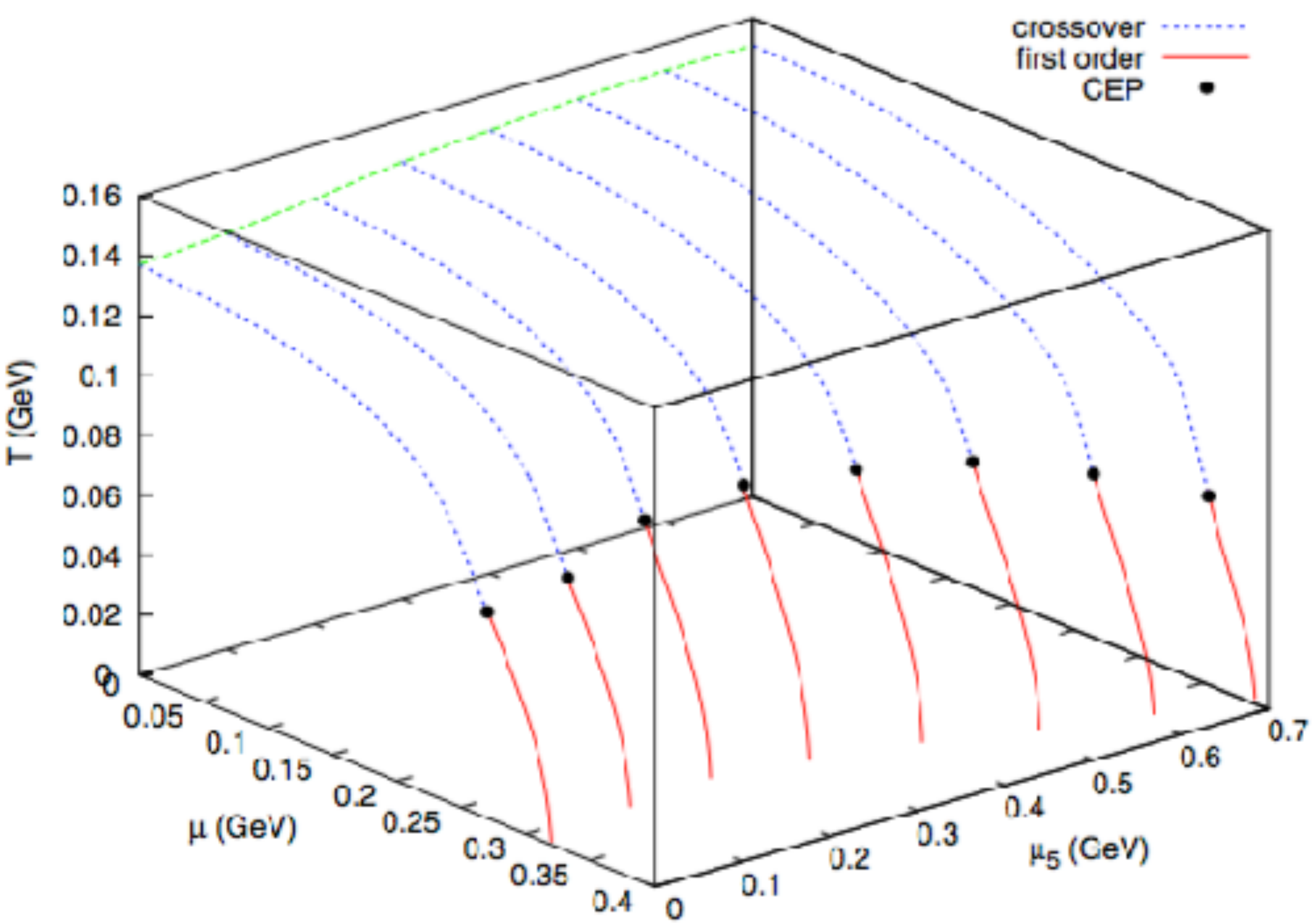
(Received 20 November 2013; published 19 February 2015)

Rank-2 confining separable model
Gluon propagator



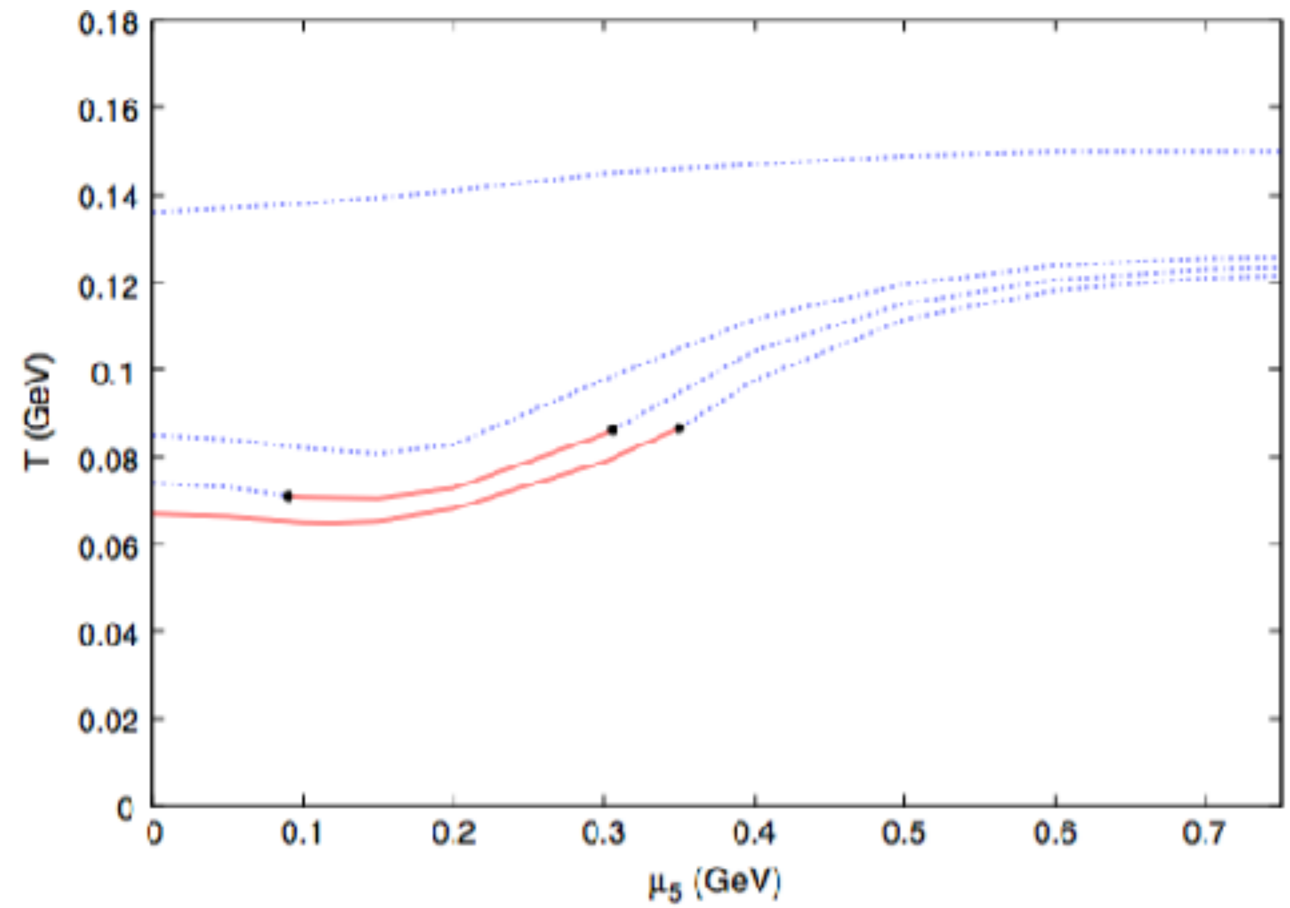


FIG. 6 (color online). The phase diagram in the $T - \mu_5$ plane when μ is at 0, 253.5, 265, and 273 MeV (from top to bottom). The solid line represents the first-order phase transition, while the dashed line represents the crossover.

PHYSICAL REVIEW D 91, 056003 (2015)

Chiral phase transition with a chiral chemical potential in the framework of Dyson-Schwinger equations

Shu-Sheng Xu,[1,7] Zhu-Fang Cui,[2,7] Bin Wang,[3] Yuan-Mei Shi,[4] You-Chang Yang,[2,5] and Hong-Shi Zong[2,6,7,*]
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(Received 9 January 2015; published 4 March 2015)

gluon propagator

$$g^2 D_{\mu\nu}(k_{nl}) = P^T_{\mu\nu} D_T(\vec{k}^2, \omega^2_{nl}) + P^L_{\mu\nu} D_L(\vec{k}^2, \omega^2_{nl}),$$

$$D_T = D_L = D_0 \frac{4\pi^2}{\sigma^6} k^2_{nl} e^{-k^2_{nl}/\sigma^2},$$

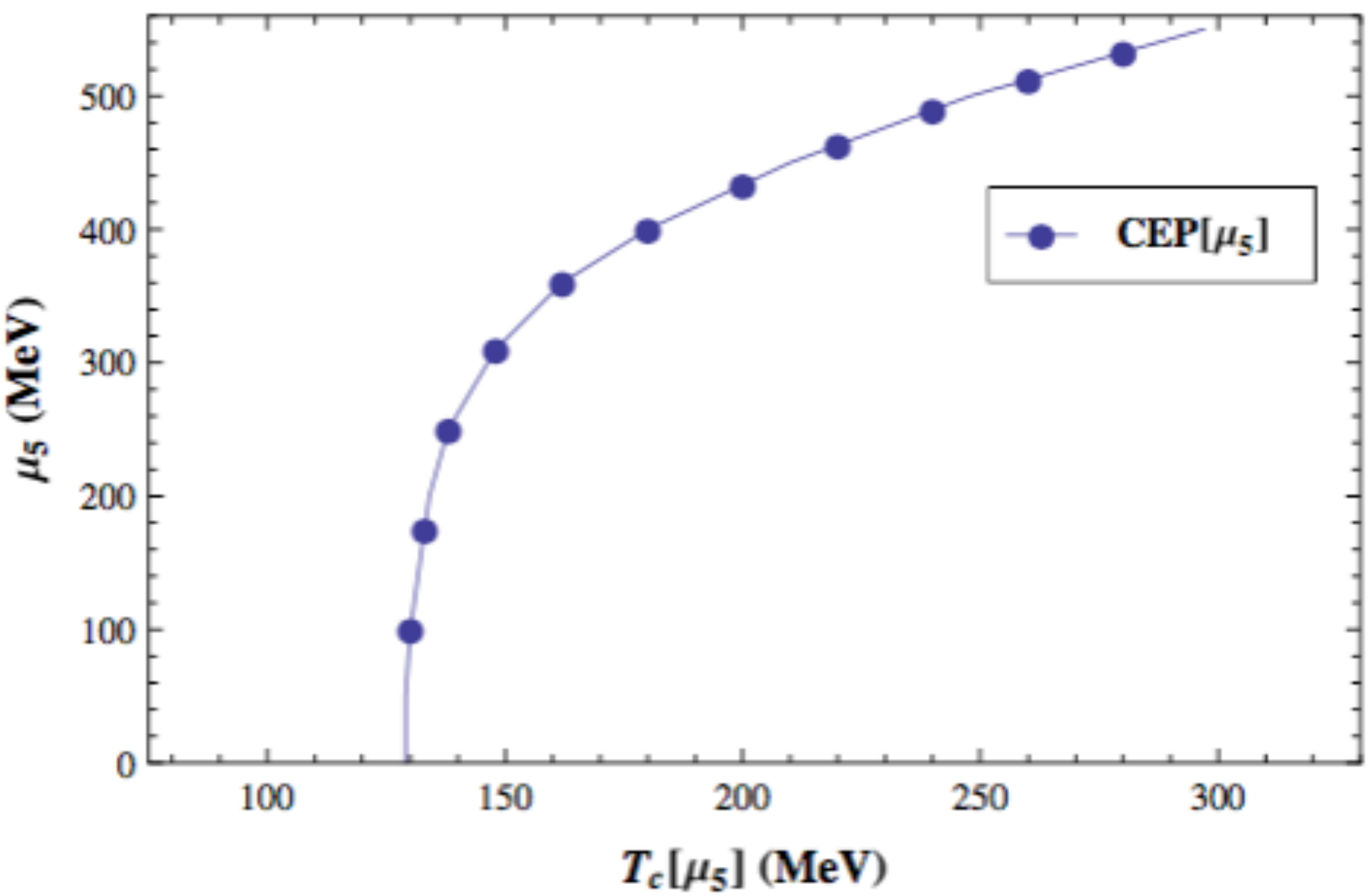


FIG. 3 (color online). The relation between μ_5 and the corresponding $T_c[\mu_5]$ in the $T - \mu$ plane.

Lattice Results

- Contrary to the case of QCD in the presence of a baryon chemical potential, which has a sign problem.

- QCD in the presence of a chiral chemical potential is **free from the sign problem** and, therefore, amenable to Monte Carlo sampling in lattice simulations

- There is hope that lattice simulations of QCD with μ_5 can be used as a benchmark platform for comparing different effective models used in the literature.

-

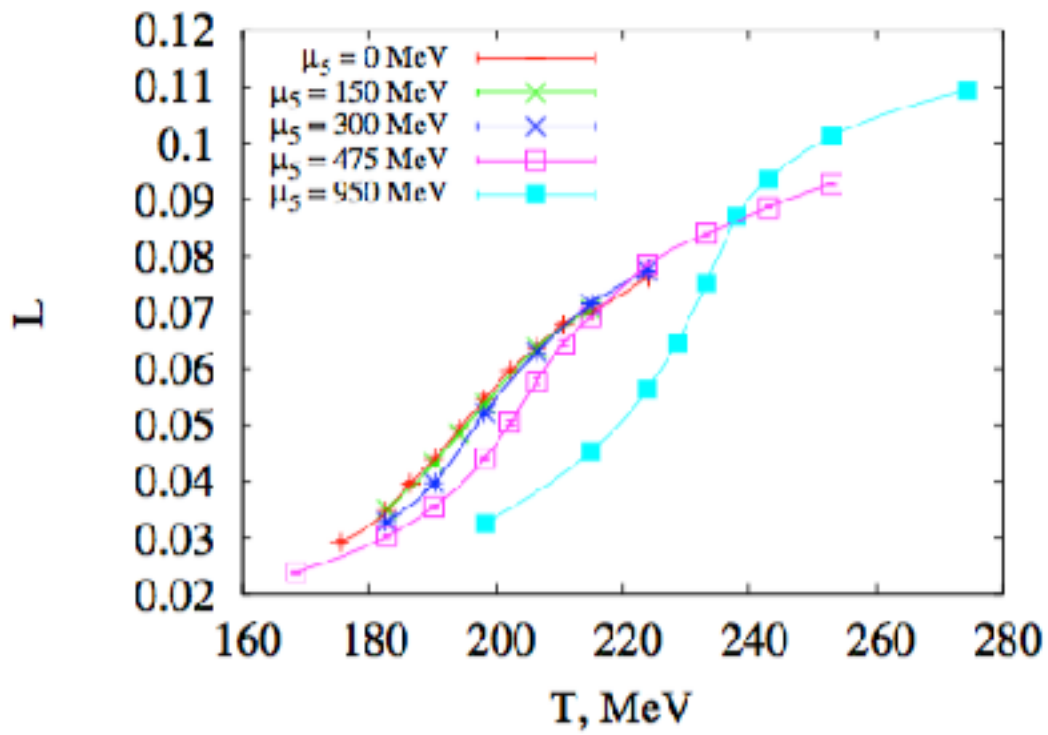
Lattice Results: Nc=2 and Nf=4

Two-color QCD with non-zero chiral chemical potential

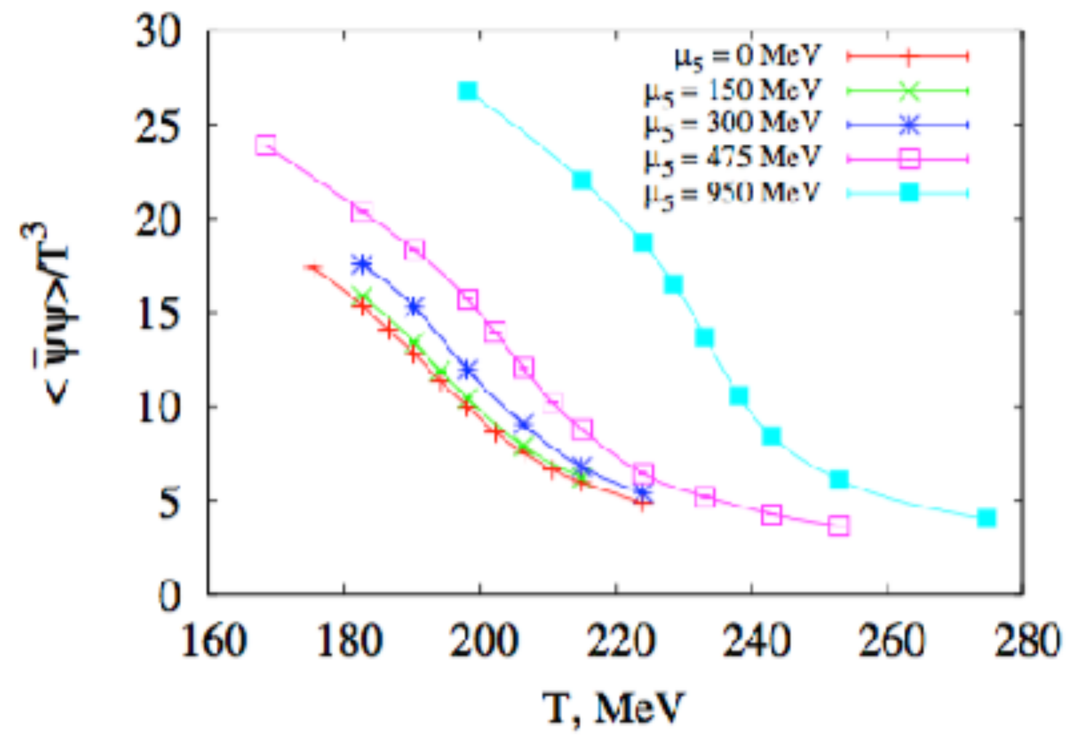
RECEIVED: *April 5, 2015*
ACCEPTED: *May 26, 2015*
PUBLISHED: *June 16, 2015*

JHEP 1506, 094 (2015)

V.V. Braguta,[a,b] V.A. Goy,[c] E.M. Ilgenfritz,[d] A.Yu. Kotov,[e,1] A.V. Molochkov,[b] M. Müller-Preussker[f] and B. Petersson[f]



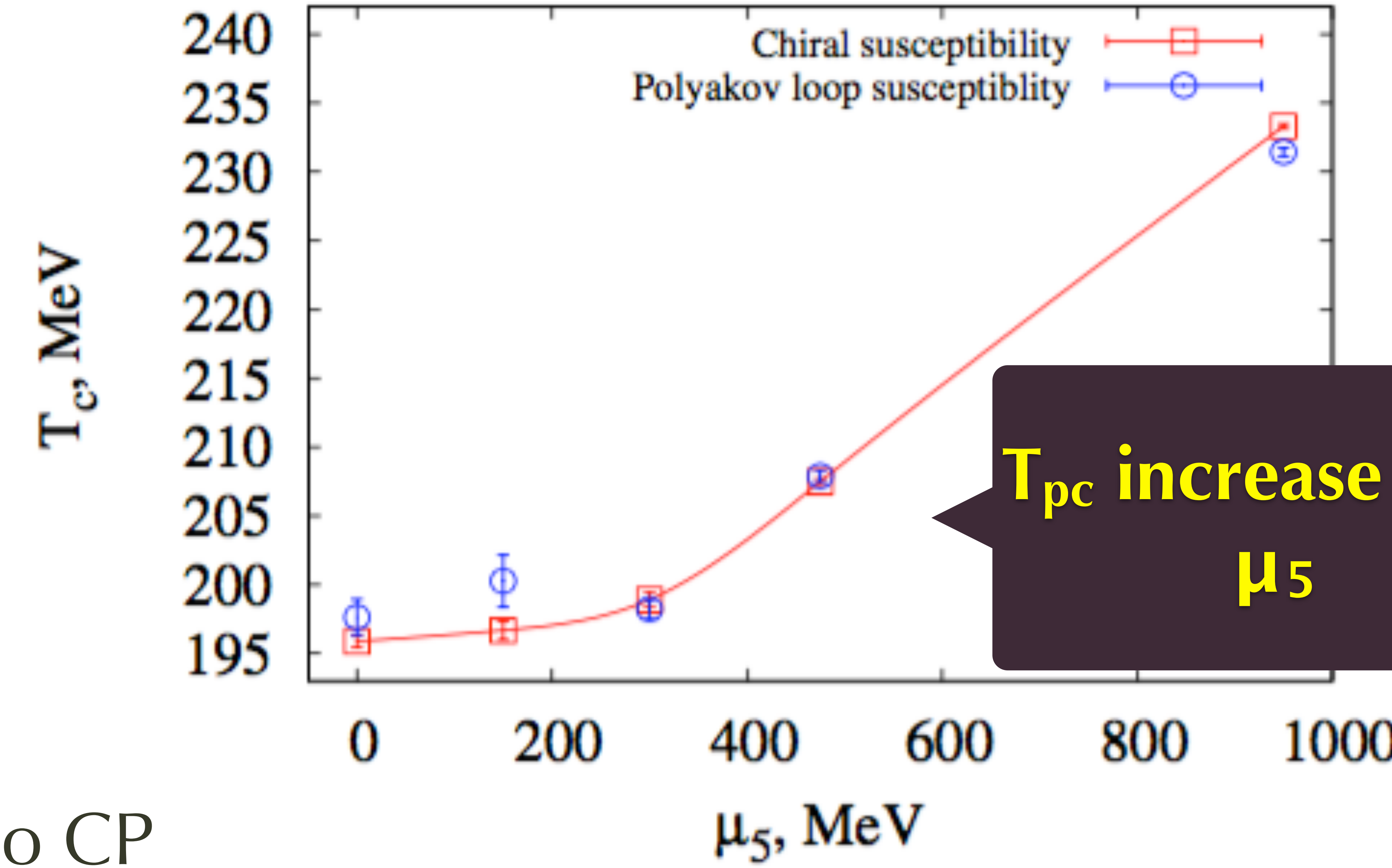
Polyakov loop



Chiral condensate

Figure 1. Polyakov loop and chiral condensate versus T for five values of μ_5. Lattice size is 6×20^3, fermion mass is $m \approx 12$ MeV. Errors are smaller than the data point symbols. The curves are to guide the eyes.

Lattice Results: Nc=2 and Nf=4



T_{pc} increase with μ_5

- No CP
- Always crossover transition!

JHEP 1506, 094 (2015)

Lattice Results: Nc=3 and Nf=2

PHYSICAL REVIEW D **93,** 034509 (2016)

Study of the QCD phase diagram with a nonzero chiral chemical potential

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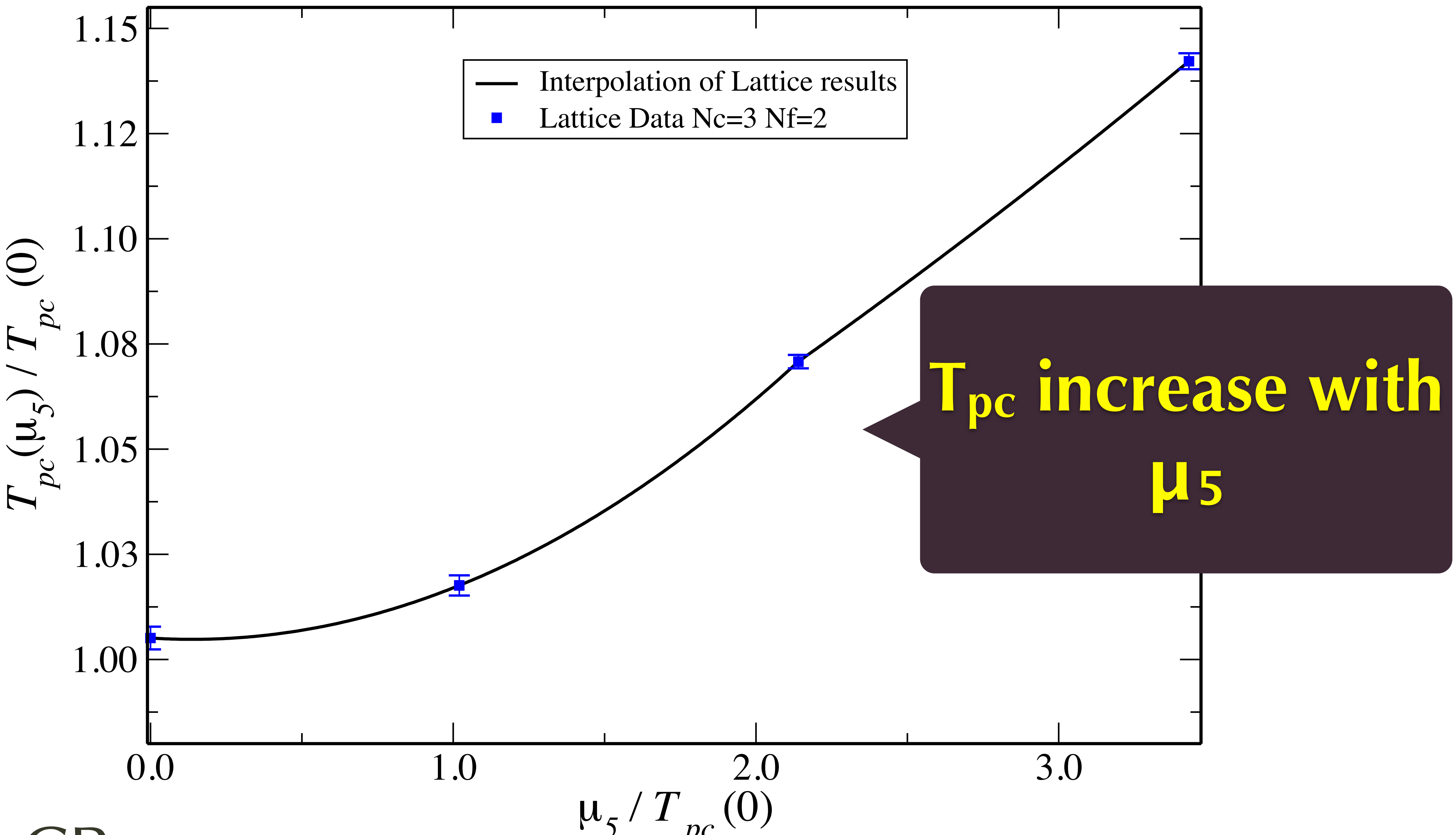
(Received 15 January 2016; published 25 February 2016)

Lattice Results: Nc=3 and Nf=2



PRD 93, 034509 (2016)

- No CP
- Always crossover transition!

Thanks Prof. Andrey Kotov for lattice unpublished data and comments!

Quark Meson Model

Critical Temperature of Chiral Symmetry Restoration for Quark Matter with a Chiral Chemical Potential

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[2] *Theoretical Physics Center for Science Facilities, Institute of High Energy Physics, Beijing 100049, China*

- Expansion for small chiral chemical potential
- nonstandard renormalisation

	RS1	RS2	RS3
$\langle\sigma\rangle$	F_π	F_π	$F_\pi\left(1+a\mu_5^2/F_\pi^2\right)$
m_σ^2	M_σ^2	$M_\sigma^2+b\mu_5^2$	$M_\sigma^2+c\mu_5^2$

e-Print: **arXiv:1602.03651 [hep-ph]**

Quark Meson Model

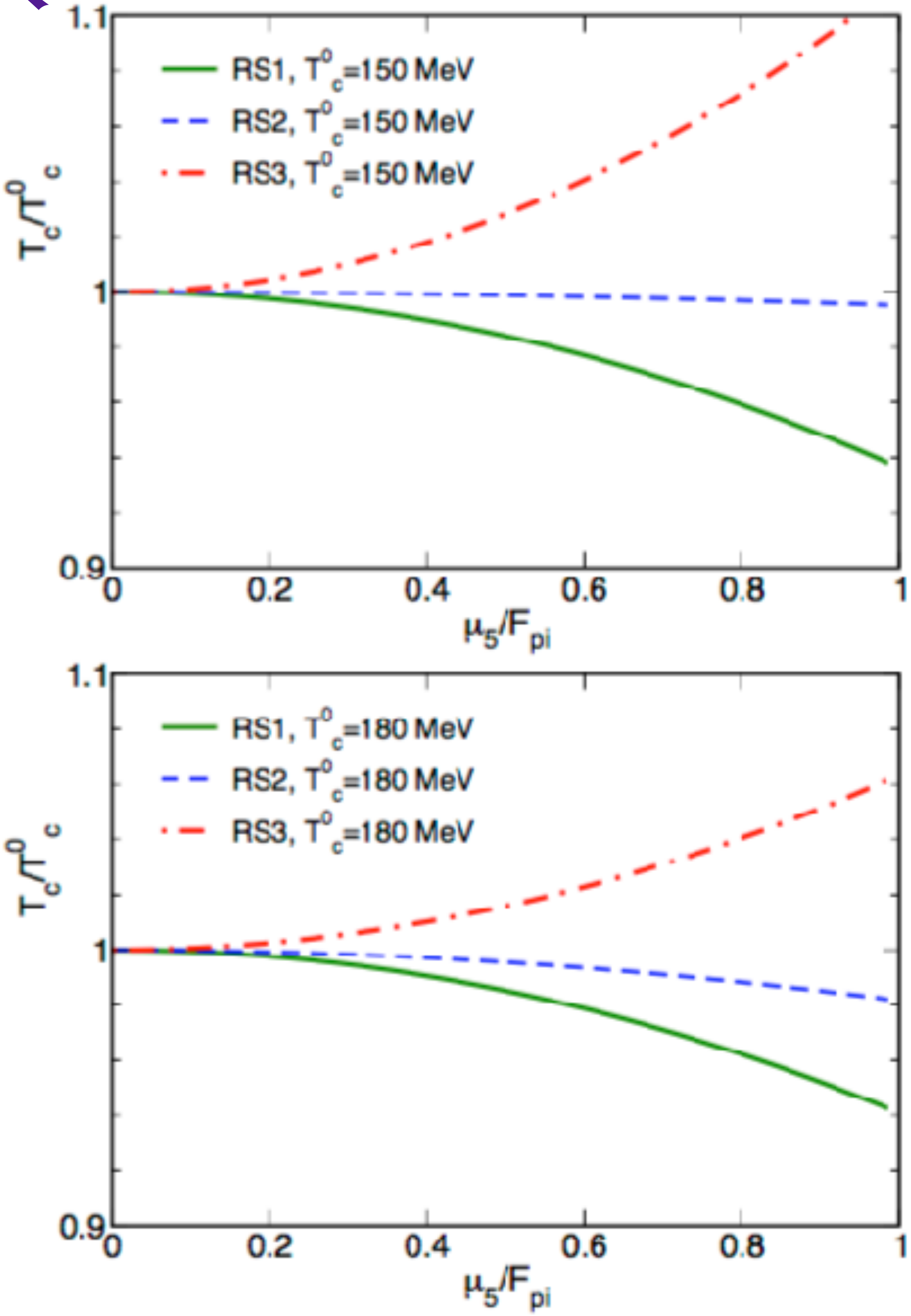


Figure 1: Critical temperature as a function of μ_5 for the three renormalization schemes discussed in this article: upper panel corresponds to $T_c^0 = 150$ MeV and lower panel to $T_c^0 = 180$ MeV.

e-Print: **arXiv:1602.03651 [hep-ph]**

Non Local NJL model

Chiral Symmetry Restoration with a Chiral Chemical Potential: the Role of Momentum Dependent Quark Self-energy

M. Ruggieri[1,*] and G. X. Peng[1,2,†]

[1] College of Physics, University of Chinese Academy of Sciences, Yuquanlu 19A, Beijing 100049, China
[2] Theoretical Physics Center for Science Facilities, Institute of High Energy Physics, Beijing 100049, China

$$S = -\beta V \frac{\sigma^2}{G} + \int \frac{d^4p}{(2\pi)^4} \bar{\psi}(p)\left[\gamma^\mu p_\mu - M(p) + \mu_5\gamma_0\gamma_5\right]\psi(p),$$

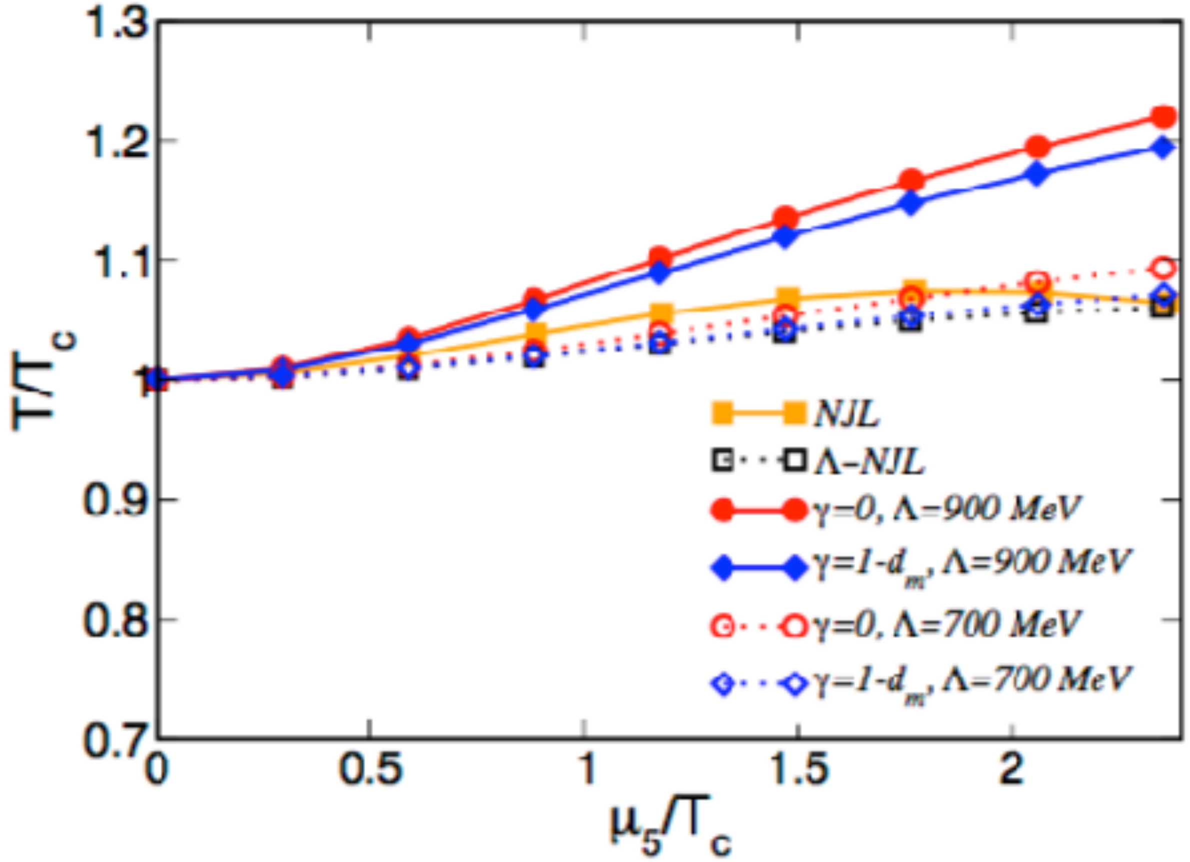
quark mass function

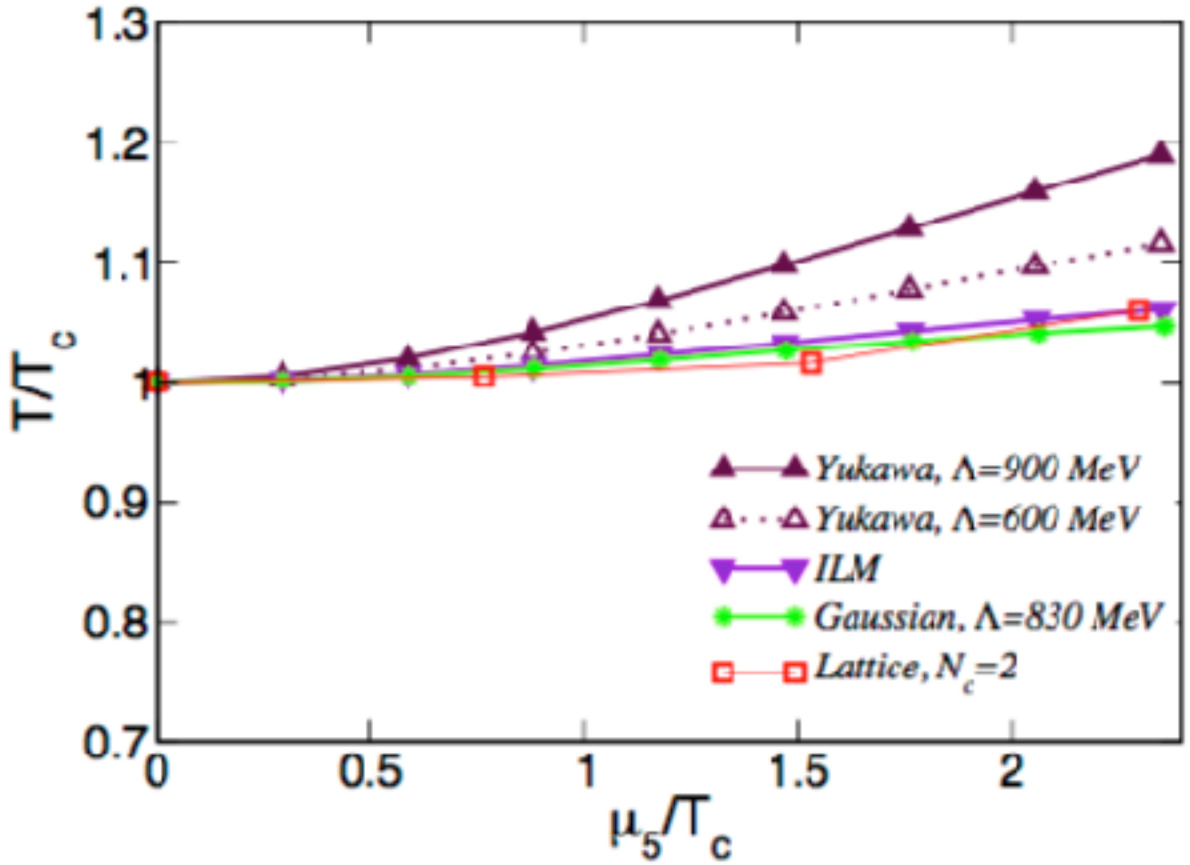
$$M(p) \equiv -2\sigma\mathcal{C}(p),$$

$$\mathcal{C}(p_E^2) = \theta(\Lambda^2 - p_E^2) + \theta(p_E^2 - \Lambda^2)\frac{\Lambda^2(\log\Lambda^2/\Lambda_{QCD}^2)^\gamma}{p_E^2(\log p_E^2/\Lambda_{QCD}^2)^\gamma}$$

e-Print: **arXiv:1602.05250 [hep-ph]**

Non Local NJL model





They conclude that the local NJL model **is not capable to capture the correct behaviour** $Tc(\mu_5)$, because of the lack of the asymptotic behavior of the constituent quark mass!

e-Print: **arXiv:1602.05250 [hep-ph]**

What's wrong with local NJL?

- **Our work identifies the source of the failure of such models and offers a way to cure the problem.**

SU(2) Nambu—Jona-Lasinio model (NJL)

$$\mathcal{L} = \bar{\psi}(i\partial\!\!\!/ - m_c + \mu_5\gamma^0\gamma^5)\psi + G\left[(\bar{\psi}\psi)^2 + (\bar{\psi}i\gamma_5\vec{\tau}\psi)^2\right]$$

good **chiral** physics, pions,...
BUT no confinement and no AF

G, Λ and m_c → m_π, f_π and $\langle\bar{\psi}\psi\rangle$

$$\Omega(M,T,\mu_5) = \Omega_0(M,\mu_5) - 2N_f N_c T \sum_{s=\pm1}\int \frac{d^3k}{(2\pi)^3} \ln\left[1 + e^{-\omega_s(k)/T}\right]$$

$$M = m_c - 2G\langle\bar{\psi}\psi\rangle$$

Y. Nambu and G. Jona-Lasinio, Phys. Rev. **122**, 345 (1961)

SU(2) Nambu—Jona-Lasinio model (NJL)

$$\Omega_0(M,\mu_5) = \frac{(M-m_c)^2}{4G} - N_f N_c \sum_{s=\pm 1} \int \frac{d^3k}{(2\pi)^3} \omega_s(k)$$

UV divergent!
Needs regularization!

$$\omega_s(k) = \sqrt{(|\mathbf{k}| + s\mu_5)^2 + M^2}$$

are the eigenstates of the Dirac operator with helicity $s = \pm 1$

Gap equation → $\frac{\partial \Omega}{\partial M} = 0$

Medium Separation Scheme

$$\frac{\partial}{\partial M^2}\left[\int \frac{d^3k}{(2\pi)^3}\omega_s(k)\right] = \int_{-\infty}^{+\infty}\frac{dk_4}{2\pi}\int \frac{d^3k}{(2\pi)^3}\frac{1}{k_4^2+\omega_s^2(k)}$$

$$\pm\frac{1}{k_4^2+k^2+M_0^2}$$

$$\omega_s(k) = \sqrt{(|\mathbf{k}|+s\mu_5)^2+M^2}$$

M_0 is the vacuum quark mass

$$\frac{1}{k_4^2+\omega_s^2(k)} = \frac{1}{k_4^2+k^2+M_0^2} + \frac{k^2+M_0^2-\omega_s^2(k)}{(k_4^2+k^2+M_0^2)\left[k_4^2+\omega_s^2(k)\right]}$$

- In order to make explicit the vacuum contribution to the integral, we use three times in sequence the identity

$$\frac{1}{k_4^2+\omega_s^2(k)} = \frac{1}{k_4^2+k^2+M_0^2} - \frac{A_s(k)}{\left(k_4^2+k^2+M_0^2\right)^2} + \frac{A_s^2(k)}{\left(k_4^2+k^2+M_0^2\right)^3} - \frac{A_s^3(k)}{\left(k_4^2+k^2+M_0^2\right)^3\left[k_4^2+\omega_s^2(k)\right]}$$

$$A_s(k) = \mu_5^2 + 2sk\mu_5 + M^2 - M_0^2$$

M0 is the quark mass in the vacuum

finite integrals no cutoff

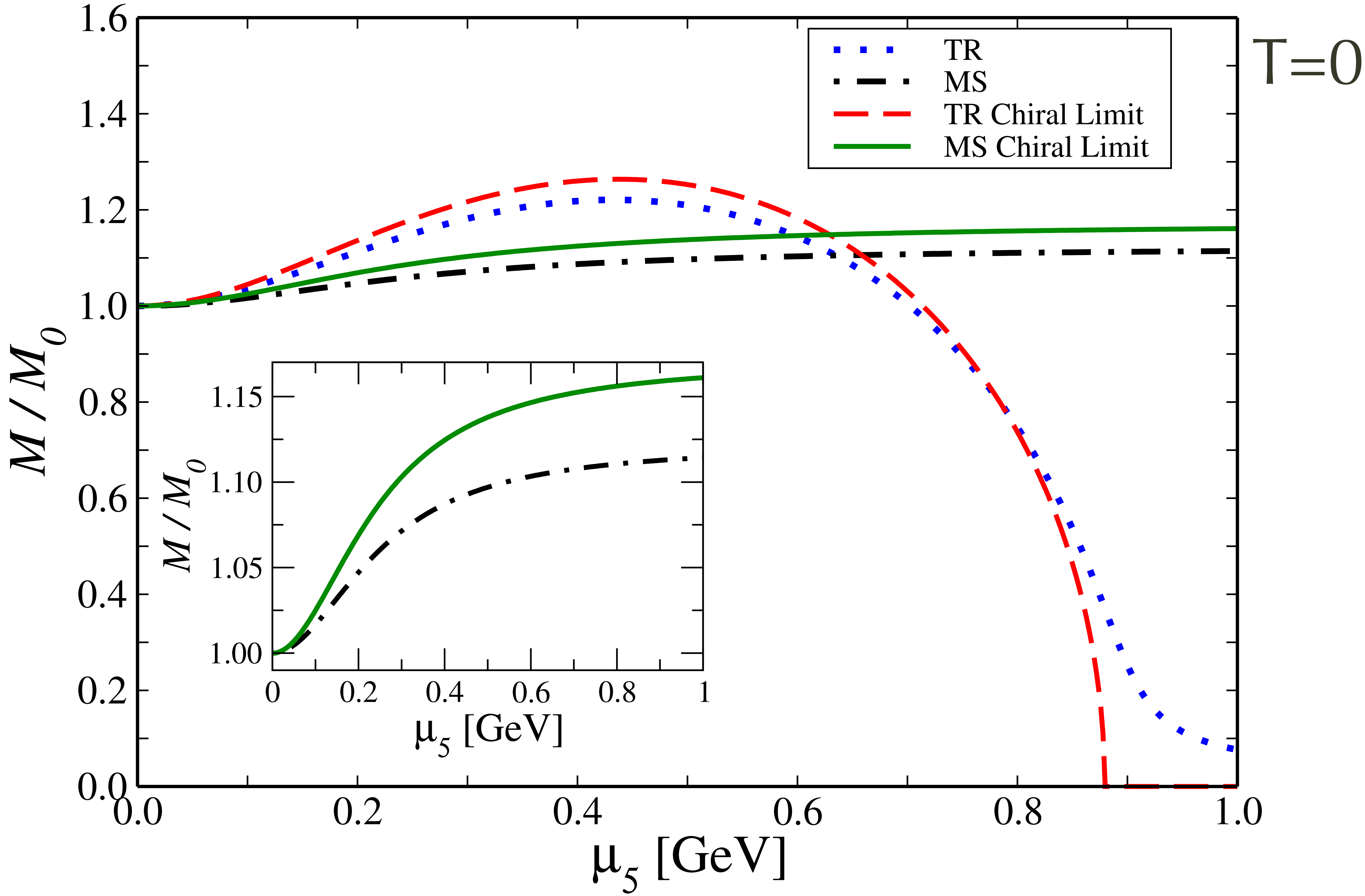
$$\frac{M - m_c}{4N_f N_c G M} = I_{\rm quad}(\Lambda, M_0) + \left(2\mu_5^2 - M^2 + M_0^2\right) I_{\log}(\Lambda, M_0) - \frac{2\mu_5^2 + M^2 - M_0^2}{8\pi^2} + \frac{M^2 - 2\mu_5^2}{8\pi^2} \ln\left(\frac{M^2}{M_0^2}\right) - \sum_{s=\pm1} \int \frac{d^3k}{(2\pi)^3} \frac{1}{\omega_s(k)} \frac{1}{e^{\omega_s(k)/T} + 1},$$

where $I_{\rm quad}(\Lambda, M_0)$ and $I_{\log}(\Lambda, M_0)$ denote the quadratically and logarithmically UV divergent integrals:

$$I_{\rm quad}(\Lambda, M_0) = \int^{\Lambda} \frac{d^4k}{(2\pi)^4} \frac{1}{k_4^2 + k^2 + M_0^2},$$

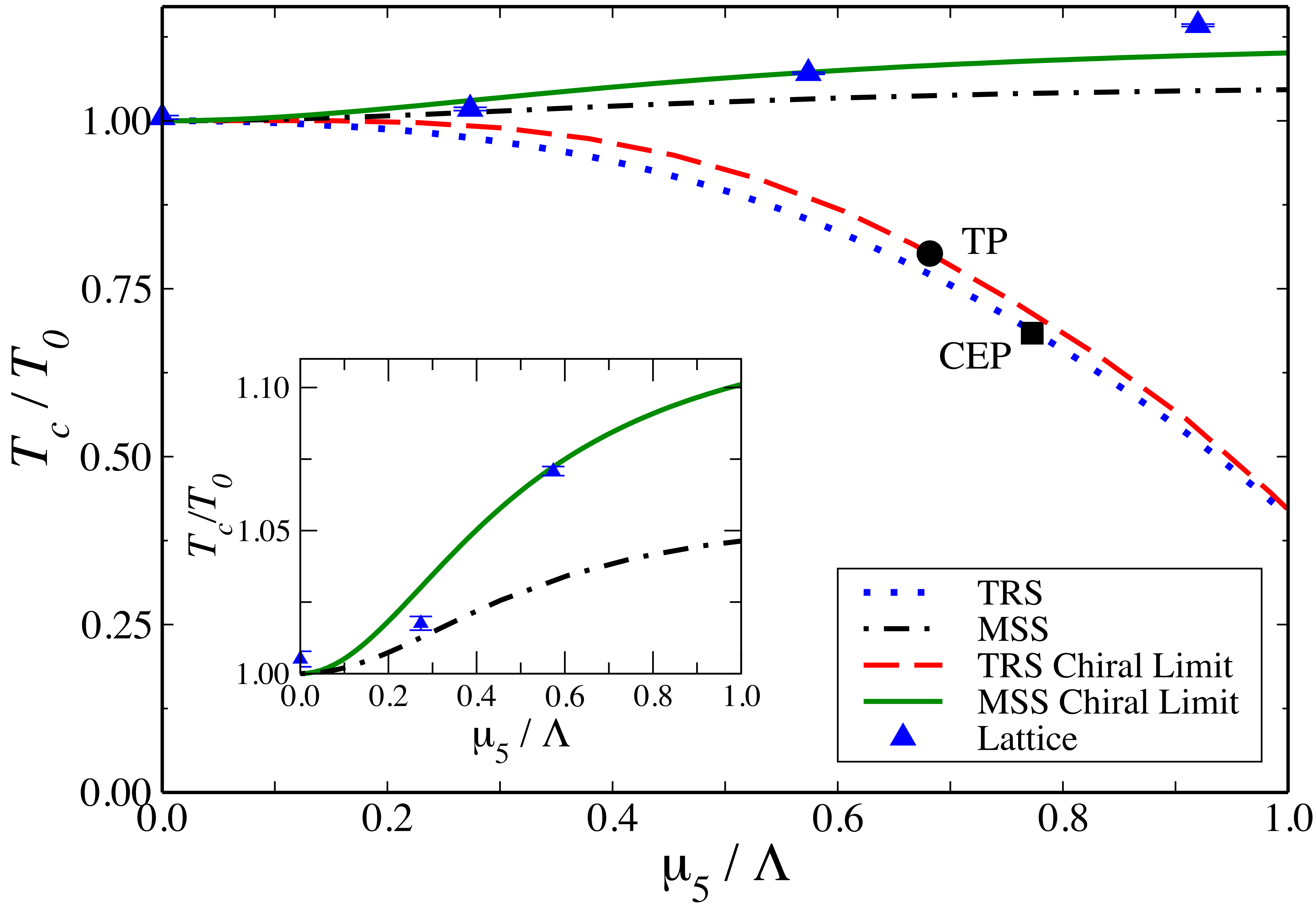
$$I_{\log}(\Lambda, M_0) = -\frac{\partial}{\partial M_0^2} I_{\rm quad}(\Lambda, M_0),$$

Medium Separation Scheme X TR



RLSF at al PRD 94, 074011 (2016)

Medium Separation Scheme X TRS

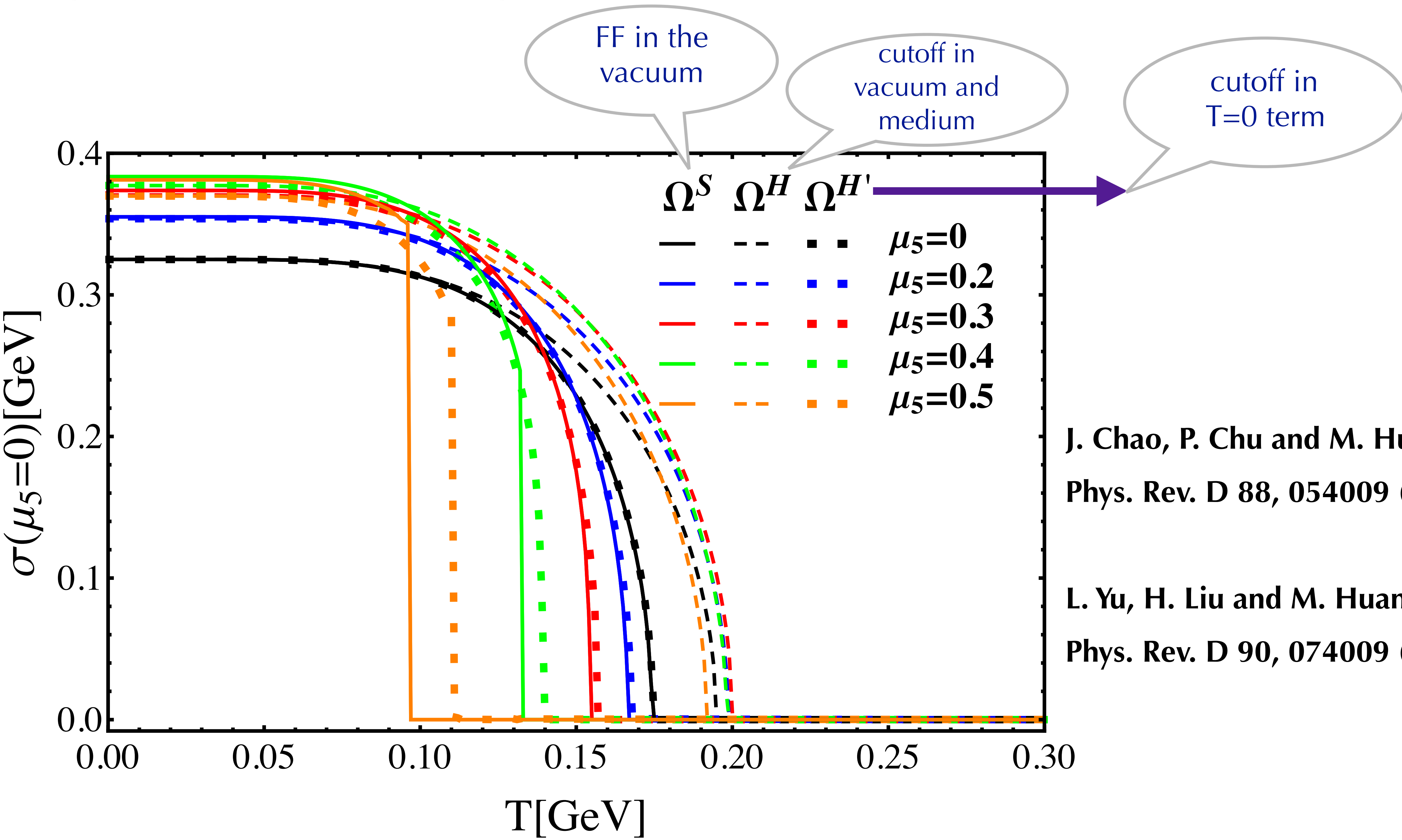


RLSF at al PRD 94, 074011 (2016)

Traditional 3D cutoff Λ (TR)

Effect of the chiral chemical potential on the chiral phase transition in the NJL model with different regularization schemes

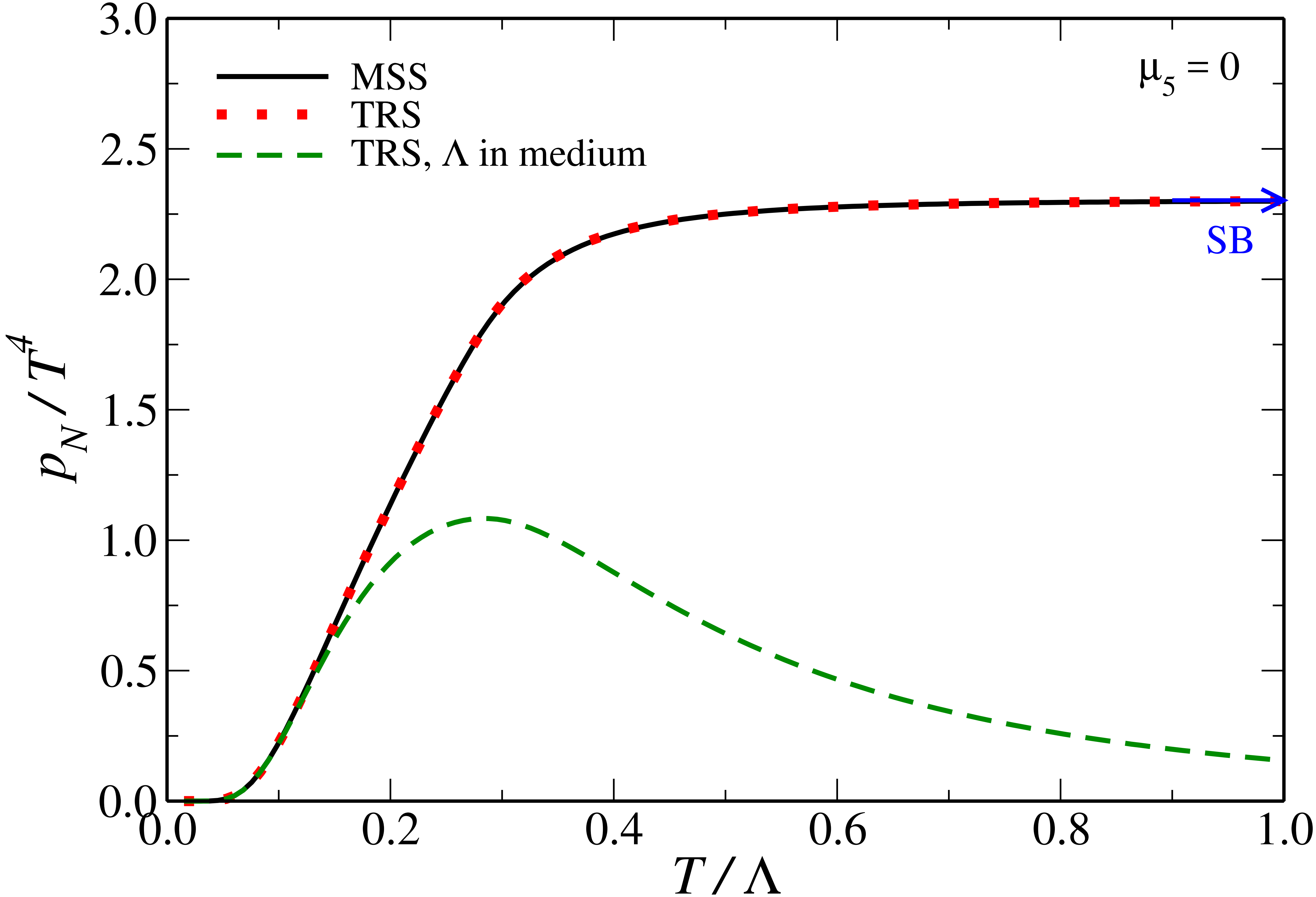
Lang Yu, Hao Liu, and Mei Huang
Phys. Rev. D **94**, 014026 – Published 21 July 2016



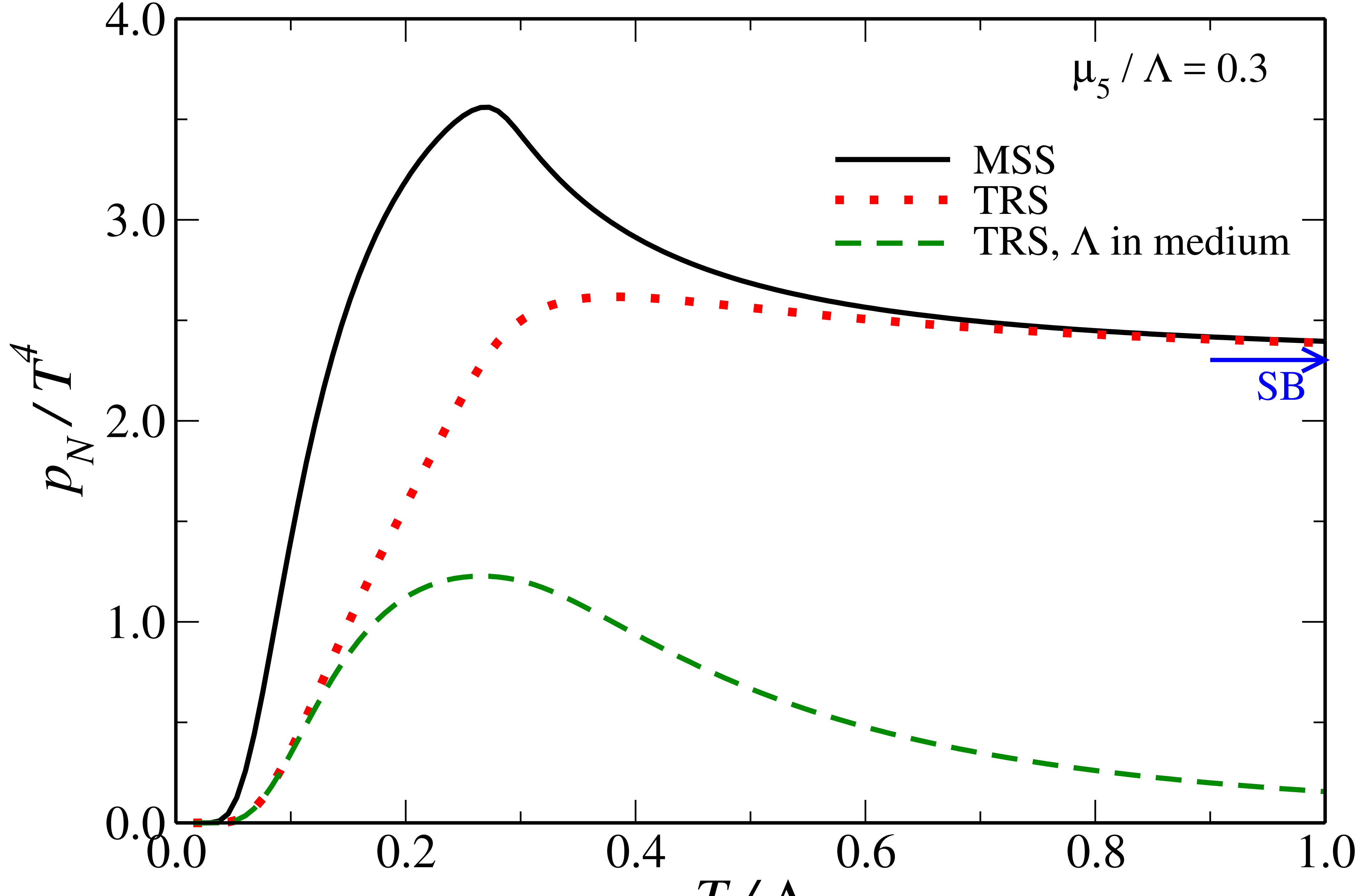
J. Chao, P. Chu and M. Huang, Phys. Rev. D 88, 054009 (2013)

L. Yu, H. Liu and M. Huang, Phys. Rev. D 90, 074009 (2014)

Stefan–Boltzmann Limit



Stefan–Boltzmann Limit



Some Remarks…

- Tc and Tpc increase with μ_5
- No Tricritical (critical) point
- we purpose a way to conciliate results for the chiral critical transition line obtained with NJL models and recent lattice results!
- We eliminated this discrepancy by a proper separation of medium effects from divergent integrals
- **All resulting divergent integrals are the same as those that appear in the vacuum**

Color Superconductivity
Nc2JL

$$\mathcal{L} = \bar{\psi}\left(i\not{D} - m_c\right)\psi + G_S\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\tau\psi\right)^2\right]$$
$$+G_D\left(\bar{\psi}i\gamma_5\tau_2 t_2 C\bar{\psi}^T\right)\left(\psi^T C i\gamma_5\tau_2 t_2\psi\right)$$

$$C = i\gamma_0\gamma_2$$

In mean Field approximation:

$$\Omega = \Omega_0 - 8T\sum_{s=\pm 1}\int\frac{d^3k}{(2\pi)^3}\ln\left\{1+\exp\left[-\frac{\sqrt{(E_k+s\ \mu)^2+\Delta^2}}{T}\right]\right\}$$

UV divergent!

$$\Omega_0 = \frac{(m-m_c)^2+\Delta^2}{4G} - 4\sum_{s=\pm 1}\int\frac{d^3k}{(2\pi)^3}\sqrt{(E_k+s\ \mu)^2+\Delta^2}$$

Gap equations - usual cutoff 3D

$$\frac{M - m_c}{2G} = M\, N_c N_f \sum_{s=\pm} \int_0^{\Lambda} \frac{dk k^2}{2\pi^2} \frac{E_k + s\mu}{E_k \omega_s} - 2M\, N_c N_f \sum_{s=\pm} \int_0^{\infty} \frac{dk k^2}{2\pi^2} \left[\frac{(E_k + s\mu)\, e^{-\beta\omega_s}}{E_k \omega_s \left(1 + e^{-\beta\omega_s}\right)} \right]$$

$$\frac{\Delta}{2G} = N_c N_f \Delta \sum_{s=\pm} \int_0^{\Lambda} \frac{dk k^2}{2\pi^2} \frac{1}{\omega_s} - 2 N_c N_f \Delta \sum_{s=\pm} \int_0^{\infty} \frac{dk k^2}{2\pi^2} \left[\frac{e^{-\beta\omega_s}}{\omega_s \left(1 + e^{-\beta\omega_s}\right)} \right]$$

MSS scheme

$$\frac{1}{k_4^2 + (E_k + s\mu)^2 + \Delta^2} = \frac{1}{k_4^2 + E_k^2 + M_0^2} + \frac{-\mu^2 - 2sE_k\mu - \Delta^2 + M_0^2}{(k_4^2 + E_k^2 + M_0^2)\left[k_4^2 + (E_k + s\mu)^2 + \Delta^2\right]}$$

$$
\begin{aligned}
\frac{M-m_c}{2G} &= M\,N_cN_f\left\{\left[2I_{\rm quad}-\left(M^2+\Delta^2-M_0^2\right)I_{\rm log}-\frac{M^2}{4\pi^2}-\frac{\left(M^2+\Delta^2\right)}{4\pi^2}\log\left(\frac{M_0^2}{M^2+M_0^2}\right)\right]\right.\\
&+\frac{\left(\mu^2+\Delta^2-M_0^2\right)^2}{8\pi^2\left(M^2+M_0^2\right)}-\frac{M_0^2\mu^2}{2\pi^2\left(M^2+M_0^2\right)}+\frac{\mu^2\left(\mu^2+\Delta^2-M_0^2\right)}{2\pi^2\left(M^2+M_0^2\right)}\\
&-\frac{15}{32\pi^2}\sum_{s=\pm1}\int_0^1 dx\,(1-x)^2\int_0^\infty\frac{k^2B_s^3}{\left[B_sx+E_k^2+M_0^2\right]^{\frac{7}{2}}}dk\\
&-\left.\frac{15}{16}\mu\int_0^\infty\frac{dk\,k^2}{2\pi^2}\frac{1}{E_k}\left[\sum_{s=\pm1}s\int_0^1 dx\,(1-x)^2\frac{B_s^3}{\left[B_sx+E_k^2+M_0^2\right]^{\frac{7}{2}}}\right]\right\}\\
&-2M\,N_cN_f\sum_{s=\pm}A_{1,T}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\Delta}{2G} &= 2N_cN_f\Delta\left\{\left[I_{\rm quad}-\frac{\left(M^2+\Delta^2-M_0^2-2\mu^2\right)}{2}I_{\rm log}-\frac{M^2}{8\pi^2}-\frac{\left(M^2+\Delta^2-2\mu^2\right)}{8\pi^2}\log\left(\frac{M_0^2}{M^2+M_0^2}\right)\right]\right.\\
&-\left.\frac{M_0^2\mu^2}{4\pi^2\left(M^2+M_0^2\right)}+\frac{\left(\mu^2+\Delta^2-M_0^2\right)^2}{16\pi^2\left(M^2+M_0^2\right)}-\frac{15}{64\pi^2}\sum_{s=\pm1}\int_0^1 dx\,(1-x)^2\int_0^\infty\frac{k^2B_s^3}{\left[B_sx+E_k^2+M_0^2\right]^{\frac{7}{2}}}dk\right\}\\
&-2N_cN_f\Delta\sum_{s=\pm}A_{2,T}
\end{aligned}
$$

$$
A_{1,T} = \int\frac{d^3k}{(2\pi)^3}\left[\frac{\left(E_k+s\mu\right)e^{-\beta\omega_s}}{E_k\omega_s\left(1+e^{-\beta\omega_s}\right)}\right]
$$

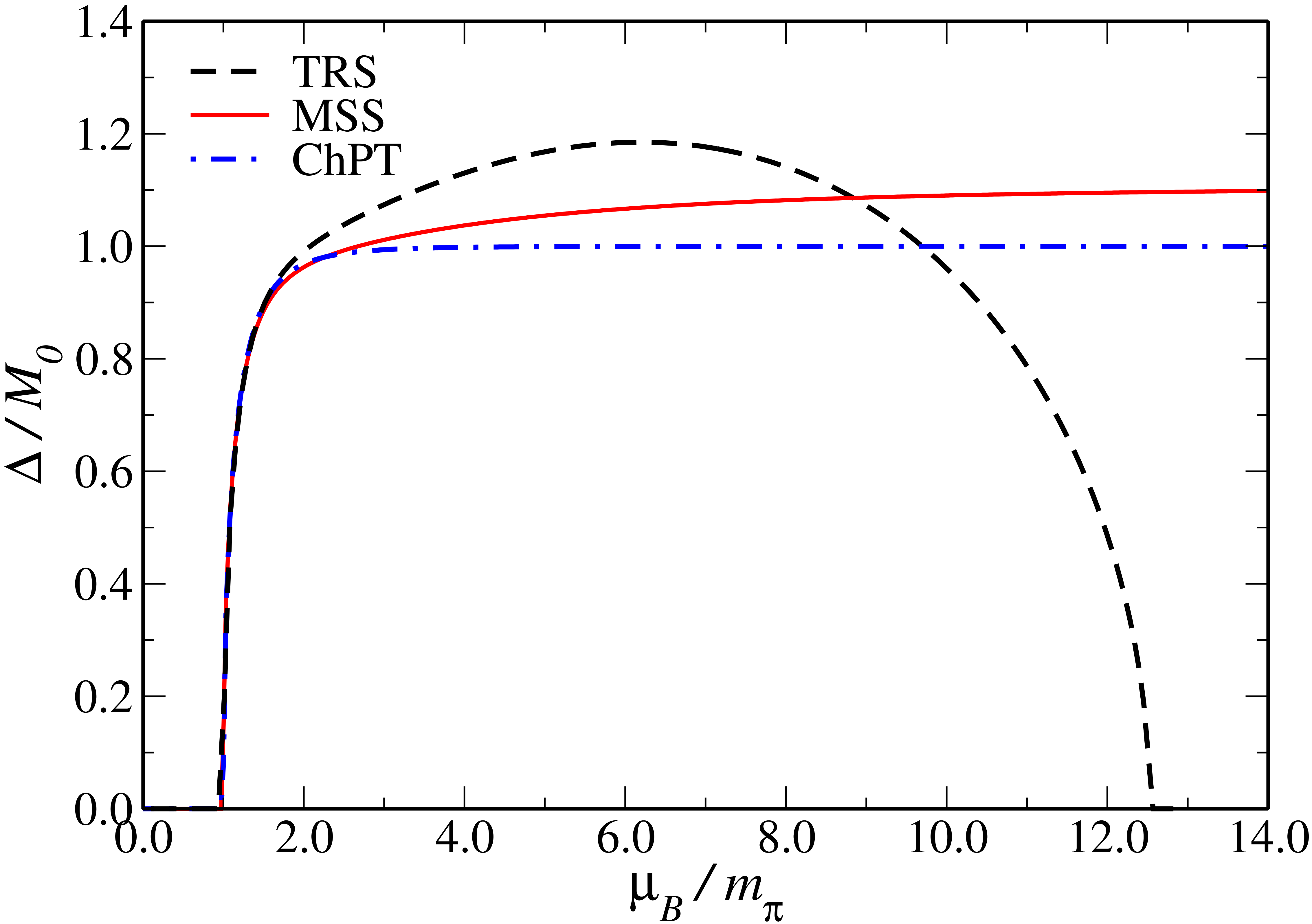
$$
A_{2,T} = \int\frac{d^3k}{(2\pi)^3}\left[\frac{e^{-\beta\omega_s}}{\omega_s\left(1+e^{-\beta\omega_s}\right)}\right]
$$

$$
I_{\rm quad} = \int_\Lambda\frac{d^3k}{(2\pi)^3}\frac{1}{\sqrt{k^2+M_0^2}}
$$

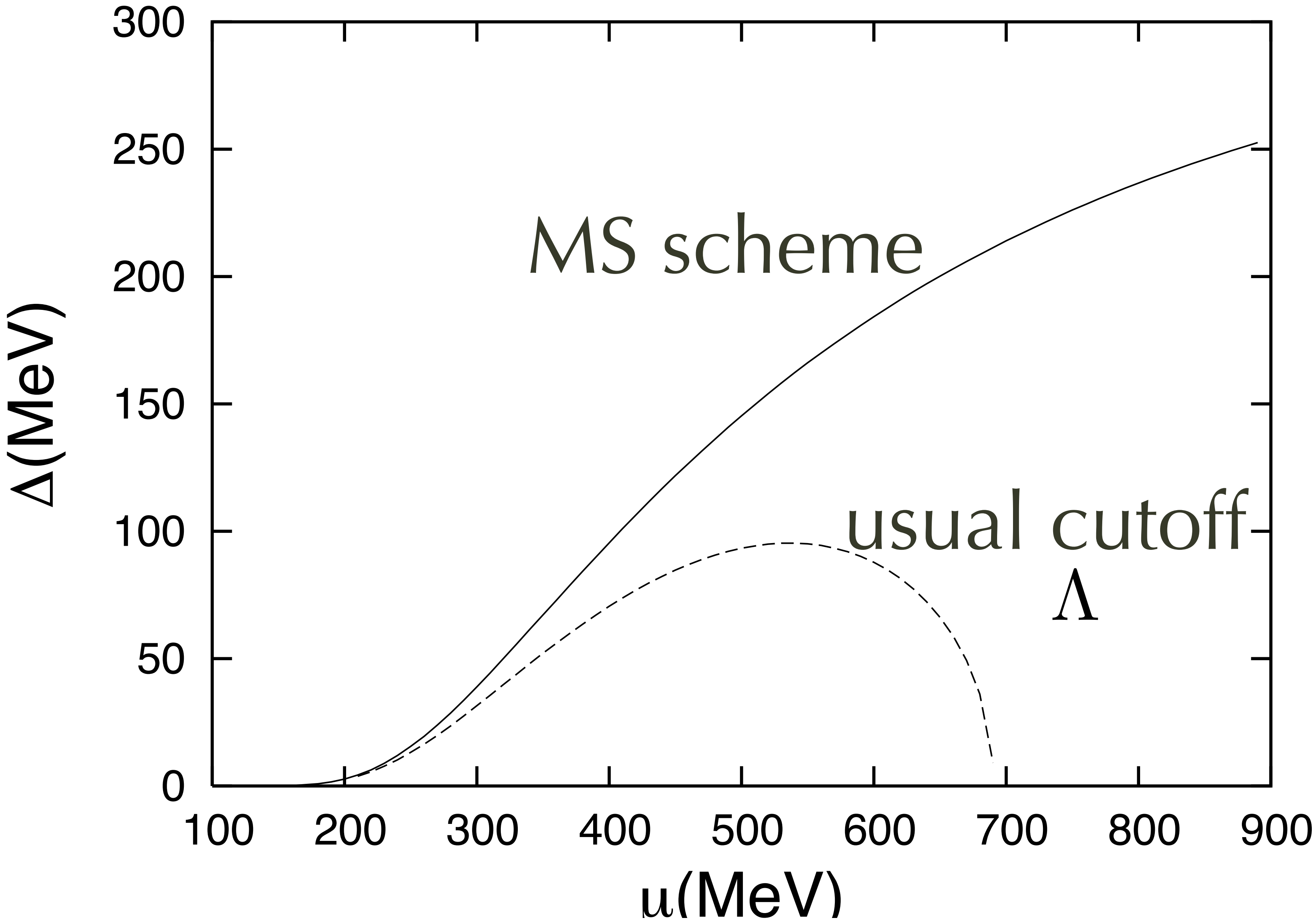
$$
I_{\rm log} = \int_\Lambda\frac{d^3k}{(2\pi)^3}\frac{1}{\left(k^2+M_0^2\right)^{\frac{3}{2}}}
$$

$$
B_s = \mu^2+2sE_k\mu+\Delta^2-M_0^2,\qquad s=\pm1
$$

Nc=2 Color Superconductivity T=0

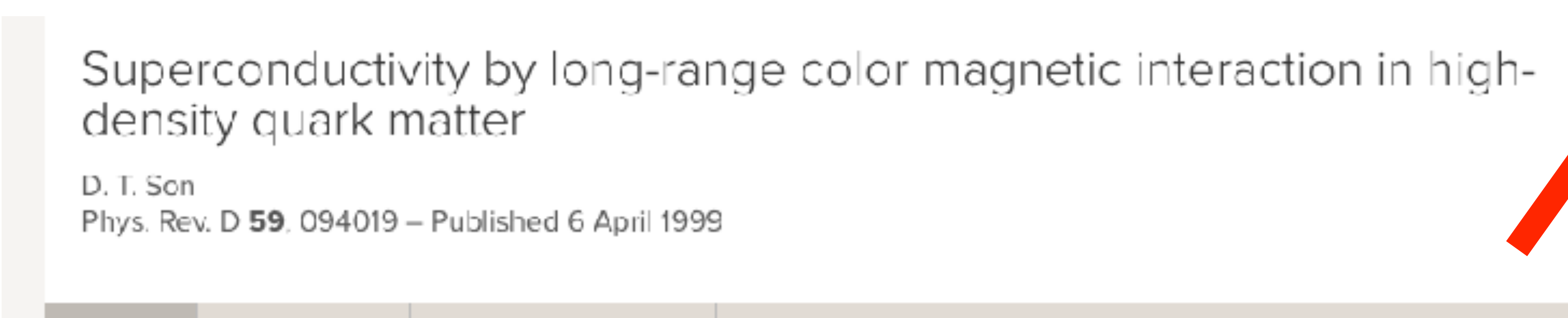


SU(2) NJL (Nc=3)



RLSF, Phys.Rev. C73 (2006), 018201

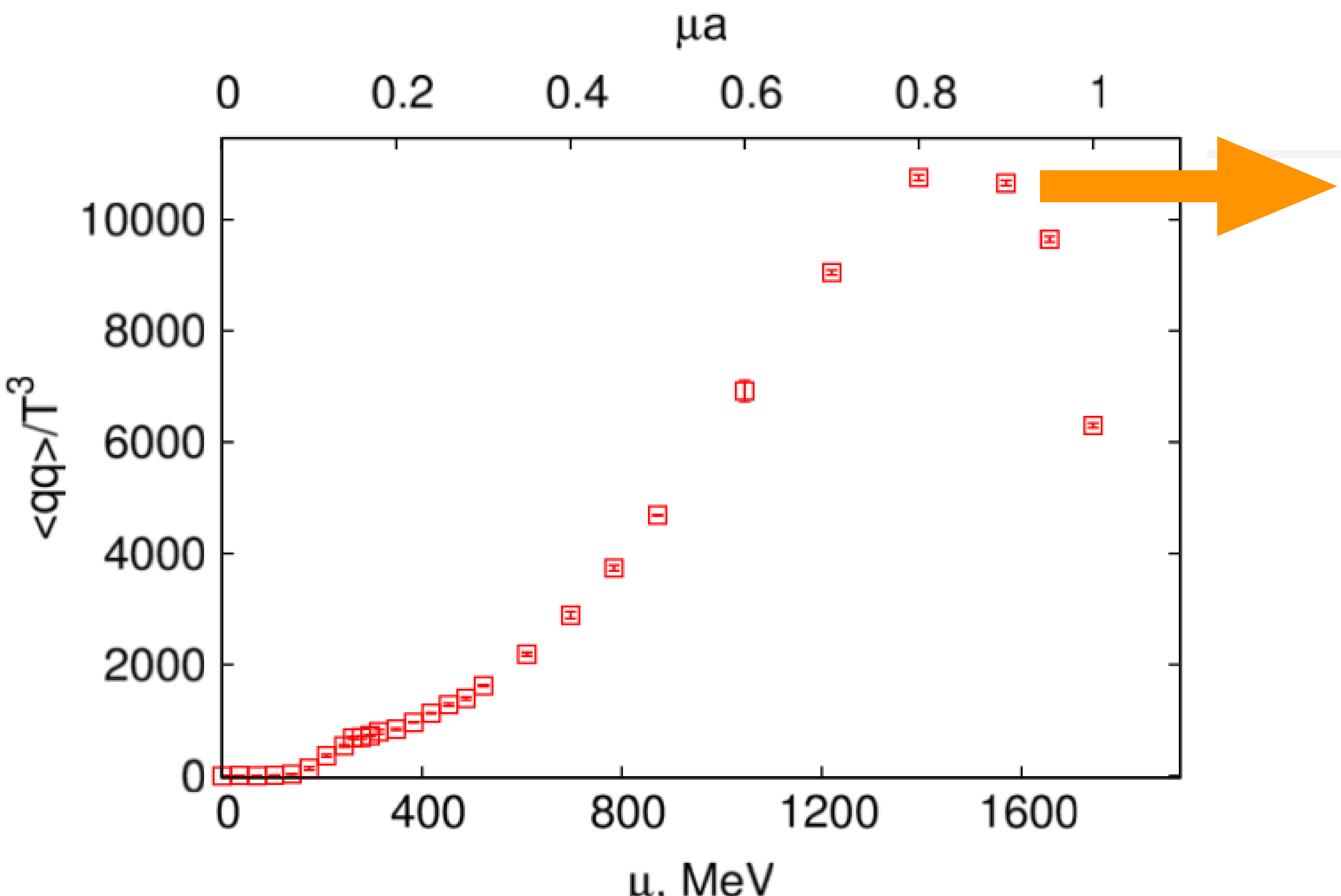
$$\Delta \sim \frac{\mu}{g^5} \exp\left(-\frac{3\pi^2}{\sqrt{2}g}\right)$$



Nc=2 no signal problem!

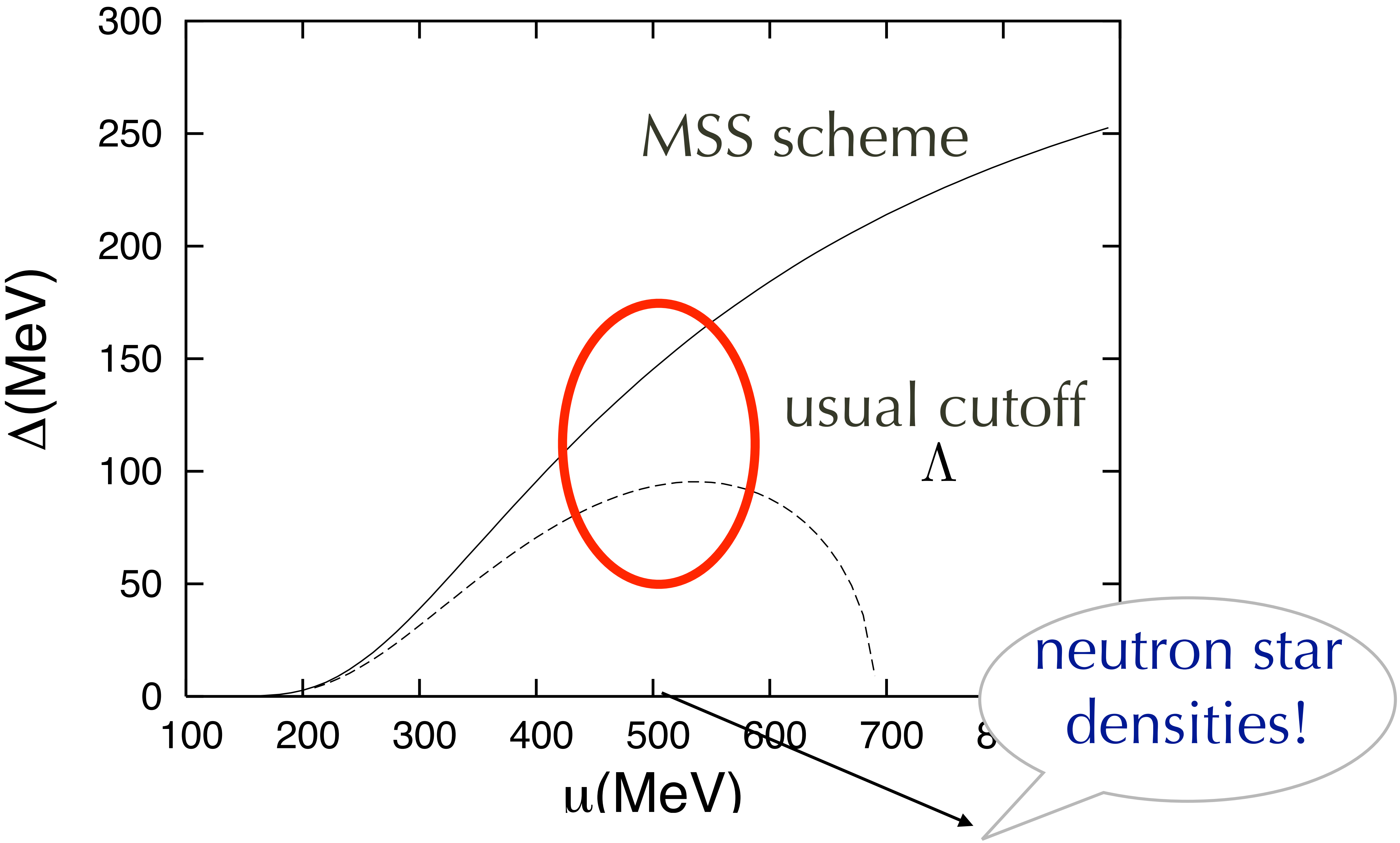
Study of the phase diagram of dense two-color QCD within lattice simulation

V. V. Braguta, E.-M. Ilgenfritz, A. Yu. Kotov, A. V. Molochkov, and A. A. Nikolaev
Phys. Rev. D **94**, 114510 – Published 15 December 2016



This behavior might be connected with a saturation effect and therefore can be considered as a lattice artifact.

SU(2) NJL (Nc=3)



RLSF, Phys.Rev. C73 (2006) 018201

gapless 2SC

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Physics Letters B

Volume 564, Issues 3–4, 10 July 2003, Pages 205-211

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Gapless two-flavor color superconductor

Igor Shovkovy [1], Mei Huang [2]

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Abstract

A new, stable gapless two-flavor color superconducting phase that appears under conditions of local charge neutrality and β-equilibrium is revealed. In this phase, the symmetry of the ground state is the same as in the conventional two-flavor color superconductor. In the low-energy spectrum of this phase, however, there are only two gapped fermionic quasiparticles, and the other four quasiparticles are gapless. The origin and the basic properties of the gapless two-flavor color superconductor are discussed. This phase is a natural candidate for quark matter in cores of compact stars.

SU(2) NJL

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m_c\right)\psi + G_s\left[\left(\bar{\psi}\psi\right)^2 + \left(\bar{\psi}i\gamma_5\vec{\tau}\psi\right)^2\right] \\ &\quad + G_d\left[\left(i\bar{\psi}^C\varepsilon\epsilon^b\gamma_5\psi\right)\left(i\bar{\psi}\varepsilon\epsilon^b\gamma_5\psi^C\right)\right]\end{aligned}$$

In $\beta-$ equilibrium,the diagonal matrix of quark chemical potentials is given in terms of quark, electrical and color chemical potentials,

$$\mu_{ij,\alpha,\beta} = \left(\mu\delta_{ij} - \mu_e Q_{ij}\right)\delta_{\alpha\beta} + \frac{2}{\sqrt{3}}\mu_8\delta_{ij}\left(T_8\right)_{\alpha\beta}$$

The explicit expressions for the quark chemical potentials read

$$\begin{aligned}\mu_{ur} &= \mu_{ug} = \mu - \frac{2}{3}\mu_e + \frac{1}{3}\mu_8 \\ \mu_{dr} &= \mu_{dg} = \mu + \frac{1}{3}\mu_e + \frac{1}{3}\mu_8 \\ \mu_{ub} &= \mu - \frac{2}{3}\mu_e - \frac{2}{3}\mu_8 \\ \mu_{db} &= \mu + \frac{1}{3}\mu_e - \frac{2}{3}\mu_8\end{aligned}$$

NJL + neutralities

$$\begin{aligned}
E &\equiv \sqrt{p^2+m^2} \\
E_\Delta^\pm &\equiv \sqrt{(E\pm\bar\mu)^2+\Delta^2} \\
\bar\mu &\equiv \frac{\mu_{ur}+\mu_{dg}}{2} = \frac{\mu_{ug}+\mu_{dr}}{2} = \mu-\frac{\mu_e}{6}+\frac{\mu_8}{3} \\
\delta\mu &\equiv \frac{\mu_{dg}-\mu_{ur}}{2} = \frac{\mu_{dr}+\mu_{ug}}{2} = \frac{\mu_e}{2}
\end{aligned}$$

Δ Gap equation: To obtain Δ Gap equation we make

$$\frac{\partial\Omega_{T=0}}{\partial\Delta} = \Delta-(2G_d)\,\Delta\left[4I_\Delta^i-2\theta\,(\delta\mu-\Delta)\int_{\mu^-}^{\mu^+}\frac{dp}{\pi^2}\frac{p^2}{E_\Delta^-}\right]=0$$

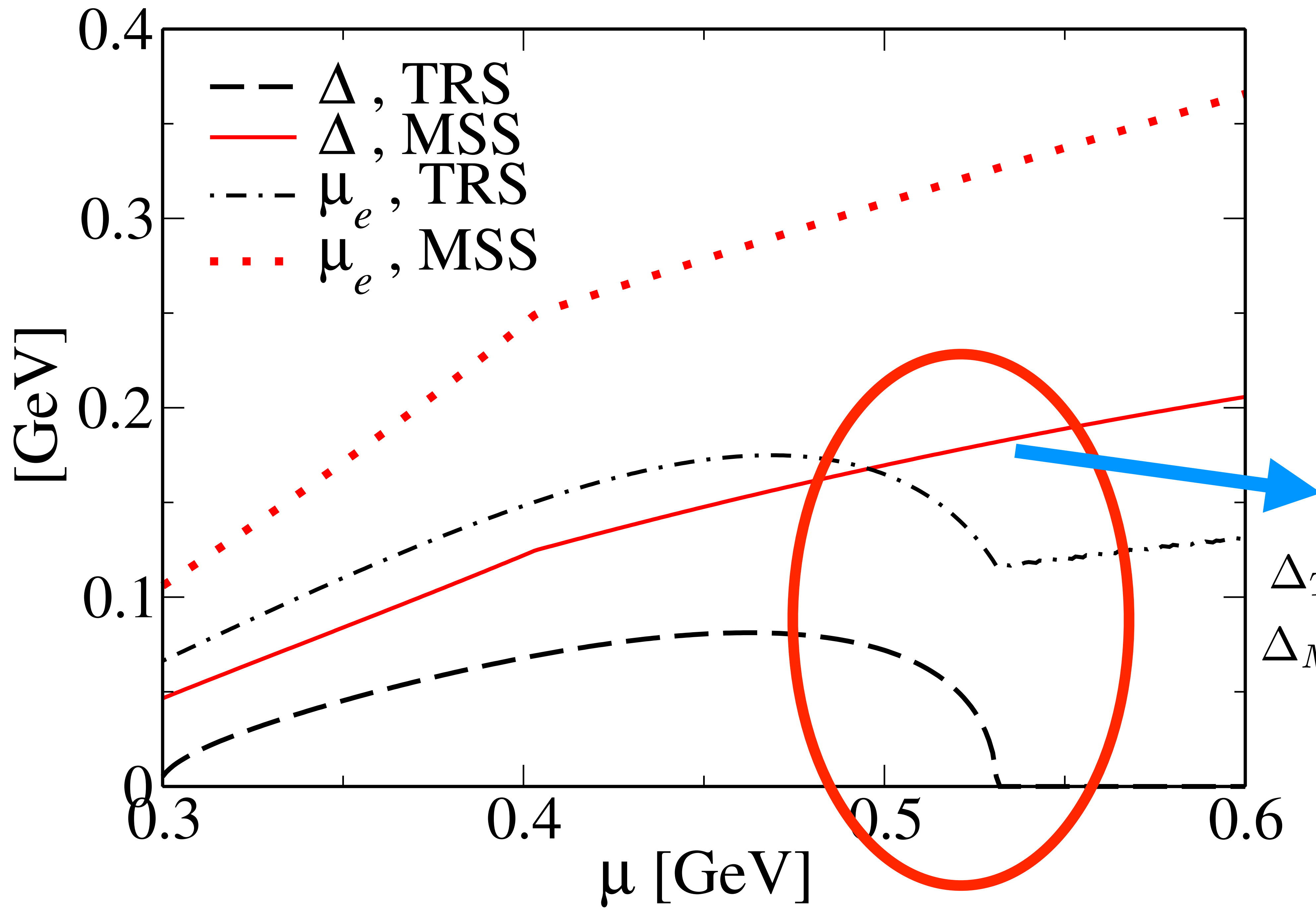
where I_Δ^i in the chirally symmetric phase is given in each case by

$$\begin{aligned}
I_\Delta^{TRS} &= \int_\Lambda\frac{d^3p}{(2\pi)^3}\left(\frac{1}{E_\Delta^+}+\frac{1}{E_\Delta^-}\right) \\
I_\Delta^{MSS} &= 2I_{\rm quad}-\left(\Delta^2-M_0^2-2\mu^2\right)I_{\rm log} \\
&+\left[\frac{3\left(M_0^2-\bar\mu^2-\Delta^2\right)^2}{4}-3M_0^2\bar\mu^2\right]I_{\rm fin,1}+2I_{\rm fin,2}
\end{aligned}$$

$$\begin{aligned}
n_8 &= 0 \\
n_e &= 0
\end{aligned}$$

MSS X TRS

$$G_D = 0.75 G_s$$



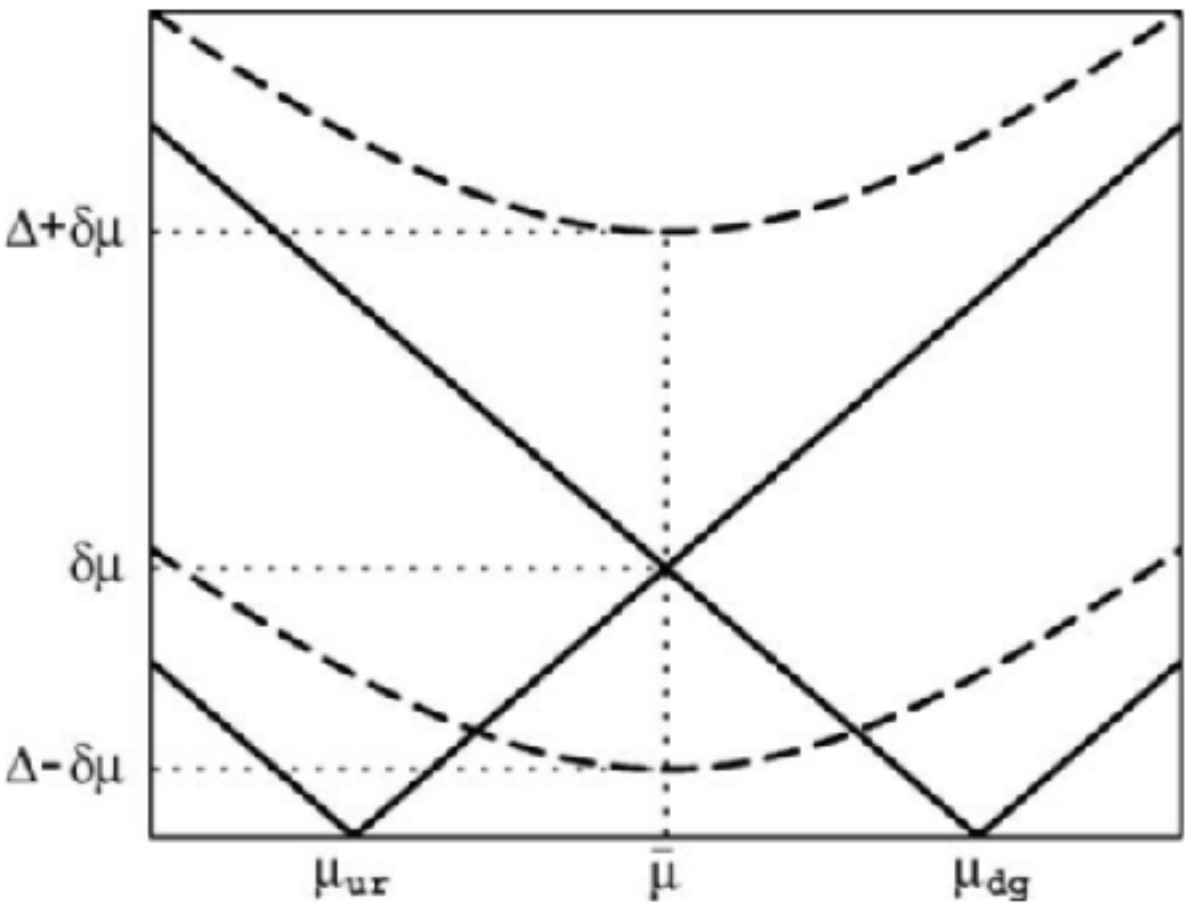
$$\mu = 0.532\, GeV$$

$$\Delta_{TRS} = 0$$

$$\Delta_{MSS} = 0.182\, GeV$$

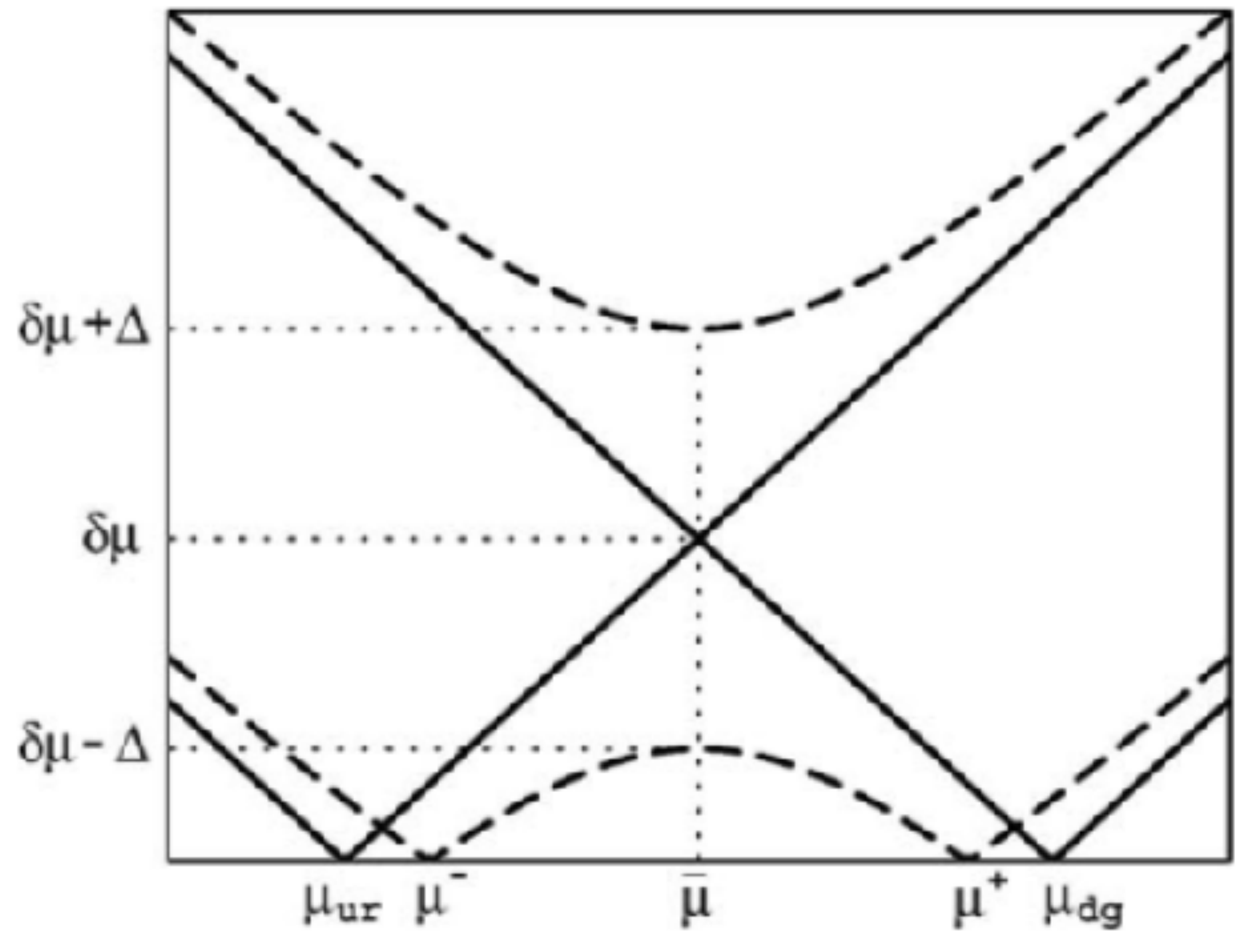
Quasiparticle dispersion relations

$\Delta > \delta\mu$



2SC

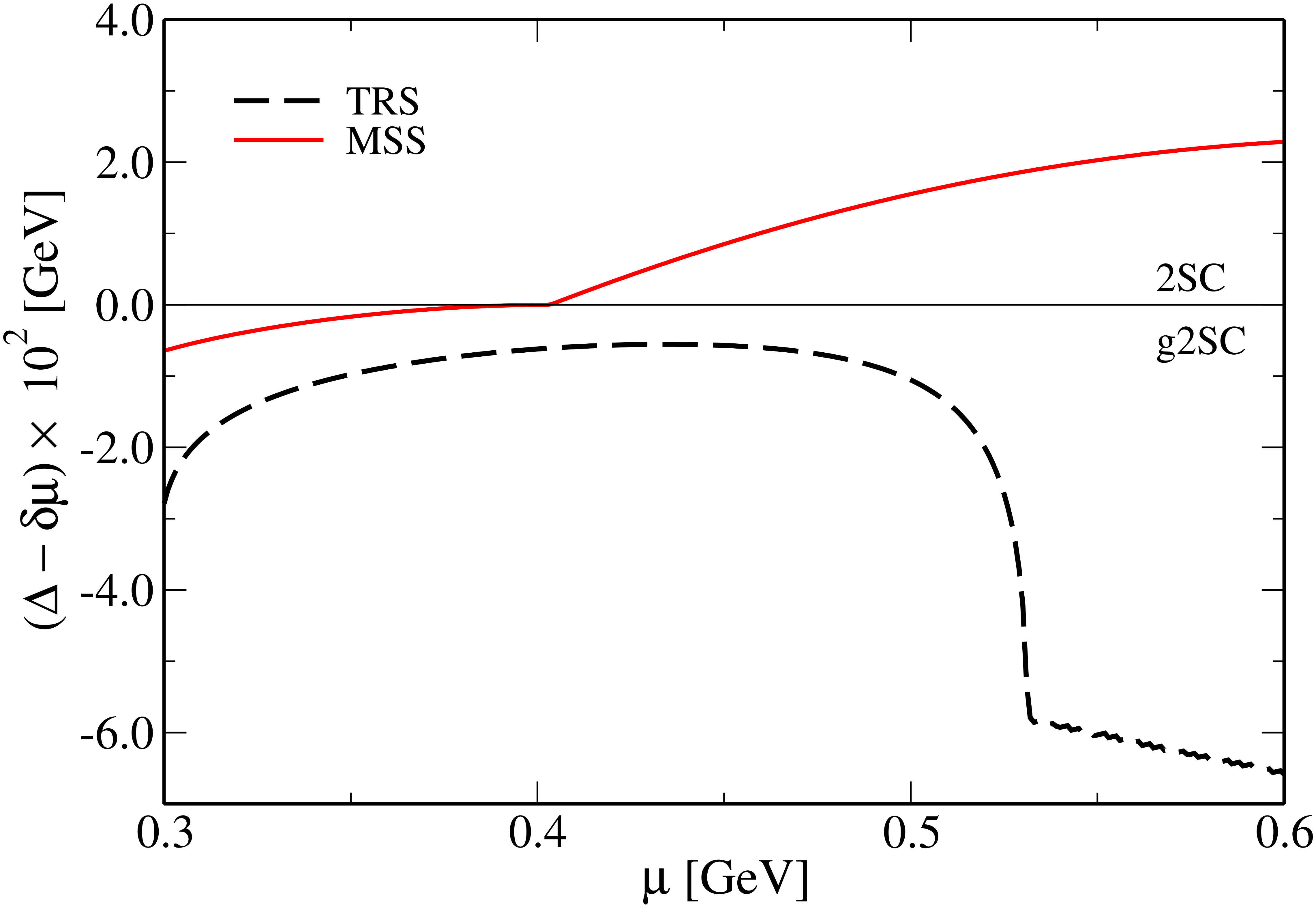
$\Delta < \delta\mu$



g2SC

M. Huang, I. Shovkovy / Nuclear Physics A 729 (2003) 835–863

g2SC X 2SC



4.0
2.0
0.0
-2.0
-4.0
-6.0
TRS
MSS
2SC
g2SC
$(\Delta - \delta\mu) \times 10^2$ [GeV]
0.3
0.4
0.5
0.6
μ [GeV]

SU(3) + CFL + GR

Equations of State of different phases of dense quark matter
E.J. Ferrer (CUNY, CCNY). 2017. 12 pp.
Published in **J.Phys.Conf.Ser. 861 (2017) no.1, 012020**
DOI: 10.1088/1742-6596/861/1/012020

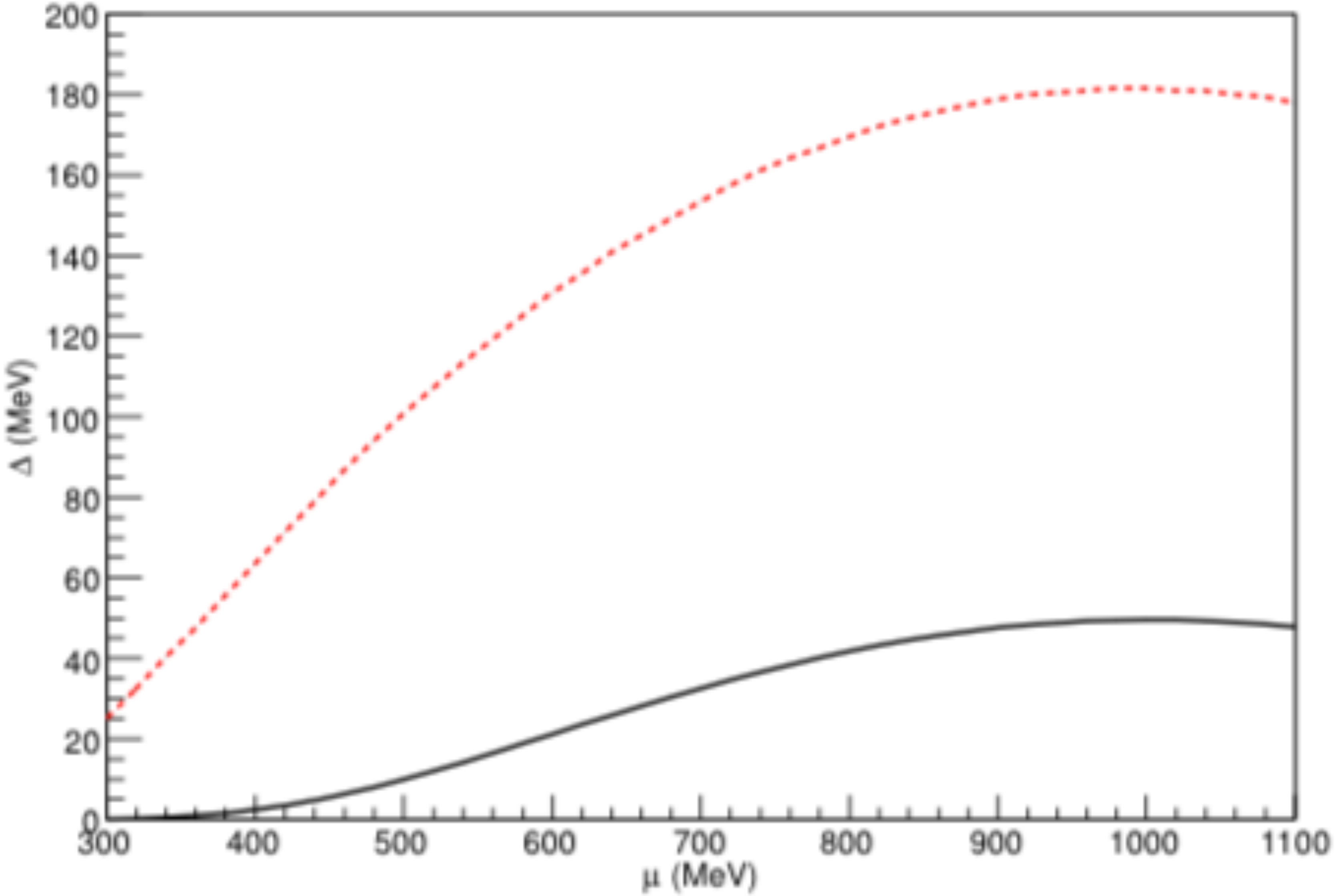
Gaussian Form Factor

$$\Omega_{CFL}^{NJL} = -\frac{1}{4\pi^2}\int_0^\infty dpp^2 e^{-p^2/\Lambda^2}(16|\epsilon| + 16|\bar{\epsilon}|) +$$
$$-\frac{1}{4\pi^2}\int_0^\infty dpp^2 e^{-p^2/\Lambda^2}(2|\epsilon'| + 2|\overline{\epsilon'}|) + \frac{3\Delta_{CFL}^2}{G} + B,$$

$$\varepsilon = \pm\sqrt{(p-\mu)^2 + \Delta_{CFL}^2}, \quad \bar{\varepsilon} = \pm\sqrt{(p+\mu)^2 + \Delta_{CFL}^2},$$
$$\varepsilon' = \pm\sqrt{(p-\mu)^2 + 4\Delta_{CFL}^2}, \quad \overline{\varepsilon'} = \pm\sqrt{(p+\mu)^2 + 4\Delta_{CFL}^2},$$

SU(3) + CFL + GR

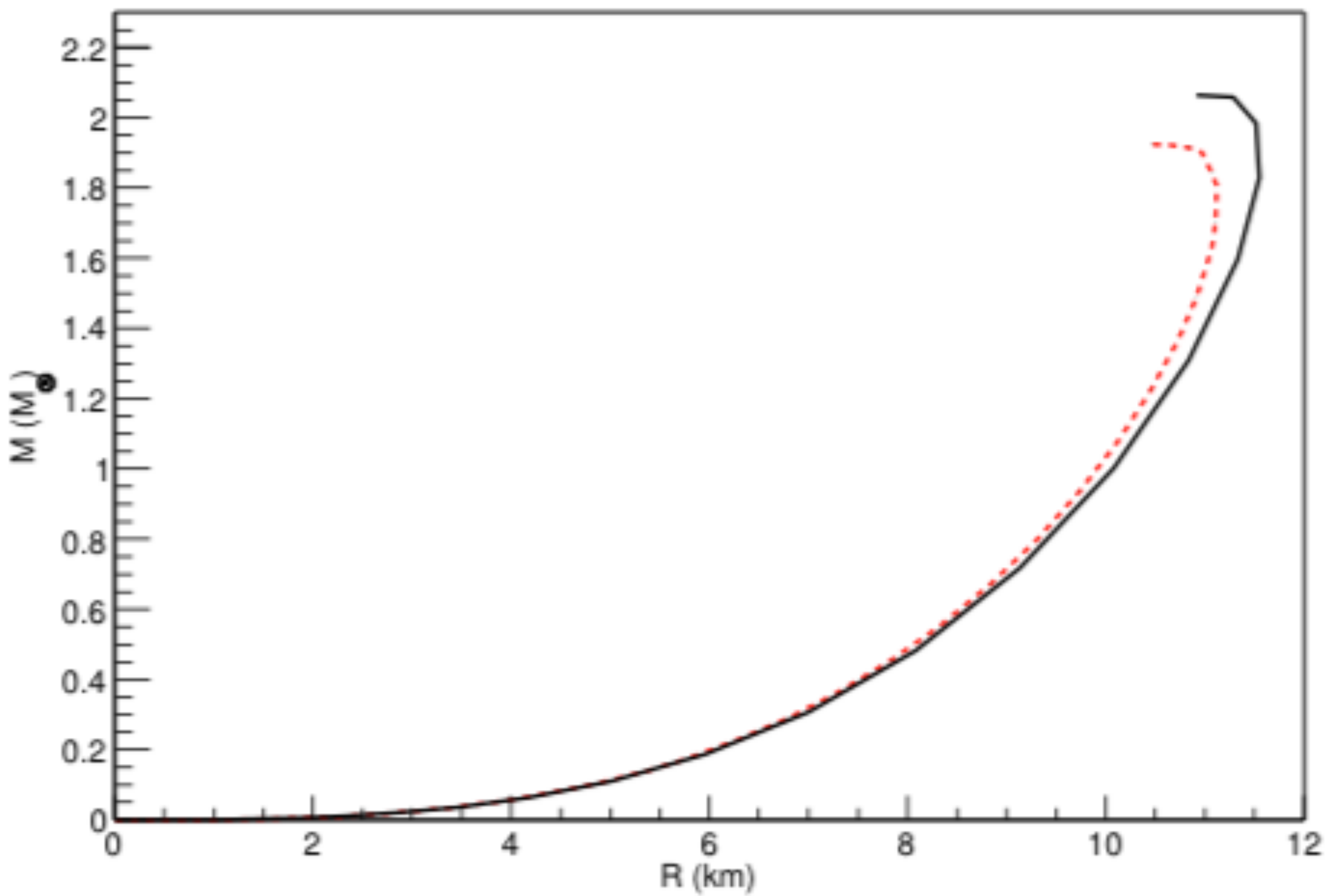
G=4.32 GeV^{-2} (full line) and G=7.10 GeV^{-2} (dashed line).



$$GR = e^{\frac{-p^2}{\Lambda^2}},\ \Lambda = 1\,GeV$$

These parameters do not describe SU(3) NJL model in the vacuum!

SU(3) + CFL + GR



G=4.32 GeV^{-2} (full line) and G=7.10 GeV^{-2} (dashed line).

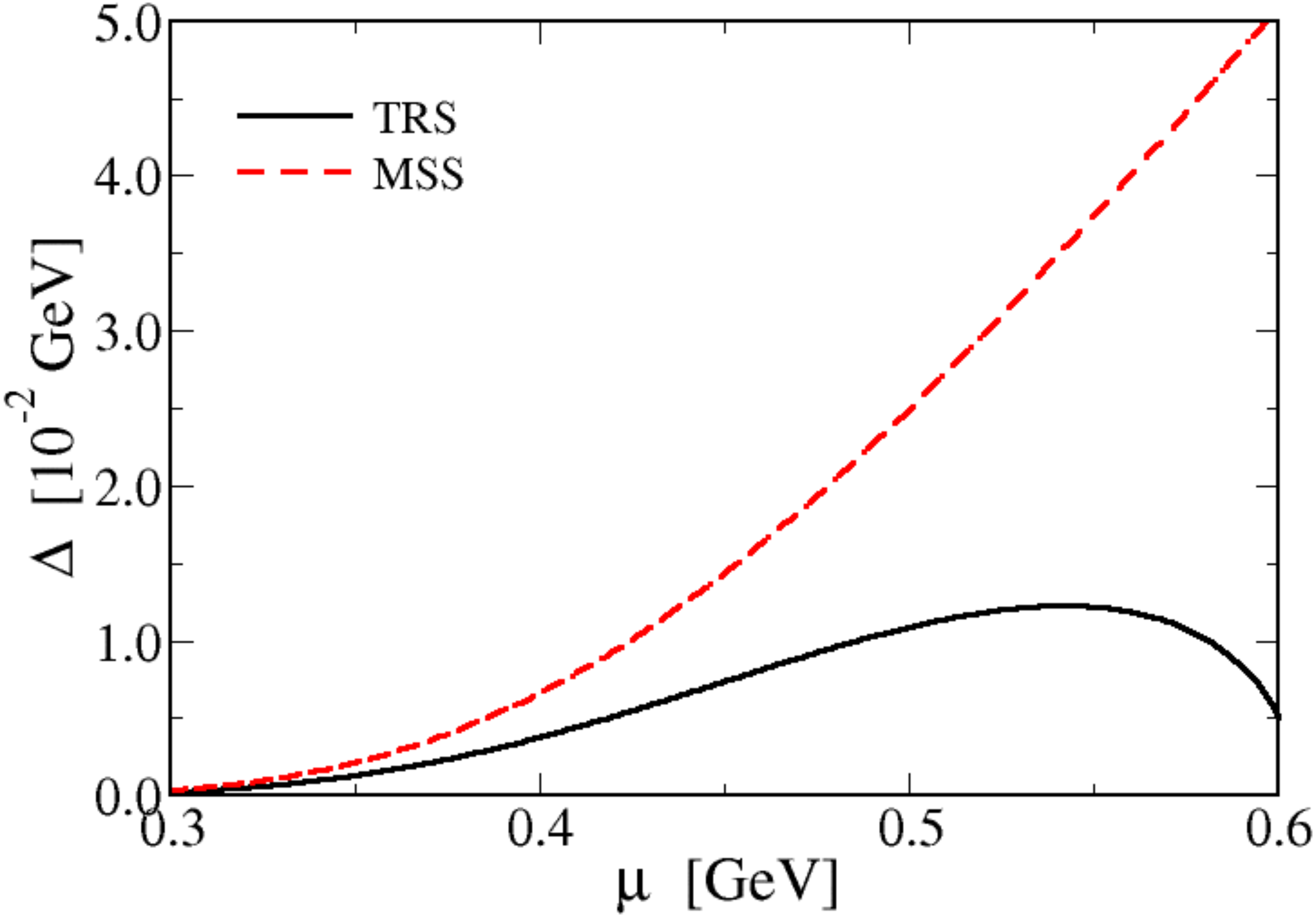
SU(3) + CFL + MSS

$G_D = 1.25\, G_s$ → $\Delta(TRS) = 10\, MeV$

Table 3.1
Three sets of parameters and related quark and meson properties in the three-flavor NJL model

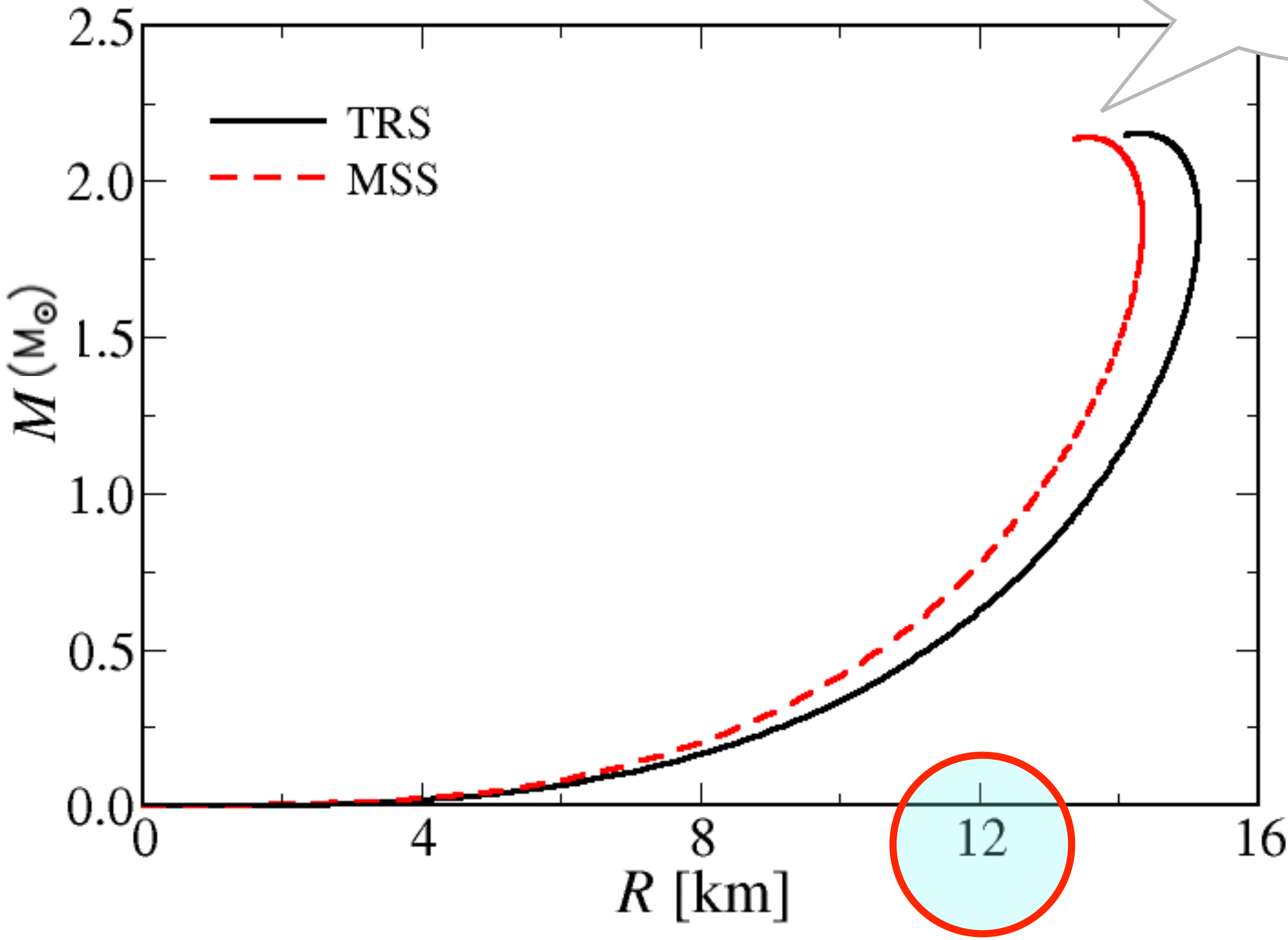
	RKH [145]	HK [71]	LKW [122]	Empirical [99]
Λ (MeV)	602.3	631.4	750	
$G\Lambda^2$	1.835	1.835	1.82	
$K\Lambda^5$	12.36	9.29	8.9	
$m_{u,d}$ (MeV)	5.5	5.5	3.6	3.5–7.5
m_s (MeV)	140.7	135.7	87	110–210
G_V/G	—	—	1.1	
$M_{u,d}$ (MeV)	367.7	335	361	
M_s (MeV)	549.5	527	501	
$(\langle\bar{u}u\rangle)^{1/3}$ (MeV)	−241.9	−246.9	−287	
$(\langle\bar{s}s\rangle)^{1/3}$ (MeV)	−257.7	−267.0	−306	
B (MeV/fm^3)	291.7	295.5	350.0	
f_π (MeV)	92.4	93.0	93	92.4 [102]
m_π (MeV)	135.0	138	139	135.0, 139.6
m_K (MeV)	497.7	496	498	493.7, 497.7
m_η (MeV)	514.8	487	519	547.3
$m_{\eta'}$ (MeV)	957.8	958	963	957.8
$m_{\rho,\omega}$ (MeV)	—	—	765	771.1, 782.6
m_{K^*} (MeV)	—	—	864	891.7, 896.1
m_ϕ (MeV)	—	—	997	1019.5

SU(3) + CFL + MSS



MSS prediction for Msun X Radio

Small radii!!!



Other applications…

Charmed mesons with a symmetry-preserving contact interaction

Fernando E. Serna, Bruno El-Bennich, and Gastão Krein
Phys. Rev. D **96**, 014013 – Published 19 July 2017

ABSTRACT

A symmetry-preserving treatment of a vector-vector contact interaction is used to study charmed heavy-light mesons. The contact interaction is a representation of nonperturbative kernels used in Dyson-Schwinger and Bethe-Salpeter equations of QCD. The Dyson-Schwinger equation is solved for the u, d, s and c quark propagators and the bound-state Bethe-Salpeter amplitudes respecting spacetime-translation invariance and the Ward-Green-Takahashi identities associated with global symmetries of QCD are obtained to calculate masses and electroweak decay constants of the pseudoscalar π, K, D and D_s and vector ρ, K^*, D^*, and D_s^* mesons. The predictions of the model are in good agreement with available experimental and lattice QCD data.

Charmed mesons at finite temperature and chemical potential
Fernando E. Serna, Gastão Krein (Sao Paulo, IFT). Dec 1, 2016. 6 pp.
Published in **EPJ Web Conf. 137 (2017) 13015**
DOI: 10.1051/epjconf/201713713015
Conference: C16-08-28.3 Proceedings
e-Print: **arXiv:1612.00473 [nucl-th]** | **PDF**

Medium Separation Scheme

Our results and methodology used in this work will certainly also interest researchers working not only with QCD (and RHIC type of experiments), but also in other contexts:

- non-relativistic versions of the type of models
- study of systems like superconducting and fermionic gases in condensed matter and atomic physics

Perspectives

Medium Separation Scheme in:

- Color superconductivity at finite T **and B**
- Quark Matter under compact stars conditions
- gapless CFL (gCFL)
- NJL + mesonic fluctuations + T + B

Thank you for your attention!