The LSMq to explore the QCD phase diagram

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Outline.

- Motivation
- The Linear Sigma Model
- Effective potential
- Fixing parameters
- The QCD Phase Diagram
- Results

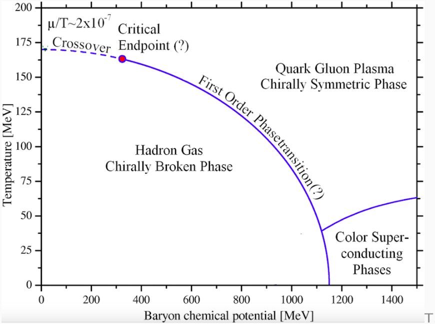
Motivation

A grand opportunity Era.

- By colliding heavy ion nuclei, RHIC and LHC are making little "Big Bang matter" (the universe microseconds after the Big Bang).
- Using current detectors (PHENIX, STAR, ALICE, ATLAS and CMS) scientist are answering questions about the microsecond-old universe that cannot be addressed by any conceivable astronomical observation. And
- The properties of the matter that filled the early universe turn out to be INTERESTING (Quark-Gluon Plasma).
- What's next? Hadronic matter under extreme conditions, not only high temperatures, also high densities. We are waiting for NICA, FAIR, J-PARC!!!

Critical End Point

Crossover/ Second order phase transition

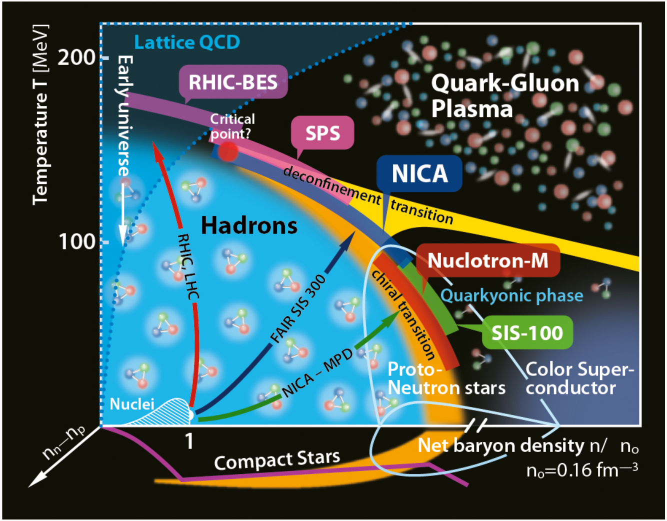


First order phase transition

Boeckel *et al.*, arXiv:1105.0832 [astro-ph.CO]

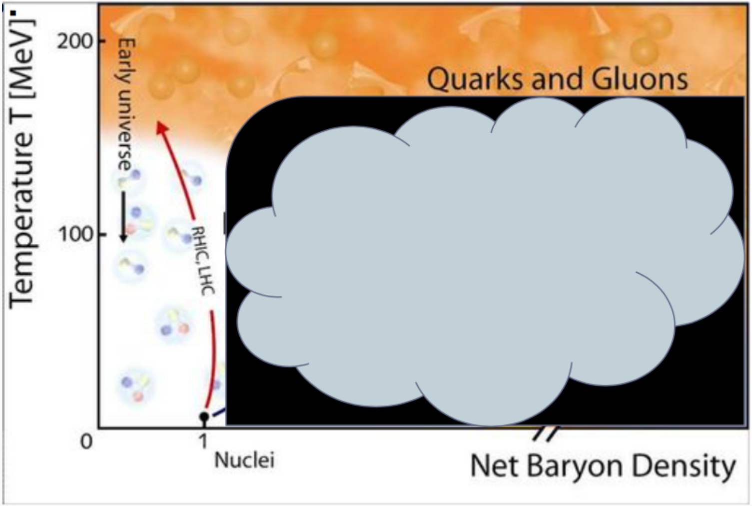
- Phase transition ⇔ Symmetry restored/broken.
- Deconfinement and/or Chiral symmetry restoration.

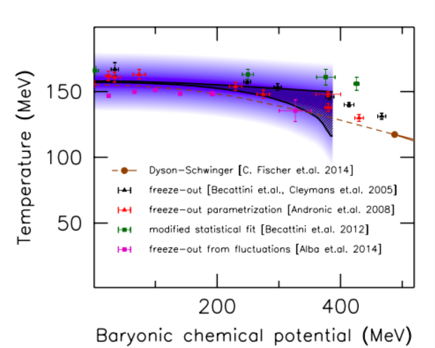
Facilities



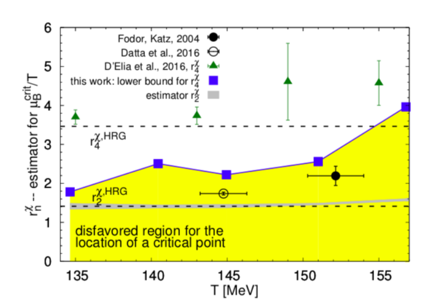
CompStar

Our current knowledge





Jana Günter,Wuppertal-Budapest Collaboration (CPOD-2017)



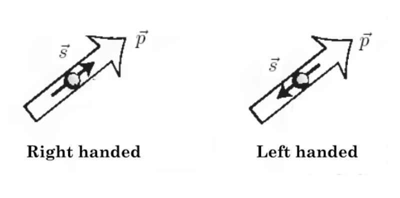
A. Bazavov *et al.* (Bielefeld-BNL-CCNU collaboration), Phys. Rev. D **95**, no. 5, 054504 (2017)

Symmetries.

QCD with massless quarks: Chiral symmetry.

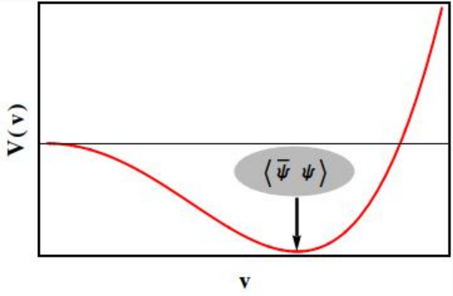
- $\mathcal{L}^0_{QCD} = \overline{\psi}(x) i\gamma_\mu \partial^\mu \psi + \mathcal{L}_{quark-gluon} + \mathcal{L}_{glue}$
- $\psi = \psi_R + \psi_L$

$$\begin{aligned} \psi_R &= \frac{1}{2}(1+\gamma_5)\psi, \\ \psi_L &= \frac{1}{2}(1-\gamma_5)\psi \end{aligned}$$

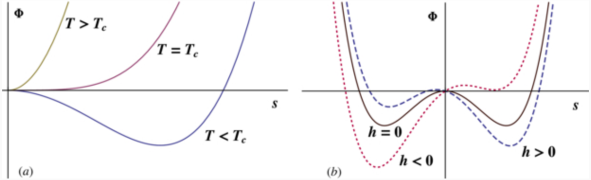


Spontaneous breaking of the chiral symmetry.

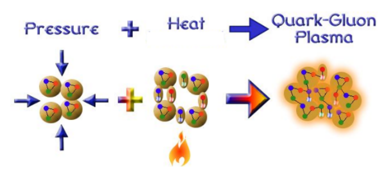
$$Q_j^A|0\rangle \neq 0 \;\Leftrightarrow\; \langle\overline{\psi}\psi\rangle \neq 0$$



Chiral symmetry restoration.

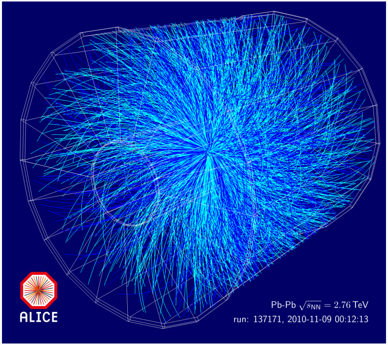


where $\phi \rightarrow V^{eff}(v, T)$ is the effective potential.

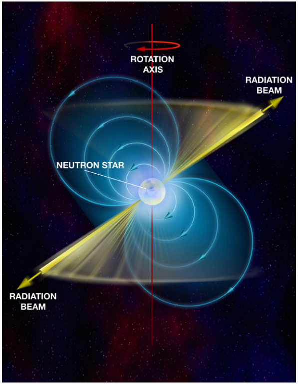


Hadronic matter in extreme conditions.

High Temperature.



High Density.



The Linear Sigma Model

Linear Sigma Model coupled to quarks.

- Effective model for low-energy QCD.
- Renormalizable theory.
- Implement ideas of chiral symmetry ($SU(2)_L \times SU(2)_R \to O(4)$).
- Effects of quarks and mesons on the chiral phase transition.
- Spontaneous symmetry breaking $O(4) \to O(3)$.

Linear Sigma Model coupled to quarks.

Lagrangian

$$\begin{aligned}\mathcal{L} &= \frac{1}{2}(\partial_\mu\sigma)^2 + \frac{1}{2}(\partial_\mu\vec{\pi})^2 + \frac{a^2}{2}(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{4}(\sigma^2 + \vec{\pi}^2)^2 \\ &+ i\bar{\psi}\gamma^\mu\partial_\mu\psi - g\bar{\psi}(\sigma + i\gamma_5\vec{\tau}\cdot\vec{\pi})\psi,\end{aligned}$$

where ψ is an SU(2) isospin doublet, $\vec{\pi} = (\pi_1, \pi_2, \pi_3)$ is an isospin triplet and σ is an isospin singlet. λ is the boson's self-coupling and g is the fermion-boson coupling. $a^2 > 0$ is the mass parameter. To allow for

spontaneous symmetry breaking

$$\sigma \to \sigma + v,$$

v can later identified as the order parameter of the theory.

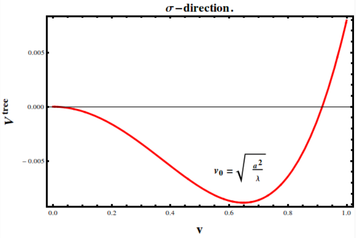
Linear Sigma Model coupled to quarks.

After the shift

$$\mathcal{L} = -\frac{1}{2}[\sigma\partial_\mu^2\sigma] - \frac{1}{2}\left(3\lambda v^2 - a^2\right)\sigma^2$$
$$- \frac{1}{2}[\vec{\pi}\partial_\mu^2\vec{\pi}] - \frac{1}{2}\left(\lambda v^2 - a^2\right)\vec{\pi}^2 + \frac{a^2}{2}v^2$$
$$- \frac{\lambda}{4}v^4 + i\bar{\psi}\gamma^\mu\partial_\mu\psi - gv\bar{\psi}\psi + \mathcal{L}_I^b + \mathcal{L}_I^f,$$

with masses

$$m_\sigma^2 = 3\lambda v^2 - a^2,$$
$$m_\pi^2 = \lambda v^2 - a^2,$$
$$m_f = gv.$$



Finite temperature and density effective potential

- Classical potential

$$V^{\text{tree}}(v) = -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4,$$

The curvature of the classical potential is equal to the sigma mass squared

$$\frac{d^2V^{\text{tree}}}{dv^2} = 3\lambda v^2 - a^2 = m_\sigma^2,$$

$$V^{\text{tree}} = -\frac{a^2}{2}v^2 + \frac{\lambda}{4}v^4 \to -\frac{(a^2+\delta a^2)}{2}v^2 + \frac{(\lambda+\delta\lambda)}{4}v^4.$$

These counter-terms are needed to make sure that the phase transition at the critical temperature T_c for $\mu_B = 0$ is second order and that this transition is first order at the critical baryon density $\mu_B^c = 0$ for $T = 0$.

Effective potential

Finite temperature and density effective potential

- 1-loop boson contribution

$$V^{(1)\mathrm{b}}(v, T) = T\sum_n \int \frac{d^3k}{(2\pi)^3} \ln D(\omega_n, \vec{k})^{1/2},$$

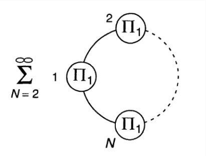
- 1-loop fermion contribution

$$V^{(1)\mathrm{f}}(v, T, \mu_q) = -T\sum_n \int \frac{d^3k}{(2\pi)^3} \mathrm{Tr}[\ln S(\tilde{\omega}_n - i\mu_q, \vec{k})^{-1}],$$

- We work within the imaginary-time formalism of thermal field theory

Beyond mean field approximation.

Next term in the perturbative series is the ring diagrams (Dolan & Jackiw, Phys. Rev. D**12** 3320 (1974)).



Le Bellac.

Screening properties of the plasma. The ring diagrams term is given by

$$V^{\text{Ring}}(v, T, \mu_q) = \frac{T}{2} \sum_n \int \frac{d^3k}{(2\pi)^3} \ln(1 + \Pi(m_b, T, \mu_q) D(\omega_n, \vec{k})),$$

with the self-energy

$$\Pi = \lambda \frac{T^2}{2} - N_f N_c g^2 \frac{T^2}{\pi^2} [Li_2(-e^{\mu/T}) + Li_2(-e^{-\mu/T})]$$

High temperature approximation

- Boson field

$$V_{\mathrm{HT}}^{(1)\mathrm{b}}(v,T)+V^{\mathrm{Ring}}(v,T,\mu_q)=-\frac{m_b^4}{64\pi^2}\left[\underline{\ln\left(\frac{4\pi a^2}{m_b^2}\right)}-\gamma_E+\frac{1}{2}\right]$$

$$-\underline{\frac{m_b^4}{64\pi^2}\ln\left(\frac{m_b^2}{(4\pi T)^2}\right)}-\frac{\pi^2T^4}{90}+\frac{m_b^2T^2}{24}-\underline{\underline{\frac{m_b^3T}{12\pi}}}$$

$$+\frac{T}{12\pi}\left(\underline{\underline{m_b^3}}-(m_b^2+\Pi(T,\mu_q))^{3/2}\right)$$

- Fermion Field

$$V_{\mathrm{HT}}^{(1)\mathrm{f}}(v,T,\mu_q)=\frac{m_f^4}{16\pi^2}\left[\underline{\ln\left(\frac{4\pi a^2}{m_f^2}\right)}-\gamma_E+\frac{1}{2}\right]+\frac{m_f^4}{16\pi^2}\left[\underline{\ln\left(\frac{m_f^2}{(4\pi T)^2}\right)}\right.$$

$$\left.-\psi^0\left(\frac{1}{2}+\frac{\mathrm{i}\mu}{2\pi T}\right)-\psi^0\left(\frac{1}{2}-\frac{\mathrm{i}\mu}{2\pi T}\right)\right]-8m_f^2T^2\left[\mathrm{Li}_2(-e^{\mu_q/T})\right.$$

$$\left.+\mathrm{Li}_2(-e^{-\mu_q/T})\right]+32T^4\left[\mathrm{Li}_4(-e^{\mu_q/T})+\mathrm{Li}_4(-e^{-\mu_q/T})\right].$$

High temperature approximation

- Effective potential

$$
\begin{aligned}
V^{\text{eff}}_{\text{HT}}(v, T, \mu_q) = &-\frac{(a^2+\delta a^2)}{2} v^2 + \frac{(\lambda+\delta\lambda)}{4} v^4 \\
&+ \sum_{b=\sigma,\bar{\pi}} \Bigg\{ -\frac{m_b^4}{64\pi^2}\left[\ln\left(\frac{a^2}{4\pi T^2}\right) - \gamma_E + \frac{1}{2}\right] \\
&- \frac{\pi^2 T^4}{90} + \frac{m_b^2 T^2}{24} - \frac{(m_b^2 + \Pi(T,\mu_q))^{3/2} T}{12\pi} \Bigg\} \\
&+ \sum_{f=u,d} \Bigg\{ \frac{m_f^4}{16\pi^2}\Bigg[\ln\left(\frac{a^2}{4\pi T^2}\right) - \gamma_E + \frac{1}{2} \\
&- \psi^0\left(\frac{1}{2} + \frac{i\mu_q}{2\pi T}\right) - \psi^0\left(\frac{1}{2} - \frac{i\mu_q}{2\pi T}\right)\Bigg] \\
&- 8 m_f^2 T^2 \left[\text{Li}_2(-e^{\mu_q/T}) + \text{Li}_2(-e^{-\mu_q/T})\right] \\
&+ 32 T^4 \left[\text{Li}_4(-e^{\mu_q/T}) + \text{Li}_4(-e^{-\mu_q/T})\right] \Bigg\}.
\end{aligned}
$$

Low temperature approximation

- 1-loop boson contribution

$$V_{\text{LT}}^{(1)\text{b}}(v, T, \mu_b) = \frac{1}{2\pi^2}\int dk\, k^2\left\{\sqrt{k^2+m_b^2}+2T\ln\left(1-e^{-(\sqrt{k^2+m_b^2}-\mu_b)/T}\right)\right\}.$$

- Low temperature approximation: The general idea consists on developing a Taylor series around $T = 0$

Low temperature approximation

- Boson field

$$V^{(1)\mathrm{b}}_{\mathrm{LT}}(v, T, \mu_b) = V^{\mathrm{b}}_0(v, \mu_b) + \frac{\pi^2 T^2}{12}\frac{\partial^2}{\partial T^2}V^{\mathrm{b}}_0(v, \mu_b) + \frac{7\pi^4 T^4}{1260}\frac{\partial^4}{\partial T^4}V^{\mathrm{b}}_0(v, \mu_b).$$

$$V^{(1)\mathrm{b}}_0(v, \mu_b) = -\frac{m_b^4}{64\pi^2}\Big[\ln\Big(\frac{4\pi a^2}{(\mu_b + \sqrt{\mu_b^2 - m_b^2})^2}\Big) - \gamma_E + \frac{1}{2}\Big] + \frac{\mu_b\sqrt{\mu_b^2 - m_b^2}}{96\pi^2}(2\mu_b^2 - 5m_b^2)$$

- Fermion Field

$$V^{(1)\mathrm{f}}_{\mathrm{LT}}(v, T, \mu_q) = V^{\mathrm{f}}_0(v, \mu_q) + \frac{\pi^2 T^2}{6}\frac{\partial^2}{\partial T^2}V^{\mathrm{f}}_0(v, \mu_q) + \frac{\pi^4 T^4}{360}\frac{\partial^4}{\partial T^4}V^{\mathrm{f}}_0(v, \mu_q)$$

$$V^{(1)\mathrm{f}}_0(v, \mu_q) = \frac{m_b^4}{16\pi^2}\Big[\ln\Big(\frac{4\pi a^2}{(\mu_q + \sqrt{\mu_q^2 - m_f^2})^2}\Big) - \gamma_E + \frac{1}{2}\Big] - \frac{\mu_q\sqrt{\mu_q^2 - m_f^2}}{24\pi^2}(2\mu_q^2 - 5m_f^2)$$

Low temperature approximation

- Effective potential

$$
\begin{aligned}
V_{\mathrm{LT}}^{\mathrm{eff}} =& -\frac{(a^2+\delta a^2)}{2}v^2 + \frac{(\lambda+\delta\lambda)}{4}v^4 - \sum_{i=\sigma,\bar{\pi}}\left\{\frac{m_i^4}{64\pi^2}\left[\ln\left(\frac{4\pi^2 a^2}{\mu_b+\sqrt{\mu_b^2-m_i^2}}\right)\right.\right. \\
& \left. - \gamma_E + \frac{1}{2}\right] - \frac{\mu_b\sqrt{\mu_b^2-m_i^2}}{24\pi^2}(2\mu_b^2-5m_i^2) - \frac{T^2\mu_b}{12}\sqrt{2\mu_b^2-5m_i^2} \\
& \left. - \frac{\pi^2 T^4\mu_b}{180}\frac{(2\mu_b^2-3m_i^2)}{(\mu_b^2-m_i^2)^{3/2}}\right\} + N_c\sum_{f=u,d}\left\{\frac{m_f^4}{16\pi^2}\left[\ln\left(\frac{4\pi^2 a^2}{\mu_q+\sqrt{\mu_q^2-m_f^2}}\right)\right.\right. \\
& \left. - \gamma_E + \frac{1}{2}\right] - \frac{\mu_q\sqrt{\mu_q^2-m_f^2}}{24\pi^2}(2\mu_q^2-5m_f^2) - \frac{T^2\mu_q}{6}\sqrt{\mu_q^2-m_f^2} \\
& \left. - \frac{7\pi^2 T^4\mu_q}{360}\frac{(2\mu_q^2-3m_f^2)}{(\mu_q^2-m_f^2)^{3/2}}\right\}
\end{aligned}
$$

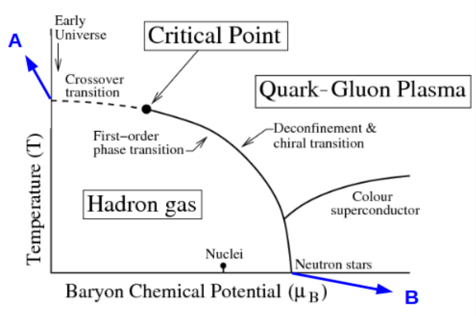
Fixing parameters

Free parameters of LSMq

- The square mass parameter a^2

$$a = \sqrt{\frac{m_\sigma^2 - 3m_\pi^2}{2}}$$

- The two coupling constants λ and g



Free parameters of LSMq

- The two coupling constants λ and g
  At point (A)

$$m_\pi^2(0, T_0^c, \mu_q = 0) = -a^2 + \Pi(T_0^c, \mu_q = 0) = 0$$

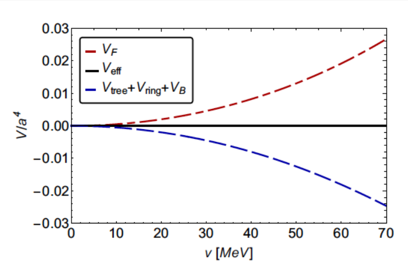
  At point (B)

$$m_\pi^2(v_1, 0, \mu_q^c) = \lambda v_1 - a^2 + \Pi(0, \mu_q^c) = 0$$

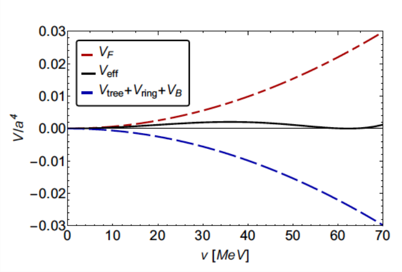
$$\frac{\partial V^{\text{eff}}}{\partial v}(v = 0, T = 0, \mu_q = \mu_q^c) = 0, \quad \frac{\partial V^{\text{eff}}}{\partial v}(v = v_1, T = 0, \mu_q = \mu_q^c) = 0,$$
$$V^{\text{eff}}(v = 0, T = 0, \mu_q = \mu_q^c) = V^{\text{eff}}(v = v_1, T = 0, \mu_q = \mu_q^c)$$

The QCD phase diagram

Identifying the order phase transition

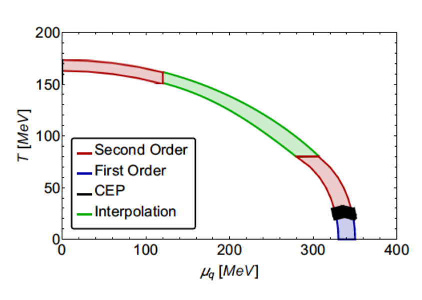


2nd order phase transition.

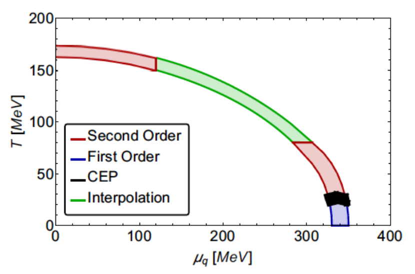


1st order phase transition.

QCD phase diagram ($\mu_q = \mu_b$)

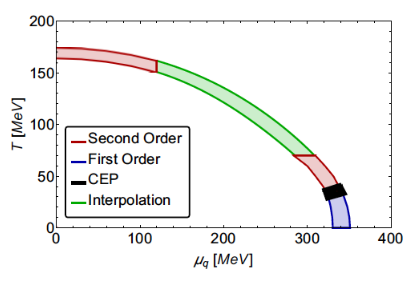


$m_\sigma = 139$ MeV

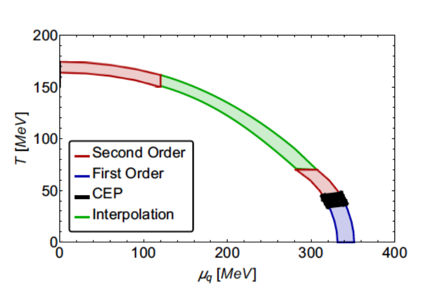


$m_\sigma = 0$ MeV

QCD phase diagram ($\mu_q = 2\mu_b$)

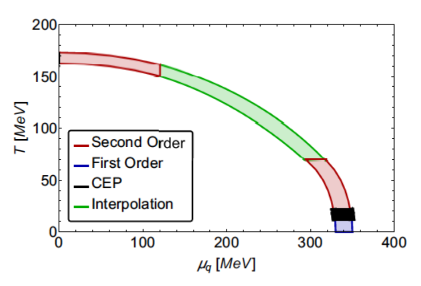


$m_\sigma = 139$ MeV

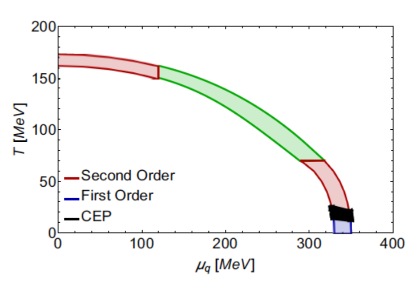


$m_\sigma = 0$ MeV

QCD phase diagram ($\mu_q = 0.5\mu_b$)



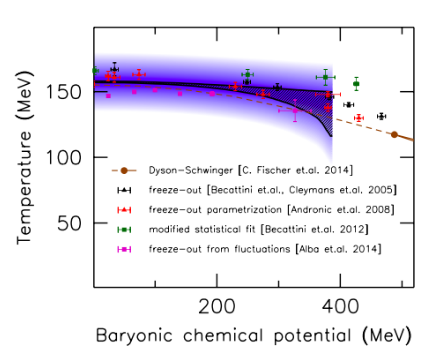
$m_\sigma = 139$ MeV



$m_\sigma = 0$ MeV

Results

LQCD results



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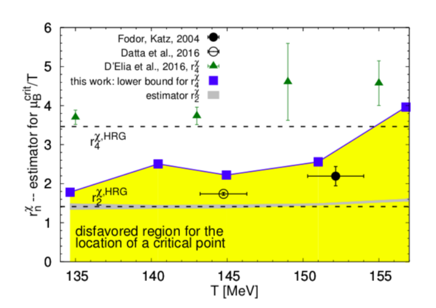

A. Bazavov *et al.* (Bielefeld-BNL-CCNU collaboration), Phys. Rev. D **95**, no. 5, 054504 (2017)

CEP. Latest results

Reference	Technique	T_{CEP}	μ_{CEP}
arXiv:1602.00062	DSEs	0.85 T_c	1.11 T_c
arXiv:1605.08430	nonlocal PNJL	69.9 MeV	319.1 MeV
arXiv:1611.06669	FRG	5.1 MeV	286.7 MeV
arXiv:1612.06673	LQCD	155 MeV	285 MeV
QM2015 and CPOD2016	LQCD	-	>2 T_{CEP}
QM2017	LQCD	145-155 MeV	>2 T_{CEP}
arXiv:1702.06731	ADS/CFT	112 MeV	612 MeV
arXiv:1705.09124	-	119-162 MeV	252-258 MeV
Sci.Rep. 7 (2017) 45937	NJL	38 MeV	245 MeV
This work	LSMq	18-45 MeV	315-349 MeV

Thanks!!!