Magnetic corrections to π-π scattering lengths
in the linear sigma model

Camburi, Sao Paulo, 2018: Many manifestations of non perturbative QCD

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- R. Zamora,Centro de Investigación y desarrollo de Ciencias Aeroespaciales (CIDCA), FACH

This talk is based on the article:

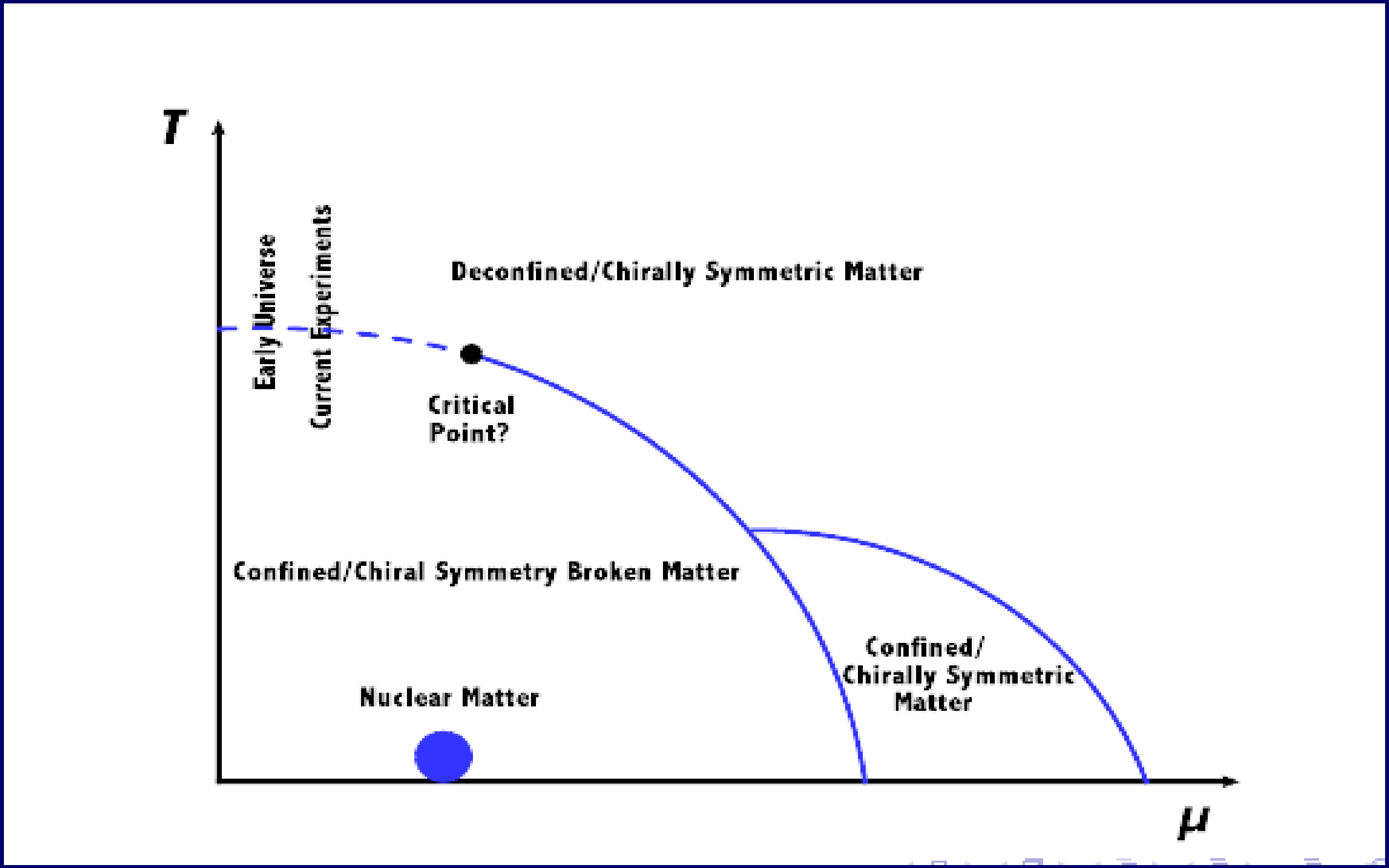
- “Magnetic Corrections to π-π scattering lengths
- In the linear sigma model ”

M. Loewe. L. Monje, and R. Zamora

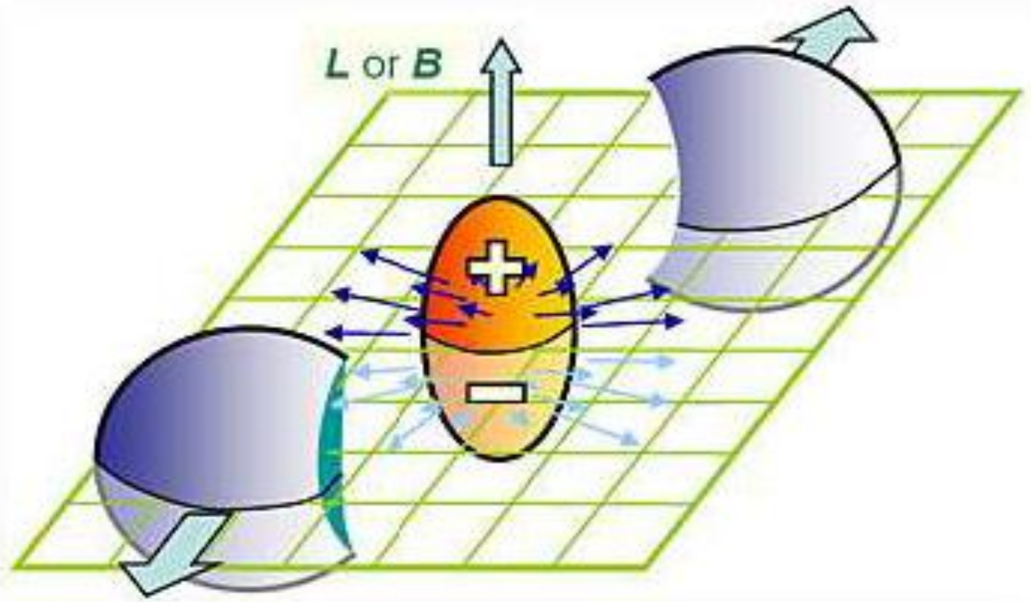
Phys. Rev D97, 056023 (2018)

I acknowledge support from Fondecyt under grant 1170107

As it is well known we expect the occurence of deconfinement (and chiral symmetry restoration) phase transitions, induced by thermal and density effects.

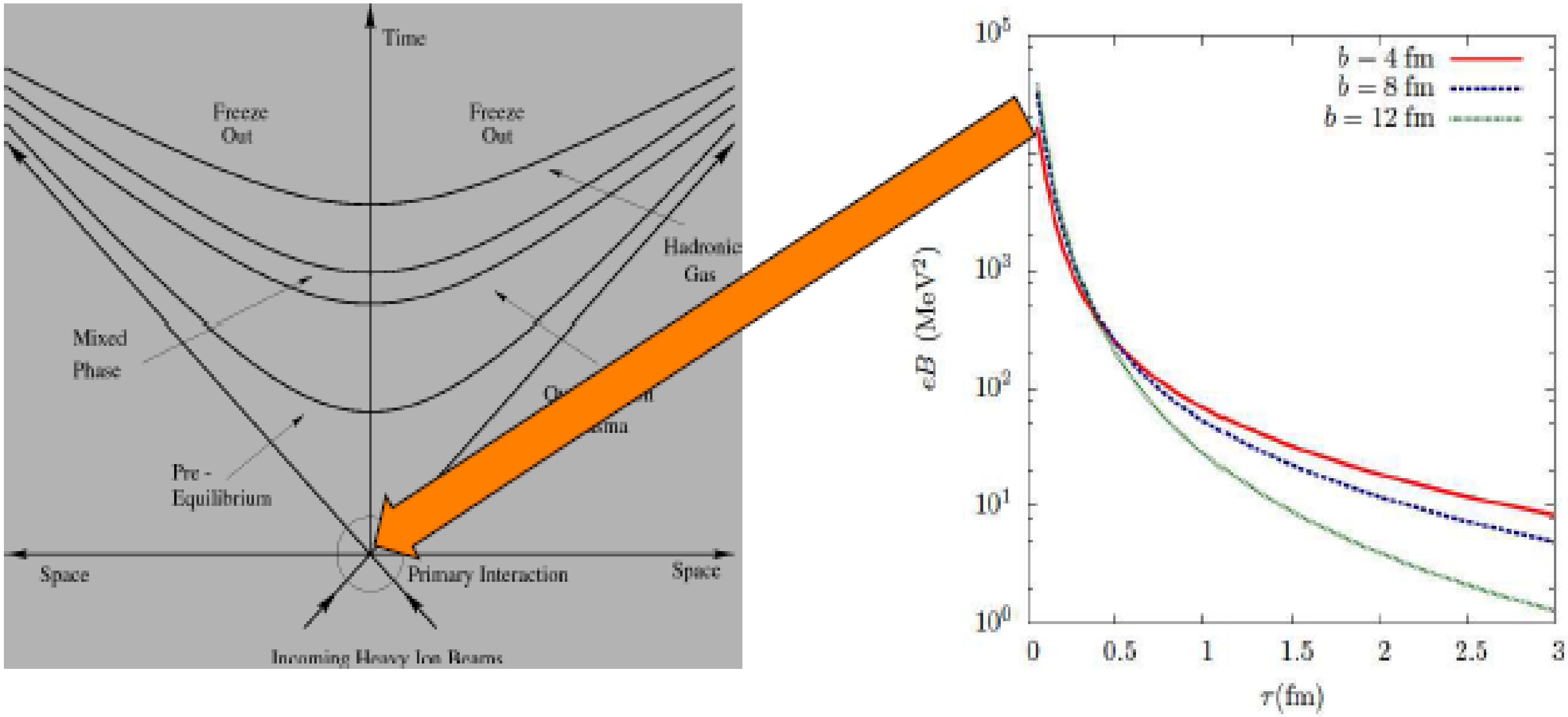


Peripheral Heavy ion collisions



Time evolution of a uniform magnetic field in a heavy-ion collision

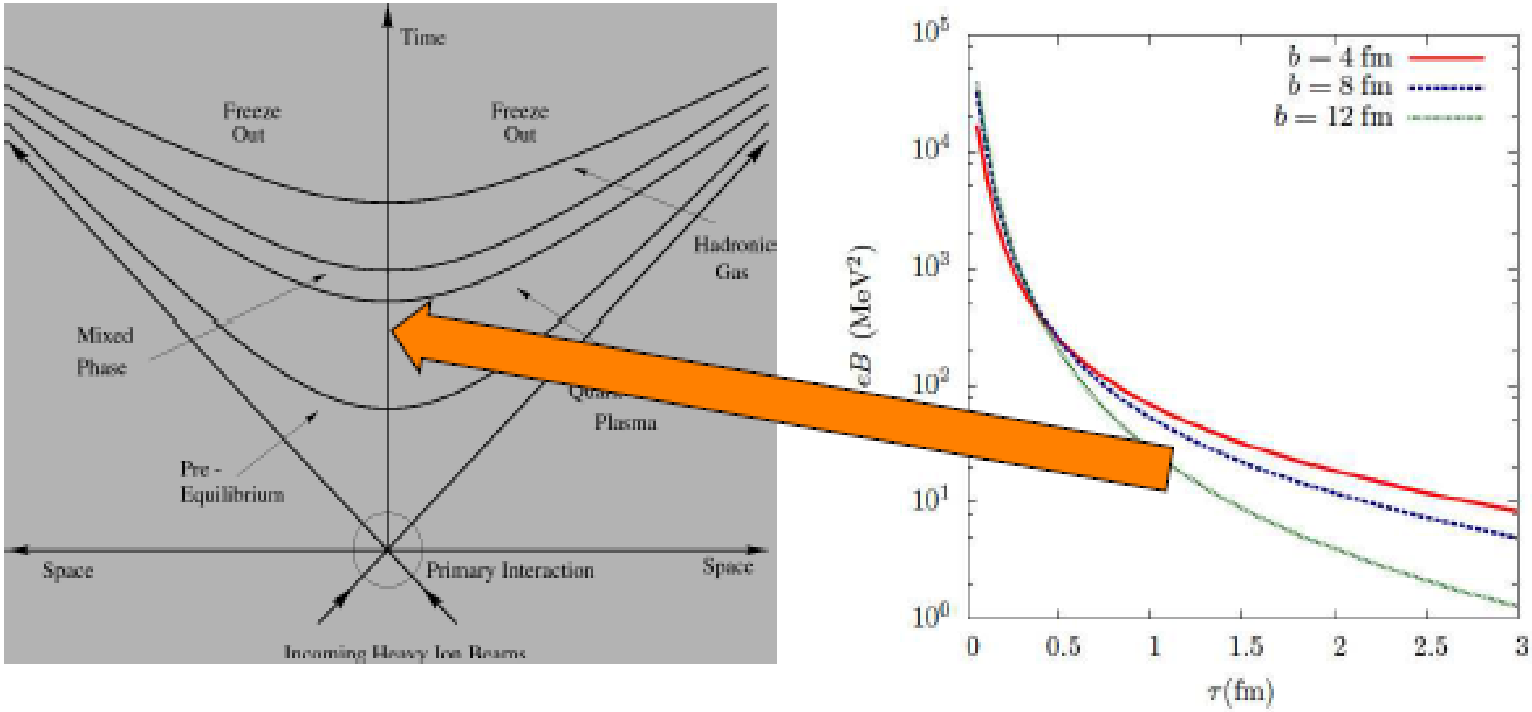
- Very intense field at early collision times



D. E. Kharzeev, L. D. McLerran, H. J. Warringa, Nucl. Phys. **A** 803 (2008) 227-253

Time evolution of a constant B field in a heavy-ion collision

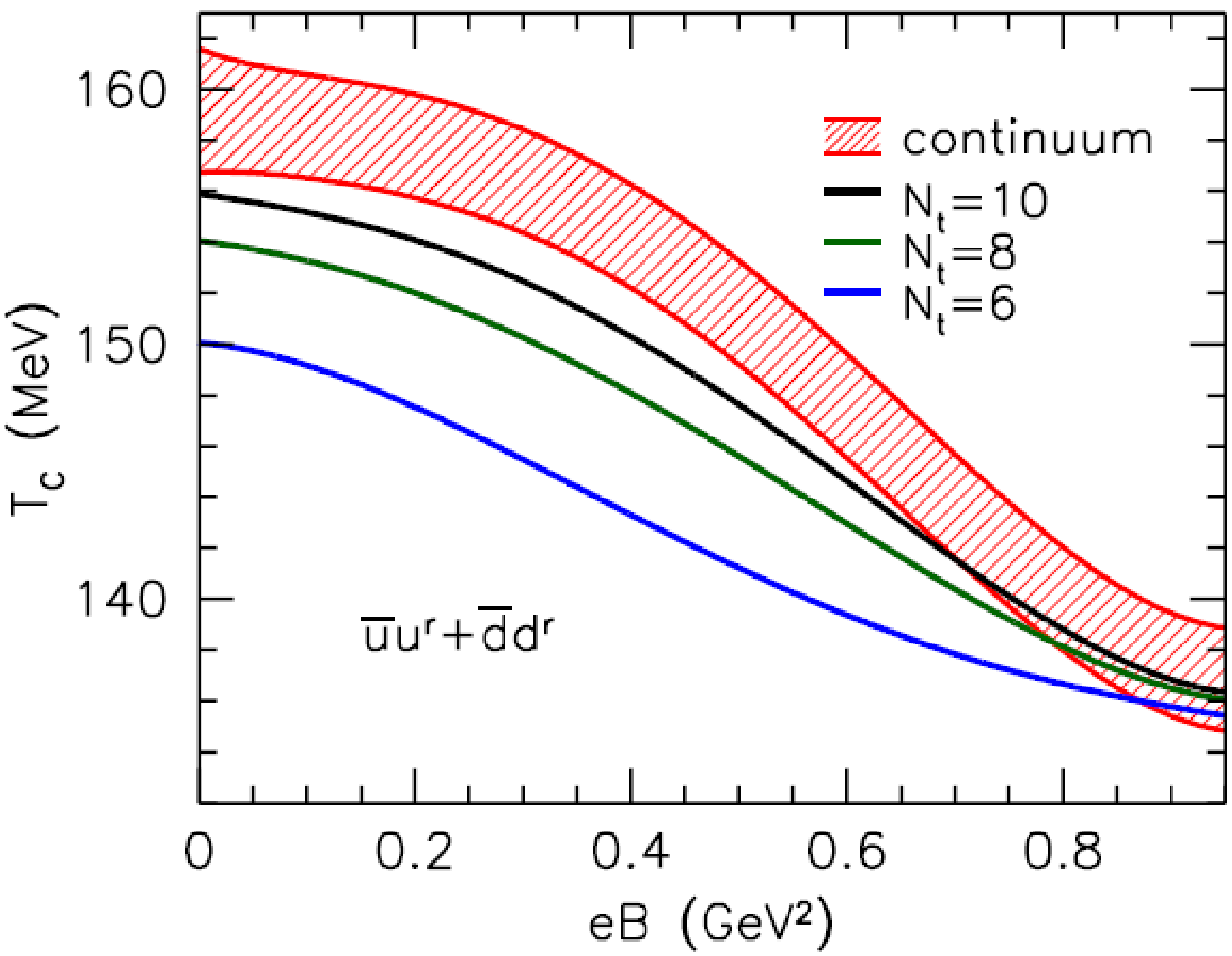
- Magnetic field rapidly decreasing function of collision time



D. E. Kharzeev, L. D. McLerran, H. J. Warringa, Nucl. Phys. **A** 803 (2008) 227-253

Latest lattice results for T_c [G. S. Bali et al., JHEP 02 (2012) 044]

Anticatalysis



During the last years we have discussed several aspects related to the phase diagram of QCD, when thermo-magnetic effects are present (including also density effects):

1) Evolution of the CEP, 2) the occurence of
a quarkyonic phase, 3) evolution of quark and gluon condensates, 4) emergence of anti-magnetic catalysis, etc.

Collaborators: C. A. Dominguez, A. Ayala, Ana Julia Mizher, R. Zamora, L. Hernández, A. Raya, S. Hernández, J. P. Carlomagno, J. C. Rojas, C. Villavicencio, including also students and other collaborators.

For this purpose we have used different techniques as, for example; QCD Sum Rules, effective models like the linear sigma model or the NJL model (local and non local), skyrmion model, etc.

The present work belongs to this general picture.

During this workshop, I became aware about the existence of two interesting articles (my thanks to C. Fisher) were IMC was also discussed in a different context. Ritus formalism for propagators inside the DSE equations.

N. Mueller and J. Pawlowski: PRD 91 (2015) 116010

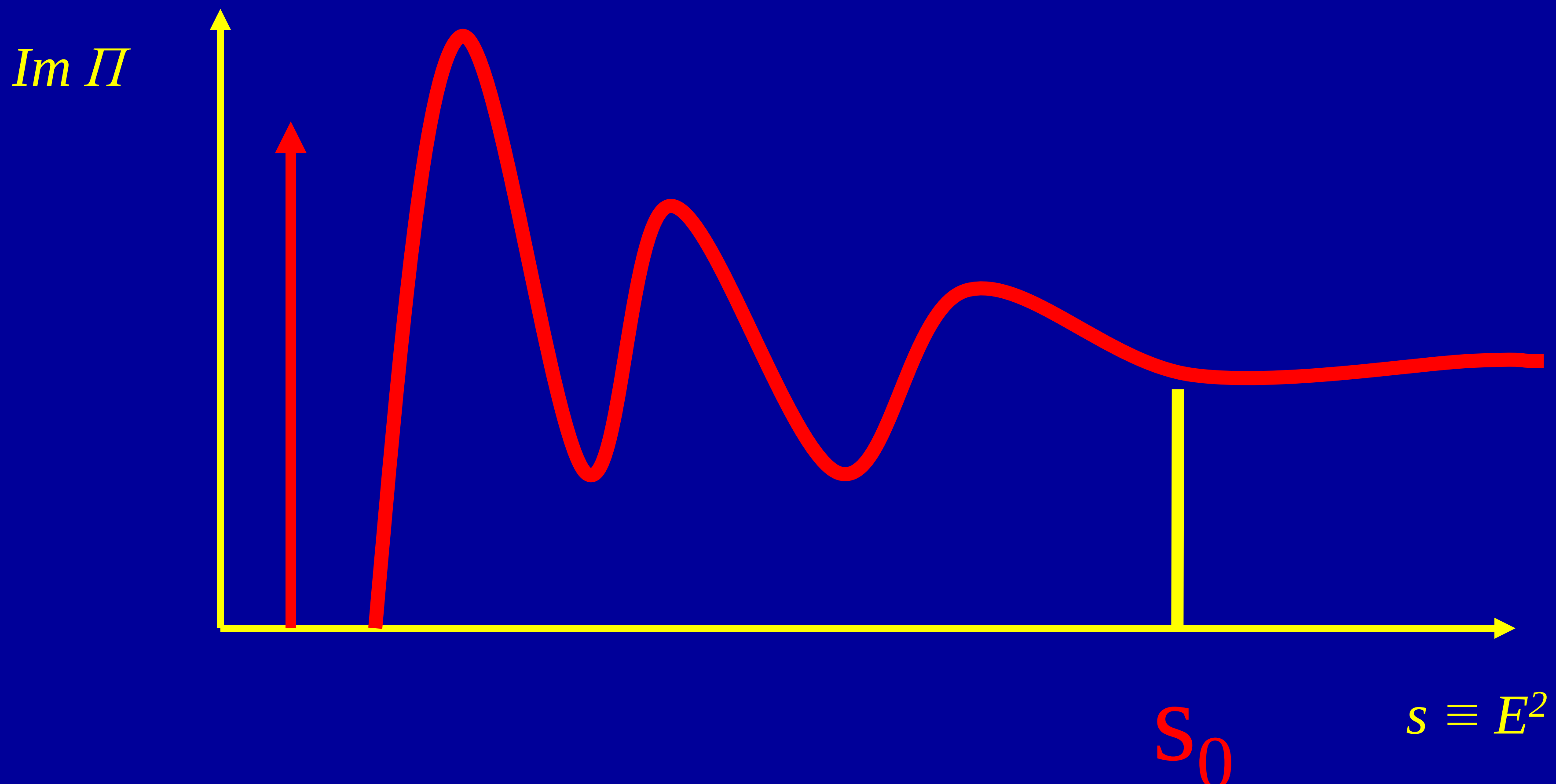
N. Mueller, J. A. Bonnet, and C. Fischer: PRD 89 (2014) 094023

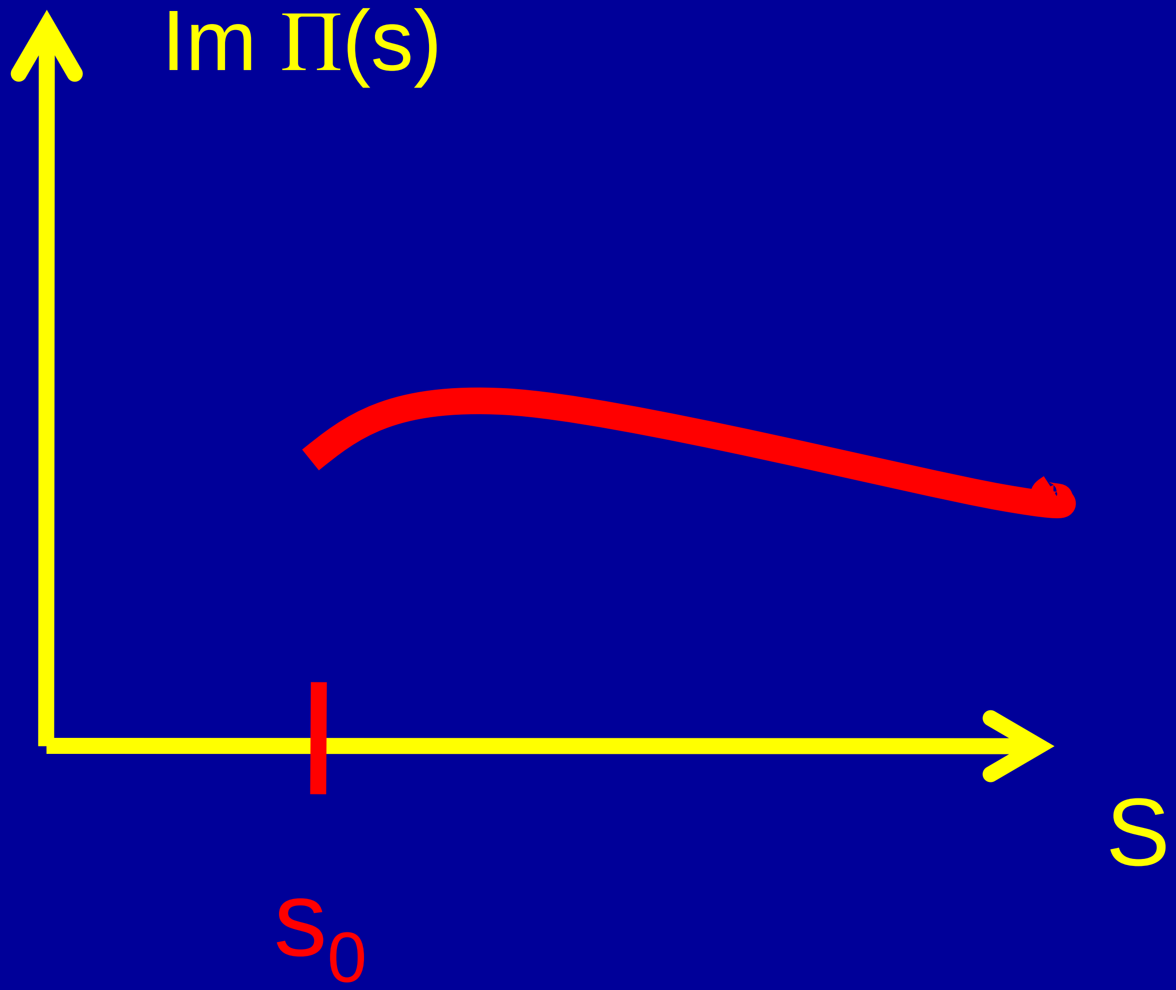
In the past, we have used the thermal evolution of the continuum threshold (QCD Sum Rules Approach) as an effective deconfinement order parameter.

(A. Ayala, C. A. Dominguez and M. Loewe: QCD Sum Rules at finite temperature: a Review, **Adv.High Energy Phys. 2017 (2017) 9291623**)

In any hadronic channel you may explore you will find the following picture

Realistic Spectral Function





Im Π(s)
s
s0

Connection between Polyakov loop and continuum threshold

"Comparison between the continuum threshold and the Polyakov loop as deconfinement order parameters"

J. P. Carlomagno and M. Loewe

Phys. Rev D95, 036003 (2017)

The Polyakov loop is defined as

$$\phi(\vec{x}) = \frac{1}{N}\mathrm{Tr}L = N^{-1}\mathrm{Tr}\mathcal{P}\exp\left(ig\int_0^\beta A_0(\vec{x},\tau)d\tau\right)$$

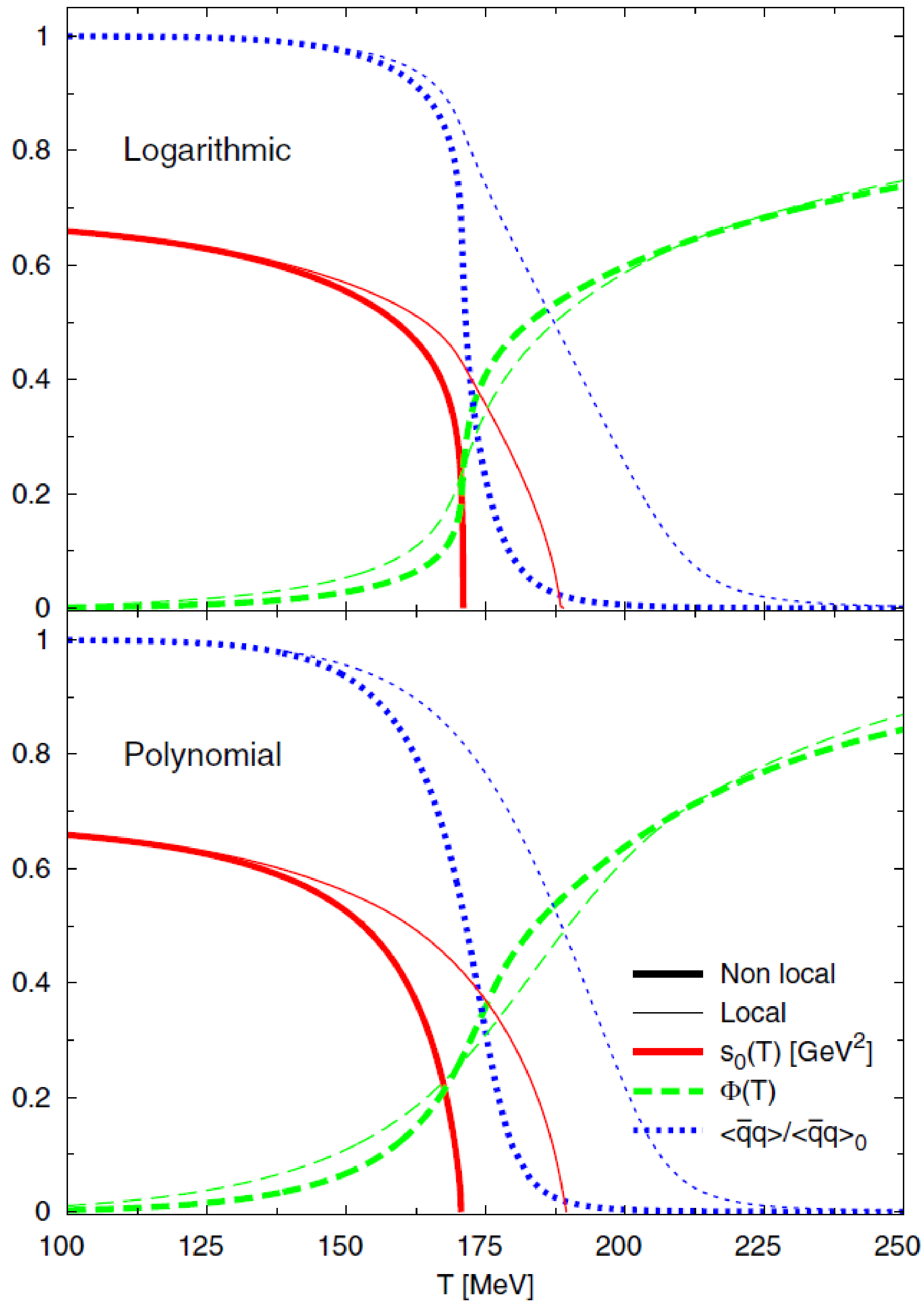
Under global Z(N) transformations it transforms as

$$\phi \to e^{i\varphi}\phi.$$

The vacuum has an N-degeneration

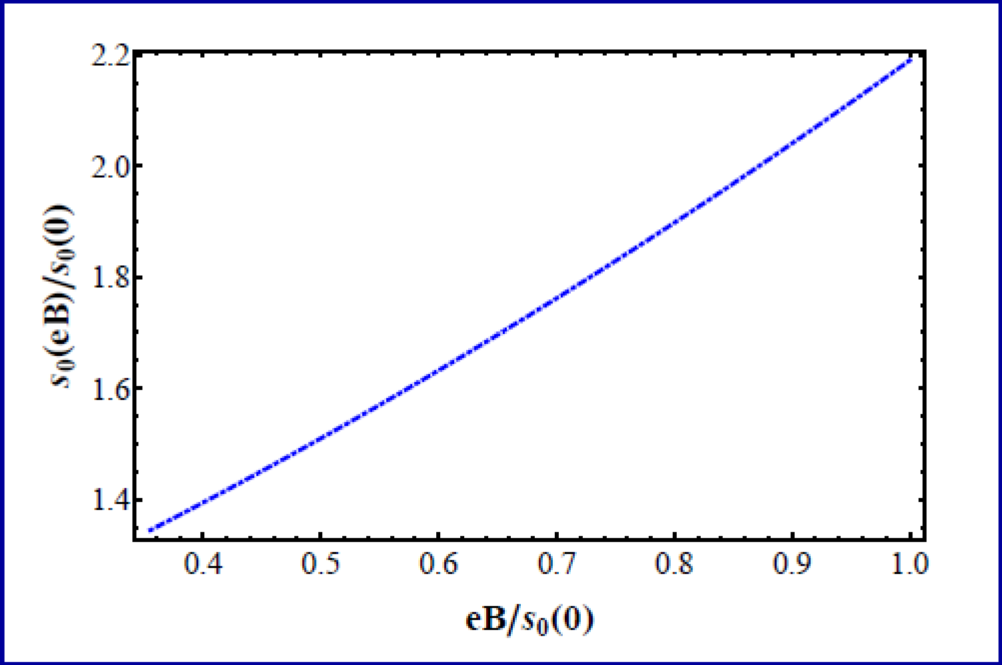
$$\langle\phi\rangle = \exp\left(\frac{i2\pi j}{N}\right)\phi_0 \quad , \quad j = 0, 1, ..., (N-1),$$

Each election of j breaks the Z(N) symmetry



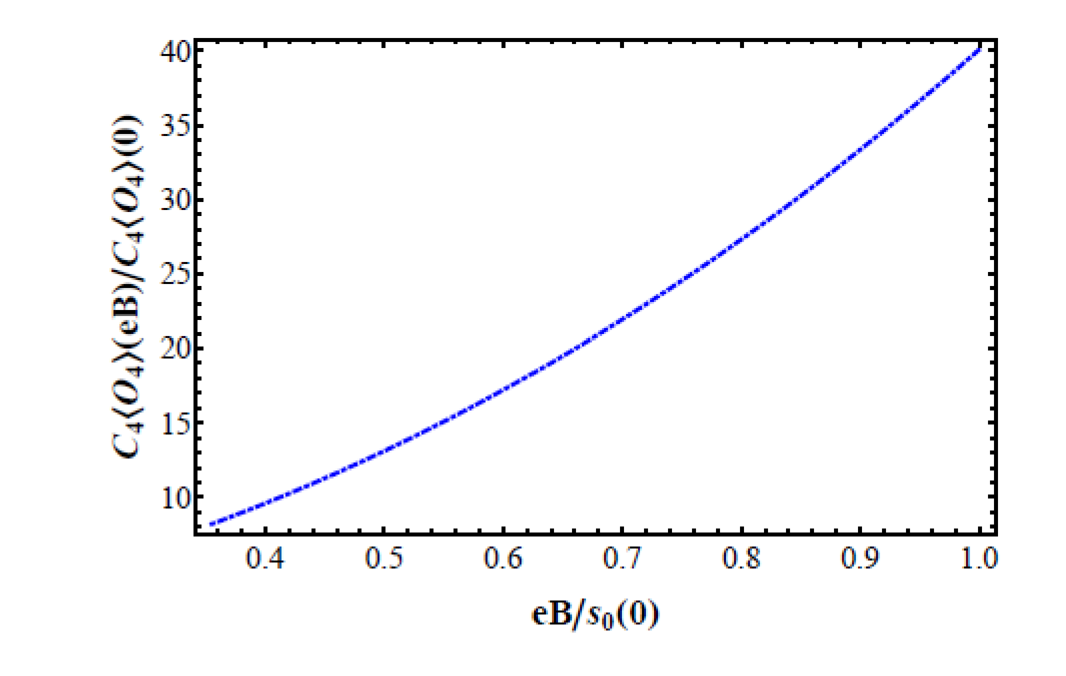
Continuum threshold (red solid line), Trace of the Polyakov loop (green dashed line) and the Normalized quark condensate (blue dotted line) as function of temperature for the non local (thick line) and local PNJL (thin line) at zero chemical potential for logarithmic (upper panel) and polynomial (lower pannel) effective potentials.

However: the magnetic field does not agree and pushes the continuum threshold to the right: A. Ayala, L. A. Hernandez, C. A. Dominguez, M. Loewe, J. C. Rojas, C. Villavicencio, PRD 92 (2015) 016006



Concerning the behavior of the gluon condensate

As an interesting result: Growing of the gluon condensate as function of an external magnetic field (no temperature effects)



In general, a π-π scattering amplitude has the form (greek indeces refer to Isospin indeces)

$$T_{\alpha\beta\,\gamma\delta} = A(s,t,u)\delta_{\alpha\beta}\delta_{\delta\gamma} + A(t,s,u)\delta_{\alpha\gamma}\delta_{\beta\gamma} + A(u,t,s)\delta_{\alpha\delta}\delta_{\beta\gamma}$$

Using the following projectors

$$P_0 = \frac{1}{3}\delta_{\alpha\beta}\delta_{\gamma\delta},$$

$$P_1 = \frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\delta} - \delta_{\alpha\delta}\delta_{\beta\gamma}\right),$$

$$P_2 = \frac{1}{2}\left(\delta_{\alpha\gamma}\delta_{\beta\delta} + \delta_{\alpha\delta}\delta_{\beta\gamma} - \frac{2}{3}\delta_{\alpha\beta}\delta_{\gamma\delta}\right)$$

In this way we find the following Isospin dependent amplitudes

$$T^0 = 3A(s,t,u) + A(t,s,u) + A(u,t,s),$$
$$T^1 = A(t,s,u) - A(u,t,s),$$
$$T^2 = A(t,s,u) + A(u,t,s).$$

The elastic regime refers to the region $4M_\pi^2 < s < 16M_\pi^2$

See J. Gasser and H. Leutwyler: Ann. Phys. 158, 142 (1984); J. Donoghue, E. Golowich, and B. Holstein: The Dynamics of the Standard Model, Cambridge 1996

Scattering lengths were introduced as appropriate paramaters for the description of nucleon-nucleon and pion nucleon scattering in the good old times.

Consider a partial wave expansion, projected into some isospin channel I.

$$T^I(s,t,u) = 32\pi \sum_{\ell=0}^{\infty} (2\ell+1) P_\ell(\cos\theta) T_\ell^I(s)$$

Coming close to the threshold

$$T_\ell^I = \left(\frac{s}{s-4m_\pi^2}\right)^{1/2} \frac{1}{2i} \left(e^{2i\delta_\ell^I(s)} - 1\right)$$

In fact, the previous expression can be expanded as

$$\Re\left(T_\ell^I\right) = \left(\frac{p^2}{m_\pi^2}\right)^\ell \left(a_\ell^I + \frac{p^2}{m_\pi^2} b_\ell^I + \cdots\right)$$

$$p^2 \equiv (s - 4m_\pi^2)/4$$

Where a_l^I are the so called scattering lengths. The next coefficients, b_l^I are the scattering slopes. It can be shown that

$$a = \lim_{p \to 0} \frac{\tan \delta_0}{p} = -\lim_{p \to 0} \frac{\delta_0}{p}$$

This definition can be extended to the other angular momentum channels

$$a_\ell = \lim_{p\to 0} \frac{\tan\delta_\ell}{p} = -\lim_{p\to 0}\frac{\delta_\ell}{p}$$

In general we have the following order

$$|a_0| > |a_1| > |a_2| > \cdots$$

At very low energies, scattering is dominated by the s-channel amplitude (isotropic scattering) and the total cross section is given by $\sigma = 4\pi\, a_0^2$. Notice that the Martin inequalities are satisfied

A. Martin, Nuovo Cimento A47 (1967) 265

$$a^I_{l+2} \leqslant a^I_l \frac{(l+1)(l+2)}{4(2l+3)(2l+5)}$$

π-π scattering lengths were measured first by L. Rossellet et al, PRD 15, 574 (1977) using pions coming from the decay

$$K^+ \to \pi^+ \pi^- e^+ \nu$$

The interaction occurs between two real pions, the only hadrons in the final state.The quantum states of the di-pion in the decay are L = 0, I = 0 and L =1, I = 1.
The invariant mass distribution of the di-pions has a maximum relatively close to the π-π threshold.

Later: B. Adeva et al, Phys. Lett. B 704 (2011) 24 (Dirac experiment)

The DIRAC experiment at CERN has achieved a sizeable production of $\pi^+\pi^-$ atoms and has significantly improved the precision on its lifetime determination. From a sample of 21 227 atomic pairs, a 4% measurement of the S-wave $\pi\pi$ scattering length difference $|a_0 - a_2| = (0.2533^{+0.0080}_{-0.0078}|_{\text{stat}}{}^{+0.0078}_{-0.0073}|_{\text{syst}})M^{-1}_{\pi^+}$ has been attained, providing an important test of Chiral Perturbation Theory.

A pionium atom, with a radius of 378 fm, decays into a pair of neutron pions.

$$\Gamma_{2\pi^0} = \frac{2}{9}\alpha^3 p^\star (a_0 - a_2)^2 (1+\delta) M_{\pi^+}^2$$

$$p^\star = \sqrt{M_{\pi^+}^2 - M_{\pi^0}^2 - (1/4)\alpha^2 M_{\pi^+}^2}$$

More recentely: X.-H Liu, F.-K. Guo and E. Epelbaum, Eur. Phys. J. C73, 2284, (2013) were able to extract the scattering lengths from heavy qurakonium decays

Charge-exchange rescattering $\pi^+\pi^- \rightarrow \pi^0\pi^0$ leads to a cusp effect in the $\pi^0\pi^0$ invariant mass spectrum of processes with $\pi^0\pi^0$ in the final state which can be used to measure $\pi\pi$ S-wave scattering lengths. Employing a non-relativistic effective field theory, we discuss the possibility of extracting the scattering lengths in heavy quarkonium $\pi^0\pi^0$ transitions. The transition $\Upsilon(3S) \rightarrow \Upsilon(2S)\pi^0\pi^0$ is studied in details. We discuss the precision that can be reached in such an extraction for a certain number of events.

$$\psi'(P_{\psi'}) \to \pi^0(p_1)\pi^0(p_2)J/\psi(p_3),$$

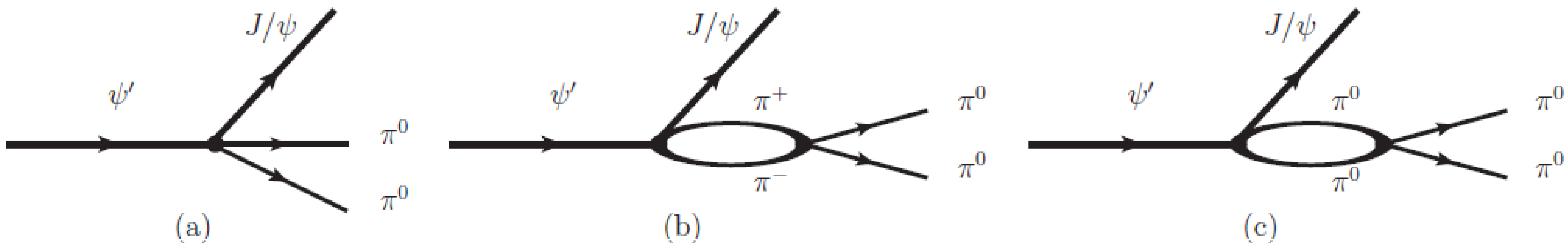


Figure 1: $\psi' \to J/\psi\pi^0\pi^0$ via tree diagram and $\pi\pi$ rescattering diagrams.

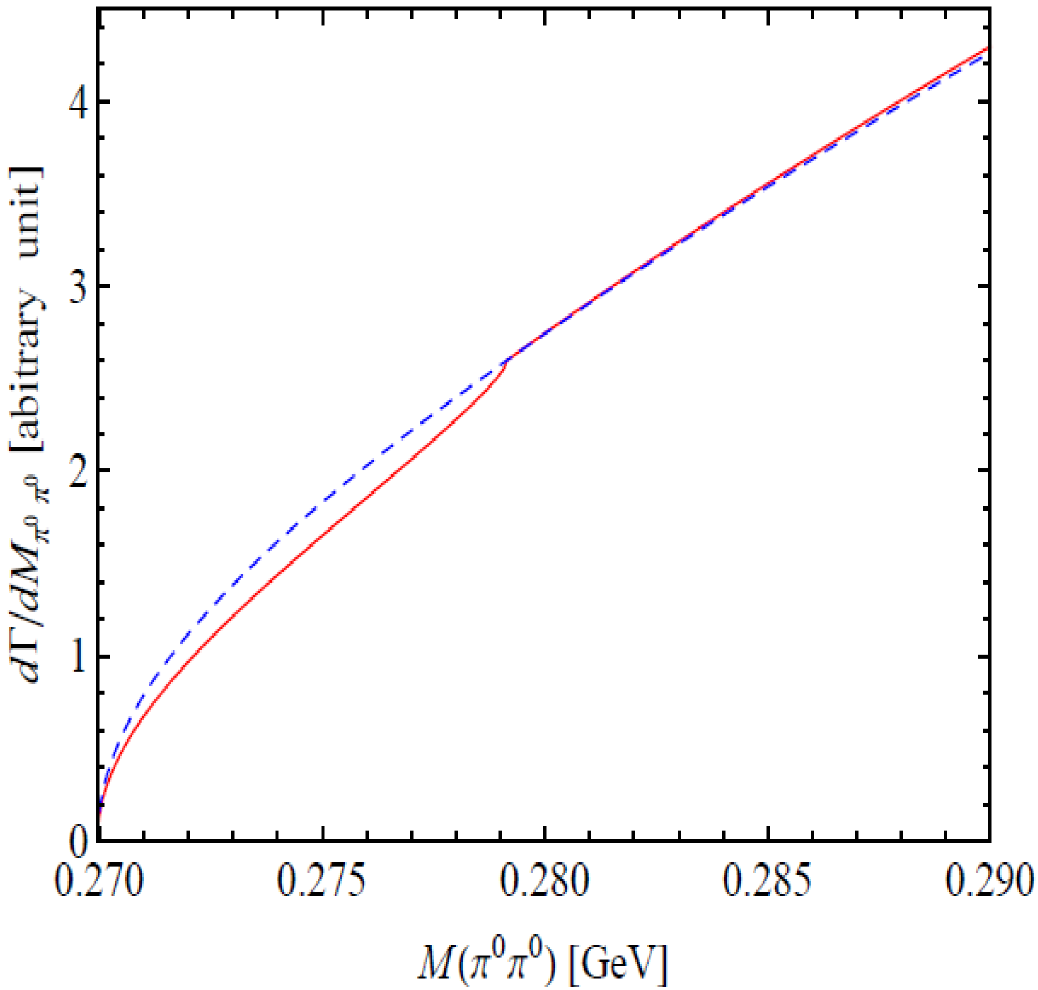


Figure 4: The cusp effect at the $\pi^+\pi^-$ threshold in the reaction $\Upsilon(3S) \to \Upsilon(2S)\pi^0\pi^0$ calculated in the NREFT framework (solid line). The dashed line shows the result without charge-exchange rescattering.

Linear sigma model. Gell-Mann-Levy 1960

$$\mathcal{L} = \bar{\psi}[i\gamma^\mu \partial_\mu - m_\psi - g(s + i\vec{\pi}\cdot\vec{\tau}\gamma_5)]\psi + \frac{1}{2}[(\partial\vec{\pi})^2 + m_\pi^2 \vec{\pi}^2] + \frac{1}{2}[(\partial\sigma)^2 + m_\sigma^2 s^2] - \lambda^2 v s(s^2 + \vec{\pi}^2) - \frac{\lambda^2}{4}(s^2 + \vec{\pi}^2)^2 + (\varepsilon c - v m_\pi^2)s.$$

The σ field has been shifted

$$\sigma = s + v.$$

$$\langle s \rangle = 0$$

$$v = \langle \sigma \rangle$$

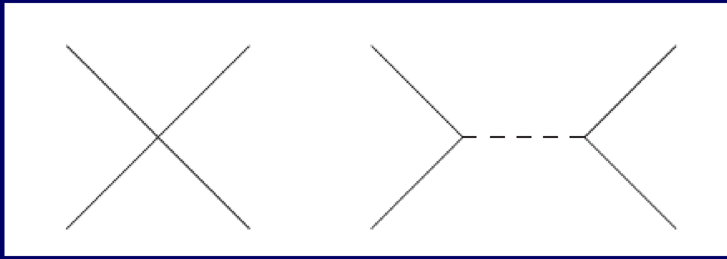
cσ is the term that breaks the SU(2)xSU(2) symmetry explicitly

All masses are determined by v, the expectation value of the σ field

$$m_\psi = gv \qquad m_\pi^2 = \mu^2 + \lambda^2 v^2 \qquad m_\sigma^2 = \mu^2 + 3\lambda^2 v^2$$

Perturbation theory at the tree level allows to identify $f_\pi = v$

Tree level diagrams for the π-π scattering lengths



$$T^0(s,t,u) = -10\lambda^2 - \frac{12\lambda^4 v^2}{s - m_\sigma^2} - \frac{4\lambda^4 v^2}{t - m_\sigma^2} - \frac{4\lambda^4 v^2}{u - m_\sigma^2}$$

$$T^1(s,t,u) = \frac{4\lambda^4 v^2}{u - m_\sigma^2} - \frac{4\lambda^4 v^2}{t - m_\sigma^2},$$

$$T^2(s,t,u) = -4\lambda^2 - \frac{4\lambda^4 v^2}{t - m_\sigma^2} - \frac{4\lambda^4 v^2}{u - m_\sigma^2}.$$

	Experimental results	Chiral perturbation theory	Linear sigma model
a_0^0	0.218 ± 0.02	$\frac{7m_\pi^2}{32\pi f_\pi^2} = 0.16$	$\frac{10m_\pi^2}{32\pi f_\pi^2} = 0.22$
b_0^0	0.25 ± 0.03	$\frac{m_\pi^2}{4\pi f_\pi^2} = 0.18$	$\frac{49m_\pi^2}{128\pi f_\pi^2} = 0.27$
a_0^2	-0.0457 ± 0.0125	$\frac{-m_\pi^2}{16\pi f_\pi^2} = -0.044$	$\frac{-m_\pi^2}{16\pi f_\pi^2} = -0.044$
b_0^2	-0.082 ± 0.008	$\frac{-m_\pi^2}{8\pi f_\pi^2} = -0.089$	$\frac{-m_\pi^2}{8\pi f_\pi^2} = -0.089$
a_1^1	0.038 ± 0.002	$\frac{m_\pi^2}{24\pi f_\pi^2} = 0.030$	$\frac{m_\pi^2}{24\pi f_\pi^2} = 0.030$
b_1^1	...	0	$\frac{m_\pi^2}{48\pi f_\pi^2} = 0.015$

Schwinger bosonic propagator

$$i\Delta(k) = \int_0^\infty \frac{ds}{\cos(qBs)} e^{is\left(k_\parallel^2 - k_\perp^2 \frac{\tan(qBs)}{qBs} - m_\pi^2 + i\epsilon\right)}$$

Weak field expansion of the propagator (Mexican Group)

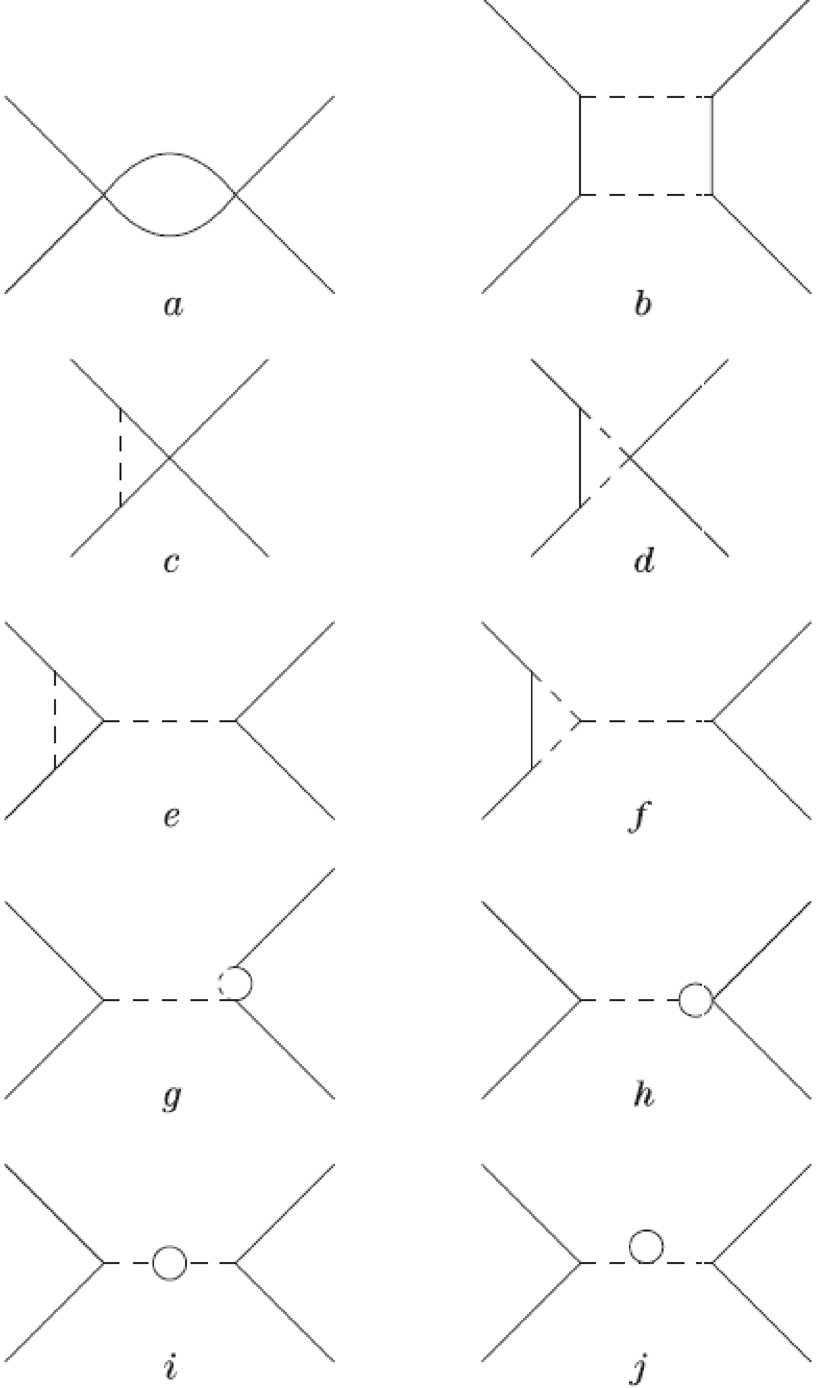
A. Ayala, A. Sanchez, G. Piccinelli, and S. Sahu: PRD 71, 023004 (2005)

$$i\Delta(k) = \frac{i}{k^2 - m_\pi^2 + i\epsilon} - \frac{i(qB)^2}{(k^2 - m_\pi^2 + i\epsilon)^3} - \frac{2i(qB)^2 k_\perp^2}{(k^2 - m_\pi^2 + i\epsilon)^4}.$$

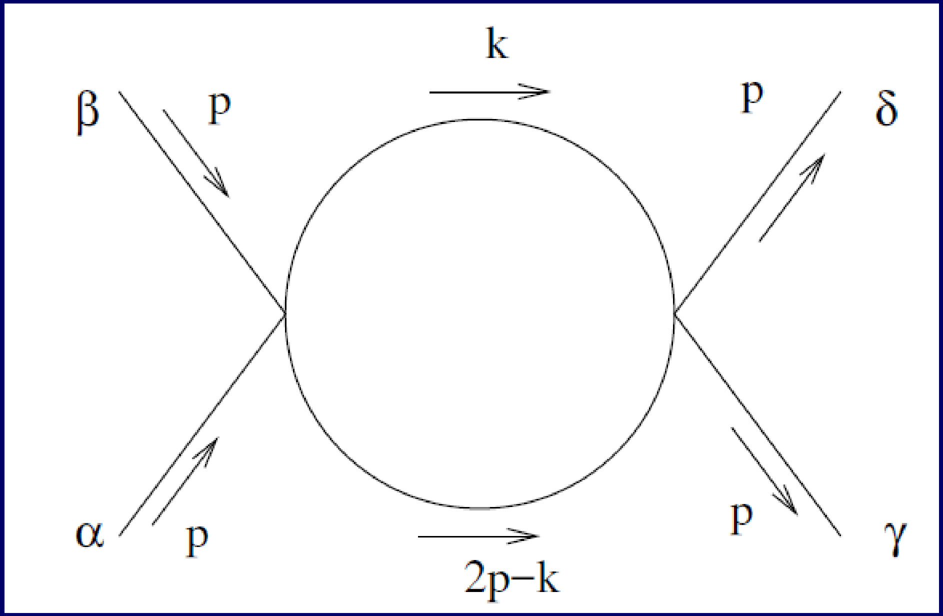
$$iD^B(k) \xrightarrow{eB \to 0} \frac{i}{k_{||}^2 - k_\perp^2 - m^2 + i\epsilon} - \frac{i(qB)^2}{\left(k_{||}^2 - k_\perp^2 - m^2 + i\epsilon\right)^3} - \frac{2i(qB)^2 k_\perp^2}{\left(k_{||}^2 - k_\perp^2 - m^2 + i\epsilon\right)^4}$$

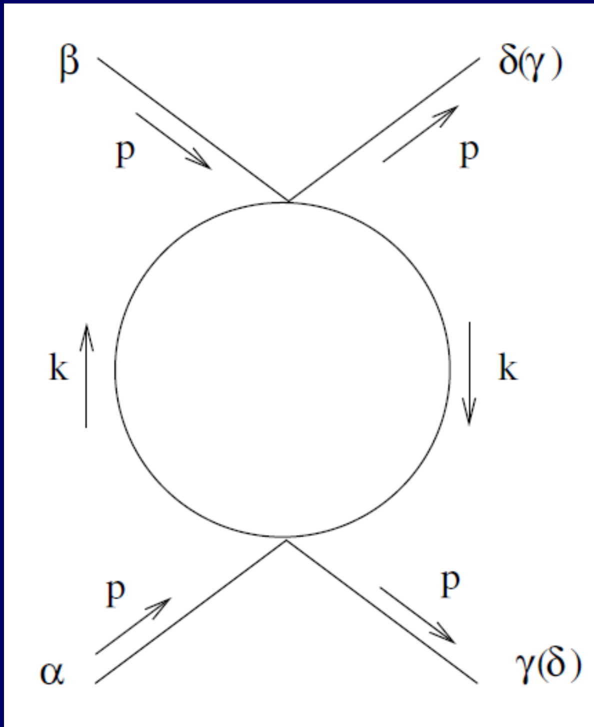
Relevant one loop diagrams in the s-channel.

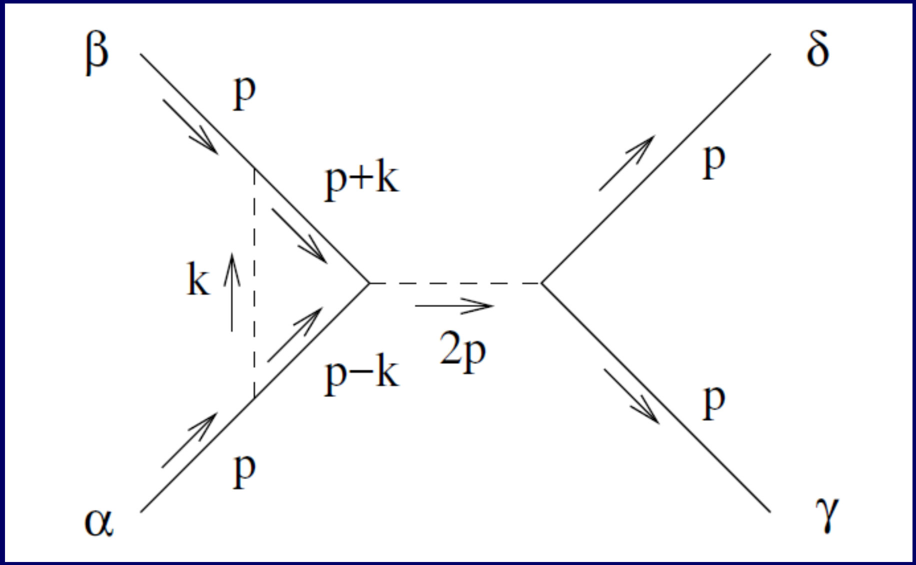
For each one of these diagrams, we have to add the corresponding diagramas in the t- and u- channels



Some s-channel and t-channel equivalent diagrams






β
p
p+k
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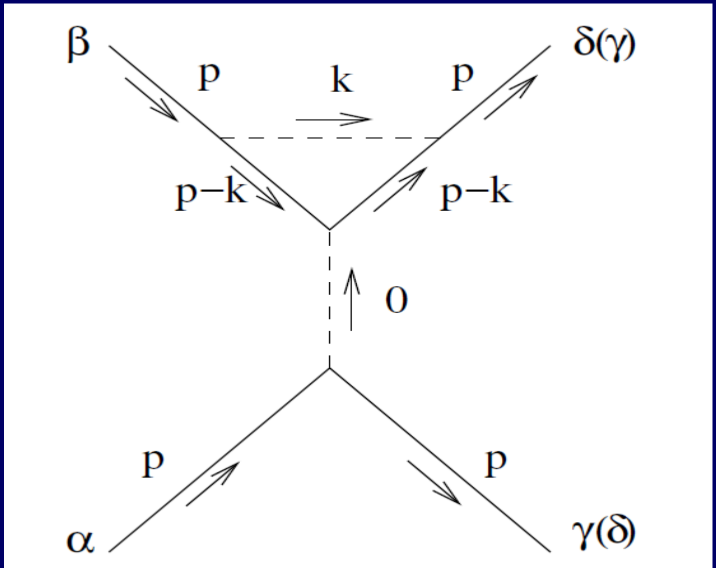
p−k
2p
p
α
δ
p
p
γ


β
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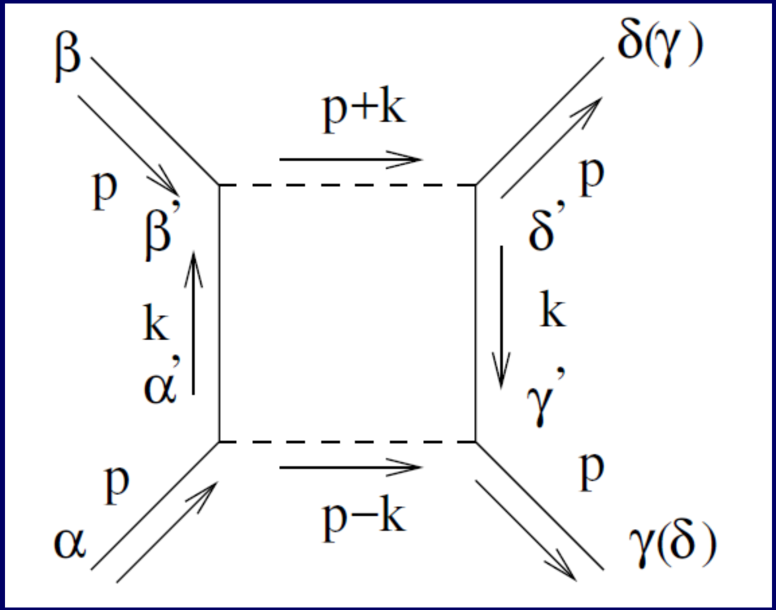
γ(δ)


β
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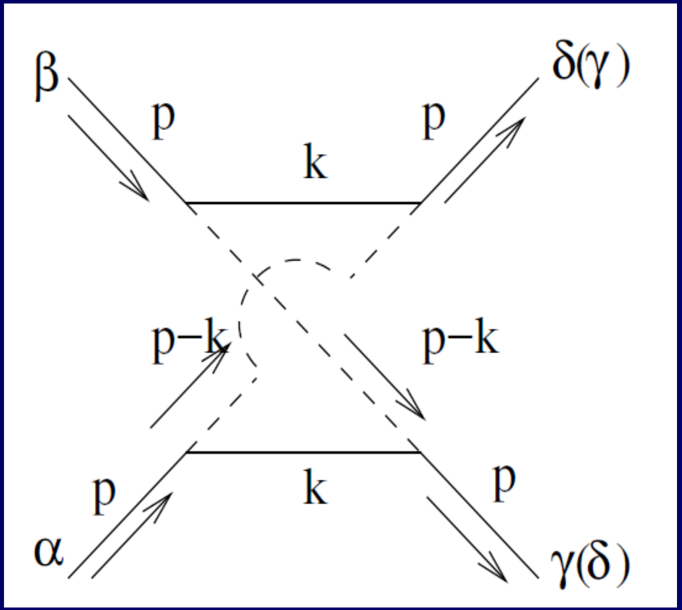
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We have used the approximation of considering the σ-meson extremely massive. Its propagator is contracted to a point.

$$\Delta_\sigma \approx -i/m_\sigma^2$$

We have checked numerically the validity of this approximation by computing box diagrams. The error is of the order $100\text{x}[Bnum - Bapprox]/Bnum \approx 0.001\%$. In this approximation we have to deal only with the first two types of diagrams (s- and t-channel). All surviving diagrams reduce to this set.

We work in the center of mass frame

$p = (2m_\pi, \vec{0})$

$$iM_{a,s} = -2\lambda^4 I_a \int \frac{d^4k}{(2\pi)^4} i\Delta(k_0, \vec{k}, m_\pi) \times i\Delta(k_0 - 2m_\pi, \vec{k}, m_\pi).$$

$$iM_{a,t} = -4\lambda^4 I_a \int \frac{d^4k}{(2\pi)^4} [i\Delta(k_0, \vec{k}, m_\pi)]^2,$$

Where we have introduced the Isospin factor

$$I_a = (7\delta_{\alpha\beta}\delta_{\gamma\delta} + 2\delta_{\alpha\gamma}\delta_{\beta\delta} + 2\delta_{\alpha\delta}\delta_{\beta\gamma}),$$

S-channel diagram:

For computing this diagram we followed the procedure introduced in: G. Piccinelli and A. Sánchez, Phys. Rev. D 96, 076014 (2017). It turns out that the S-channel diagram vanishes when we go to the threshold where, per definition, the scattering lengths are defined.

$$iL(p) = \int \frac{d^4k}{(2\pi)^4} D(k) D(p-k)$$

The idea is to intoduce a proper time representation

In tis way we get

$$D(k) = \int_0^\infty ds e^{is(k^2 - m_\pi^2 + i\epsilon)},$$

$$iL(p) = \int \frac{d^4k}{(2\pi)^4} \int_0^\infty ds_1 ds_2 e^{-i(s_1+s_2)m_\pi^2} \times e^{is_1(p-k)^2} e^{is_2k^2}.$$

After integrating the gaussian term in the loop momentum and intoducing the variables

$$s_1 = s\frac{1-v}{2} \quad \text{and} \quad s_2 = s\frac{1+v}{2}$$

We find that the imaginary part of L is given by

$$\Im L = \int_0^1 dv \frac{1}{2\pi i} \int_{-\infty}^{\infty} \frac{ds}{s} e^{is(\frac{1}{4}(1-v^2)p^2 - m_\pi^2)}$$

Using the integral representation of the Heaviside function we get

$$\Im L = \int_0^1 dv \theta \left[\frac{1}{4}(1-v^2)p^2 - m_\pi^2 \right]$$
$$= \sqrt{1 - \frac{4m_\pi^2}{p^2}} \theta(p^2 - 4m_\pi^2),$$

which vanishes when going to the threshold

$$p = (2m_\pi, \vec{0})$$

For higher powers in the denominators, that appear precisely when magnetic terms are introduced, we may use the useful identity

$$\frac{1}{N!}\left(\frac{i\partial}{\partial\mu^2}\right)^n \Delta = \Delta^{n+1}$$

$$L_{\text{general term}} \propto (qB)^2 \left(\frac{\partial}{\partial\mu_\pi^2}\right)^2 \times \int ds_1 ds_2 \frac{d^4k}{(2\pi)^4} e^{is_1(k_0^2-\vec{k}^2-m_\pi^2+i\epsilon)} \times e^{is_2((k_0-2m_\pi)^2-\vec{k}^2-\mu_\pi^2+i\epsilon)}.$$

For the t-channel diagram we follow a strategy based on the Hurwitz zeta-function. We start with the full Schwinger propagators

$$\mathscr{I}_{H1} = \int \frac{d^4k}{(2\pi)^4}\, iD(k_0,\vec{k})\, iD'(k_0,\vec{k})$$

$$= \frac{1}{(2\pi)^4}\int \frac{d^4k\; ds\; ds'}{\cos(qBs)\cos(qBs')} e^{is(k_{||}^2 - k_\perp^2 \frac{\tan(qBs)}{qBs} - m_\pi^2 + i\epsilon)} e^{is'(k_{||}^2 - k_\perp^2 \frac{\tan(qBs')}{qBs'} - m_\pi^2 + i\epsilon)}$$

Integrating in the transverse momentum

$$\pi \int_0^\infty d(k_\perp^2) e^{-ik_\perp^2(\tan(qBs)+\tan(qBs'))/qB} = -\frac{i\pi qB}{\tan(qBs)+\tan(qBs')}$$

We get

$$iM_{a,t} = -\frac{8\lambda^4 \pi qB}{(2\pi)^4} \sum_{l=0}^{\infty} \int dk_0 dk_3 \times \frac{(-1)}{(qB(2l+1) - k_{\|}^2 + m_\pi^2 - i\epsilon)^2}$$

$$k_{\|}^2 = k_0^2 - k_3^2$$

Using Plemelj's decomposition and the mass derivative

$$iM_{a,t} = -\frac{8\lambda^4 \pi qB}{(2\pi)^4} \left(\frac{\partial}{\partial m_\pi^2}\right) \times \sum_{l=0}^{\infty} \int dk_0 dk_3 i\pi\delta(qB(2l+1) - k_{\|}^2 + m_\pi^2)$$

The integration in k_0 gives us

$$iM_{a,t} = -\frac{8\lambda^4\pi^2}{(2\pi)^4} i \left(\frac{\partial}{\partial m_\pi^2}\right) \times \int dk_3 \frac{qB}{\sqrt{2qB}} \zeta\left(\frac{1}{2}, \frac{1}{2} + \frac{k_3^2 + m_\pi^2}{2qB}\right)$$

Hurwitz-ζ function

$$\zeta(s,q) = \sum_{n=0}^{\infty} \frac{1}{(q+n)^s}$$

$$\zeta(s,a) = \frac{1}{2} a^{-s} + \frac{a^{1-s}}{s-1} + \frac{Z(s,a)}{\Gamma(s)}$$

$$Z(s,a) \sim \sum_{k=1}^{\infty} \frac{B_{2k}}{(2k)!} \frac{\Gamma(2k+s-1)}{a^{2k+s-1}}$$

Large a (Poincare) expansion

In our case

$$a = \frac{1}{2} + \frac{1}{2x}, \text{ where } x = \frac{qB}{k_3^2 + m_\pi^2}$$

Expanding around x = 0

$$iM_{a,t} = -\frac{8\lambda^4\pi^2 qB}{(2\pi)^4\sqrt{2qB}}\, i\left(\frac{\partial}{\partial m_\pi^2}\right) \times \int dk_3 \left[-\frac{\sqrt{2}}{\sqrt{x}} - \frac{x^{3/2}}{12\sqrt{2}} + O[x]^{7/2}\right]$$

In this way,
keeping terms up to order
B^2

$$iM_{a,t} = \frac{8\lambda^4\pi^2}{(2\pi)^4}\, i\left(\frac{\partial}{\partial m_\pi^2}\right)\left(\frac{(qB)^2}{12m_\pi^2}\right) = -\frac{\lambda^4 i(qB)^2}{24\pi^2 m_\pi^4}$$

Finally, the tadpole integral can be carried on using the same technique (without taking mass derivatives)

$$
\begin{aligned}
\mathscr{I}_{H2} &= \int \frac{d^4k}{(2\pi)^4}\, iD(k_0,\vec{k}),\\
&= \int \frac{d^4k}{(2\pi)^4}\frac{1}{\cos(qBs)} e^{is\left(k_{||}^2 - k_\perp^2 \frac{\tan(qBs)}{qBs} - m_\pi^2 + i\epsilon\right)}
\end{aligned}
$$

$$
\begin{aligned}
\mathscr{I}_{H2} &= 2\pi^2 qB \sum_{l=0}^{\infty} \int \frac{dk_3}{(2\pi)^4} \frac{1}{\sqrt{qB(2l+1)+k_3^2+m_\pi^2}}\\
&= 2\pi^2 \frac{qB}{\sqrt{2qB}} \int \frac{dk_3}{(2\pi)^4} \zeta\left(\frac{1}{2}, \frac{1}{2} + \frac{k_3^2+m_\pi^2}{2qB}\right),\\
\mathscr{I}_{H2} &= \frac{2\pi^2}{(2\pi)^4}\frac{(qB)^2}{24}\frac{2}{m_\pi^2} = \frac{(qB)^2}{96\pi^2 m_\pi^2}.
\end{aligned}
$$

Amplitude in the s-channel

$$A_B(s,t,u) = \frac{\left(\frac{1}{4m_\pi^2 - m_\sigma^2}\right)^2 \left(\frac{qB}{m_\pi}\right)^2 (\lambda^6 v^2)}{4\pi^2} - \frac{2\left(\frac{qB}{m_\pi^2}\right)^2 (\lambda^8 v^4)}{6\pi^2 m_\sigma^4},$$

Amplitudes in the t and u-channels

$$\begin{aligned} A_B(t,s,u) &= A_B(u,t,s) \\ &= -\frac{2\left(\frac{qB}{m_\pi^2}\right)^2 (\lambda^8 v^4)}{2\pi^2 m_\sigma^4} - \frac{2\left(\frac{qB}{m_\pi^2}\right)^2 (\lambda^8 v^4)}{6\pi^2 m_\sigma^4} \\ &\quad + \frac{2\left(\frac{qB}{m_\pi^2}\right)^2 (\lambda^8 v^4)}{6\pi^2 m_\sigma^4} - \frac{2\left(\frac{qB}{m_\pi^2}\right)^2 (5\lambda^6 v^2)}{12\pi^2 m_\sigma^4} \\ &\quad - \frac{2\left(\frac{qB}{m_\pi^2}\right)^2 (\lambda^6 v^2)}{12\pi^2 m_\sigma^2} - \frac{2\lambda^4 \left(\frac{qB}{m_\pi^2}\right)^2}{24\pi^2} + \frac{2\left(\frac{qB}{m_\pi}\right)^2 (\lambda^6 v^2)}{4\pi^2 m_\sigma^4} \end{aligned}$$

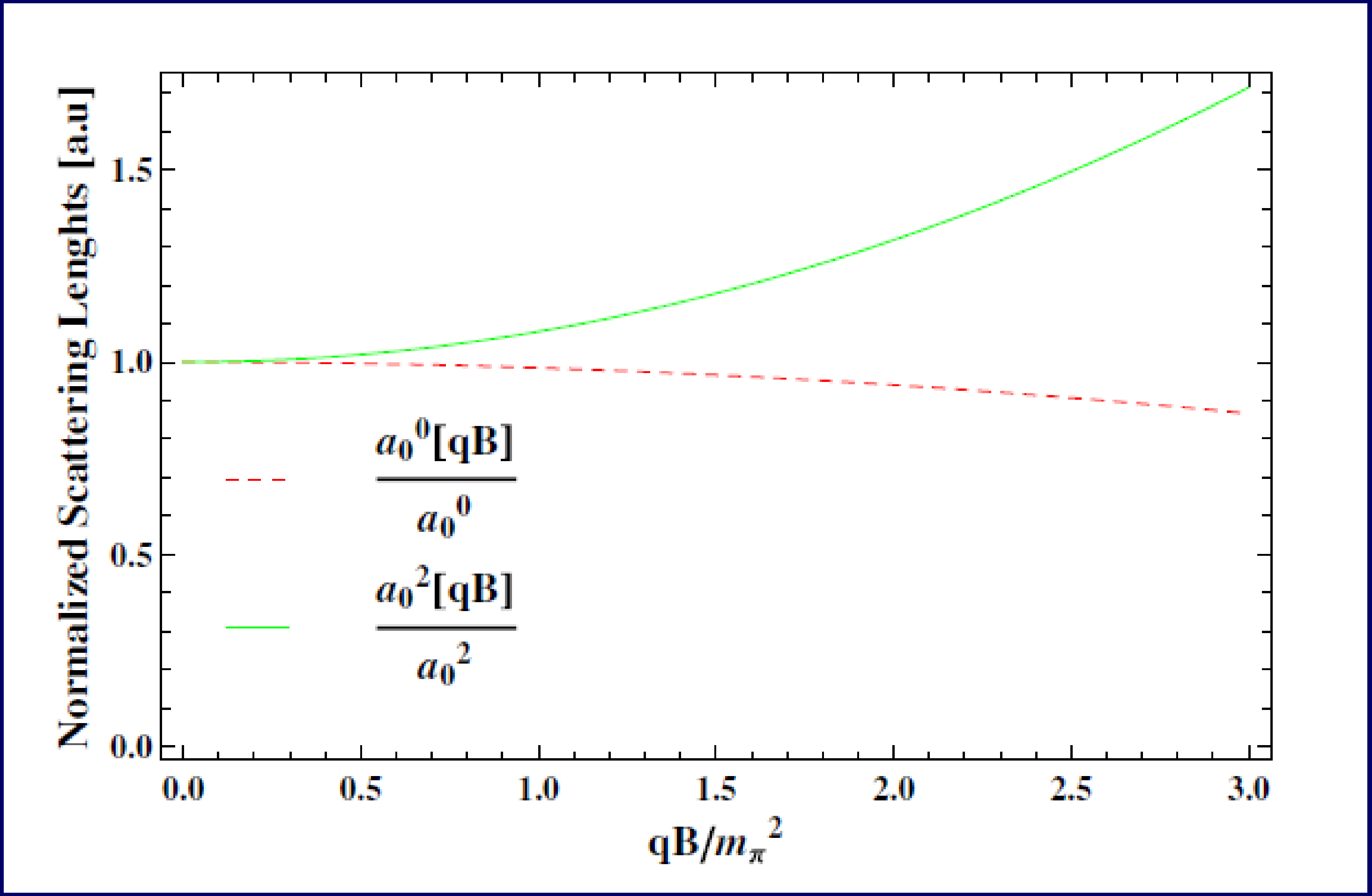
From the previous expressions we get

$$a_0^0(B) = 0.217 + \frac{3A_B(s,t,u) + 2A_B(t,s,u)}{32\pi}$$

$$a_0^2(B) = -0.041 + \frac{2A_B(t,s,u)}{32\pi}.$$

$m_\sigma = 550$ MeV $\lambda^2 = 4.26$ $v = 89$ MeV

Using m_σ= 550 MeV and m_π = 140 MeV



The thermal behavior of the scattering lengths in the linear sigma model was discussed, obtaining just the opposite behavior: M. Loewe and C. Martínez, PRD 77, 105006 (2008). The same answer is obtained in the NJL model: E. Quack et al, Phys. Lett. B348,1 (1995); N. Kayser, PRC 59,2954 (1999); M. Loewe, J. C. Rojas and J. Ruiz, PRD 78 096007 (2008)

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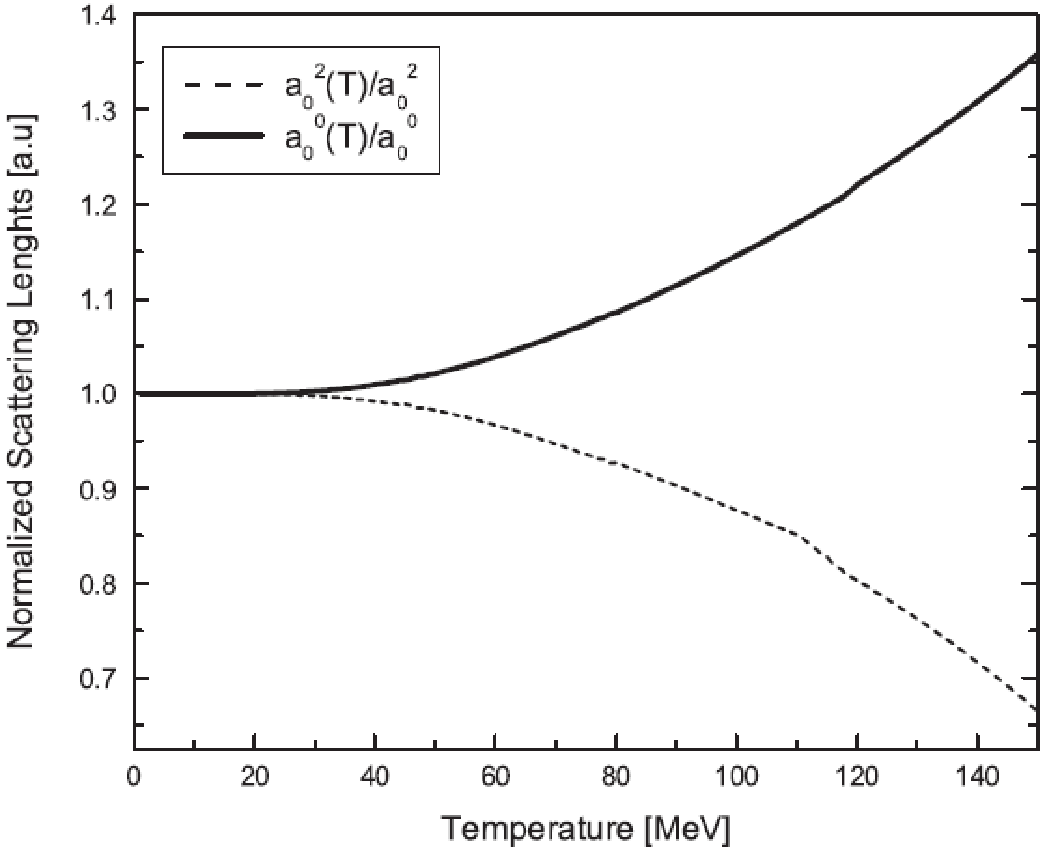


FIG. 3. Scattering lengths normalized to $T = 0$.

In the NJL model

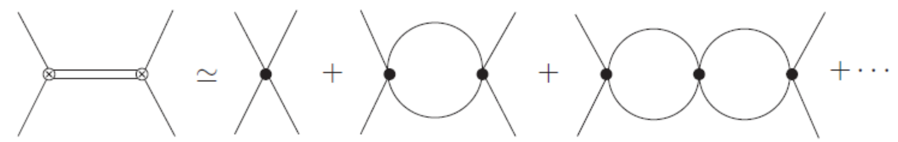
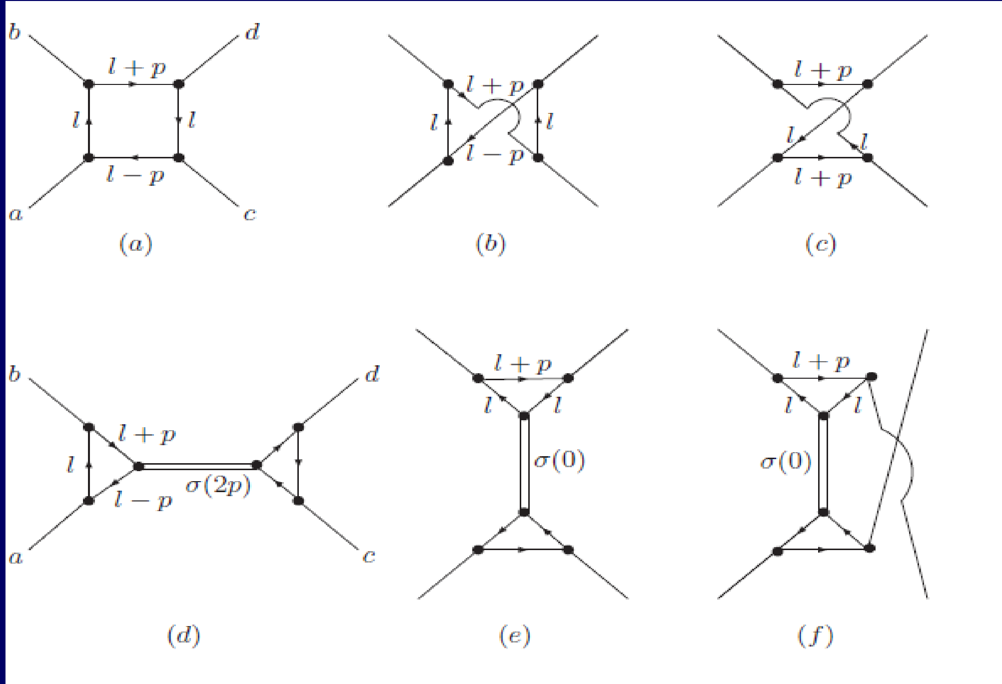


FIG. 1: Effective meson exchange modeled as a chain of quark bubbles.



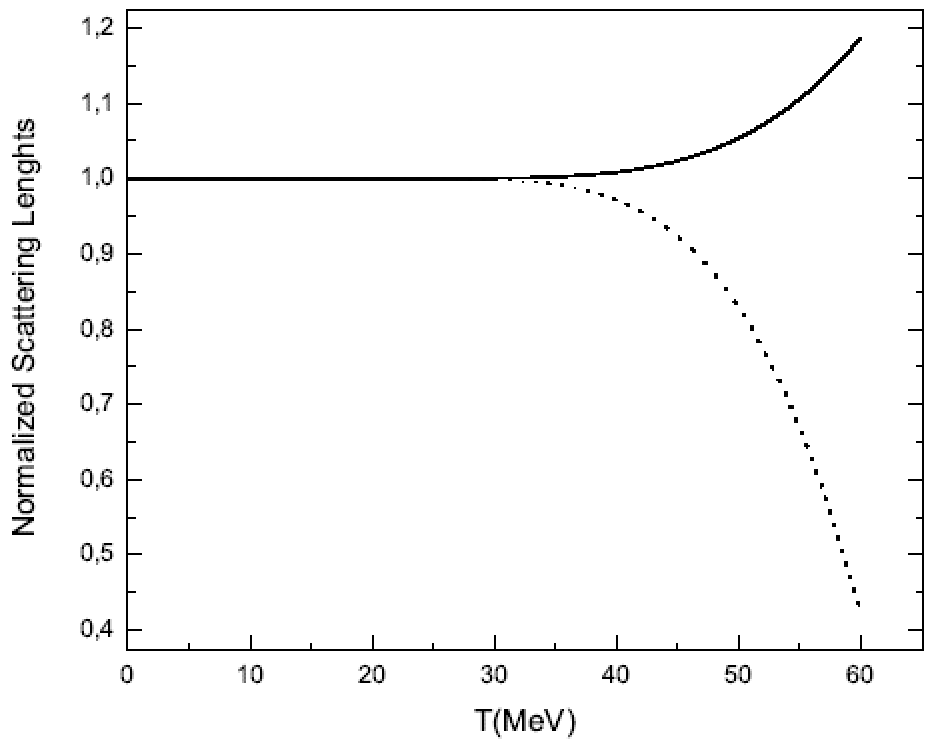


FIG. 5: Thermal dependence of the $\pi - \pi$ scattering lengths, normalized to its zero temperature value. The upper curve correspond to $a_0^{0+\beta}/a_0^0$, and the lower one to $a_2^{0+\beta}/a_2^0$. Temperatures are in MeV.

As a kind of Conclusions:

The channel I = 2 corresponds to the most symmetric state of a two-pion state in the isospin space. The increasing of the scattering lengths in I = 2 channel (due to magnetic effects) shows that the interaction between pions becomes more intense. This, in turn, can be associated to a proximity effect between pions.

Temperature effects are in competition with magnetic effects. The calculation for scattering lengths with both effects present has not been done yet.

Notice that a similar effect was found by us (A. Ayala et al, PRD 96, 034007 (2017)) in a different context (correlation lengths in NJL). The same happens with the evolution of the continuum threshold in QCD -SR