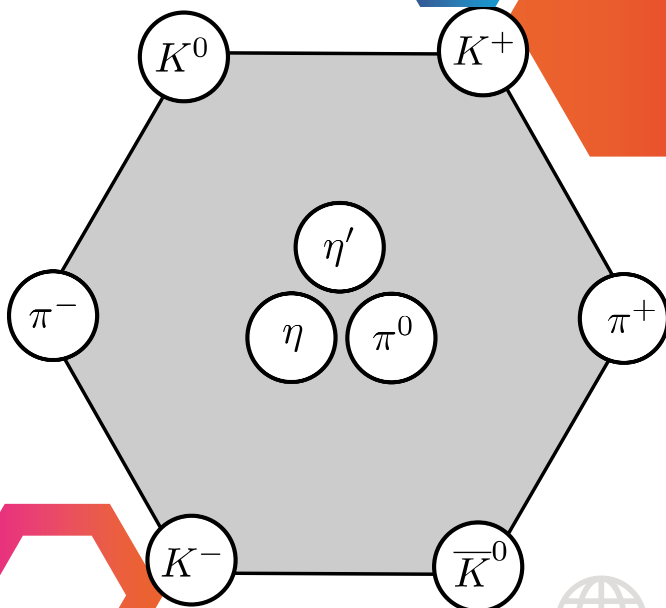


Bound-states and resonances in the DSE/BSE approach

Richard Williams
University of Gießen

RW, Fischer, Heupel PRD 93 (2016)
RW, arXiv:1804.11161

Bundesministerium für Bildung und Forschung
HIC for FAIR
Helmholtz International Center

Motivation

Extract properties of hadrons from QCD

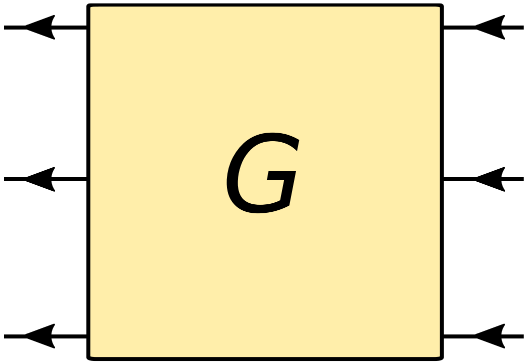
- Propagators and vertices
- Formulate description of bound-states in the continuum.

Test truncations against Hadronic Spectrum

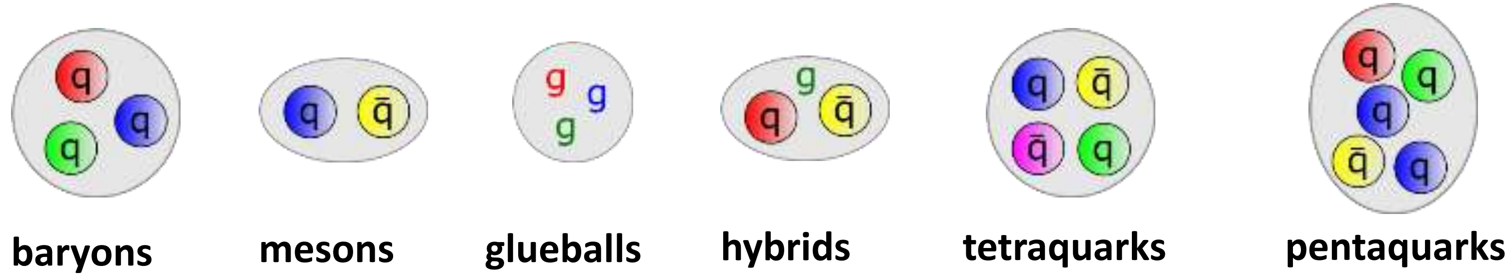
- Include/Exclude interaction terms

Interaction terms responsible for

- Binding quarks and (anti)quarks
- Unquenching effects
- Decay channels
- Splitting between parity partners …

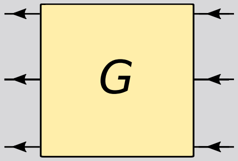


Extract from Green's functions



Hadronic states

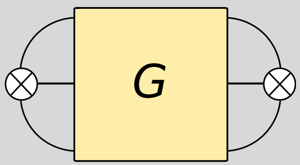
Poles

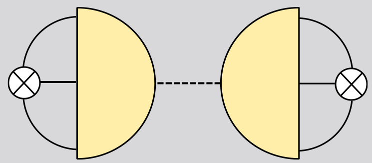


$$\mathbf{G}_{\alpha\beta\gamma;\alpha'\beta'\gamma'} = \langle 0|\, T\psi_\alpha\psi_\beta\psi_\gamma\bar{\psi}_{\alpha'}\bar{\psi}_{\beta'}\bar{\psi}_{\gamma'}\, |0\rangle$$

See Eichmann

Lattice



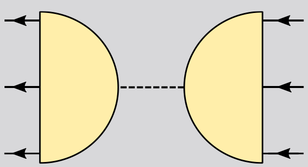


$$e^{-mt} \quad \Longleftrightarrow \quad \frac{1}{p^2+m^2}$$

gauge-invariant current correlators

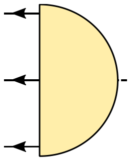
Exponential time-decay.

BSE



$$\mathbf{G} \sim \sum_\lambda \frac{\Psi^\lambda \bar{\Psi}^\lambda}{p^2+m_\lambda^2}$$

Spectral decomposition.

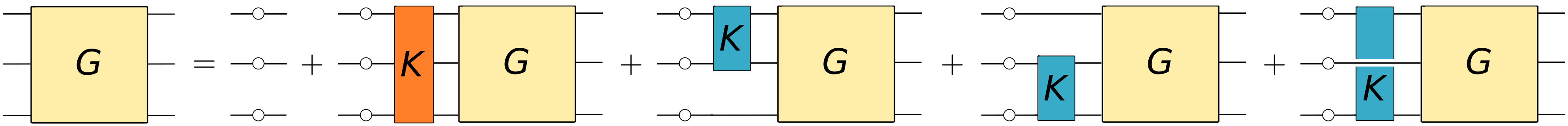


$$\Psi^\lambda_{\alpha\beta\gamma} = \langle 0|\, T\psi_\alpha\psi_\beta\psi_\gamma\, |\lambda\rangle$$

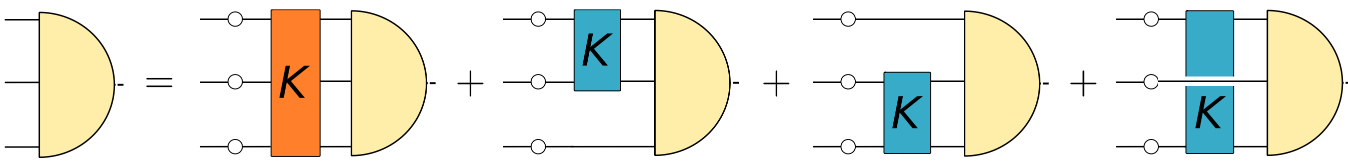
BS wavefunction

DSE and BSE

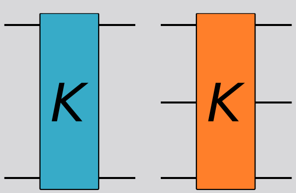
Trade one unknown **G**, for another unknown **K**



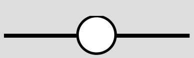
Solution (on-shell) yields Bethe-Salpeter wavefunction



See Eichmann, El-Bennich



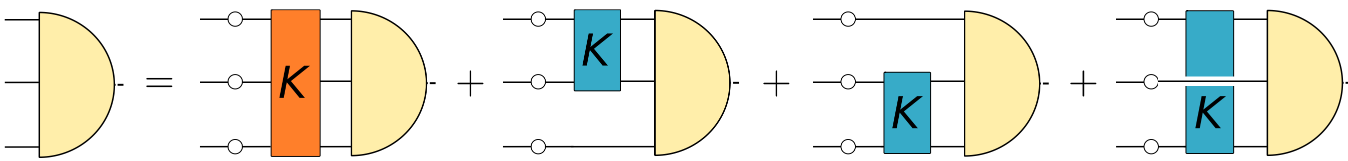
Irreducible 2-, 3-, 4-body kernels **define** equation



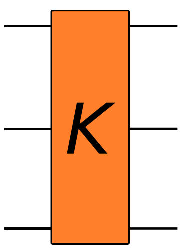
Dressed particle constituents: Green's functions

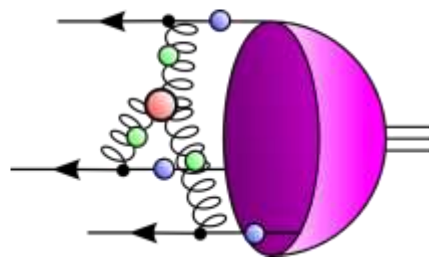
DSE and BSE

Solution (on-shell) yields Bethe-Salpeter wavefunction



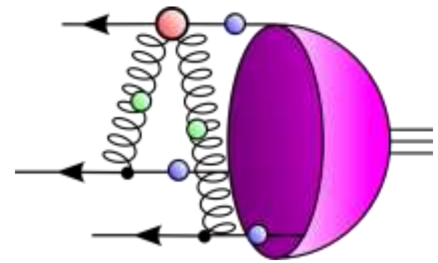
Irreducible three-body force small:



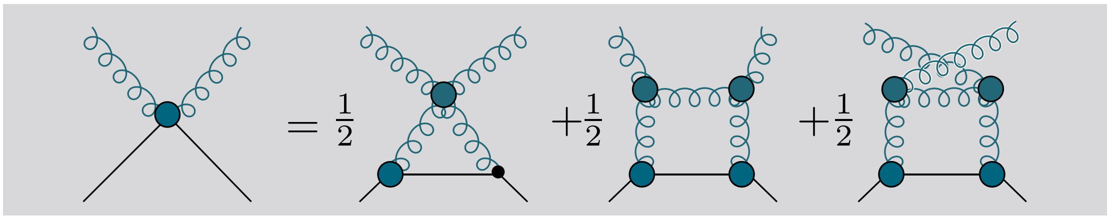


Zero by color

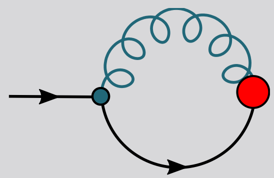
$$\epsilon_{i'j'k'} T^a_{ii'} T^b_{jj'} T^c_{kk'} i f^{abc} = 0$$



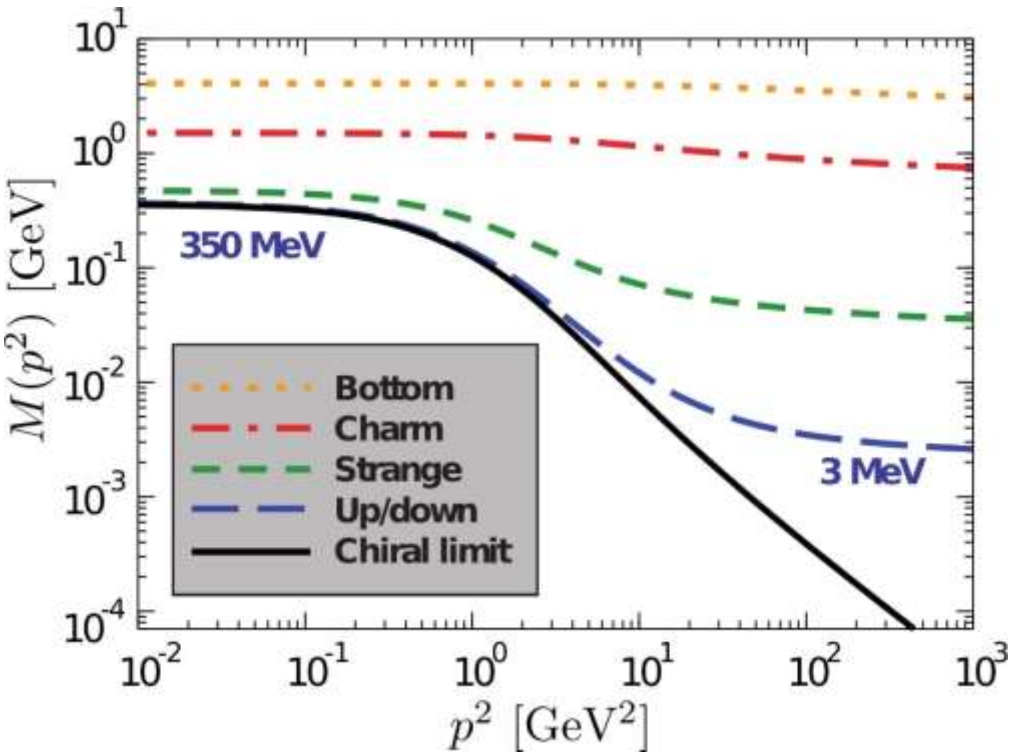
Very small



DSE

$$\frac{\delta\Gamma[\phi]}{\delta\psi} = \frac{\delta S[\phi]}{\delta\psi} +$$ 

$$S^{-1}(p) = A(p^2)\left(-i\not{p} + M(p^2)\right)$$



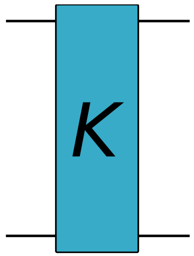
It's QCD:

- Mass function runs
- Coupling runs
- Vertices run

Everything runs!

Very difficult to disentangle in detail

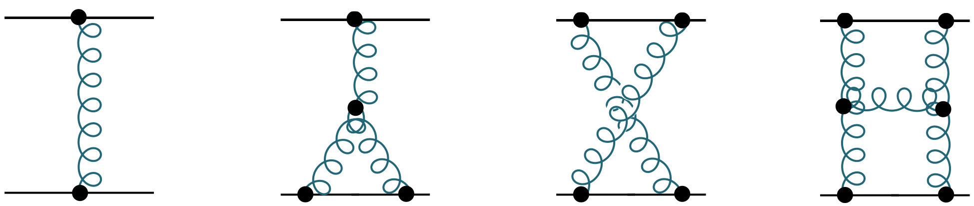
BSE



Expose corrections to the Bethe-Salpeter kernel

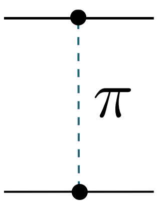
- Systematic and improvable
- Lead to meaningful inclusion of "physics"
- Preserve axial-vector Ward-Takahashi identity

Diagrammatic



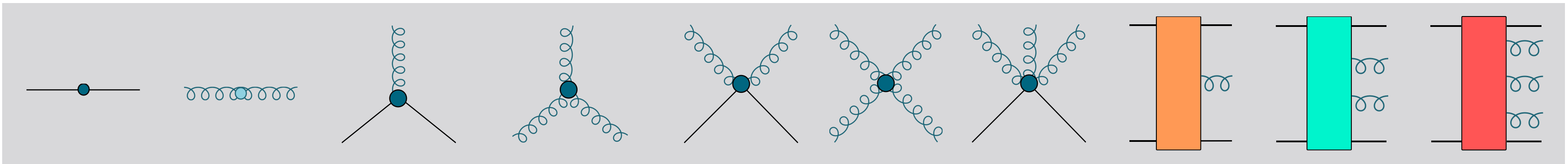
[Fischer, RW PRL 103 (2009) 122001]
[Sanchis-Alepuz, RW PLB 749 (2015) 592]
[Binosi, Chang, Papavassiliou, Qin, Roberts PRD 93 (2016) 096010]

Effective/Composite



[Fischer, Nickel, Wambach ORD 76 (2007) 094009]
[Fischer, RW PRD 78 (2008) 074006]
[Sanchis-Alepuz, Fischer, Kubrak PLB 733 (2014) 151]

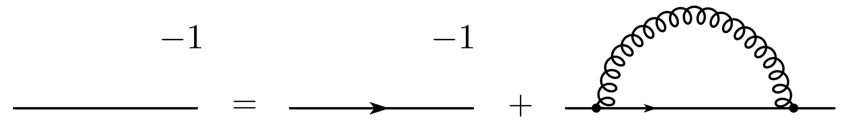
Infinite tower of coupled Green's functions to consider … *truncation*



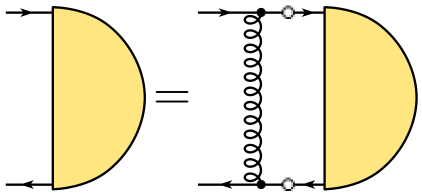
Truncation

2PI 2-loop (rainbow-ladder)

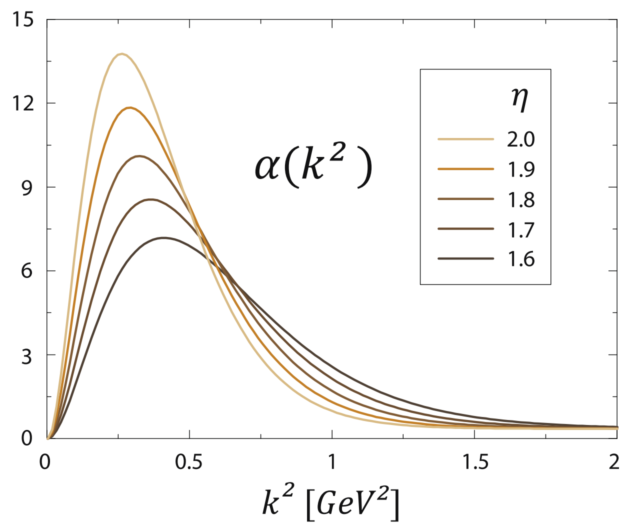
Quark DSE



Meson BSE



[Maris, Tandy PRC 60 (1999) 055214]



Euclidean space:

- Time-like properties require analytic continuation of propagators/vertices into the complex plane.

Routinely solved by standard methods

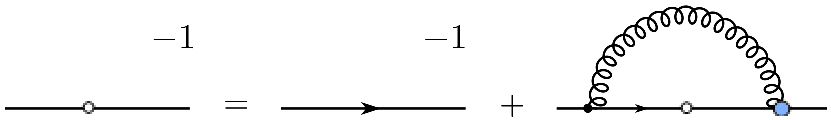
- Quark for complex momenta (Cauchy, shell-method, path deformation)
- One-loop BSE kernel independent of total momentum P

e.g. [Sanchis-Alepuz, RW, arXiv:1710.04903]

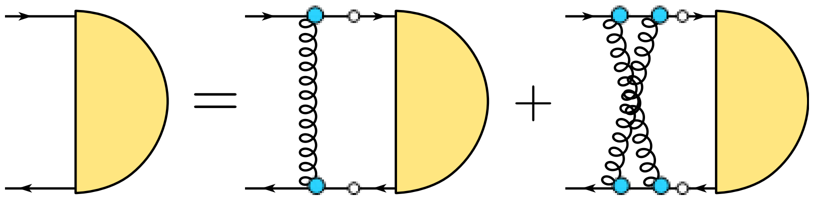
Truncation

3PI 3-loop

Quark DSE

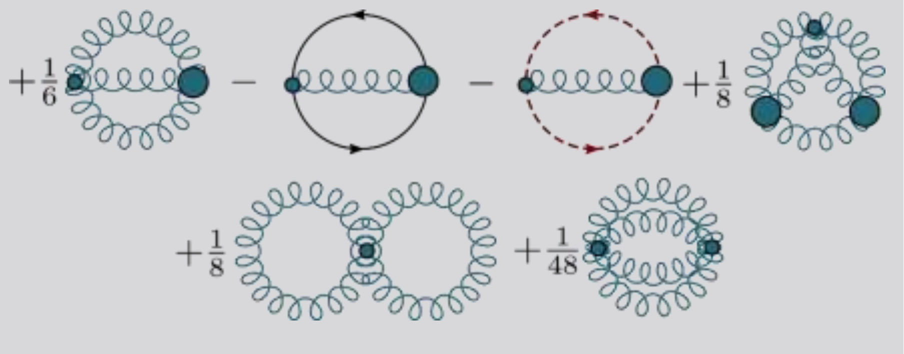


Meson BSE

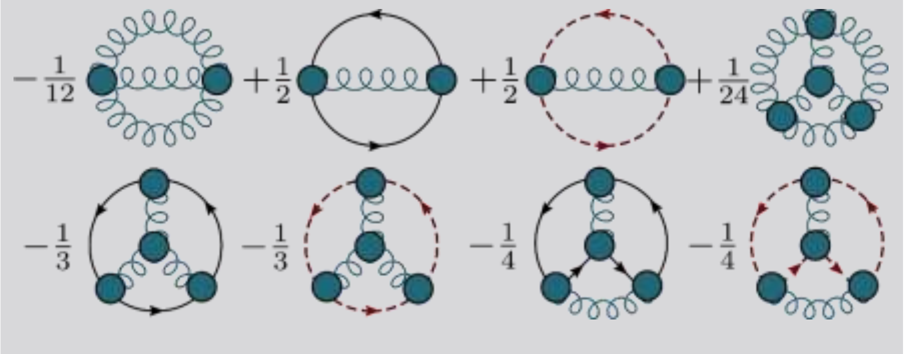


... truncate using e.g. nPI effective action

$\mathbf{\Phi^0}$: non-interacting part



$\mathbf{\Phi^{int}}$: interacting part

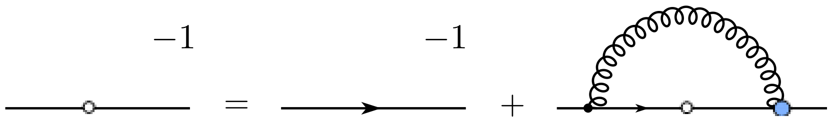


[RW, Fischer, Heupel, PRD93 (2016)]

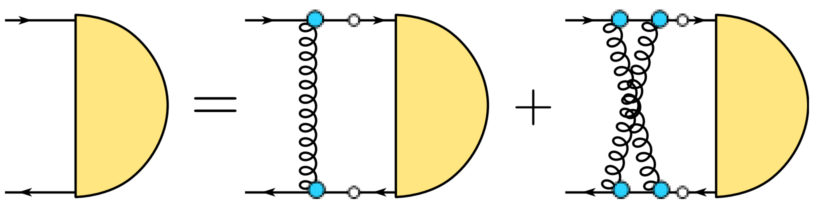
Truncation

3PI 3-loop

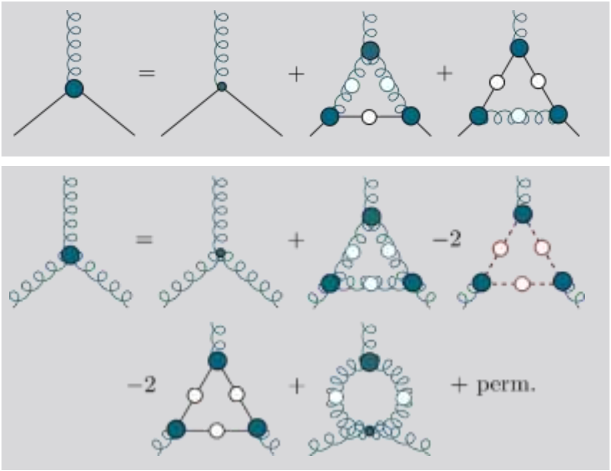
Quark DSE

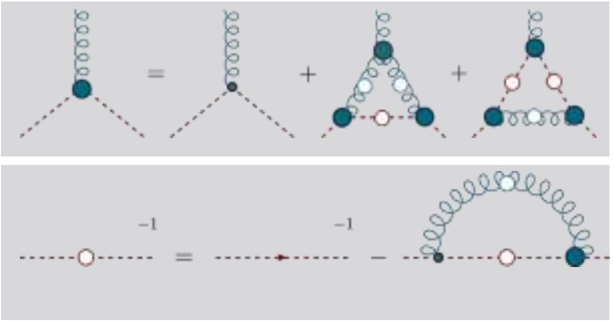


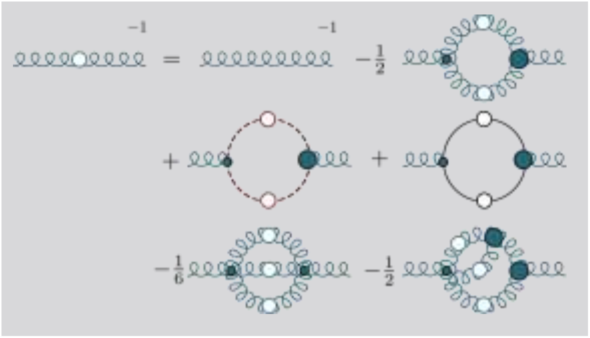
Meson BSE



truncate using e.g. nPI effective action







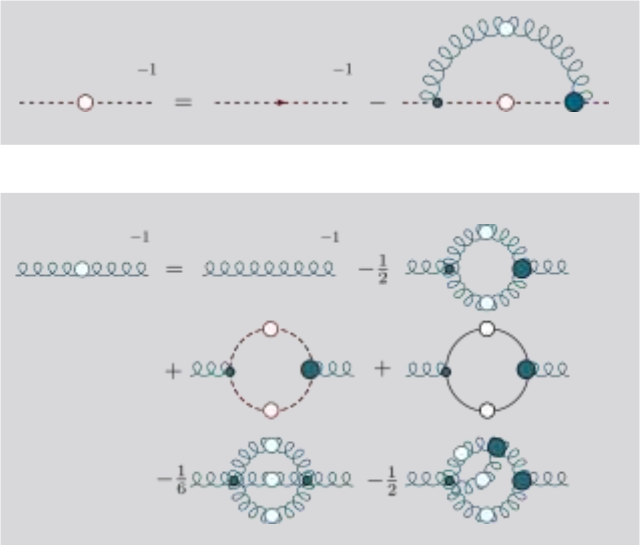
… ~19 non-linear coupled integral equations

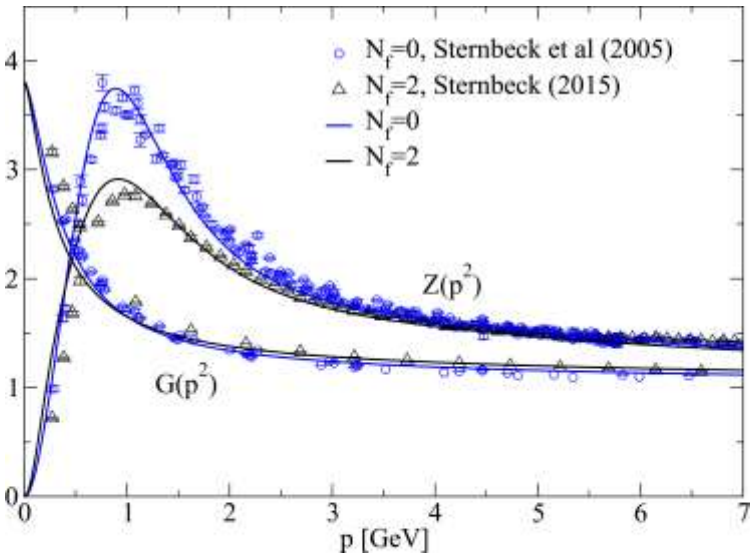
[RW, Fischer, Heupel, PRD93 (2016)]
[M. Q. Huber, EPJC77 (2017)]

Ghost/Gluon

$$D^{\mu\nu}(p) = \left(\delta^{\mu\nu} - \frac{p^\mu p^\nu}{p^2}\right)\frac{Z(p^2)}{p^2}$$

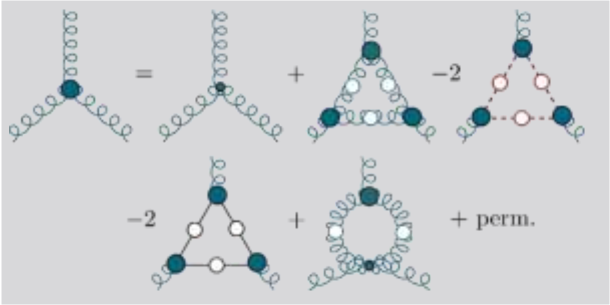
$$D_G(p) = -\frac{G(p^2)}{p^2}$$

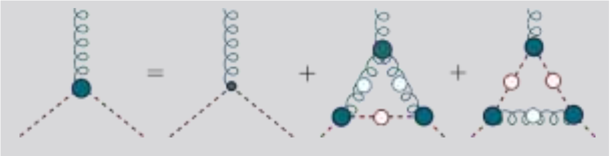




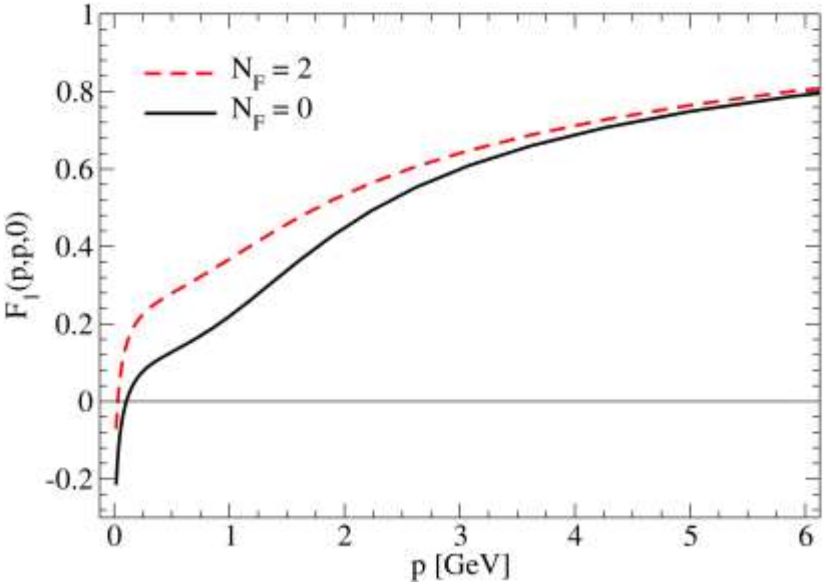
By now, convergence between different functional approaches.

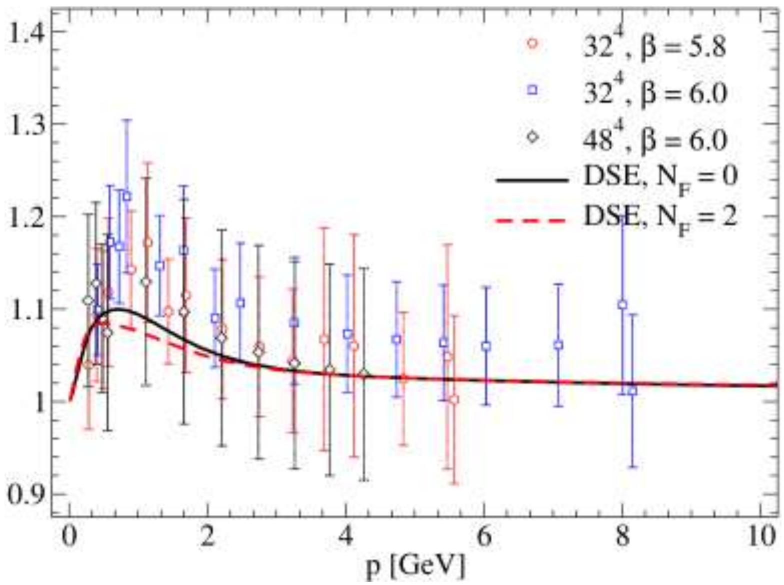
Unquenching effects due to quark loop





(tree-level tensor structure)

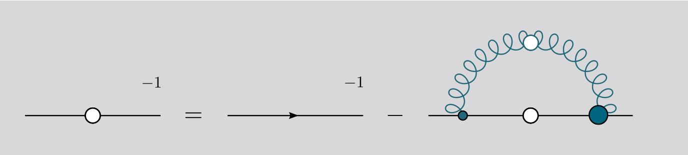




See also [Eichmann, RW, Alkofer, Vujinovic, PRD89 (2014)]
[Aguilar, Binosi, Papavassiliou, PRD89 (2014)]

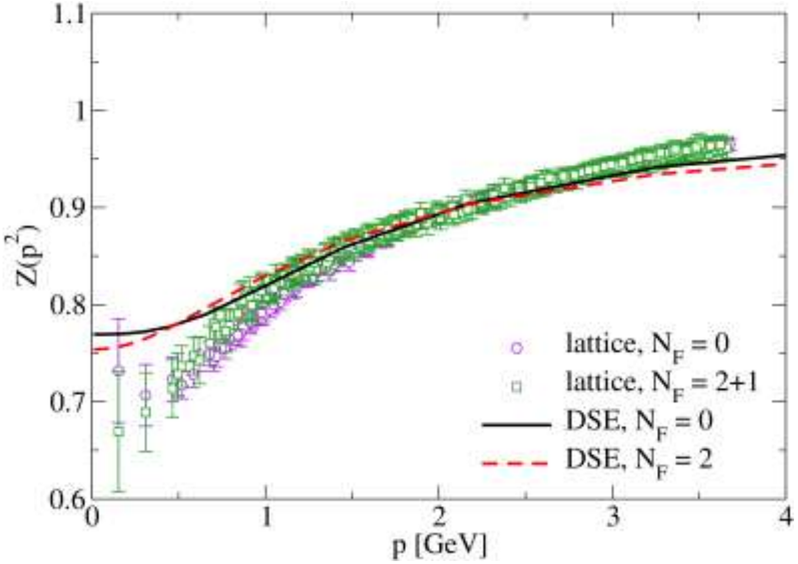
Quark

Quenched vs Unquenched

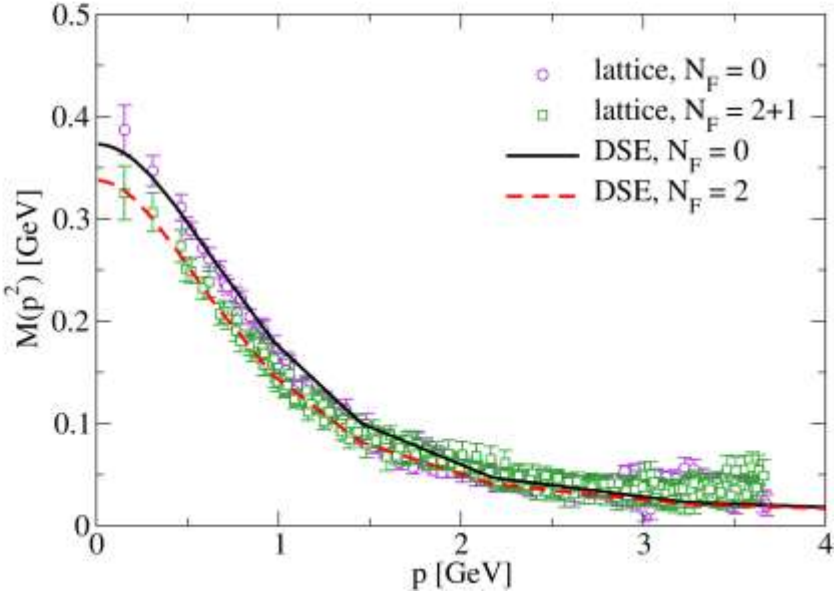


$$S^{-1}(p) = A(p^2)(-i\,\gamma\cdot p \;+ M(p^2))$$

Wavefunction



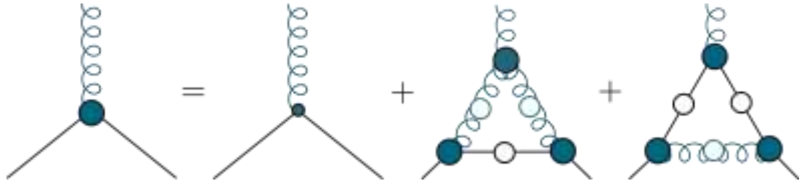
Mass function



QG Vertex

4 longitudinal and 8 transverse components

$$\Gamma^{\mu}(l,k) = \Gamma^{\mu}_{L}(l,k) + \Gamma^{\mu}_{T}(l,k)$$



Transverse part satisfies:

$$k^{\mu}\Gamma^{\mu}_{T}=0 \qquad \left(\delta_{\mu\nu} - \frac{k^{\mu}k^{\nu}}{k^{2}}\right)\Gamma^{\mu}_{T}(l,k) = \Gamma^{\mu}_{T}(l,k)$$

Longitudinal part satisfies:

$$\left(\delta_{\mu\nu} - \frac{k^{\mu}k^{\nu}}{k^{2}}\right)\Gamma^{\mu}_{L}(l,k) \neq 0$$

Not vanishing. Mixes.

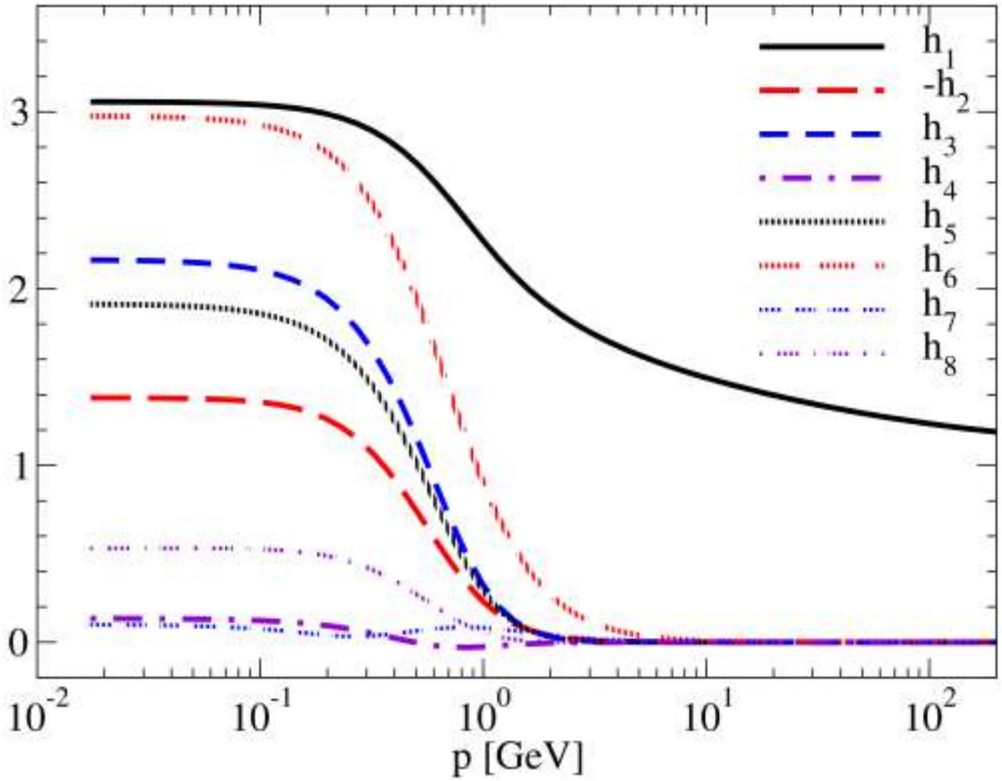
In Landau gauge, the ***transversely projected*** combination enters.

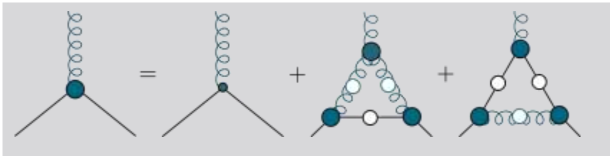
$$\left(\delta_{\mu\nu} - \frac{k^{\mu}k^{\nu}}{k^{2}}\right)\Gamma^{\mu}(l,k)$$

Constraints from STI are highly relevant but mix with transverse components

QG Vertex

8 transverse components





DCSB: important

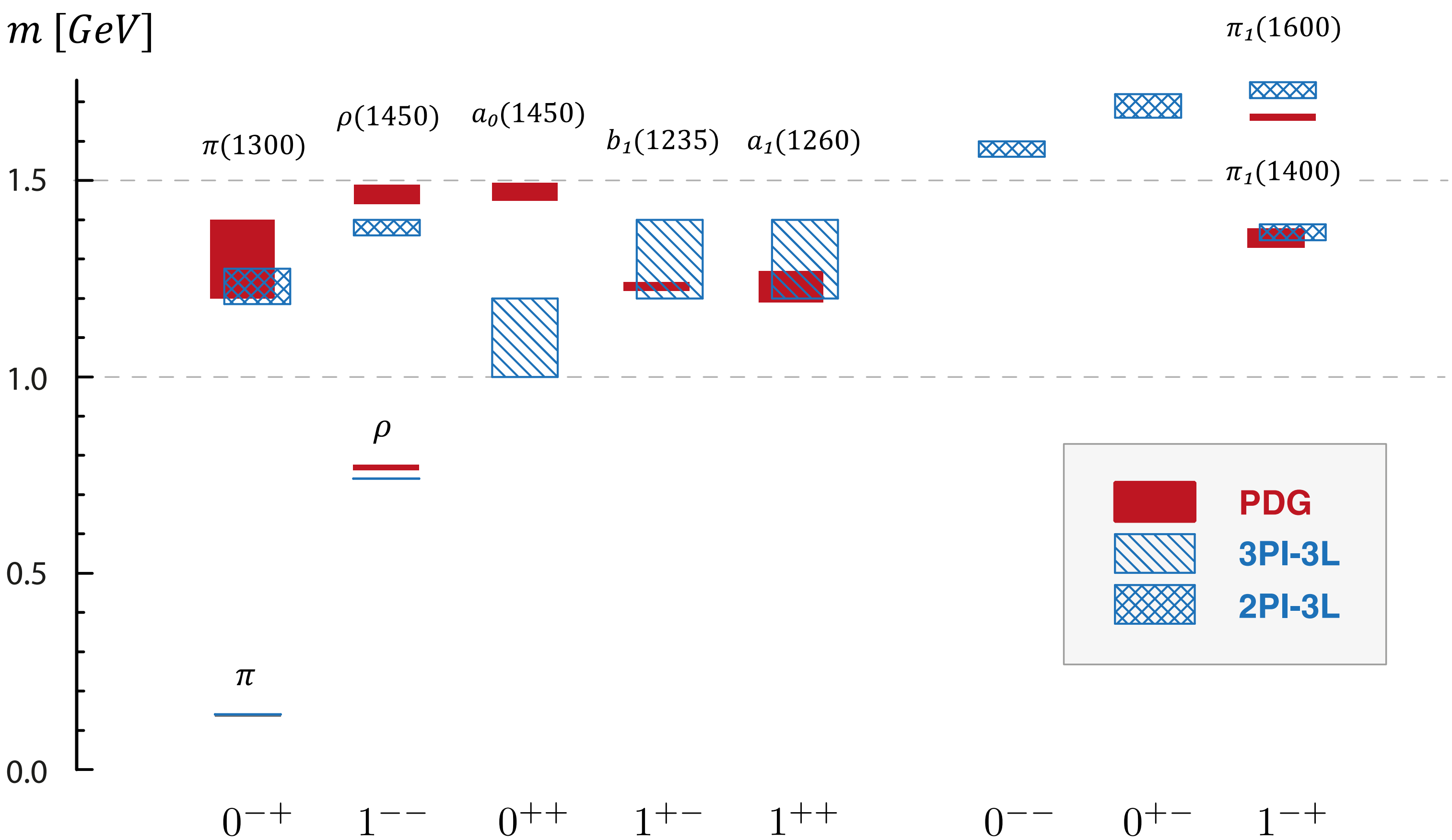
See Aguilar

Rich structure in non-classical tensor structures

[Chang, Roberts PRC 85 (2012) 052201]
[RW, Fischer, Heupel PRD 93 (2016) 034026]

$$\Gamma_{\mu}^{a}\,(l,k) = h_1\gamma_T^{\mu} + h_2 l_T^{\mu}\gamma\cdot l + h_3 i l_T^{\mu} + h_4(l\cdot \mathrm{k})\frac{i}{2}\left[\gamma_T^{\mu},\gamma\cdot l\right] + h_5\frac{i}{2}\left[\gamma^{\mu},\gamma\cdot k\right]$$
$$+h_6\frac{1}{6}\left[\gamma^{\mu},\gamma\cdot l,\gamma\cdot k\right] + h_7 t_{(kl)}^{\mu\nu}(l\cdot k)\gamma^{\nu} + h_8 t_{(kl)}^{\mu\nu}\left[\gamma^{\nu},\gamma\cdot l\right]$$

Light Spectrum

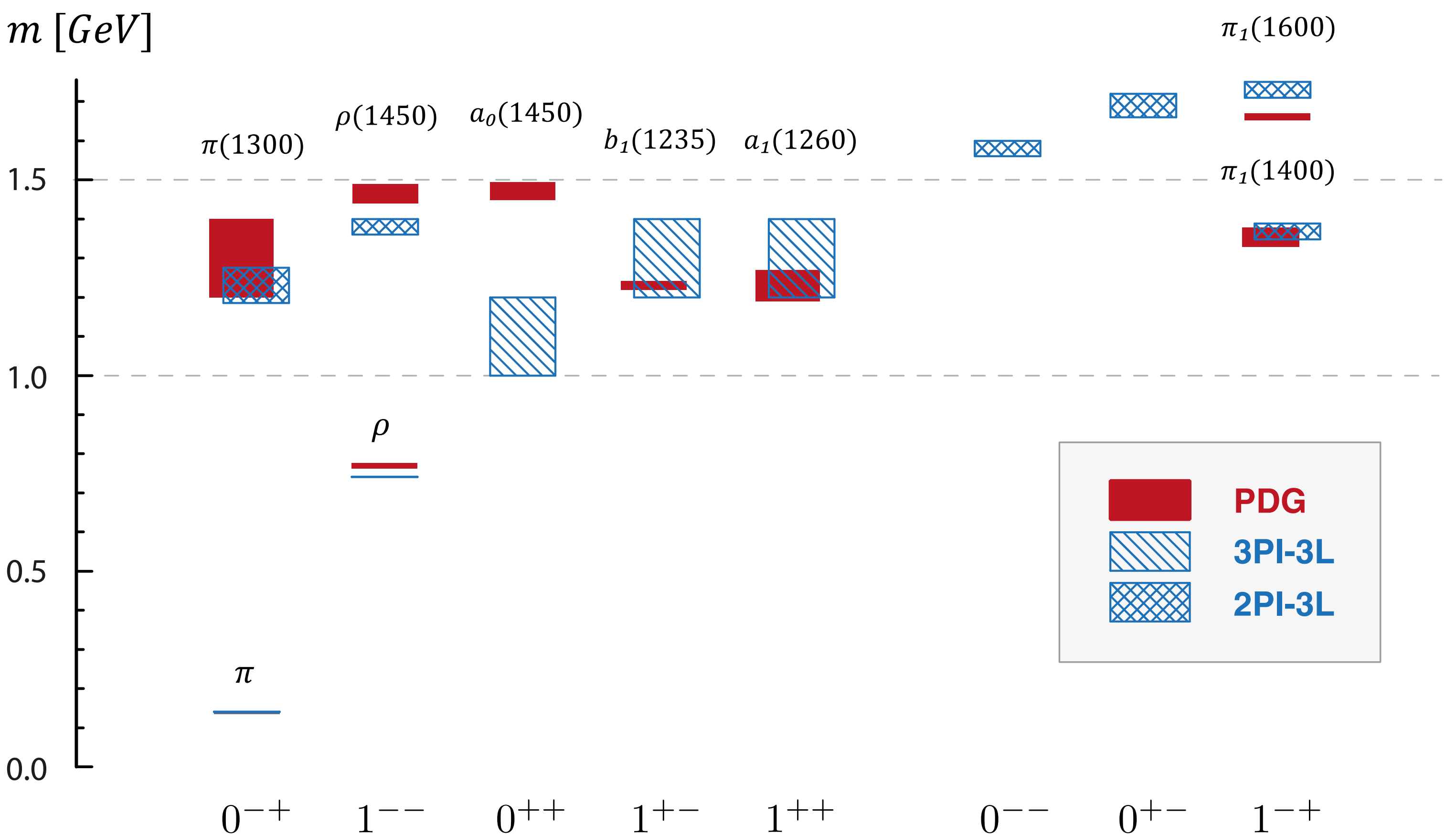


[RW, Fischer, Heupel, PRD93 (2016)]

Notable features

- Correct $\rho - a_1$ splitting. Degeneracy in axial-vectors
- Lightest $q\bar{q}$ scalar pushed above 1 GeV.

Light Spectrum



[RW, Fischer, Heupel, PRD93 (2016)]

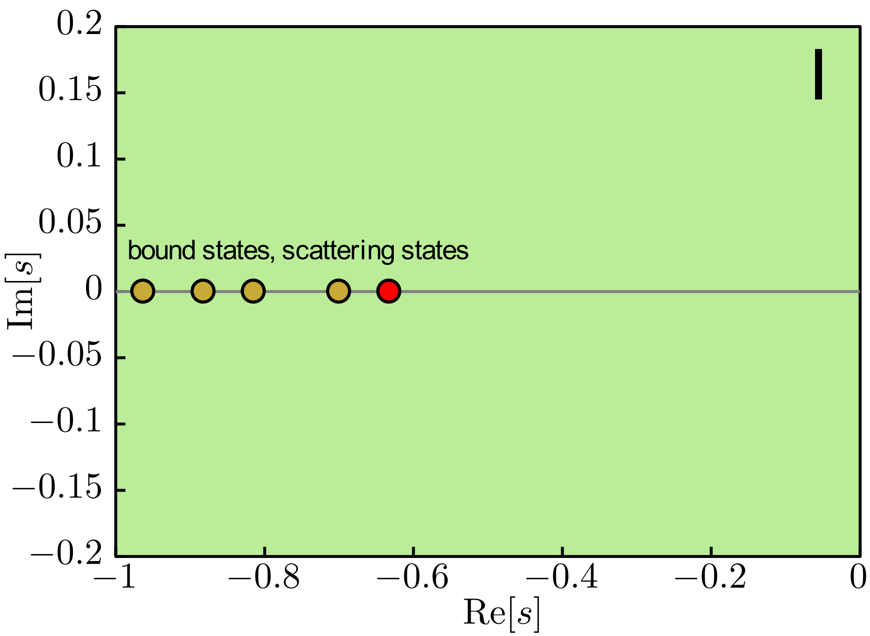
But something is missing

- Bound states **below** strong decay threshold: π, K, D, B
- Most hadrons lie **above** strong decay threshold

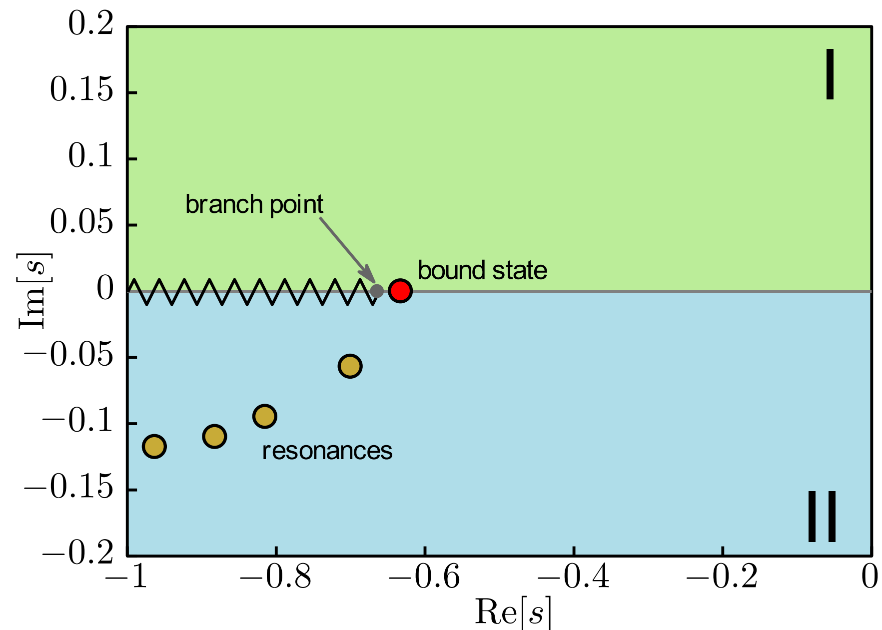
Resonances

(in)finite volume

Lattice: finite volume. No cuts.
Bound states, scattering states



Continuum: infinite volume.
Branch cuts. Bound states, resonances



(sketch)

Resonances

- Appear as poles on the “unphysical sheet” (labelled II).
- Information reconstructed on the Lattice via Lüscher formalism.

Resonances

Consider: function $V(s)$ that exposes "pole" of correlation function e.g. two-point correlator on the lattice, vertex function etc.

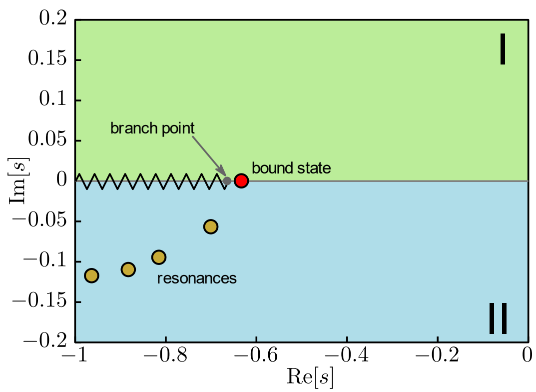
Below decay threshold

- Expect poles on the real-axis
- ***Bound state***

$$V(s) \sim \frac{1}{s + M^2}$$

Above decay threshold

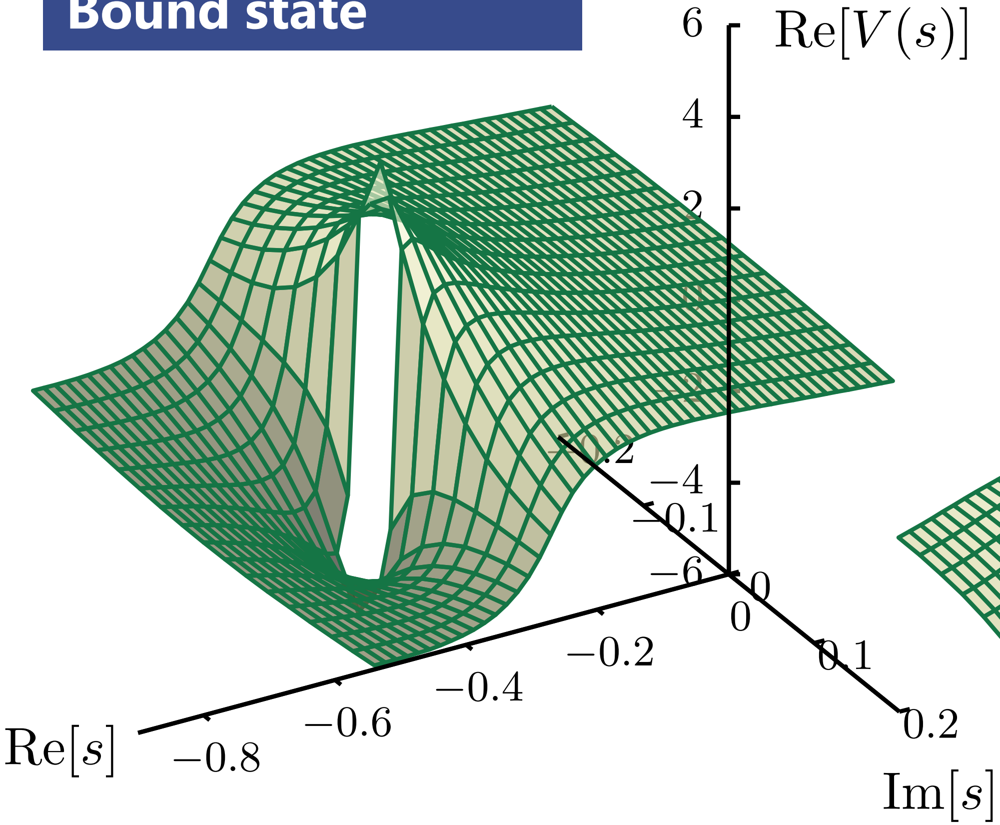
- Expect poles shifted from real-axis, in "unphysical sheet"
- ***Resonance***



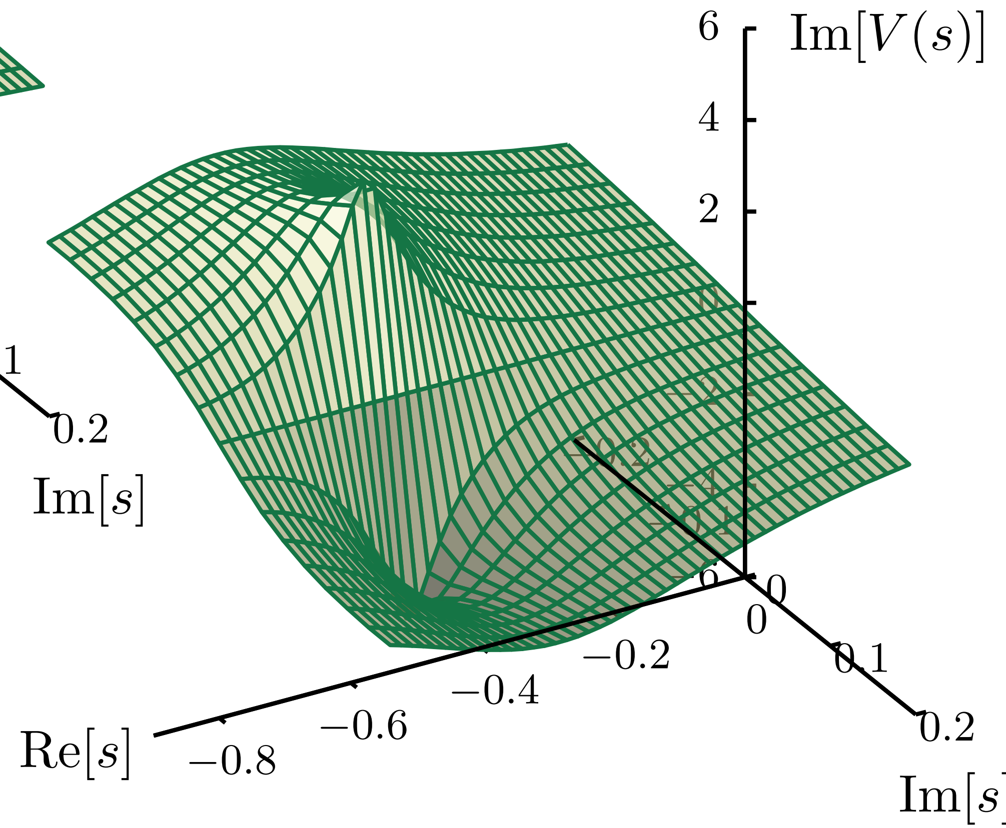
Let's visualize this:

$$V(s) \sim \frac{1}{s + \left(M - \frac{i\Gamma}{2}\right)^2}$$

Bound state

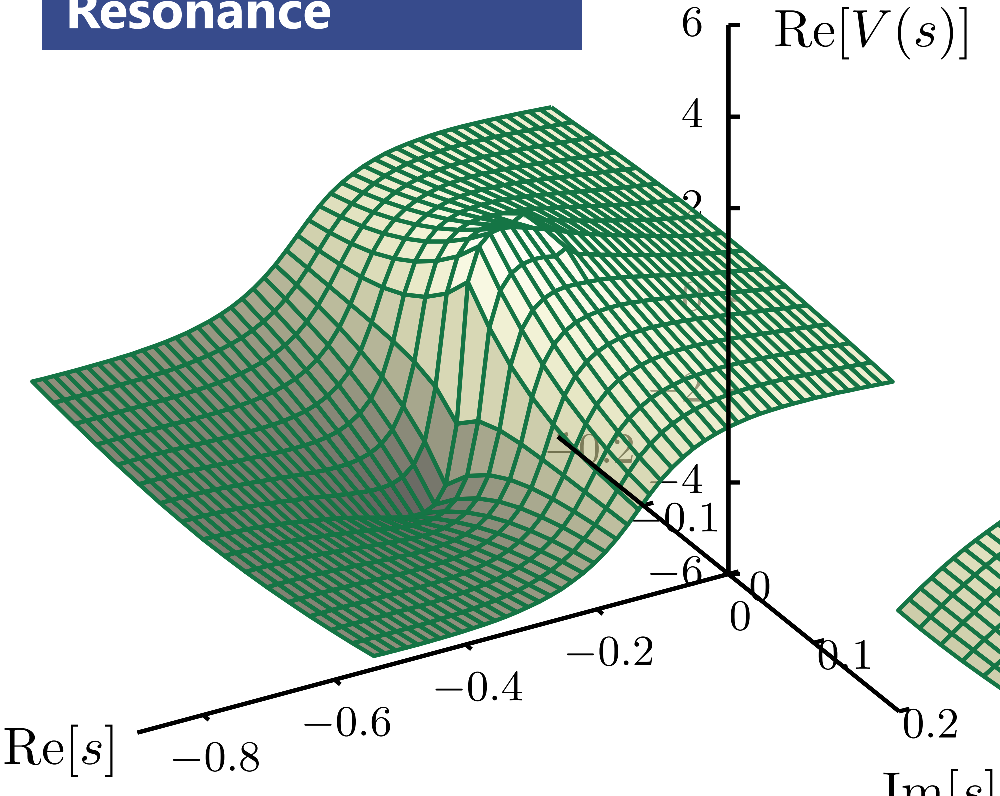


(transformed z-axis with $\sinh^{-1}$)

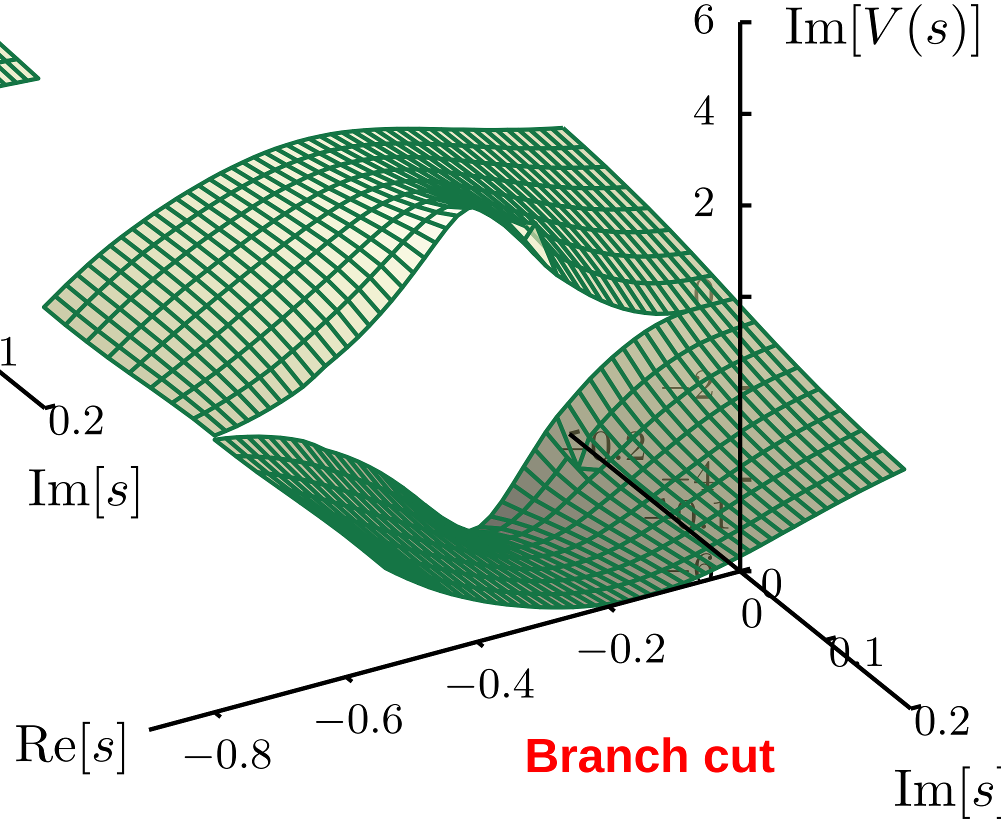


Pole readily apparent on the real-axis

Resonance



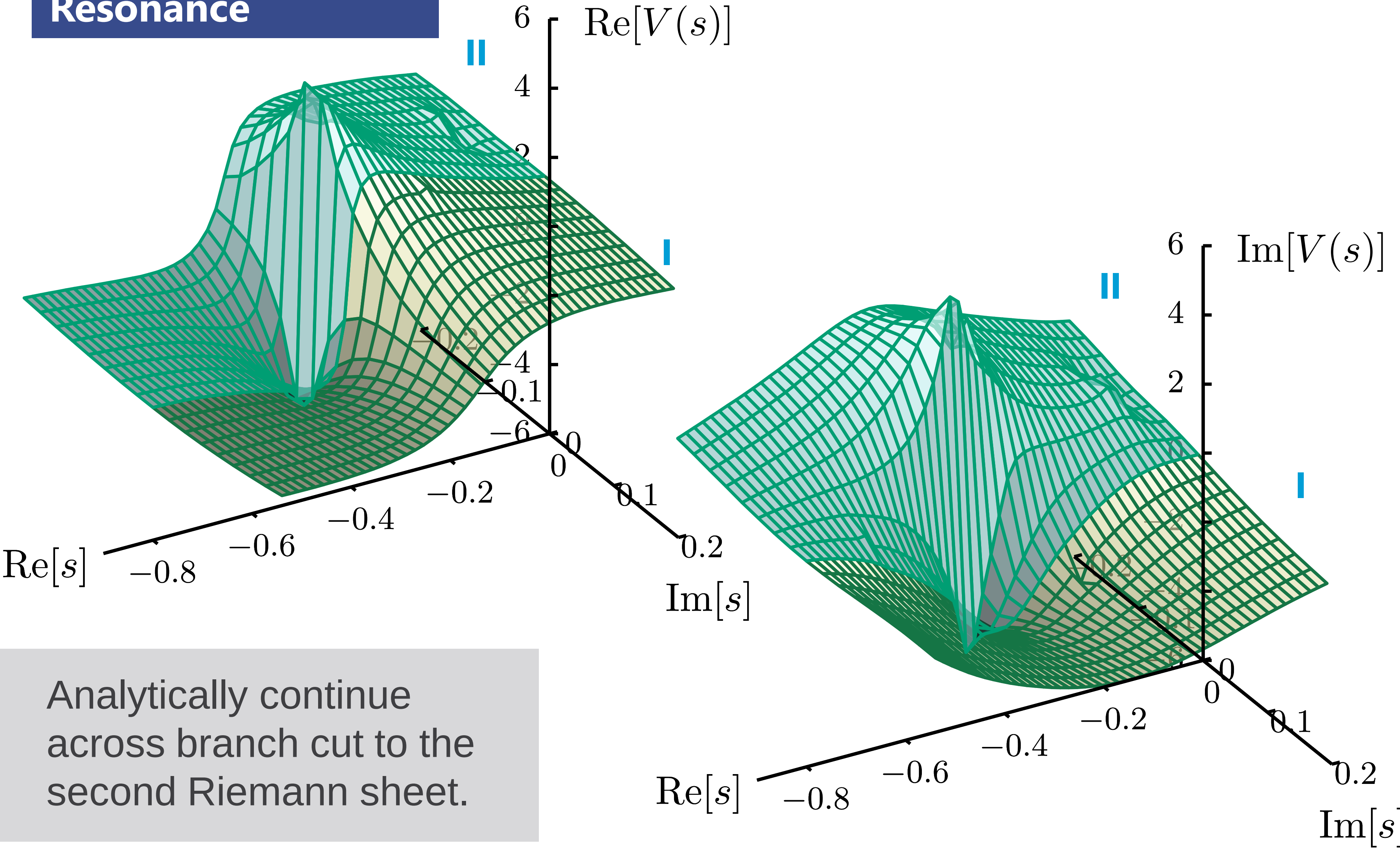
Non-analytic across BC



Branch cut

No poles on the "physical" sheet

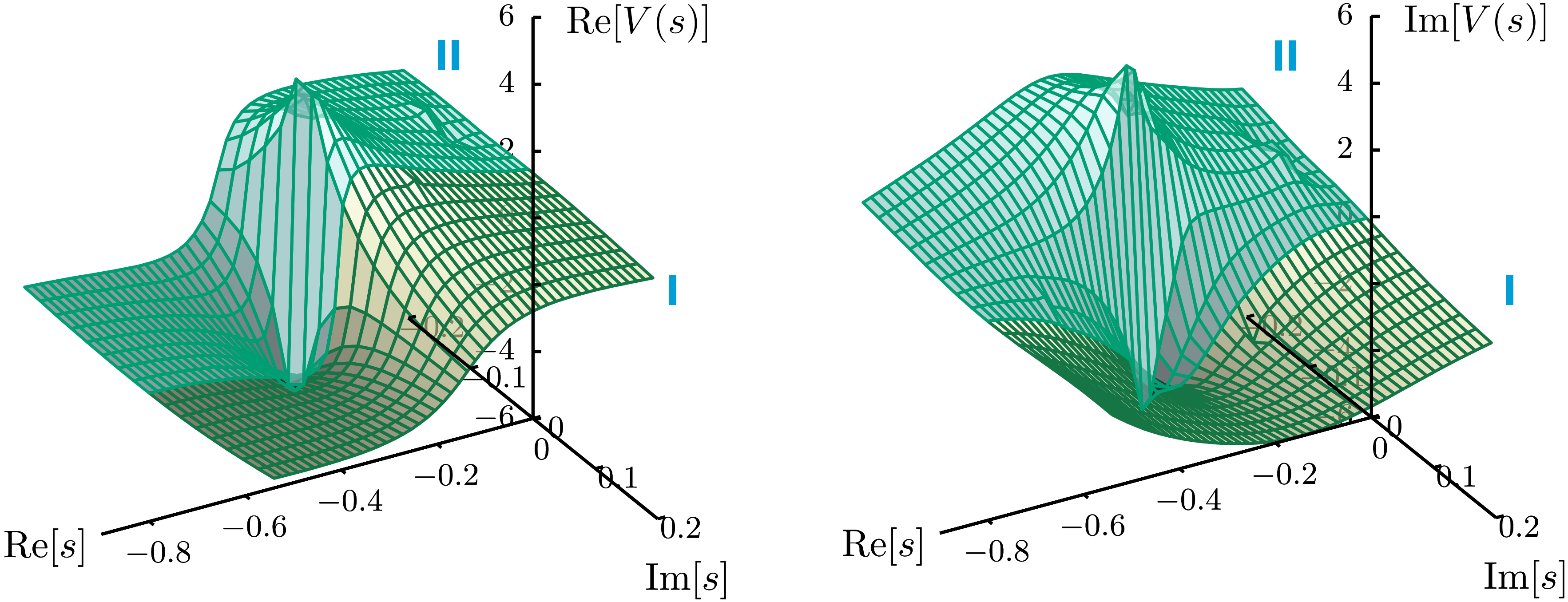
Resonance



Analytically continue across branch cut to the second Riemann sheet.

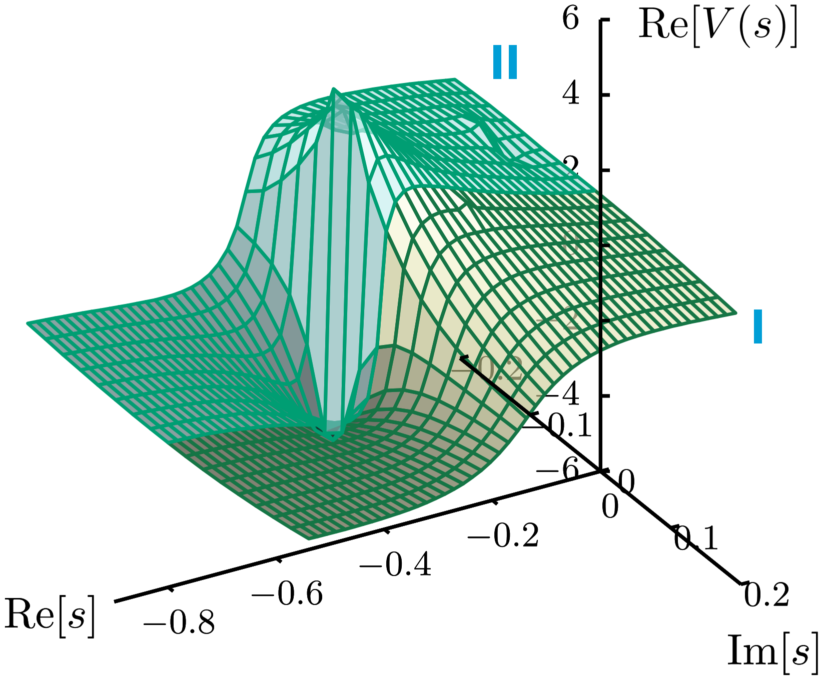
Poles on the "unphysical" sheet

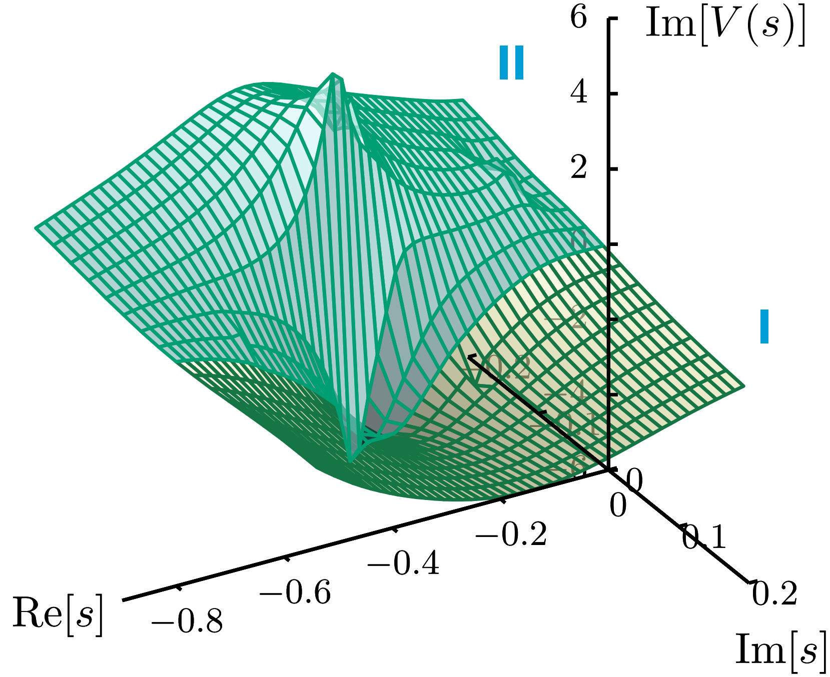
Resonance



What would we expect to see in the BSE approach?

Resonance





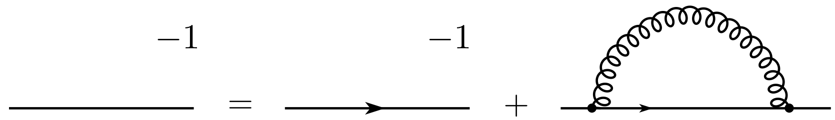
What would we expect to see in the BSE approach?

This is the Bethe-Salpeter approach! ☺

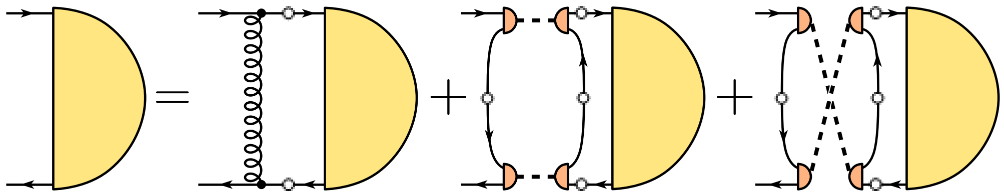
Truncation

Hadronic Unquenching

Quark DSE



incl. decay



[Watson, Cassing, FBS 35 (2004)]
[Fischer, Nickel, Wambach, PRD 76 (2007)]
[Fischer, RW, PRD 78 (2008)]

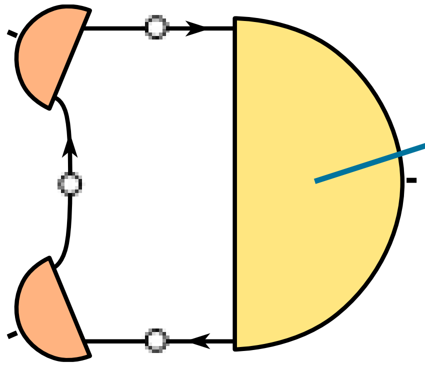
Specifically

- Two-pion decay kernel
- **Couples** to *e.g.* vector and scalar mesons.
- Does **not couple** to pseudoscalar (CP and P): *maintains chiral symmetry*

Truncation

Clarification

- Not calculating $\rho \to \pi\pi$ in impulse approximation
- Not calculating $g_{\rho\pi\pi}$

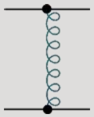


Not calculating this!

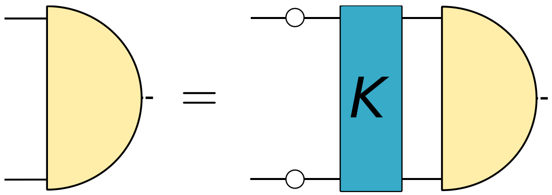
- Calculating (in)homogeneous Bethe-Salpeter equation and determining solution $P^2 = \left[i\left(M_R - \frac{i\Gamma_R}{2}\right)\right]^2$ for $\Gamma \neq 0$

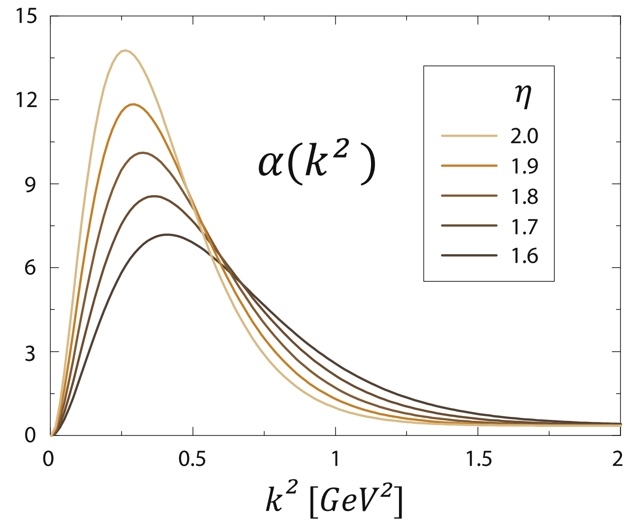
Truncation

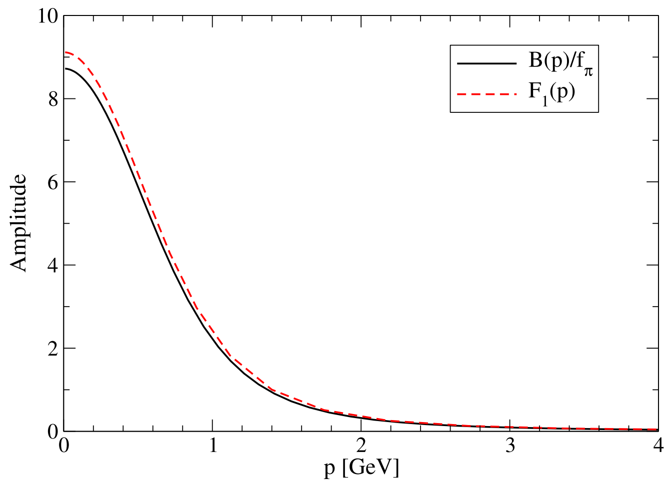
Rainbow-Ladder:
One-gluon exchange



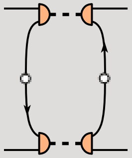
[Maris, Tandy PRC 60 (1999) 055214]







Decay/Unquenching:
Two-pion exchange



$$\Gamma_\pi(k,P) = \gamma_5 \frac{B(k^2)}{f_\pi}, \qquad D_\pi(q^2) = \left(q^2 + m_\pi^2\right)^{-1}$$

[Watson, Cassing, FBS 35 (2004)]
[Fischer, Nickel, Wambach, PRD 76 (2007)]
[Fischer, RW, PRD 78 (2008)]

Decomposition

Covariant basis for bound-state:

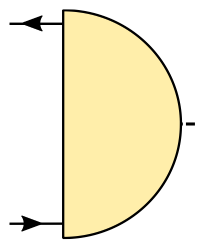
$$\Gamma^{(\rho)} = \sum_i g_i \tau_i^{(\rho)}, \qquad \chi^{(\rho)} = \sum_i h_i \tau_i^{(\rho)}$$

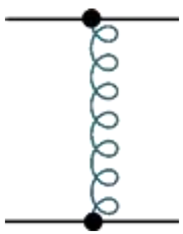
pseudoscalar	
$\tau_1 = \gamma^5$	$\tau_3 = \hat{\not{P}}\gamma^5$
$\tau_2 = \widehat{\not{p}_T}\gamma^5$	$\tau_4 = i\widehat{\not{p}_T}\hat{\not{P}}\gamma^5$

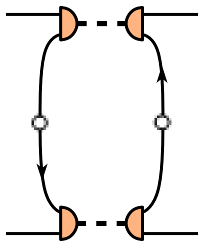
vector			
$\tau_1^\rho = \gamma_T^\rho$	$\tau_3^\rho = i\widehat{p_T}^\rho$	$\tau_5^\rho = 3\widehat{p_T}^\rho\widehat{\not{p}_T} - \gamma_T^\rho$	$\tau_7^\rho = \gamma_T^\rho\widehat{\not{p}_T} - \widehat{p_T}^\rho$
$\tau_2^\rho = \gamma_T^\rho\hat{\not{P}}$	$\tau_4^\rho = \widehat{p_T}^\rho\hat{\not{P}}$	$\tau_6^\rho = (3\widehat{p_T}^\rho\widehat{\not{p}_T} - \gamma_T^\rho)\hat{\not{P}}$	$\tau_8^\rho = i(\gamma_T^\rho\widehat{\not{p}_T} - \widehat{p_T}^\rho)\hat{\not{P}}$

Quark rotation matrix:

$$Y_{ij} = \mathrm{Tr}\left[\overline{\tau}_i^{(\rho)} S(p_+)\tau_j^{(\rho)}(p,P)S(p_-)\right],$$







Kernel trace:

$$L_{ij}^{\mathrm{RL}} = \int_k \mathrm{Tr}\left[\overline{\tau}_i^\rho(p,P)\gamma^\mu\tau_j^\rho(k,P)\gamma^\nu\right] D^{\mu\nu}(q),$$

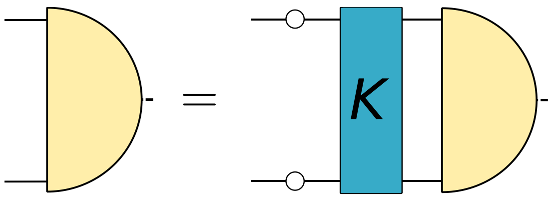
$$L_{ij}^{\pi\pi,\mathrm{s}} = \int_k\int_l \overline{J}_i^\rho(p,l,P)J_j^\rho(k,l,P)D_+^\pi D_-^\pi,$$

$$J_j^\rho(k,l,P) = \mathrm{Tr}\left[\overline{\Gamma}_\pi\tau_j^\rho(k,P)\overline{\Gamma}_\pi S(k-l)\right],$$

$$\overline{J}_i^\rho(p,l,P) = -\left[C^T J_i^\rho(-p,-l,-P)C\right]^T.$$

BSE:

$$g_i = \sum_A L_{ij}^A h_j = \sum_A L_{ij}^A Y_{jk} g_k = M_{ik} g_k,$$



Integrating over Poles

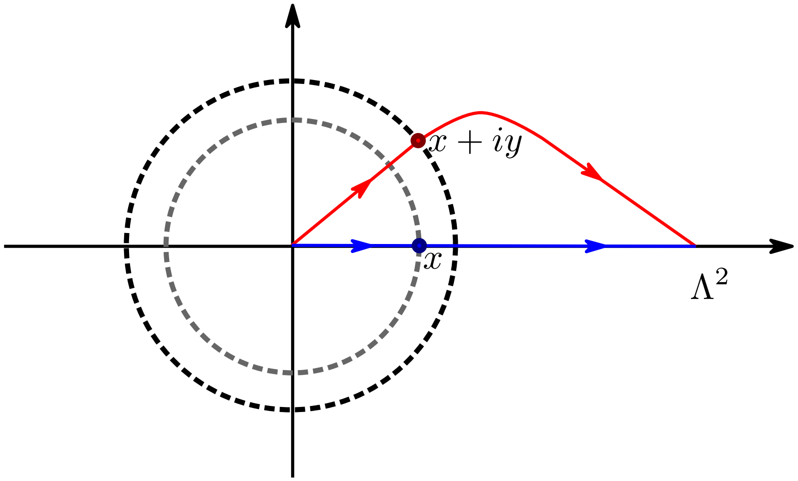
Consider: integral of the form

$$A(p^2) = \int d^4k \frac{C(k,p)}{k^2 + p^2 - 2k \cdot p + m^2}$$

With pole dependent upon the angle between k and p

- Angular integral "sweeps" out the pole.
- Radial integral should be deformed to avoid cut structure.

Applied to **quark propagator**



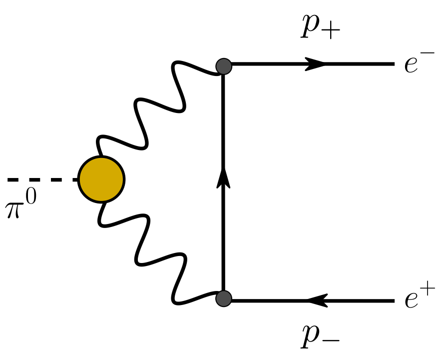
$$I(p^2) = \int dq^2 q^2 \sigma(q^2) K(q^2, p^2)$$

$$K(q^2, p^2) = \int dz \sqrt{1 - z^2} \frac{f((q-p)^2)}{(q-p)^2} K_\theta(q,p)$$

e.g. [Alkofer, Detmold, Fischer, Maris, PRD 70 (2004)]

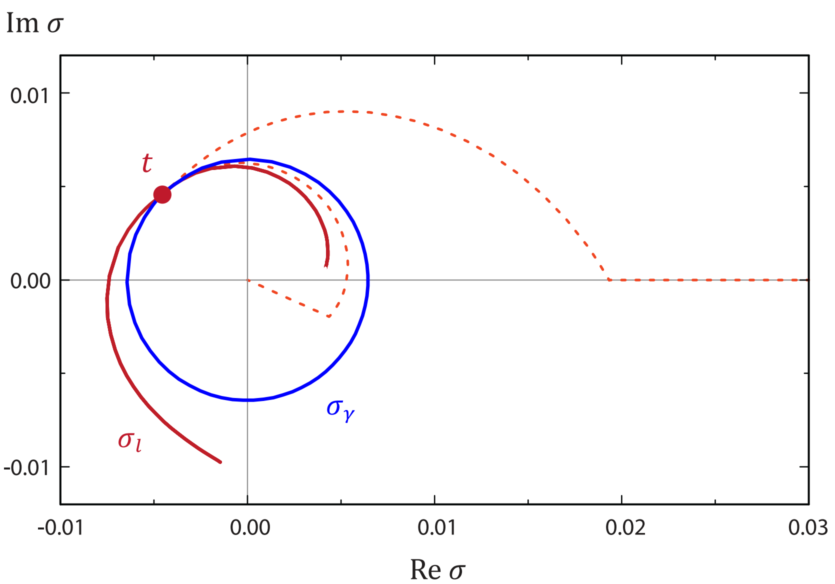
Integrating over Poles

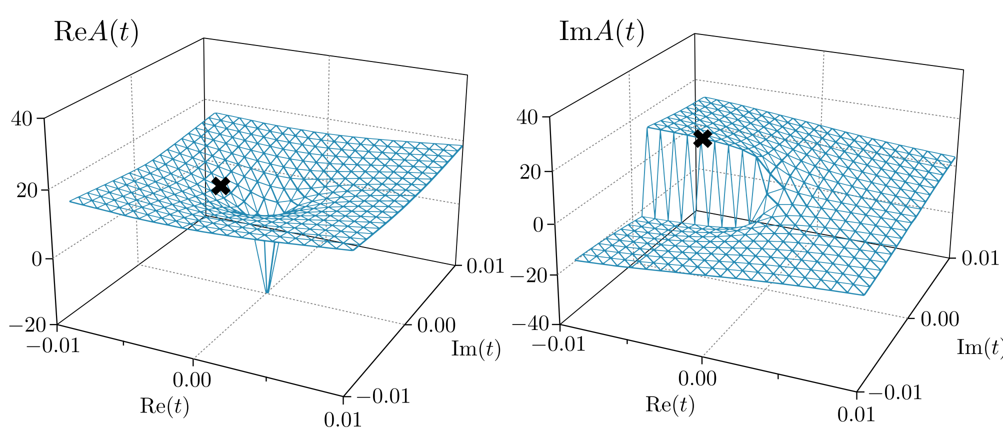
Applied to **rare pion decay** $\pi^0 \to e^+e^-$ to avoid cut structure during integration



$$\mathcal{A}(t) = \frac{1}{2\pi^2 t} \int d^4\Sigma \, \frac{(\Sigma \cdot \Delta)^2 - \Sigma^2 \Delta^2}{(p+\Sigma)^2 + m^2} \, \frac{F(Q^2, Q'^2)}{Q^2 \, Q'^2}$$

- Results in agreement with dispersion relations
- Technique has further applications



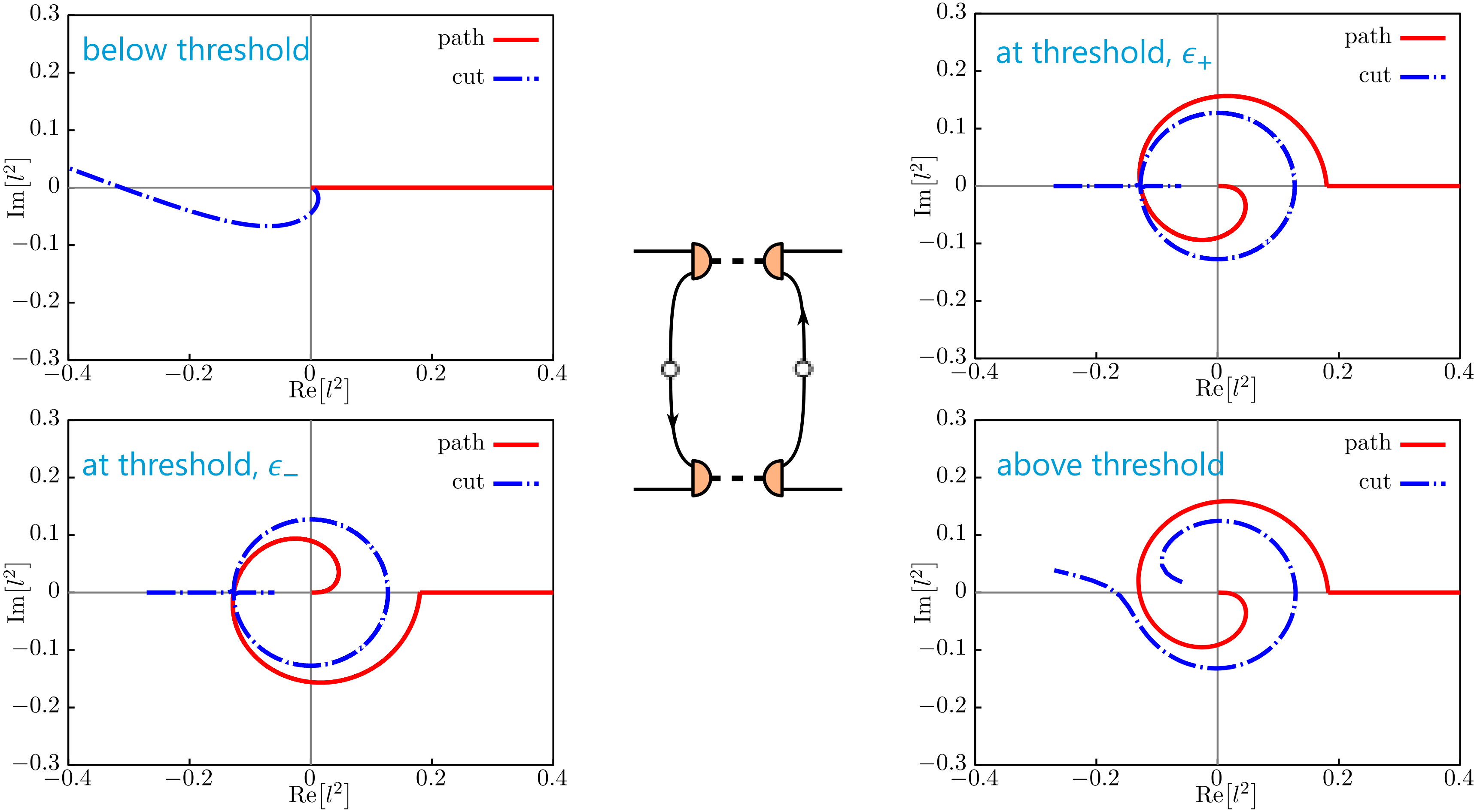


[Weil, Eichmann, Fischer, RW, PRD 96 (2017)]

Integrating over Poles

Two-pion cuts

$$l^2_{\text{cut}} = -z\sqrt{t} + \sqrt{t(z^2-1) - m_\pi^2}, \qquad t = P^2/4$$

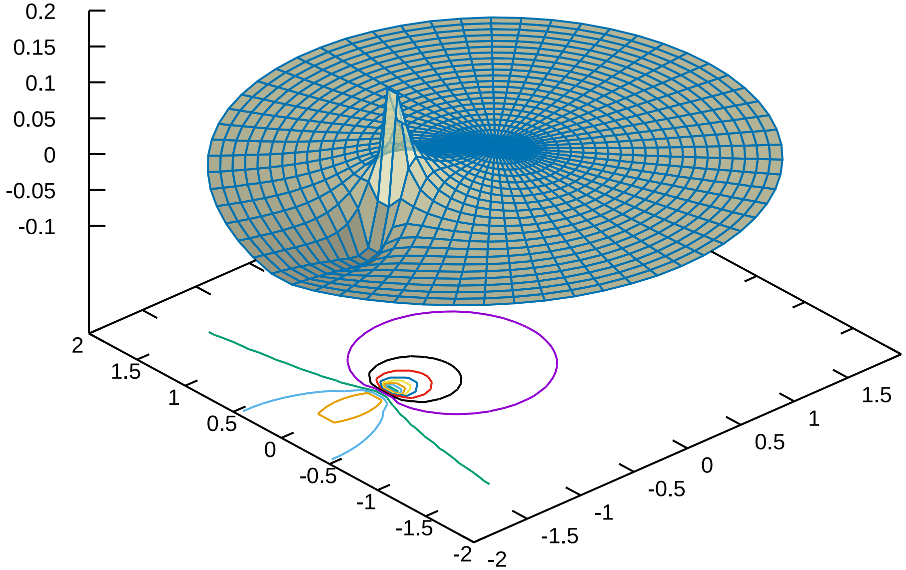


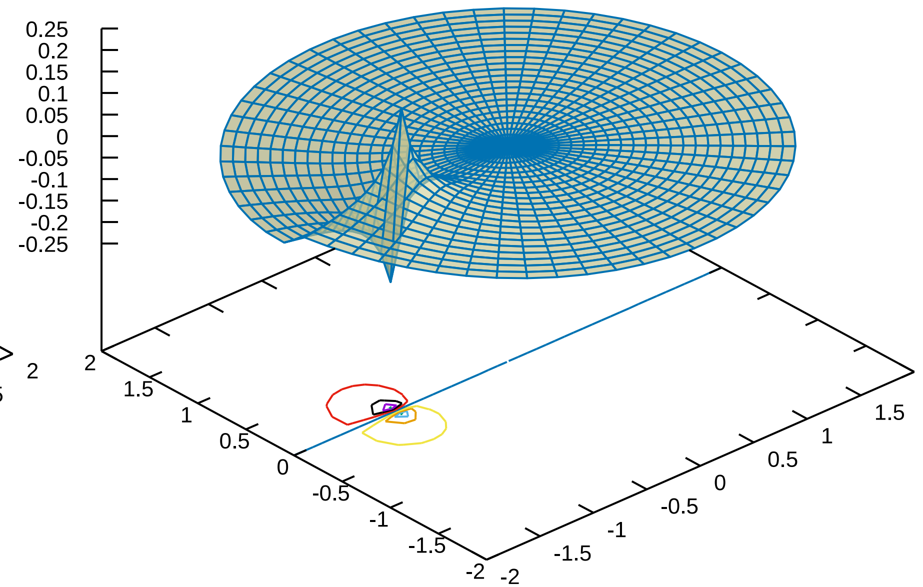
See also [Windisch, Huber, Alkofer, APPS 6 (2013)]

Two-pion integral

$$F(l,P) \propto \frac{l_T^\rho}{l_T^2}\int_k J_j^\rho(k,l,P)h_j(k,P)\ .$$

$$I(P^2) = \int_l \frac{1}{l^2\left(l_+^2+m_\pi^2\right)\left(l_-^2+m_\pi^2\right)}.$$





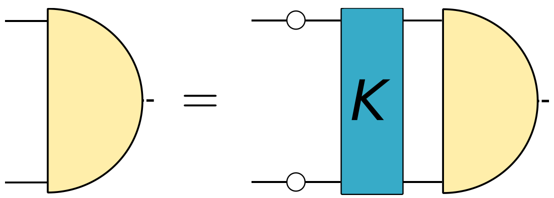
$$I(P^2) = \frac{1}{4\pi^2P^2}\left[\ln\left(\frac{a^2-1}{a^2}\right)+\frac{1}{a}\ln\left(\frac{a+1}{a-1}\right)\right],$$

$$a=\sqrt{1+4m_\pi^2/P^2}$$

Calculation

Put together:

- Solve quark for complex momenta
- Calculate one-loop RL kernel
- Calculate two-loop pi-pi kernel
- Choose appropriate path deformation



Solve BSE as eigenvalue equation for $\lambda(P^2) = 1$ *complex*

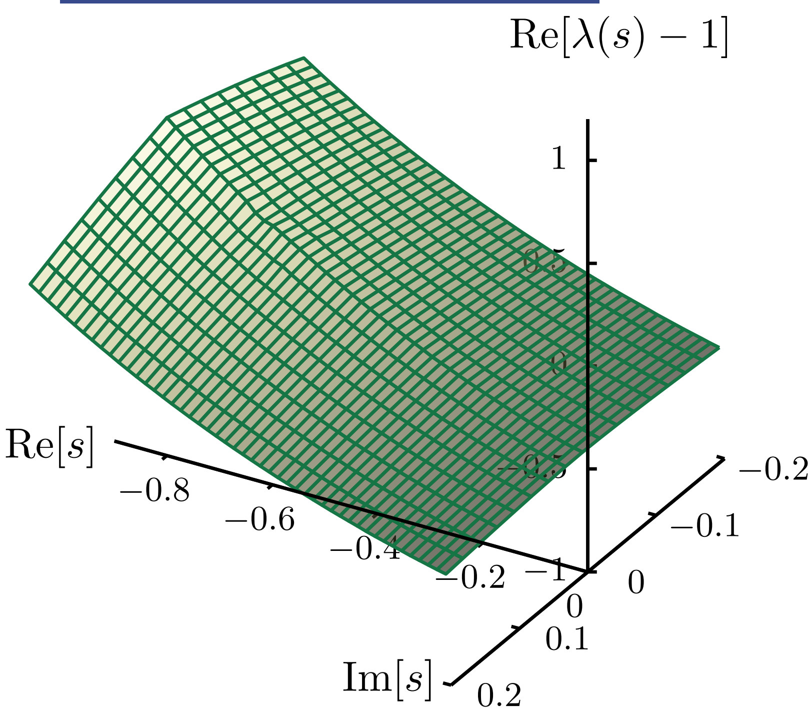
$$\Gamma = \lambda(P^2)\, K\, \Gamma, \qquad P^2 \in \mathbb{C}$$

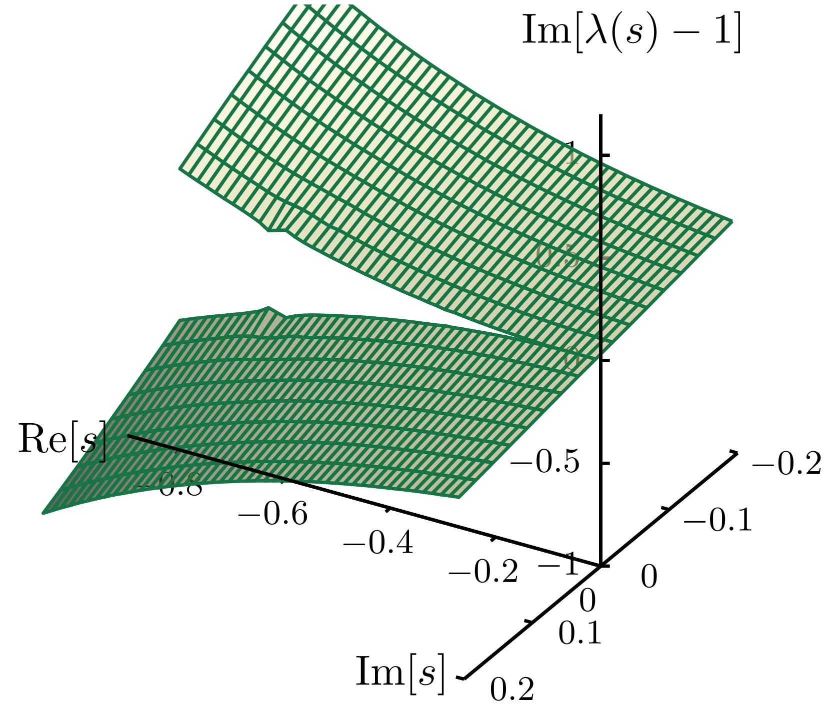
(Or solve for pole in inhomogeneous system)

$$\Gamma = I + K\, \Gamma$$

Use **right tools** for solving the (eigen)system

Eigenvalues

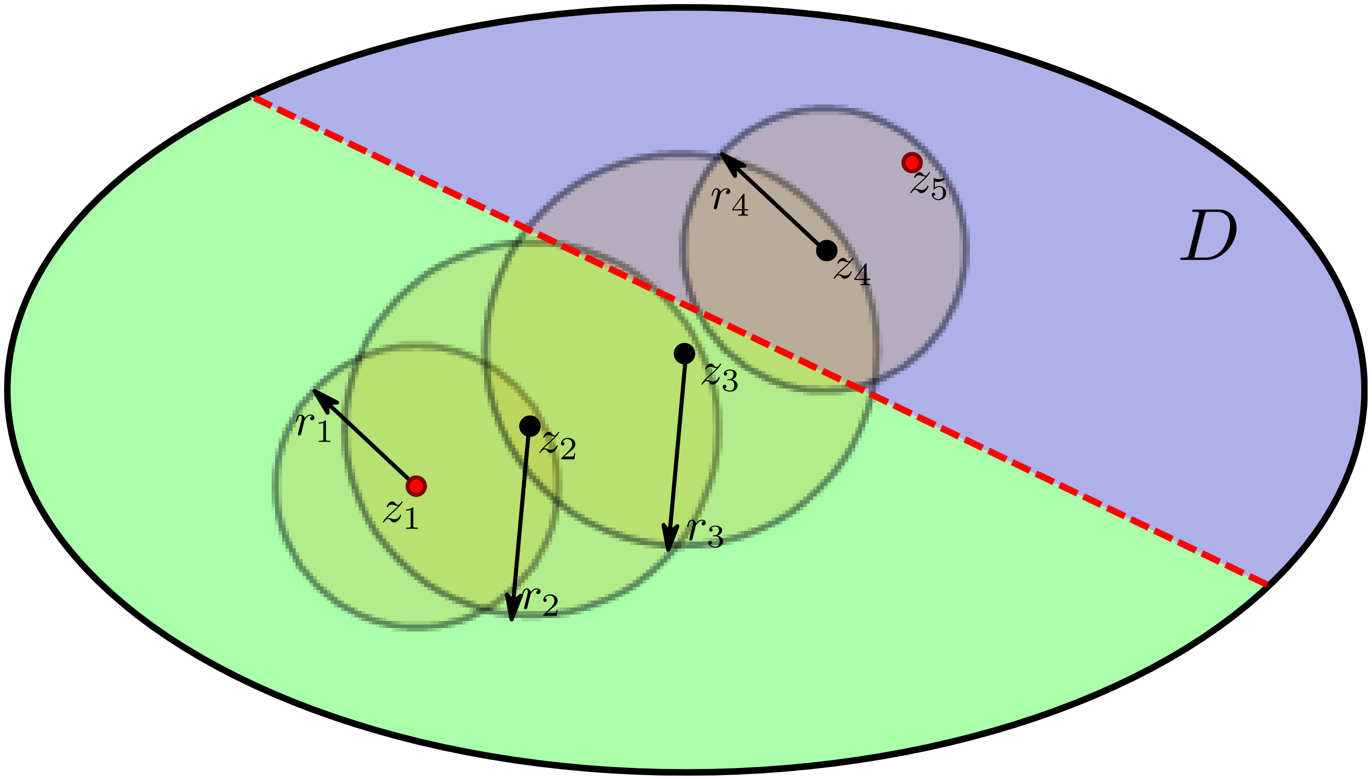




- **“tent structure” in real part**
- **Branch cut in imaginary part**

No solution on “physical sheet” where: $\lambda(s) = 1$

Analytic Continuation

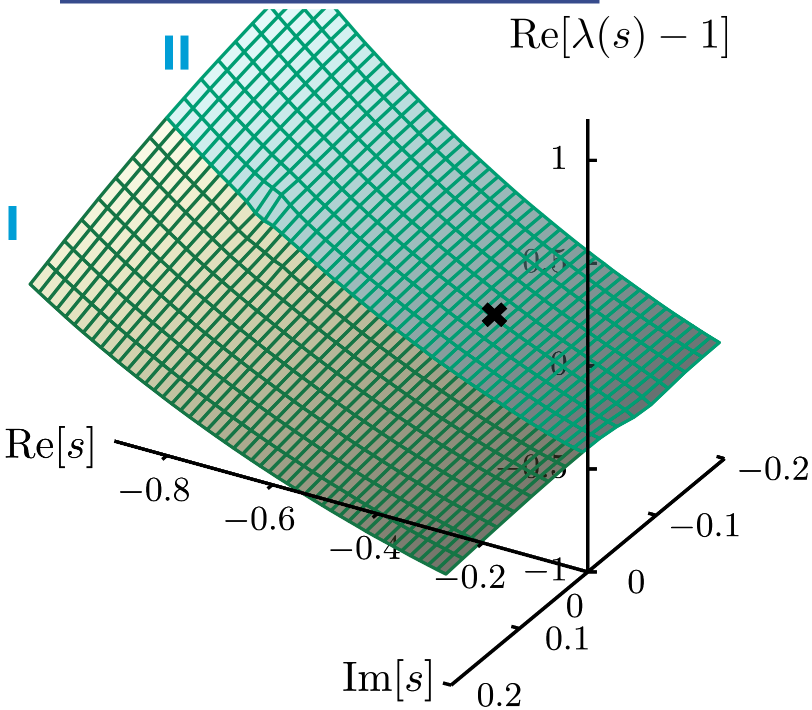


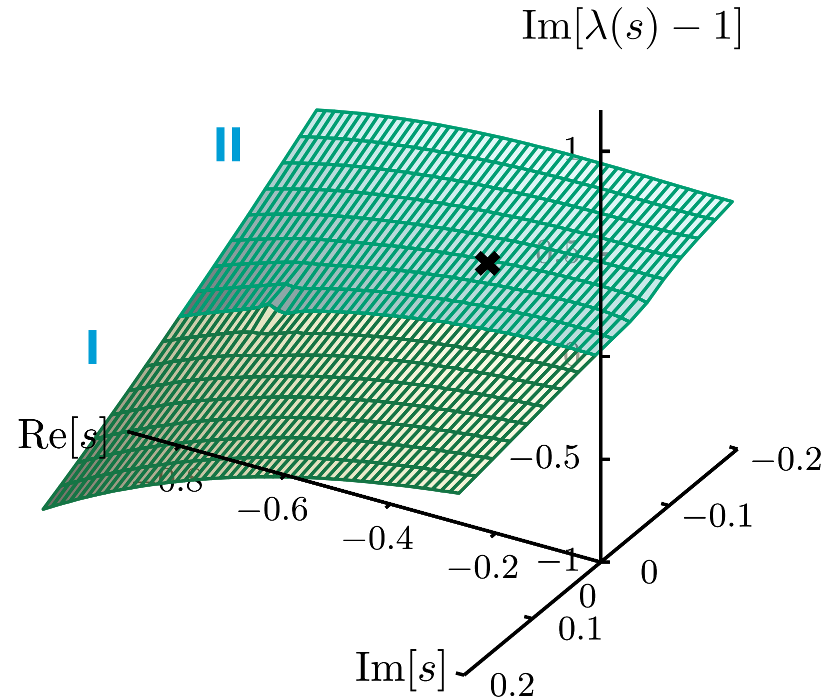
Analytic continuation (from e.g. z_1 to z_5)

- Using power series (i.e. Hadamard method)
- Pade approximants. RVP and Schlessinger point method.

[Tripolt et al, arXiv:1801.10384]

Eigenvalues





Analytically continue to find $\lambda(s) = 1$ on "unphysical sheet"

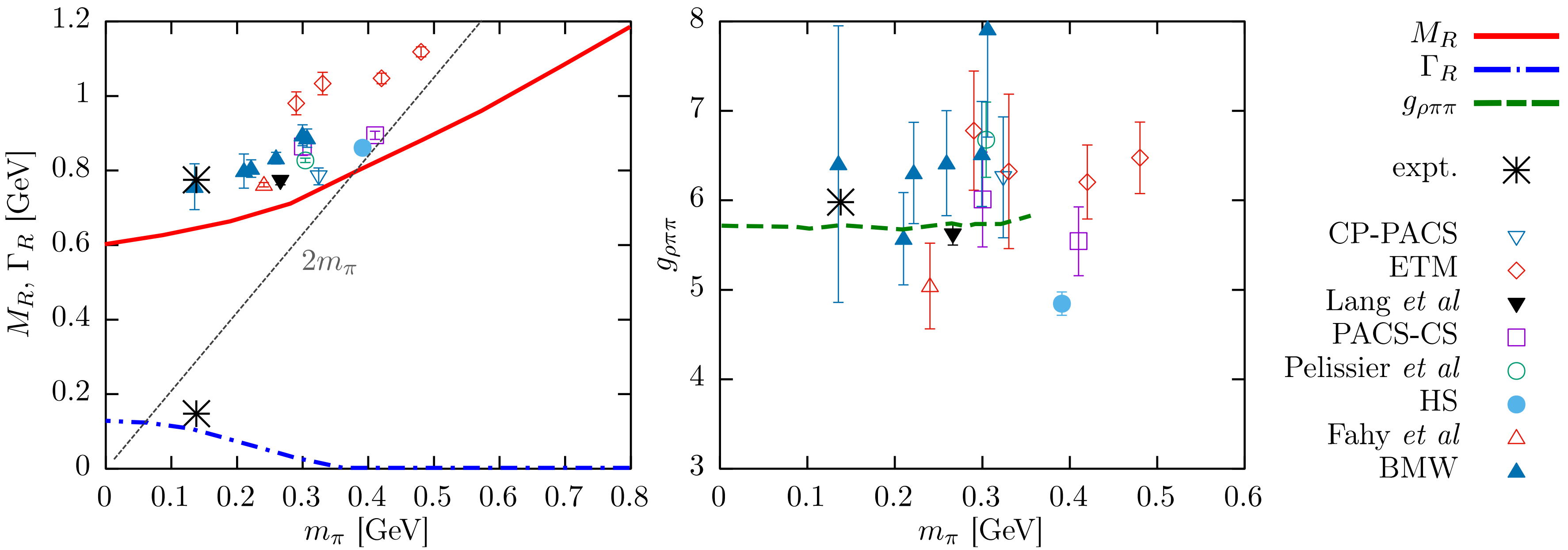
	$s\ [GeV^2]$	$M_R\ [GeV]$	$\Gamma_R\ [GeV]$
RL	-0.55	0.74	0.0
RL+decay	$-0.408 + 0.065i$	0.64	0.1

$$s = P^2 = \left[i\left(M_R - \frac{i\Gamma_R}{2}\right)\right]^2$$

Repulsive corrections BRL

[Fischer and RW, PRL 103 (2009)]

Mass dependence



Here: strong coupling constant $g_{\rho\pi\pi} \sim 5.7$ **(experimental value** $g_{\rho\pi\pi} \sim 6.0$**)**

RL: (impulse approximation) $g_{\rho\pi\pi} \sim 5.2$

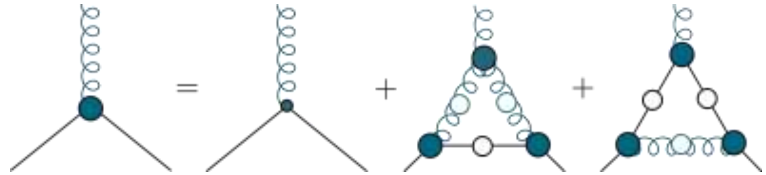
[Jarecke, Maris, Tandy, PRC67 (2003)]
[Mader, Eichmann, Blank, Krassnigg, PRD84 (2011)]

$$\Gamma_R = \frac{p^3}{M_R^2}\,\frac{g_{\rho\pi\pi}^2}{6\pi}, \qquad p = \sqrt{M_R^2/4 - m_\pi^2}\,,$$

Summary

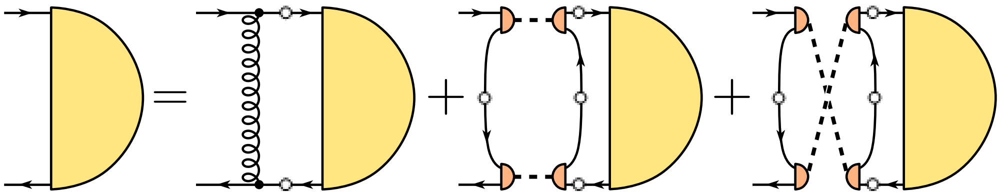
- **Bound-states with coupled 3 point fns.**

  [RW, Fischer, Heupel PRD 93 (2016)]



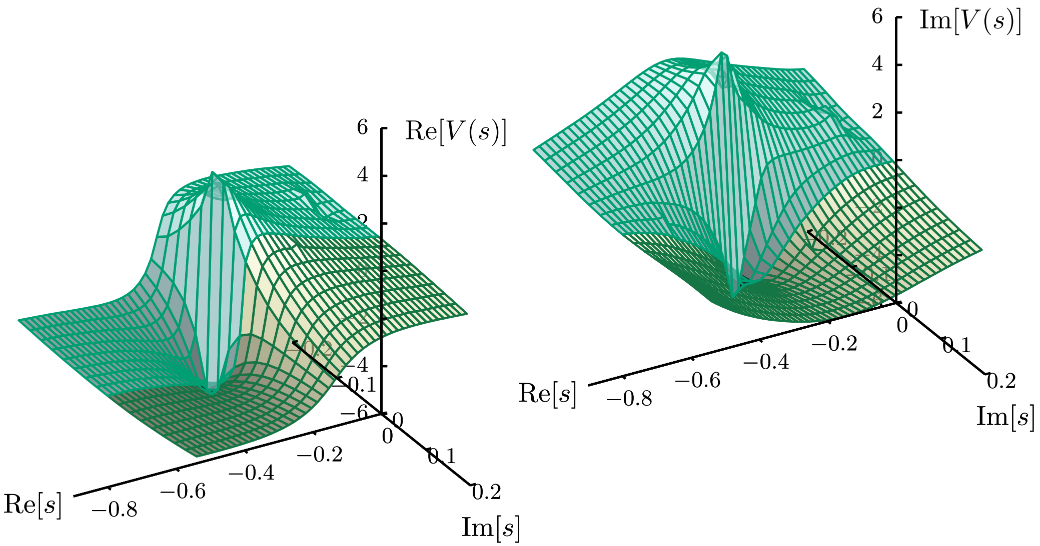
- **Resonances in BSE!**

  [RW, arXiv:1804.11161]



Next Steps

- Extend to other bound-states
  - Baryons
  - Tetraquarks **See Fischer**
- Solidify truncation + … *more*



Review

Eichmann, Sanchis-Alepuz, RW, Alkofer, Fischer 1606.9602 Prog. Part. Nucl. Phys. (in press)

Summary

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  [RW, Fischer, Heupel PRD 93 (2016)]

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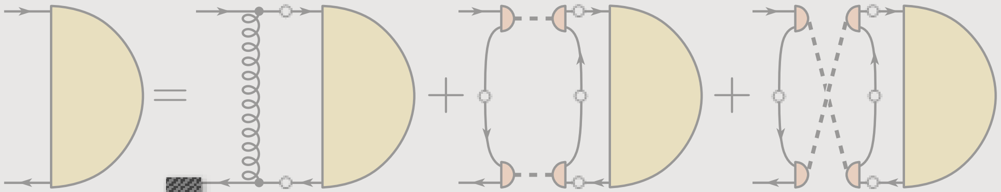
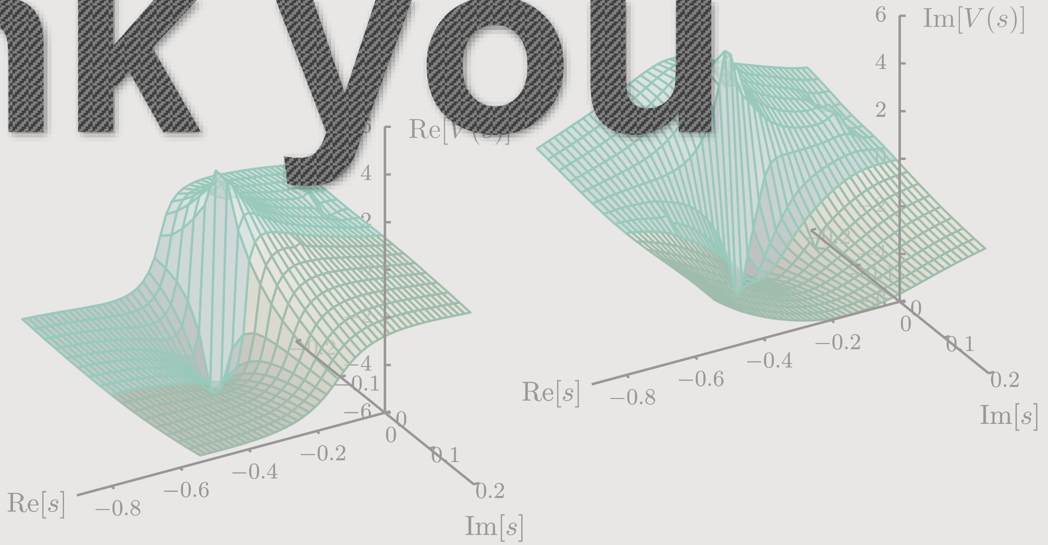


Thank you



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