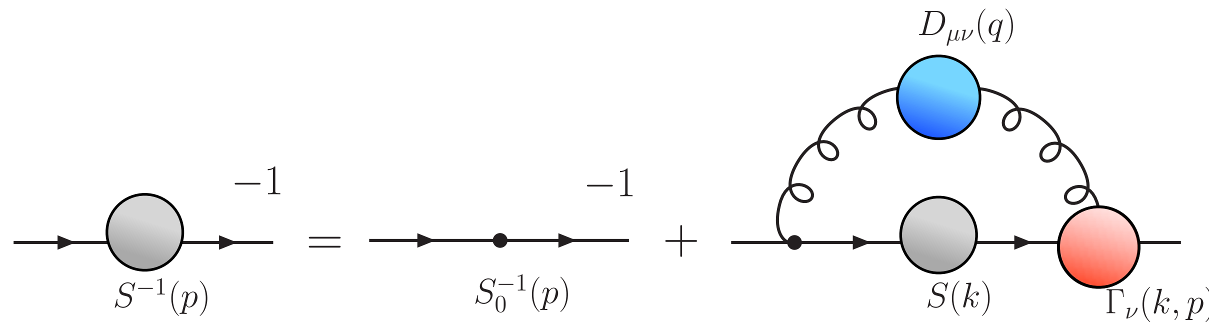
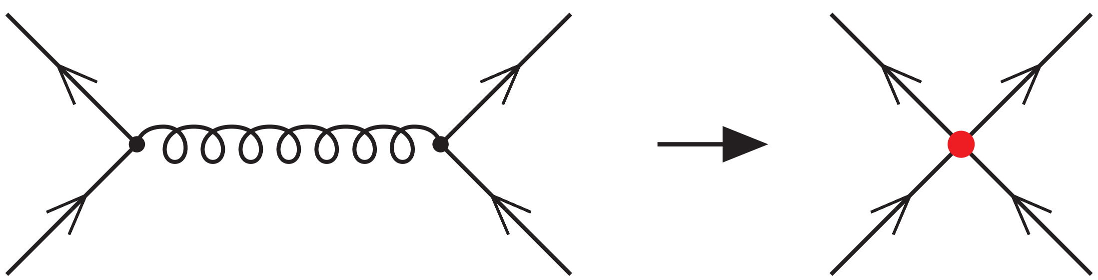
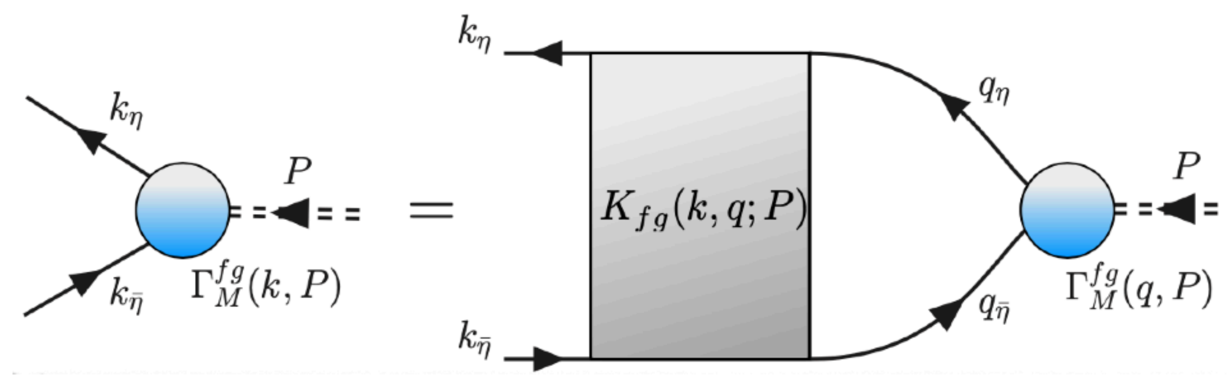






Contact interaction model in hadron physics: Theoretical insight and applications

Fernando E. Serna

University of Sucre, Colombia

IF-USP - QCD seminar

São Paulo - June 17, 2024

Quantum Chromodynamics (QCD)

- The strong force is the responsible to held neutron and proton inside the atomic nucleus.

- It is around 100 times stronger than the electromagnetic.

- Quantum Chromodynamics (QCD) is the fundamental theory that describes the strong interactions.

- It QCD is a local, non-Abelian gauge theory whose basic degrees of freedom are quarks and gluons.

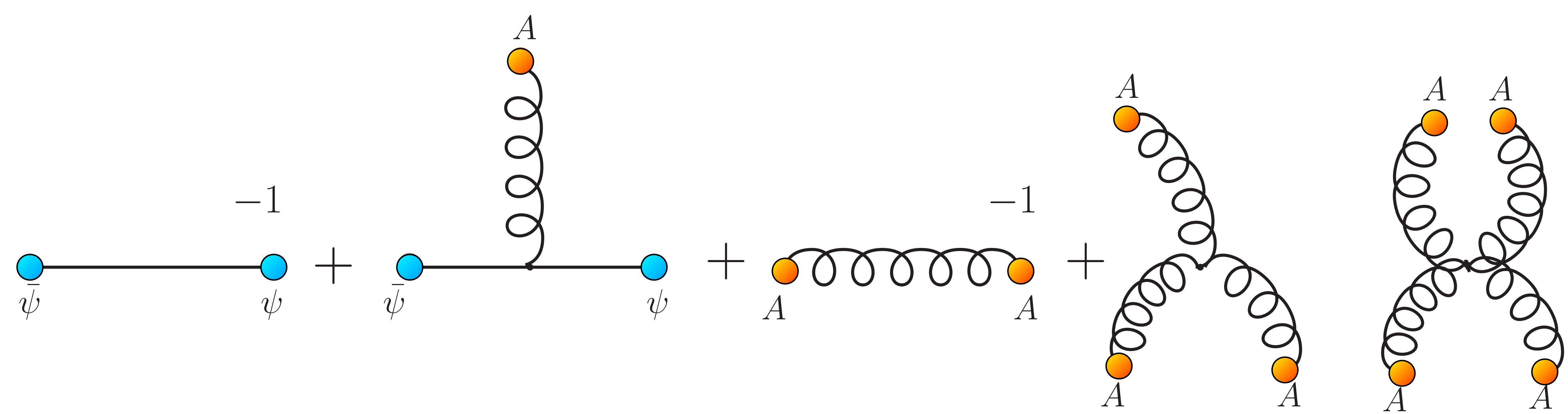
Quantum Chromodynamics (QCD)

- QCD is characterized by the classical action:

$$S_{\rm QCD} = \int dx \left[\delta^{cc'} \bar{\psi}^c_j (\gamma^\mu \partial_\mu + m_f) \psi^{c'}_j - g t^{cc'}_a \bar{\psi}^c_j \gamma^\mu A^a_\mu \psi^{c'}_j + \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a \right]$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

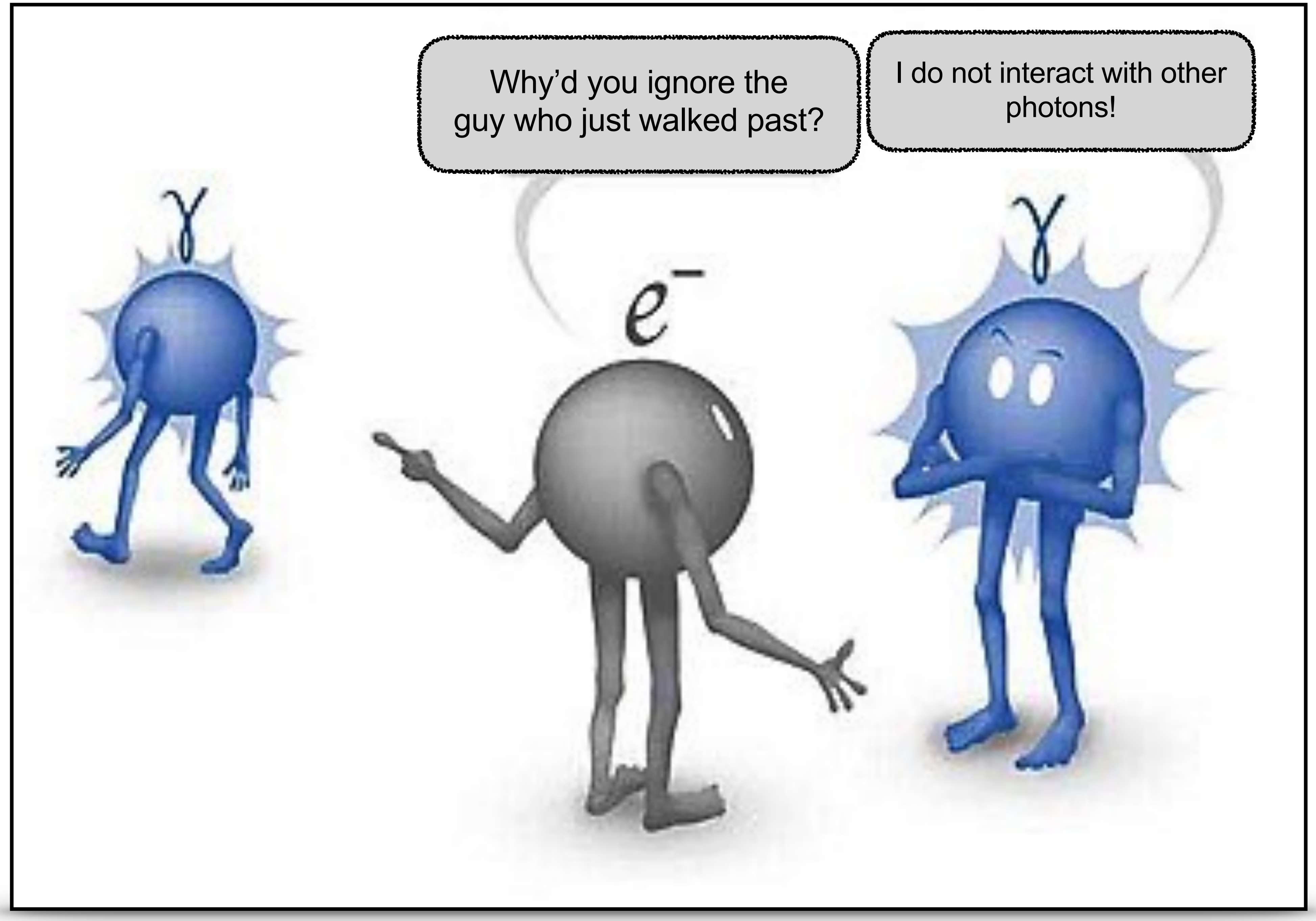
- $g f^{abc} A^b_\mu A^c_\nu$ generates gluon self-interactions.

$S_{\rm QCD}$ = 

Quantum Chromodynamics (QCD)

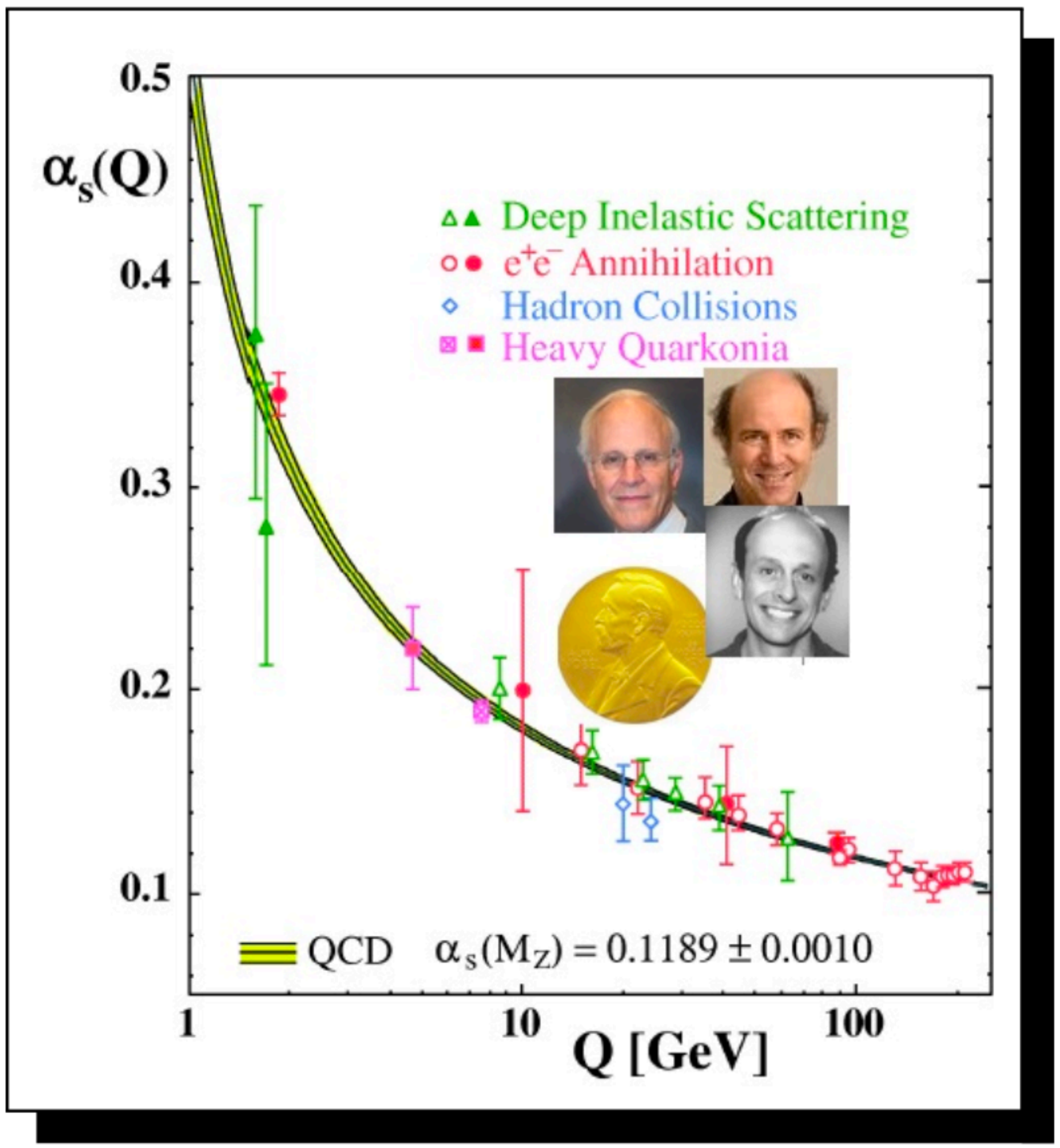
QED

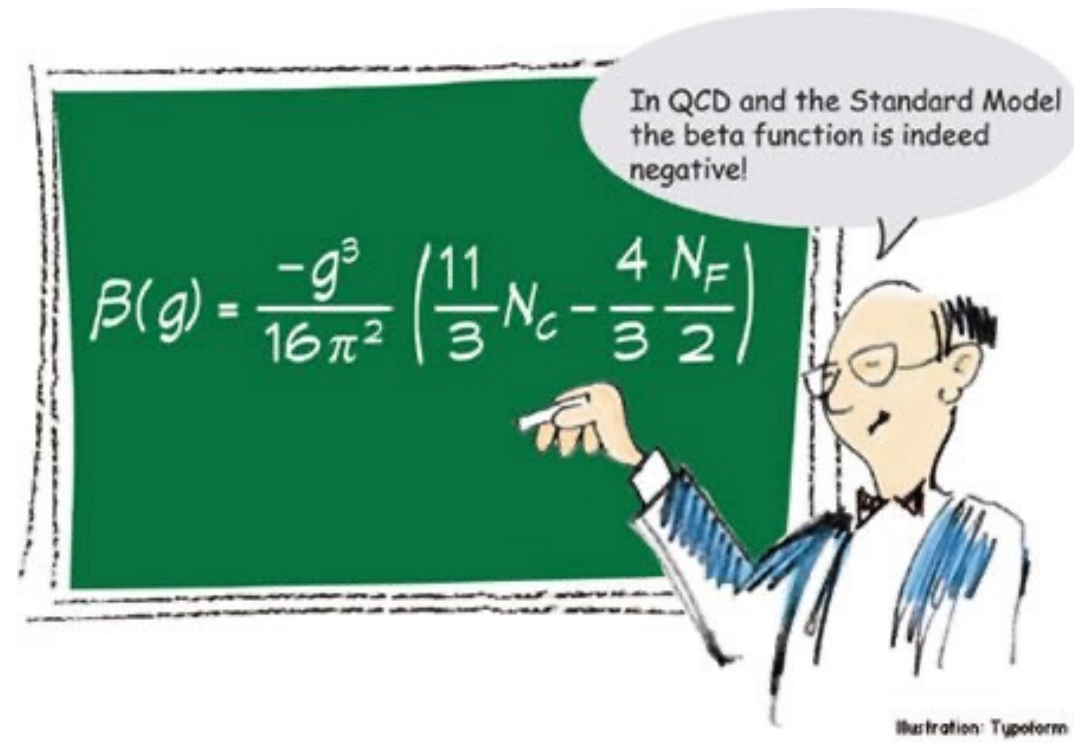
$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$



Quantum Chromodynamics (QCD)

- At large momentum transfers:
  - **Asymptotic freedom:** Quarks behave quasi-free





$$\alpha_s(Q^2) = \frac{12\pi}{(11N_c - 2N_f)\log\left(\frac{Q^2}{\Lambda_{QCD}}\right)}$$

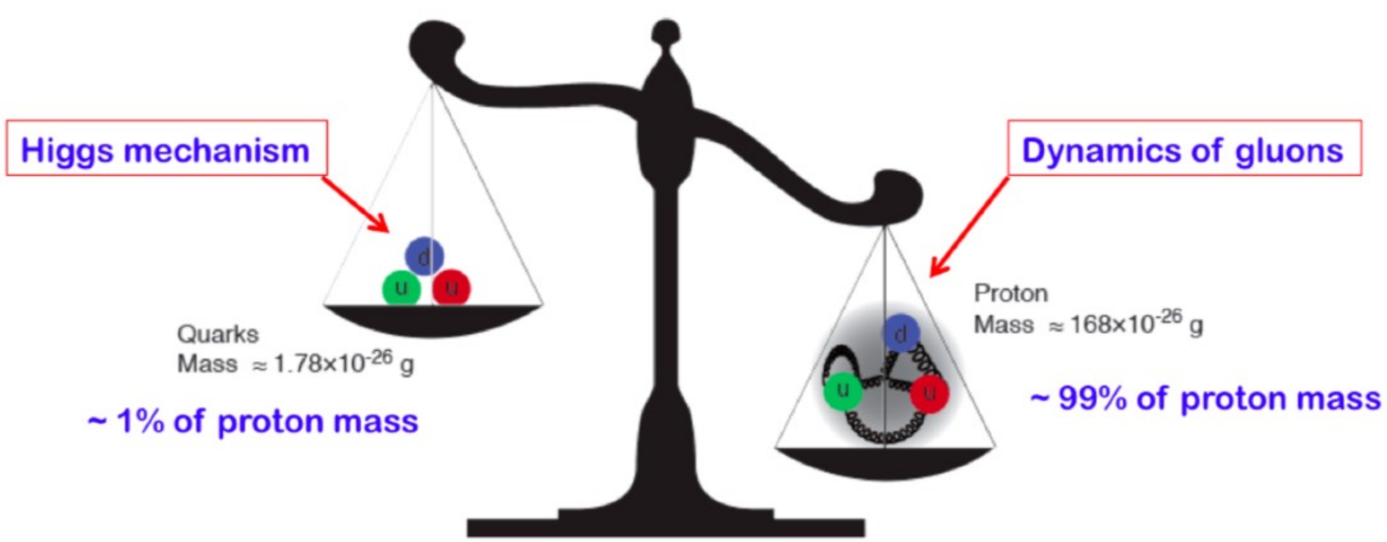
Quantum Chromodynamics (QCD)

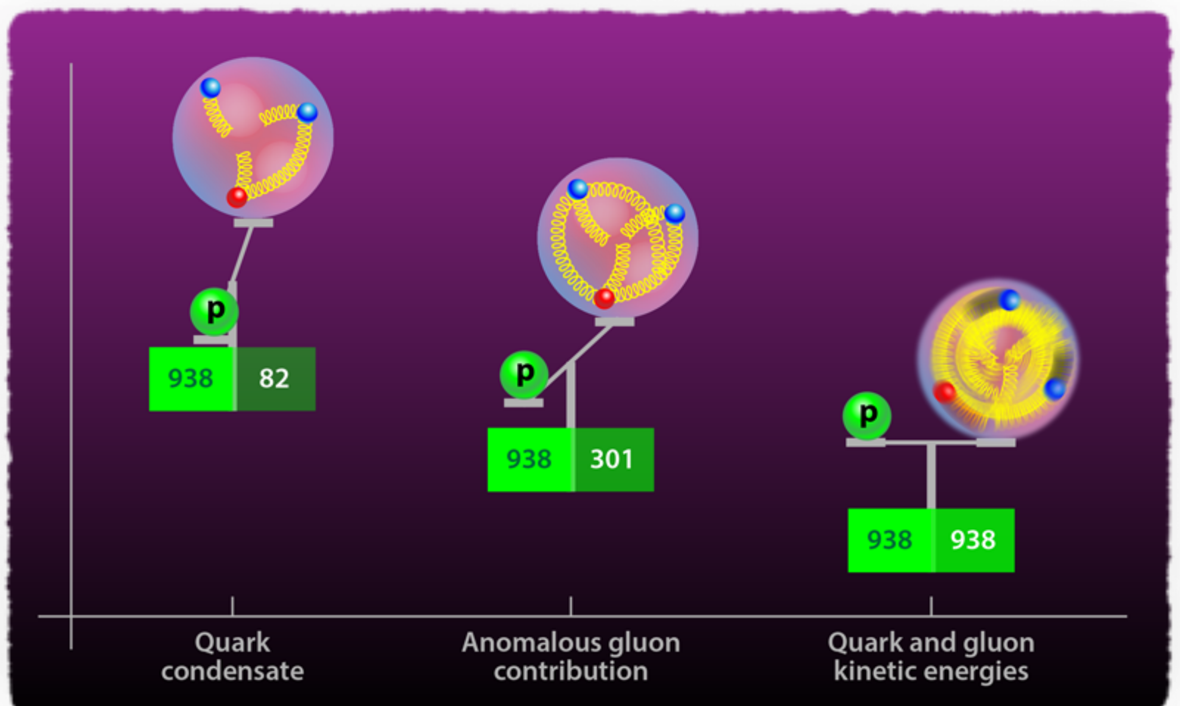
- Strong interactions are singularly characterized by two phenomena:

  - **Color confinement**

  - **Mass generating phenomenon ⇒ Dynamical Chiral Symmetry Breaking (DCSB)**

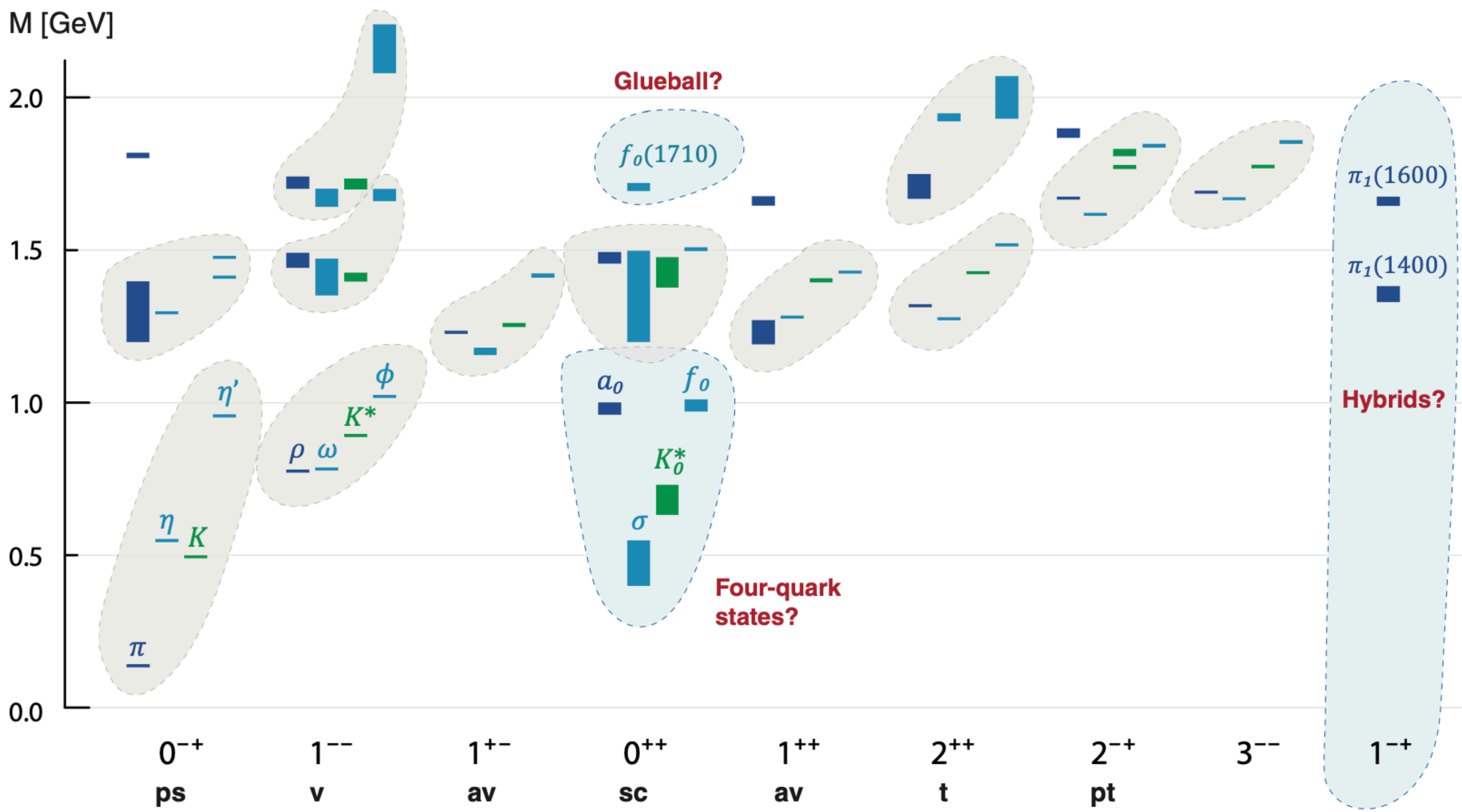
- DCSB is the most important mass generating mechanism for visible matter in our universe.





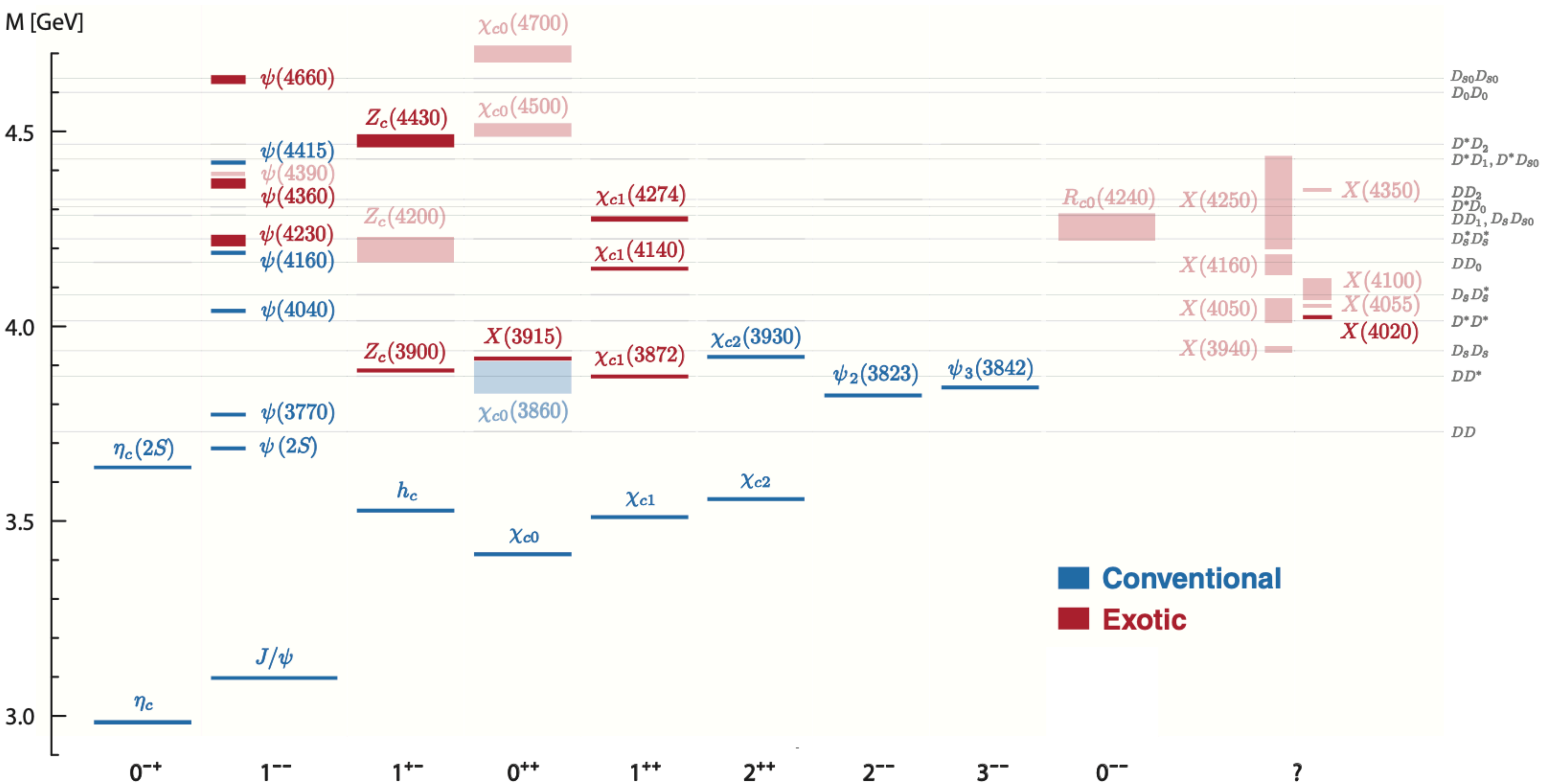
PRL. 121, 212001 (2018)

Many hadrons



Light and strange meson spectrum

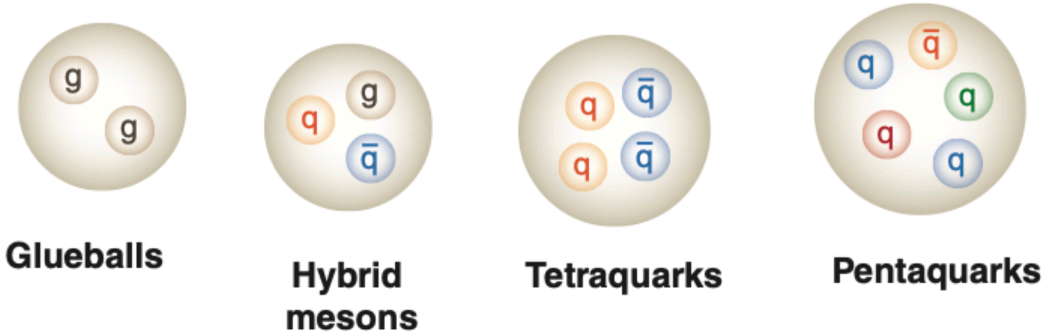
Many hadrons

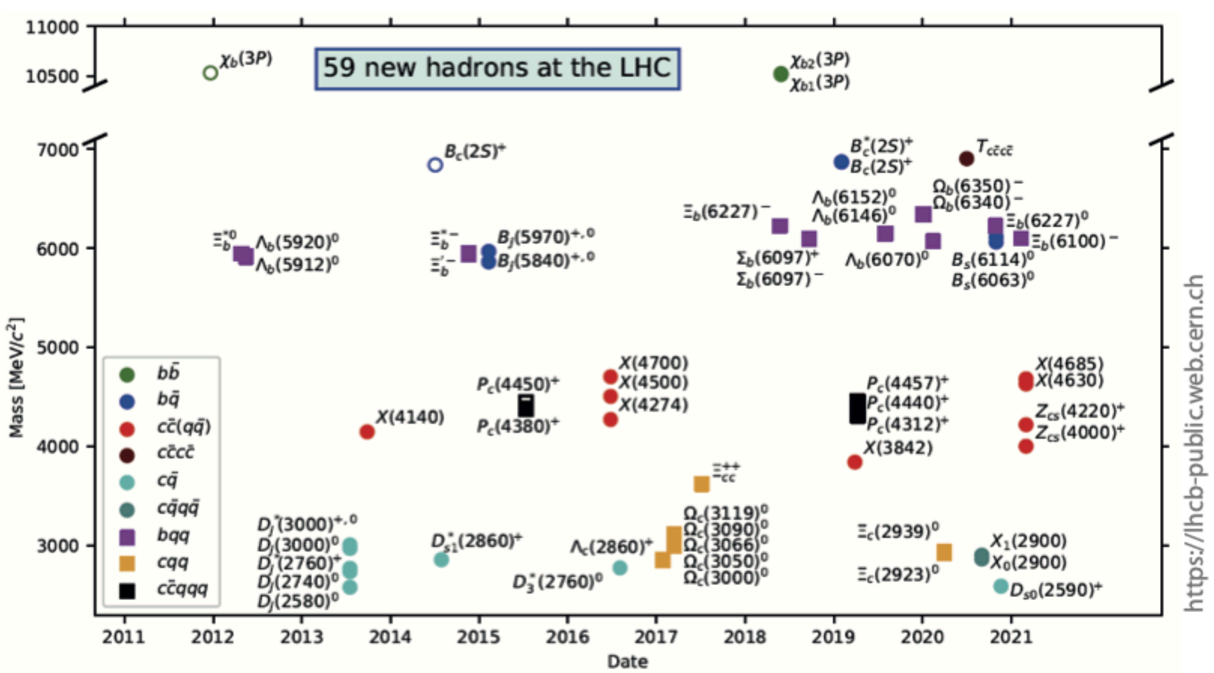


Charmonium spectrum

Many open questions

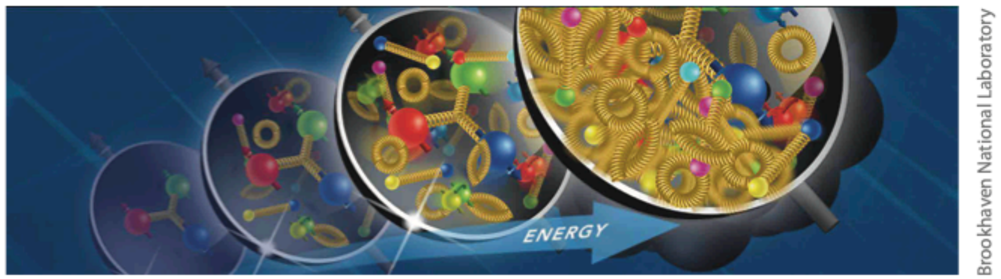
- **Understanding exotic hadrons:**
  Hadron spectroscopy at LHC, Belle II, BES III, PANDA, JLab, ELSA, ...

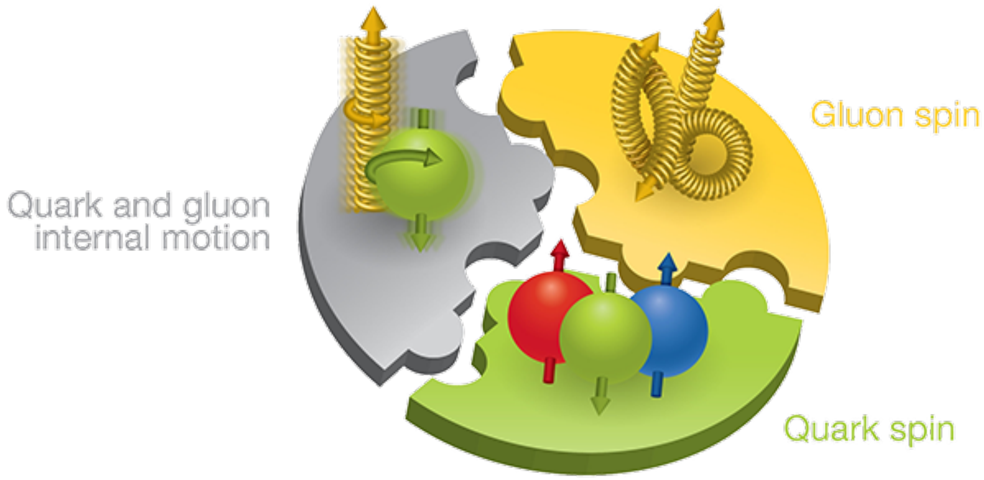




- **Mass generation and confinement?**

- **Quark-gluon structure of hadrons and nuclei:**
  Hadron tomography at EIC, JLab, COMPASS/AMBER, ...



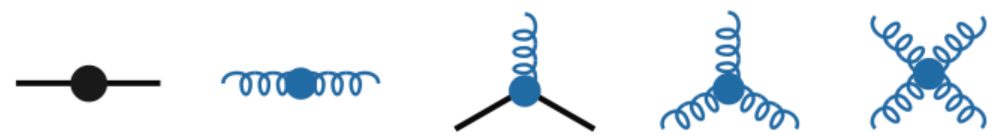


Proton spin puzzle

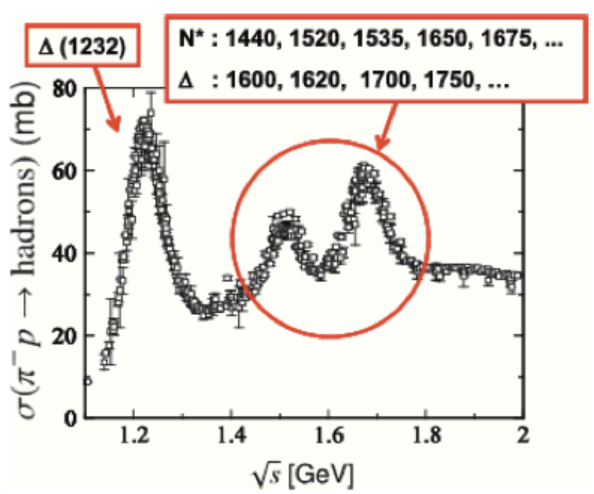
Theory tools

- Non-perturbative physics requires special tools.

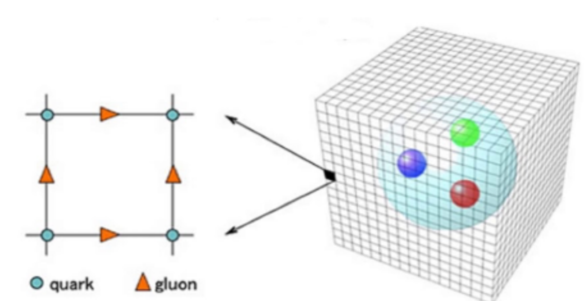
Functional methods
(DSEs & BSEs, FRG, ...)



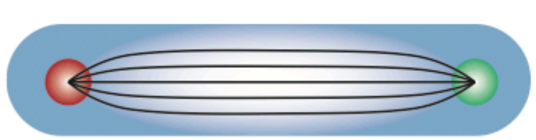
Amplitude analyses



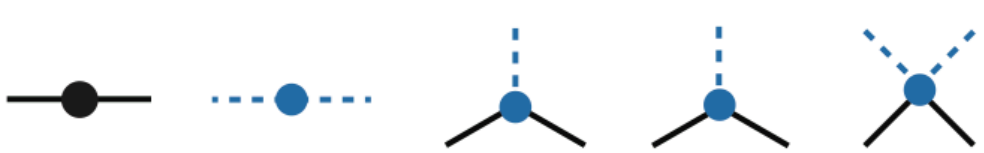
Lattice QCD



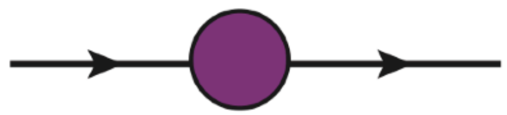
Phenomenological models



Effective theories
(ChPT, ...)

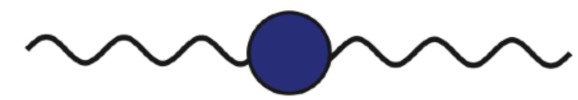


$$\langle\Omega|T\{\psi(x)\overline{\psi}(y)\}|\Omega\rangle := iS(x-y)$$



Quark propagator

$$\langle\Omega|T\{A^a_\mu(x)A^b_\nu(y)\}|\Omega\rangle := -i\Delta^{ab}_{\mu\nu}(x-y)$$

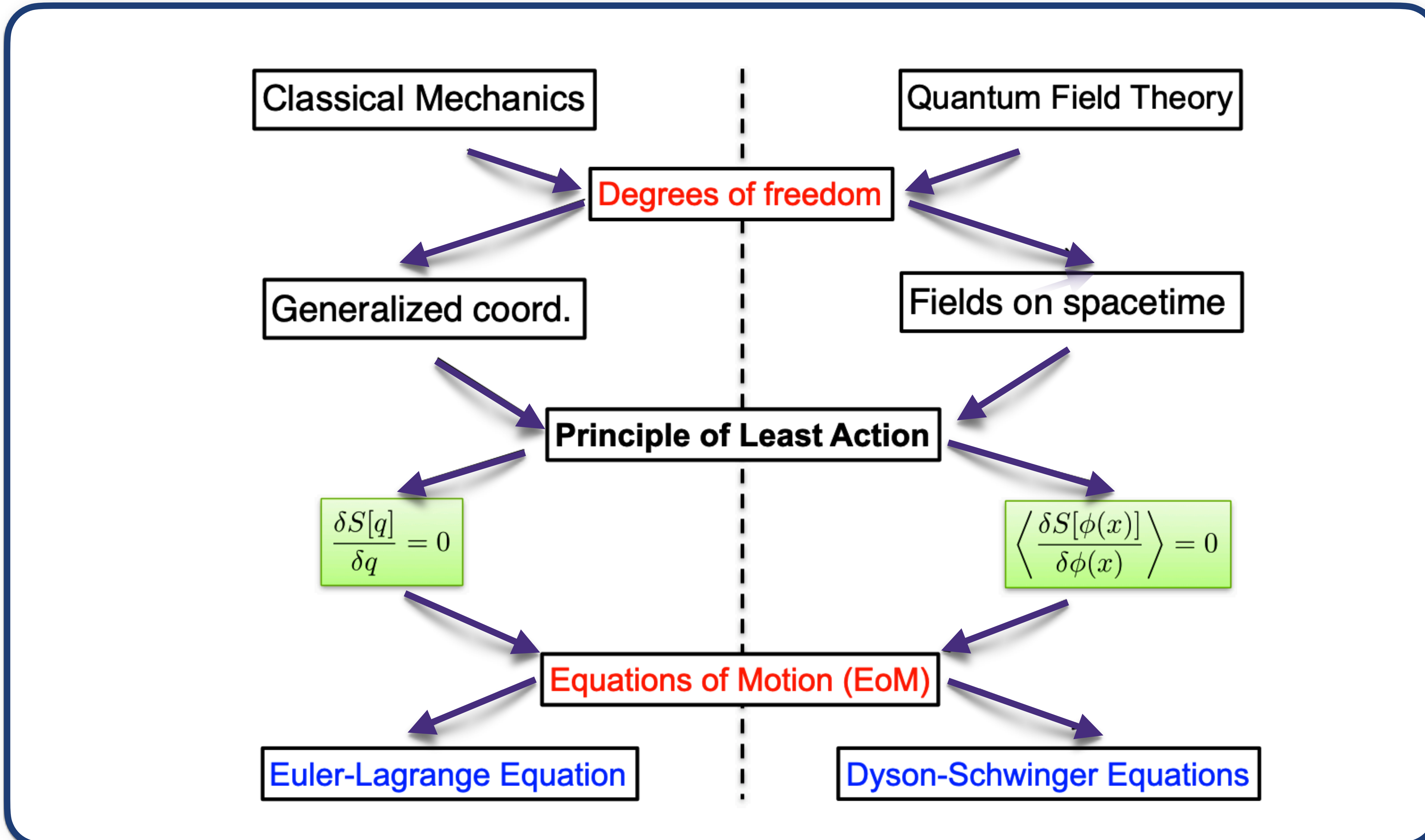


Gluon propagator

Non-perturbative continuum tools for QCD

Non-perturbative continuum tools for QCD

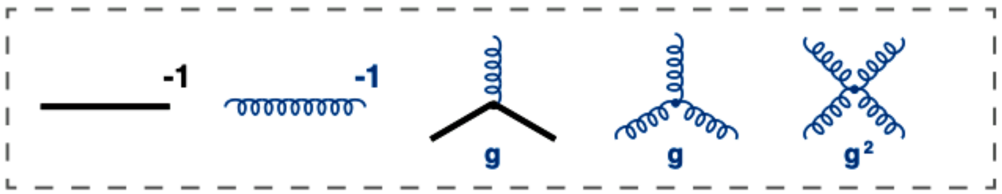
- For QCD we have first principles:



QCD's Dyson-Schwinger Equations (DSEs)

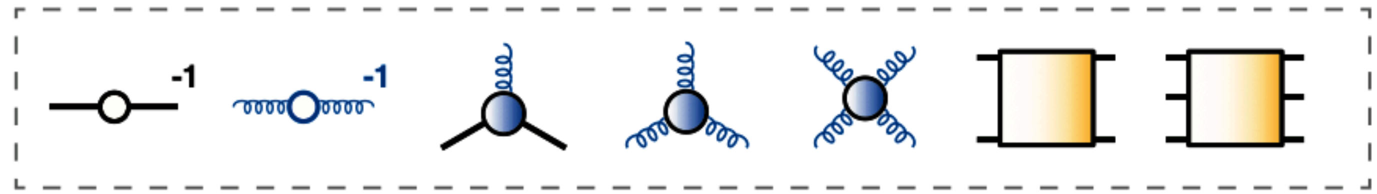
QCD's classical action:

$$S = \int d^4x \left[\bar{\psi}\,(\not{\partial} + ig\not{A} + m)\,\psi + \tfrac{1}{4}\,F^{\,a}_{\mu\nu}\,F^{\mu\nu}_{a}\right]$$

$=$ 

Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar{\psi}, A]\, e^{-S} \quad = \quad e^{-\Gamma}$$

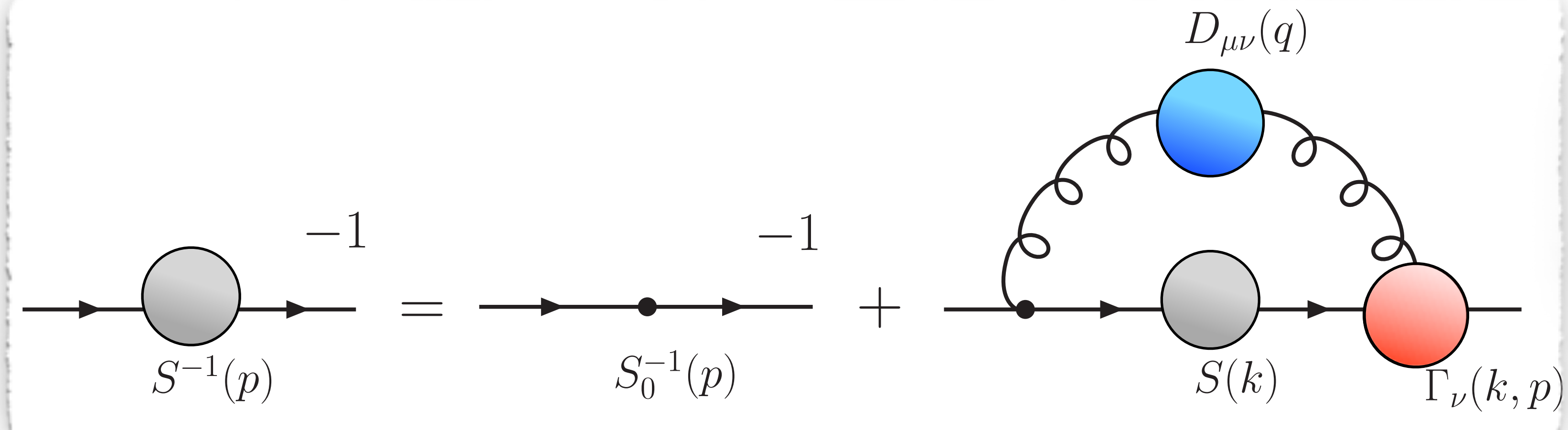


DSE = Quantum equation of motion: obtained from path integral, relate n-point functions

- Infinitely many coupled equations
- Continuum methods: Reproduce perturbation theory, but non-perturbative
- Systematic truncations: neglect higher n-point functions to obtain **closed system**

QCD's Dyson-Schwinger Equations (DSEs)

Quark Dyson-Schwinger Equation



Quark mass function

$$M_f(p^2) = \frac{B_f(p^2,\mu^2)}{A_f(p^2,\mu^2)}$$

$$S_f^{-1}(p) = Z_2^f\,(i\gamma\cdot p + m_f^{\rm bm}) + Z_1^f g^2\int^\Lambda \frac{d^4k}{(2\pi)^4} D_{\mu\nu}^{ab}(q)\frac{\lambda^a}{2}\gamma_\mu S_f(k)\Gamma_{\nu,f}^b(k,p)$$

Quark propagator

$$S_f^{-1}(p) = i\gamma\cdot pA_f(p^2,\mu) + B_f(p^2,\mu)$$

$$= A_f(p^2,\mu)[i\gamma\cdot p + M_f(p^2)]$$

- $D_{\mu\nu}^{ab}(q)$ = Dressed gluon propagator
- $\Gamma_\nu^b(k,p)$ = Dressed quark-gluon vertex
- Z_2 = Quark wave function renormalization constant
- Z_4 = Quark-gluon vertex renormalization constant

Renormalization condition:

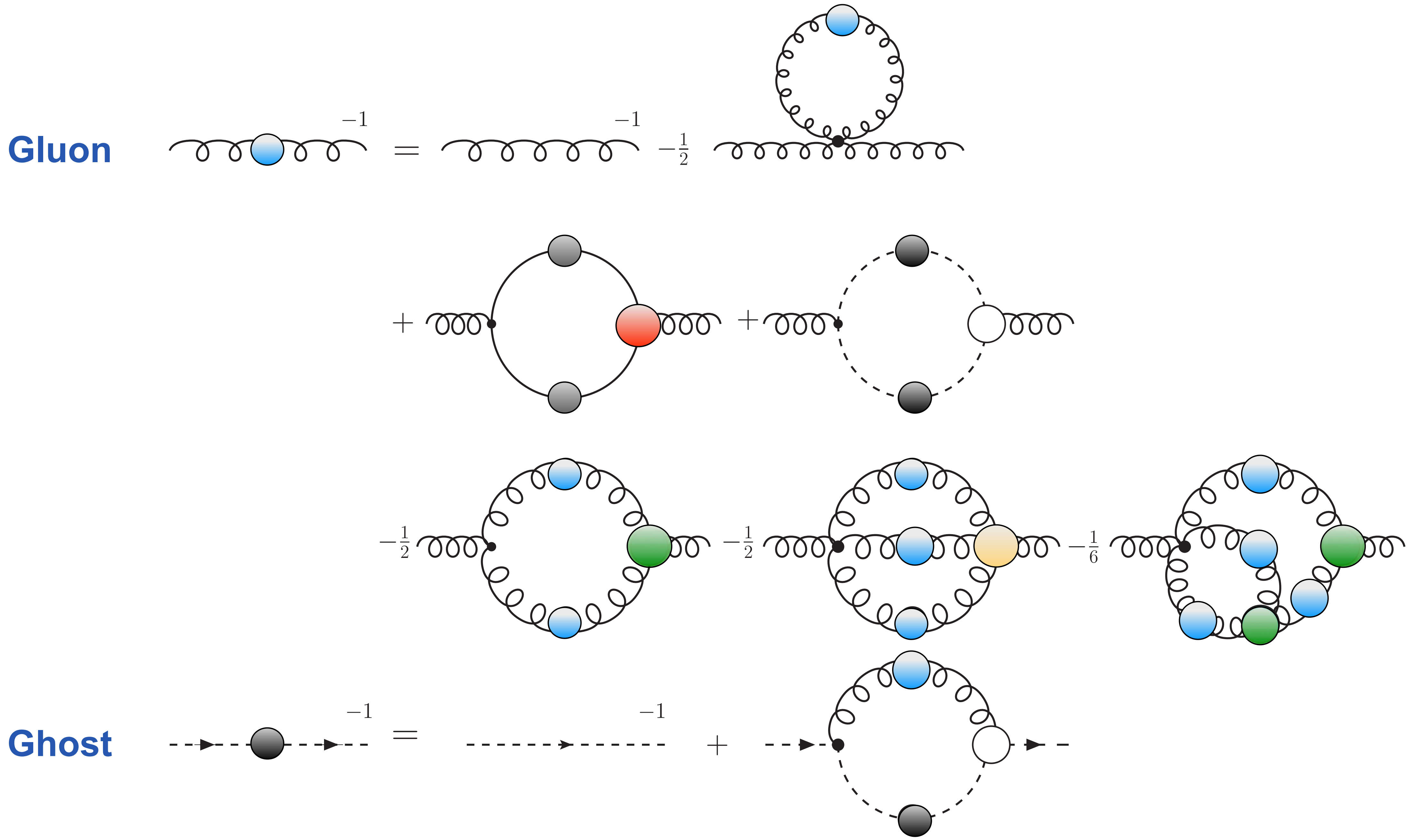
$$A_f(p^2) = A_f(p^2)\big|_{p^2=\mu^2} = 1$$

$$S_f^{-1}(p)\,\big|_{p^2=\mu^2} = i\gamma\cdot p + m_f(\mu)$$

$$Z_4^f(\mu,\Lambda)\,m_f(\mu) = Z_2^f(\mu,\Lambda)\,m_f^{\rm bm}(\Lambda)$$

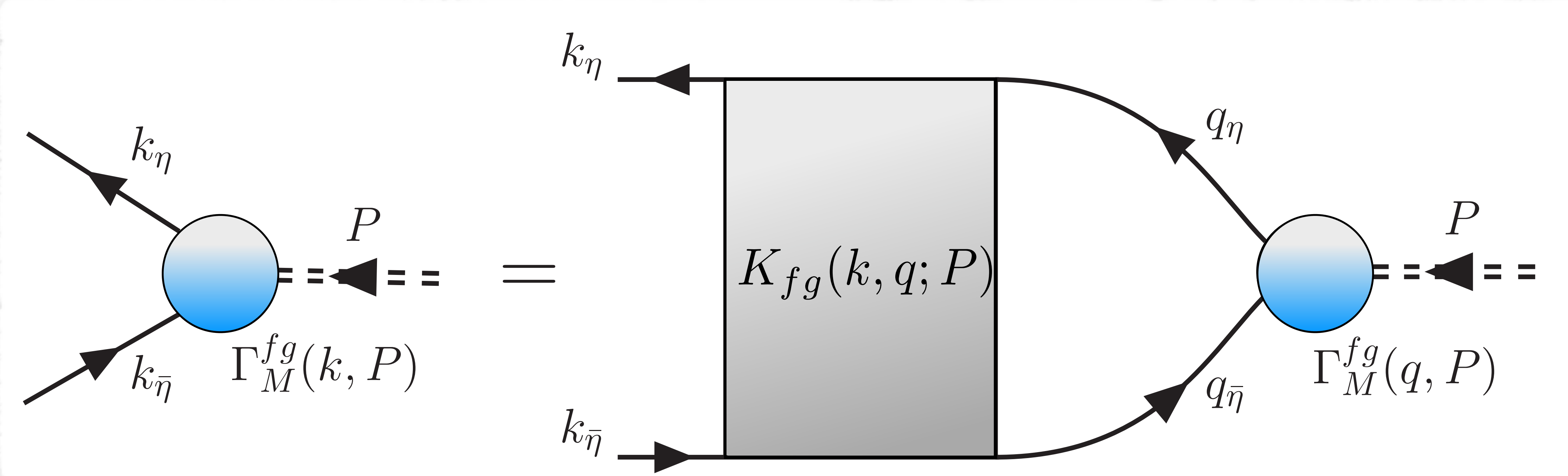
QCD's Dyson-Schwinger Equations (DSEs)

$$D^{ab}_{\mu\nu}(q) = \delta^{ab}\left[\delta_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right]\Delta_\xi(q^2) + \delta^{ab}\xi\frac{q_\mu q_\nu}{q^2}$$



Bethe-Salpeter Equation for QCD bound-states

- **BSE = Bound-state equation for meson**



$$\Gamma_M^{fg}(k,P) = \int^{\Lambda} \frac{d^4q}{(2\pi)^4} K_{fg}(k,q;P) S_f(q_\eta) \Gamma_M^{fg}(q,P) S_g(q_{\bar\eta})$$

- $K_{fg}(q,k;P)$ = Quark-antiquark scattering kernel
- $S_f(q_\eta)$ = Dressed quark propagator
- $\Gamma_M^{fg}(k,P)$ = Meson's Bethe-Salpeter Amplitude (BSA)

$$q_\eta = q + \eta P$$
$$q_{\bar\eta} = q - \bar\eta P$$
$$\eta + \bar\eta = 1$$

Bethe-Salpeter Equation for QCD bound-states

Bethe-Salpeter amplitudes

- The general form of $\Gamma_M(k,P)$ is given by

$$\Gamma_M(k,P) = \sum_{i=1}^{N} T_M^i(k,P) F_i(k,P)$$

C. H. Llewellyn-Smith, Ann. Phys. (N.Y.) 53, 521 (1969).

  - Where $T_M^i(k,P)$ are Dirac's covariants;
  - $F_M^i(k,P)$ are Lorentz invariants amplitudes;
  - N denotes the number of covariants which are different for different meson's channel.
- For the case of pseudoscalar mesons we have $N = 4$ and for vector mesons one has $N = 8$.

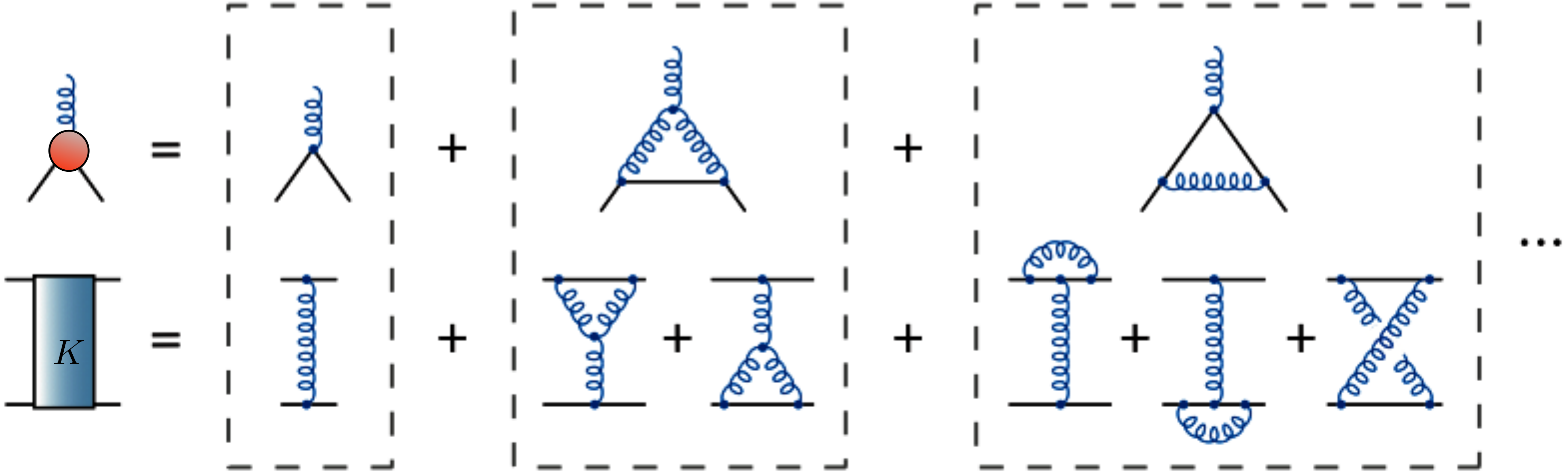
$$\Gamma_M^{fg}(k,P) = \gamma_5 \Big[iE_M^{fg}(k,P) + \gamma \cdot P \, F_M^{fg}(k,P) + \gamma \cdot k \; k \cdot P \, G_M^{fg}(k,P) + \sigma_{\mu\nu} k_\mu P_\nu \, H_M^{fg}(k,P) \Big]$$

Pseudoscalar channel

C. H. Llewellyn-Smith, Ann. Phys. (N.Y.) 53, 521 (1969).

Truncation schemes and symmetries

- **DSE/BSE:** Kernel can be derived in accordance with chiral symmetry



Truncation must preserve **AV-WTI**, which ensures that we will have massless pions in the chiral limit.

AV-WTI: $$P_\mu \Gamma^{fg}_{5\mu}(k;P) = S_f^{-1}(k_\eta)\, i\gamma_5 + i\gamma_5 S_g^{-1}(k_{\bar{\eta}})$$

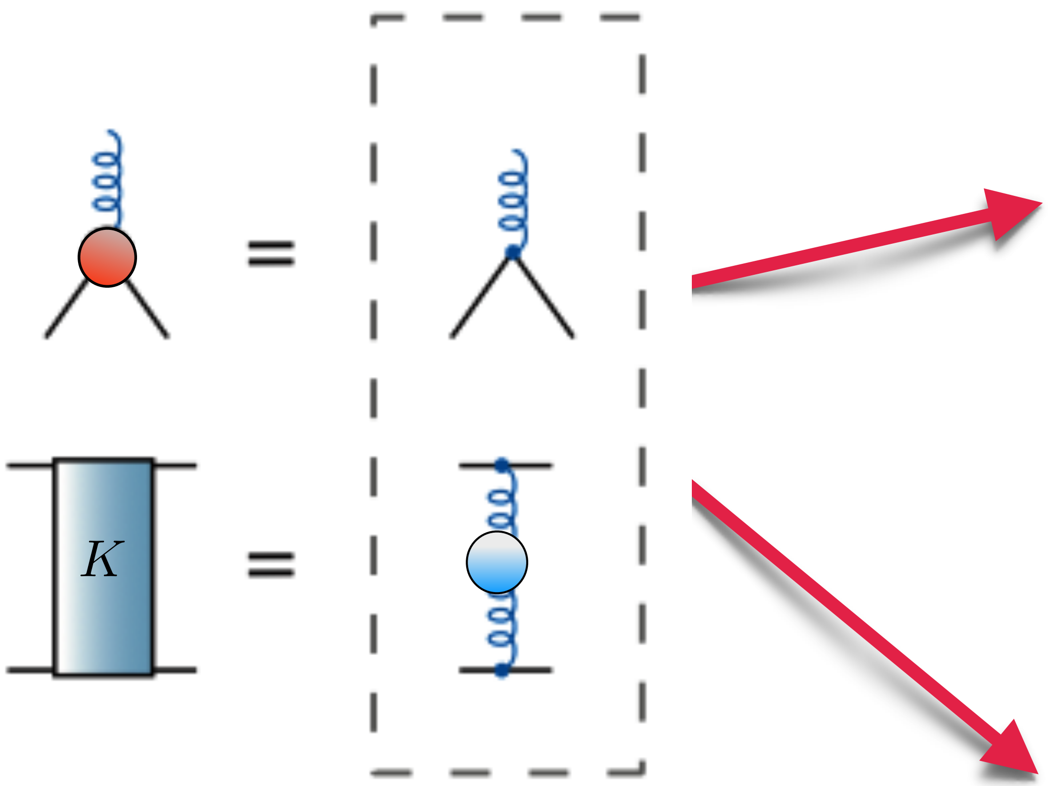
Axial vector-vertex:

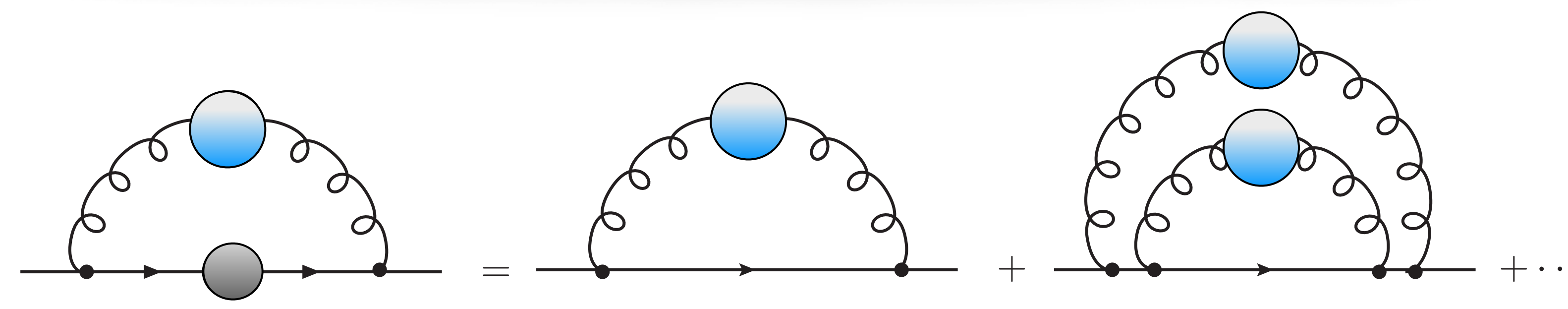
$$\Gamma^{fg}_{5\mu}(k;P) = Z_2^f \gamma_5\gamma_\mu + \int^\Lambda \frac{d^4q}{(2\pi)^4} K_{fg}(q,k;P)\, S_f(q_\eta)\Gamma^{fg}_{5\mu}(q;P) S_g(q_{\bar{\eta}})$$

Rainbow-Ladder truncation

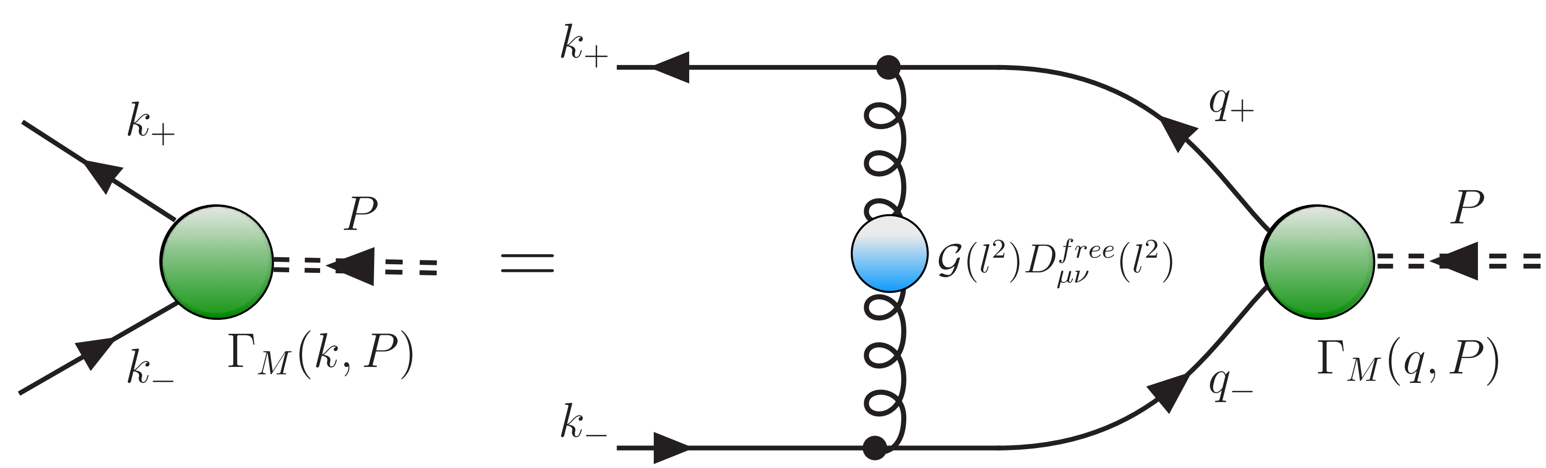
- Leading truncation

$$Z_1^f g^2 D_{\mu\nu}(q)\Gamma_{\nu,f}(k,p) = \left(Z_2^f\right)^2 \mathcal{G}(q^2) D_{\mu\nu}^{\text{free}}(q)\frac{\lambda^a}{2}\gamma_\nu$$





$$K(q,k;P) = -Z_2^2\,\mathcal{G}\left(l^2\right) D_{\mu\nu}^{\text{free}}(l)\,\gamma_\mu\frac{\lambda^a}{2}\gamma_\nu\frac{\lambda^a}{2}$$



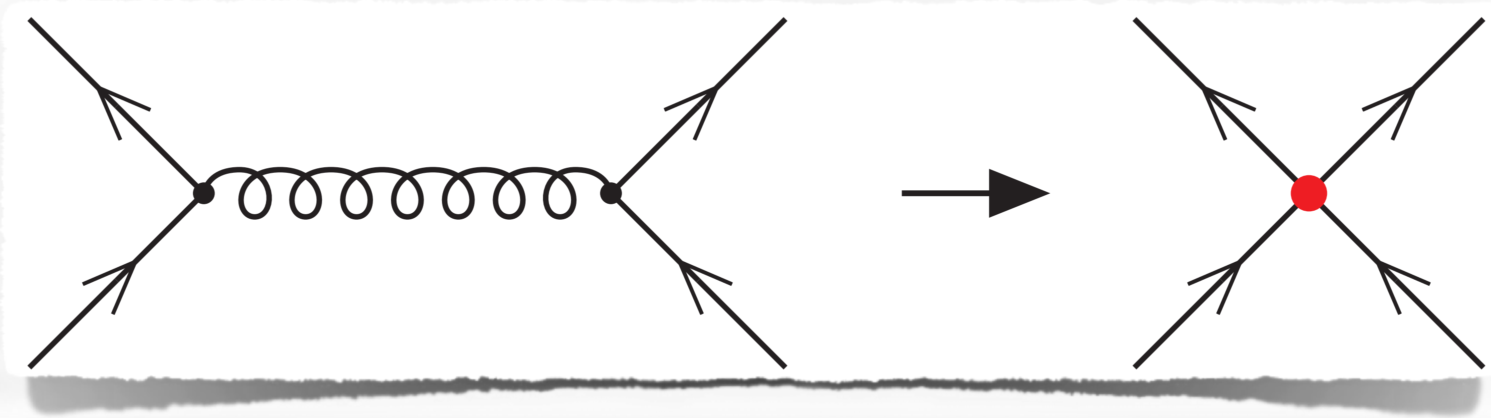
$\mathscr{G}(q)$: Effective gluon interaction

Contact interaction model

- Ansatz for gluon propagator:

$$L_{QCD} = \bar{\psi}(\gamma^{\mu}\partial_{\mu} + m_f)\psi + g\left(\bar{\psi}\frac{\lambda^a}{2}\gamma_{\mu}\psi\right)^2$$

$$D^{ab}_{\mu\nu}(q) = \frac{\delta^{ab}}{g^2}\frac{4\pi\alpha_{\mathrm{IR}}}{m_g^2}\delta_{\mu\nu}$$



- where m_g is the gluon mass scale with together α_{IR} quantify the interaction strength.

- If we plug the truncated quark-gluon vertex and gluon-propagator into the quark DSE, it simplifies to

$$S_f^{-1}(p) = i\gamma\cdot p + m_f + \frac{16\pi\alpha_{\mathrm{IR}}}{3m_g^2}\int^{\Lambda}\frac{d^4k}{(2\pi)^4}\gamma_{\mu}S_f(k)\gamma_{\mu}$$

Contact interaction model

- The scattering kernel in the BSE became:

$$K(k,q;P) = -g^2 D^{ab}_{\mu\nu}(q)\left(\frac{\lambda^a}{2}\gamma_\mu\right)\left(\frac{\lambda^b}{2}\gamma_\nu\right) = -\frac{4\pi\alpha_{\rm IR}}{m_g^2}\delta_{\mu\nu}\left(\frac{\lambda^a}{2}\gamma_\mu\right)\left(\frac{\lambda^a}{2}\gamma_\nu\right)$$

- The BSE simplifies to

$$\Gamma^{fg}_{\rm PS}(P) = -\frac{16\pi\alpha_{\rm IR}}{3m_g^2}\int\frac{d^4q}{(2\pi)^4}\gamma_\mu S_f(q+P)\Gamma^{fg}_{\rm PS}(P)S_g(q)\gamma_\mu$$

- Since we used a simple form for the gluon propagator, that does not depend on the relative momentum k.

$$\Gamma^{fg}_{\rm PS}(P) = \gamma_5\left[iE^{fg}_{\rm PS}(P) + \frac{\gamma\cdot P}{2M_{fg}}F^{fg}_{\rm PS}(P)\right]$$

Solution quark DSE

- Now, we use the general solution of the quark propagator inside the quark DSE.

$$i\gamma\cdot pA_f(p^2)+B_f(p^2)=i\gamma\cdot p+m_f+\frac{16\pi\alpha_{\rm IR}}{3m_g^2}\int\frac{d^4k}{(2\pi)^4}\frac{\gamma_\mu[-i\gamma\cdot kA_f(k)+B_f(k)]\gamma_\mu}{A_f^2(k^2)k^2+B_f^2(k^2)}$$

- Using appropriate projectors, the quark DSE decouples in:

$$\begin{aligned}A_f(p^2) &= 1+\frac{32\pi\alpha_{\rm IR}}{3p^2m_g^2}\int\frac{d^4k}{(2\pi)^4}\frac{(k\cdot p)A_f(k^2)}{k^2A_f^2(k^2)+B_f^2(k^2)}\\ B_f(p^2) &= m_f+\frac{64\pi\alpha_{\rm IR}}{3m_g^2}\int\frac{d^4k}{(2\pi)^4}\frac{B_f(k^2)}{k^2A_f^2(k^2)+B_f^2(k^2)}\end{aligned}$$

Which implies that $A_f(p^2)=1$ and then $B_f=M_f$, so that:

$$S^{-1}(p)=i\gamma\cdot p+M_f,$$

$$M_f = m_f+\frac{64\pi\alpha_{\rm IR}}{3m_g^2}\int\frac{d^4k}{(2\pi)^4}\frac{M_f}{k^2+M_f^2}$$

Solution quark DSE

- The integral in the equation of M_f is quadratically divergent.

$$\int \frac{d^4k}{(2\pi)^4}\frac{M_f}{k^2+M_f^2} \sim \int_0^\infty \frac{k^3}{k^2}dk \sim \int_0^\infty k\,dk = k^2\Big|_0^\infty = \infty.$$

- **Cutoff regularization:**

$$\int \frac{d^4k}{(2\pi)^4}\frac{M_f}{k^2+M_f^2} = \frac{M_f}{16\pi^4}\int d\Omega_4 \int_0^\Lambda \frac{k^3}{k^2+M_f^2}dk$$

$$= \frac{M_f}{16\pi^2}\left[\Lambda^2 - M_f^2 \ln\left(1+\frac{\Lambda^2}{M_f^2}\right)\right]$$

$$\longrightarrow \quad M_f = m_f + \frac{4\alpha_{\rm IR}}{3\pi m_g^2} M_f \left[\Lambda^2 - M_f^2 \ln\left(1+\frac{\Lambda^2}{M_f^2}\right)\right]$$

Solution quark DSE

- **Proper time regularization:**

$$\rightarrow \quad M_f = m_f + M_f \frac{4\alpha_{\rm IR}}{3\pi m_g^2} \int_0^\infty dk^2 \, \frac{k^2}{k^2 + M_f^2}$$

- We implement the following identity:

$$\frac{1}{k^2 + M_f^2} = \int_0^\infty d\tau \; e^{-\tau(k^2+M_f^2)} \longrightarrow \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau \; e^{-\tau(k^2+M_f^2)} = \frac{e^{-(k^2+M_f^2)\tau_{\rm uv}^2} - e^{-(k^2+M_f^2)\tau_{\rm ir}^2}}{k^2 + M_f^2}$$

Which has no poles when $q^2 \rightarrow -M_f^2$. Then, one has,

$$\int_0^\infty dk^2 \, \frac{k^2}{k^2 + M_f^2} = \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau \frac{1}{\tau^2} e^{-\tau M_f^2}$$

$$\rightarrow \quad M_f = m_f + M_f \frac{4\alpha_{\rm IR}}{3\pi m_g^2} \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau \frac{1}{\tau^2} e^{-\tau M_f^2}$$

$$\tau_{\rm ir} = \frac{1}{\Lambda_{\rm ir}} \qquad \tau_{\rm uv} = \frac{1}{\Lambda_{\rm uv}}$$

Solution quark DSE

- Now, we set $G = 4\pi\alpha_{\rm IR}/mg^2$

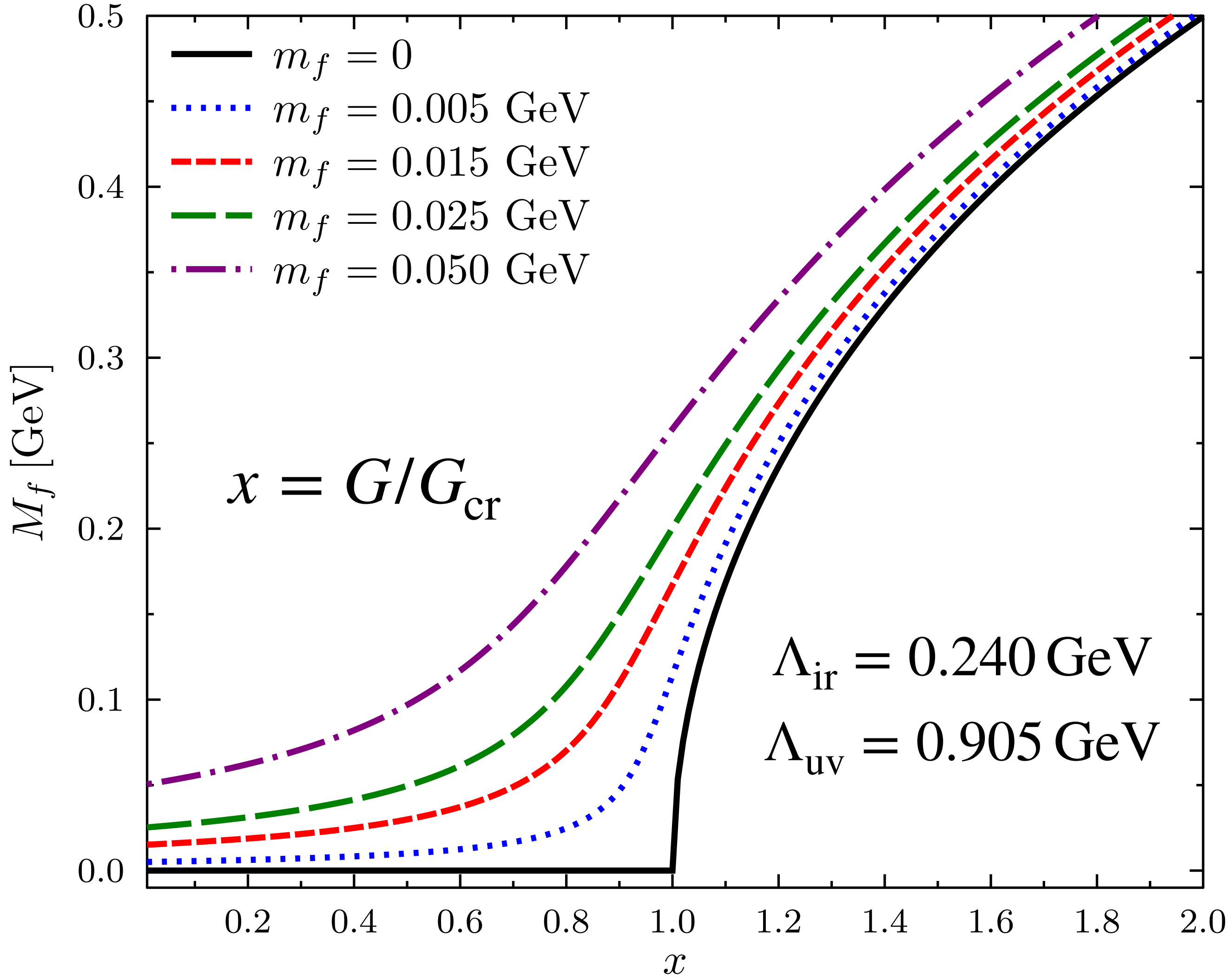
$$M_f = m_f + \frac{M_f\,G}{3\pi^2}\int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2}\frac{d\tau}{\tau^2}\,e^{-\tau M_f^2}$$

- Trivial solution for $G = 0$, but a solution also exists at large G.

- In the chiral limite we have:

$$G_{\rm cr} = \frac{3\pi^2}{(\Lambda_{\rm uv}^2 - \Lambda_{\rm ir}^2)}$$

$$G > G_{\rm cr} \longrightarrow M_f \neq 0$$



Solution BSE

- After plugging the BSA into the BSE:

$$\gamma_5\left[iE^{fg}_{\rm PS}(P)+\frac{\gamma\cdot P}{2M_{fg}}F^{fg}_{\rm PS}(P)\right] = -\frac{16\pi\alpha_{\rm IR}}{3m_g^2}\int\frac{d^4q}{(2\pi)^4}\gamma_\mu S_f(q_+)\gamma_5\left[iE^{fg}_{\rm PS}(P)\right.$$
$$\left.+\frac{\gamma\cdot P}{2M_{fg}}F^{fg}_{\rm PS}(P)\right]S_g(q_-)\gamma_\mu .$$

- Using appropriate projector we can write the BSE in matrix form:

$$\begin{bmatrix} E_{\rm PS}(P) \\ F_{\rm PS}(P)\end{bmatrix} = \frac{4\alpha_{\rm IR}}{3\pi m_g^2}\begin{bmatrix}\mathcal{K}^{EE}_{\rm PS} & \mathcal{K}^{EF}_{\rm PS} \\ \mathcal{K}^{FE}_{\rm PS} & \mathcal{K}^{FF}_{\rm PS}\end{bmatrix}\begin{bmatrix} E_{\rm PS}(P) \\ F_{\rm PS}(P)\end{bmatrix}$$

Where for instant, the first matrix element looks like

$$\mathcal{K}^{EE}_{\rm PS}(P) = 16\pi^2\int\frac{d^4q}{(2\pi)^4}\frac{[M_fM_g+(q\cdot P)+q^2]}{[(q+P)^2+M_f^2](q^2+M_g^2)}$$

Log divergent!

Solution BSE

- Feynman parameters

$$\mathcal{M}^2 = x(1-x)P^2 + M_f^2 x + M_g^2(1-x)$$

$$\longrightarrow \mathcal{K}_{\rm PS}^{EE}(P) = 16\pi^2 \int_0^1 dx \int \frac{d^4q}{(2\pi)^4} \frac{[M_f M_g + (q\cdot P) + q^2]}{[(q+xP)^2 + \mathcal{M}^2]^2}$$

- After performing the change of variable $k = q + xP$, we obtain

$$\mathcal{K}_{\rm PS}^{EE}(P) = 16\pi^2 \int_0^1 dx \int \frac{d^4k}{(2\pi)^4} \frac{[k^2 + (1-2x)(k\cdot P) - x(1-x)P^2 + M_f M_g]}{[k^2 + \mathcal{M}^2]^2}.$$

Again, the integral with $(k \cdot P)$ is zero. Then,

$$\mathcal{K}_{\rm PS}^{EE}(P) = \int_0^1 dx \left[\mathcal{C}_{01}(\mathcal{M}^2) + [L^-(x) - 2J(x)P^2]\mathcal{C}_{02}(\mathcal{M}^2)\right]$$

$$\mathcal{C}_{01}(\mathcal{M}^2) = \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau \frac{1}{\tau^2} e^{-\tau\mathcal{M}^2}$$

$$\mathcal{C}_{02}(\mathcal{M}^2) = -\frac{d\mathcal{C}_{01}(\mathcal{M}^2)}{d\mathcal{M}^2}$$

$$J(x) = x(1-x),$$

$$L(x) = M_f^2 x + M_g^2(1-x).$$

$$L^{\pm}(x) = M_f M_g \pm L(x)$$

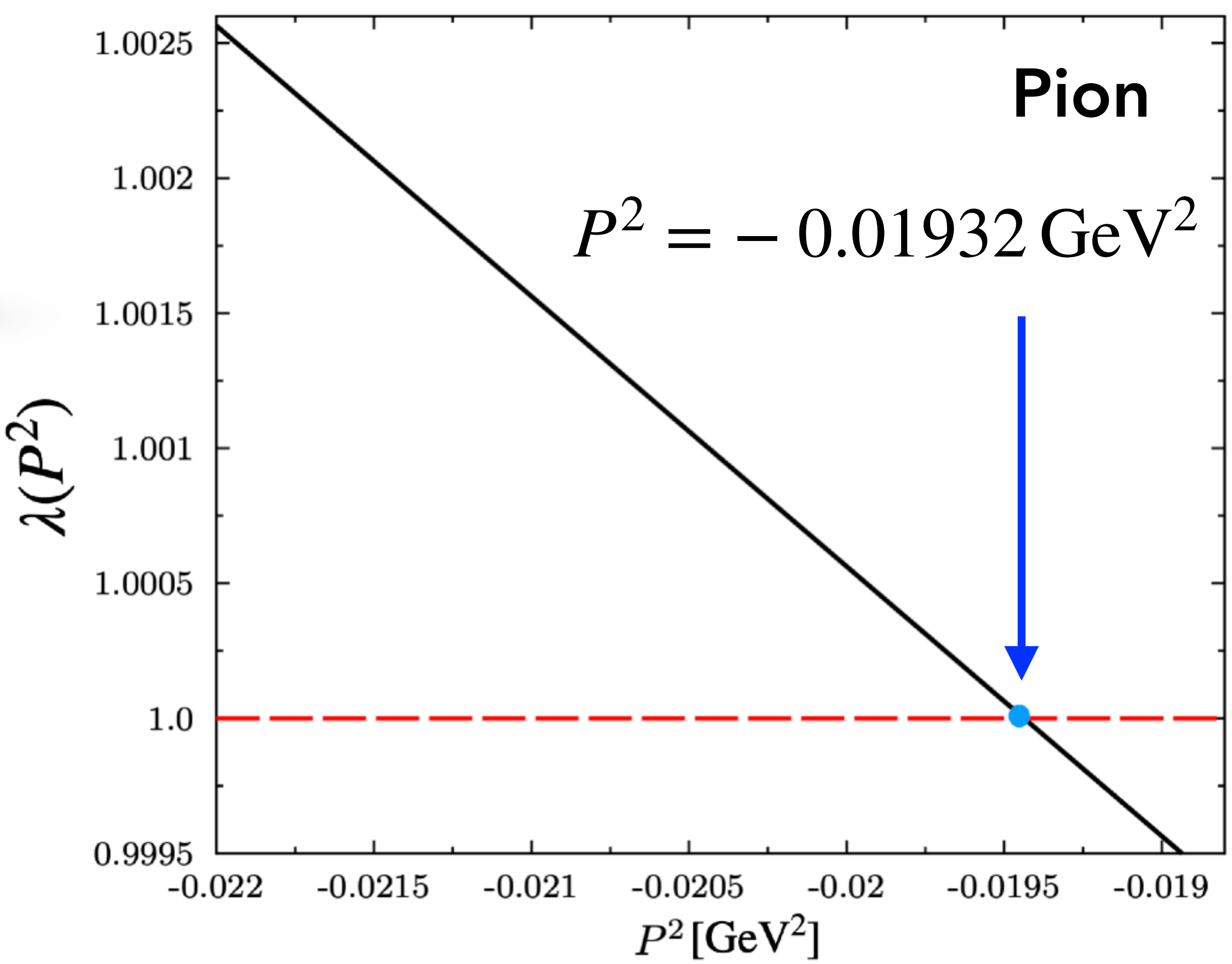
Solution BSE

- Masses and BSA obtained from BSE as an eigenvalue problem

$$\lambda(P^2)\begin{bmatrix} E_{\rm PS}(P) \\ F_{\rm PS}(P) \end{bmatrix} = \frac{4\alpha_{\rm IR}}{3\pi m_g^2}\begin{bmatrix} \mathcal{K}_{\rm PS}^{EE} & \mathcal{K}_{\rm PS}^{EF} \\ \mathcal{K}_{\rm PS}^{FE} & \mathcal{K}_{\rm PS}^{FF} \end{bmatrix}\begin{bmatrix} E_{\rm PS}(P) \\ F_{\rm PS}(P) \end{bmatrix}$$

$$\lambda(P^2 = -m_{\rm PS}^2) - 1 = 0$$

$$m_\pi = 0.139\,\text{GeV}$$



Solution BSE

- Together the mass we obtain the unnormalized BSA. We use a normalization condition.

$$\mathcal{N}_M^2 = \frac{\partial}{\partial P^2}\Pi_M(Q,P)\big|_{Q^2=P^2}$$

$$\Pi_M(Q,P) = 2N_c\int\frac{d^4q}{(2\pi)^4}\mathrm{Tr}[\bar{\Gamma}^{fg}(q,Q)S_f(q_+)\Gamma^{fg}_{M},(Q,P)S_g(q_-)]$$

In the contact model the normalization constant is:

$$\begin{aligned}\mathcal{N}_{\mathrm{PS}}^2 &= -\frac{3}{4\pi^2}\Bigg\{-2E_{\mathrm{PS}}^2\frac{\partial\mathcal{K}^{EE}_{\mathrm{PS}}(P)}{\partial P^2} + 2E_{\mathrm{PS}}F_{\mathrm{PS}}\left(\frac{1}{P^2}\mathcal{K}^{EF}_{\mathrm{PS}}(P) - 2\frac{\partial\mathcal{K}^{EF}_{\mathrm{PS}}(P)}{\partial P^2}\right)\\ &+ \frac{P^2}{M_{fg}^2}F_{\mathrm{PS}}^2\frac{\partial\mathcal{K}^{FF}_{\mathrm{PS}}(P)}{\partial P^2}\Bigg\}\Bigg|_{P^2=-m_{\mathrm{Ps}}^2}\end{aligned}$$

Solution BSE

- We can compute leptonic decay constant via:

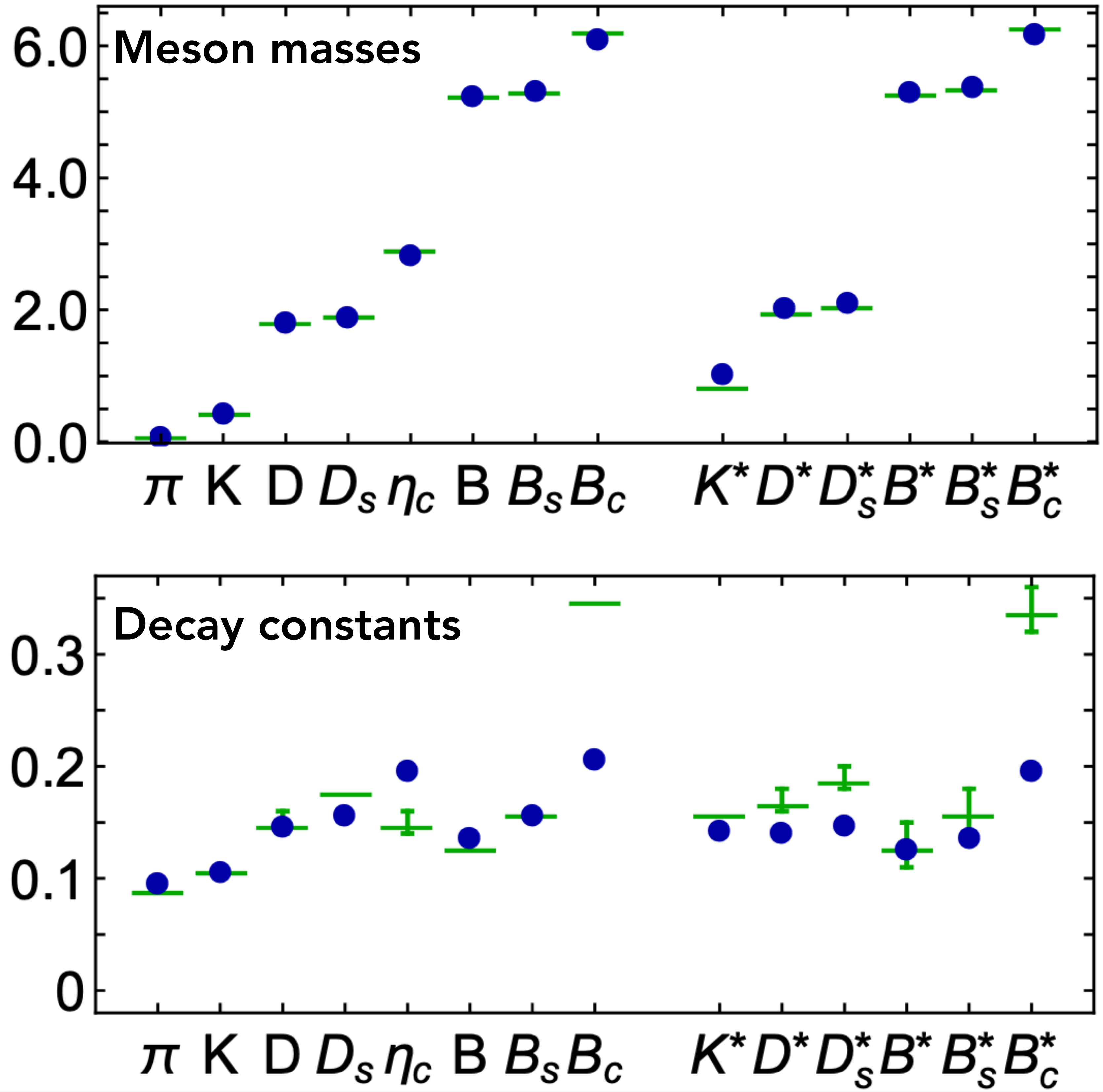
$$P^2 f_M = N_c \int \frac{d^4q}{(2\pi)^4} \mathrm{Tr}[\gamma_5 \gamma \cdot P S_f(q_+) \Gamma_M^{fg}(q,P) S_g(q_-)]$$

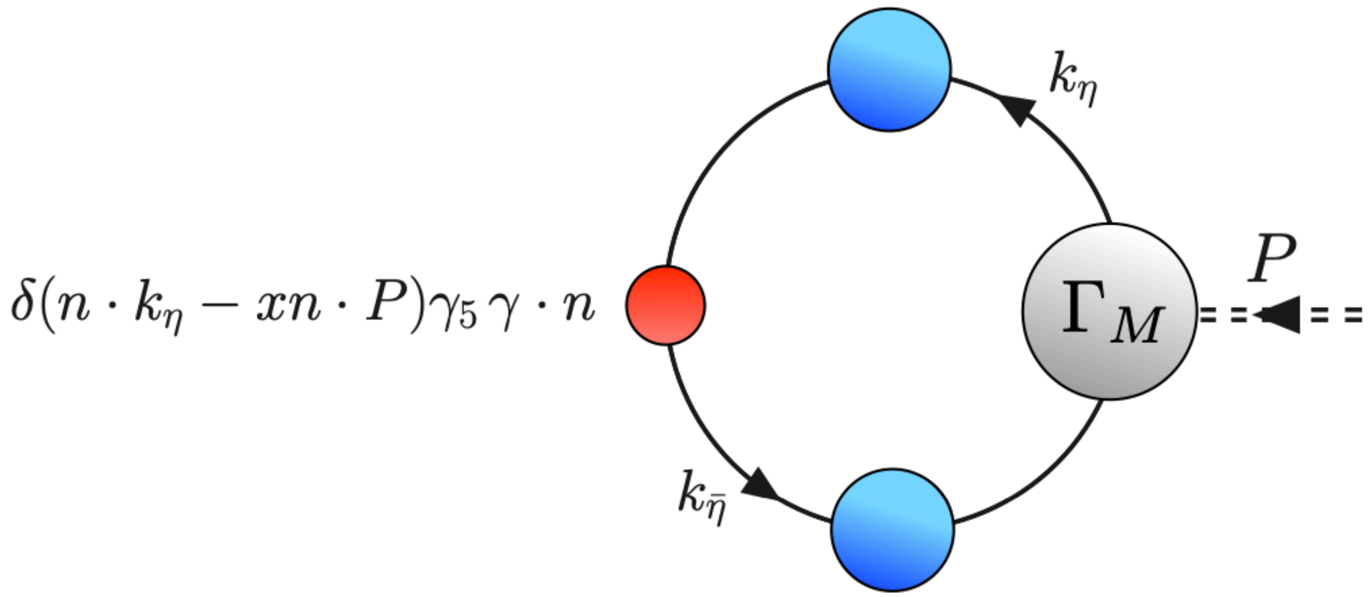
$$f_{\rm PS} = \frac{3}{4\pi^2 M_{fg}} \left[E_{\rm PS}^{\rm n} \mathcal{K}_{\rm PS}^{FE}(P) + F_{\rm PS}^{\rm n} \mathcal{K}_{\rm PS}^{FF}(P) \right] \Big|_{P^2 = -m_{\rm PS}^2}$$

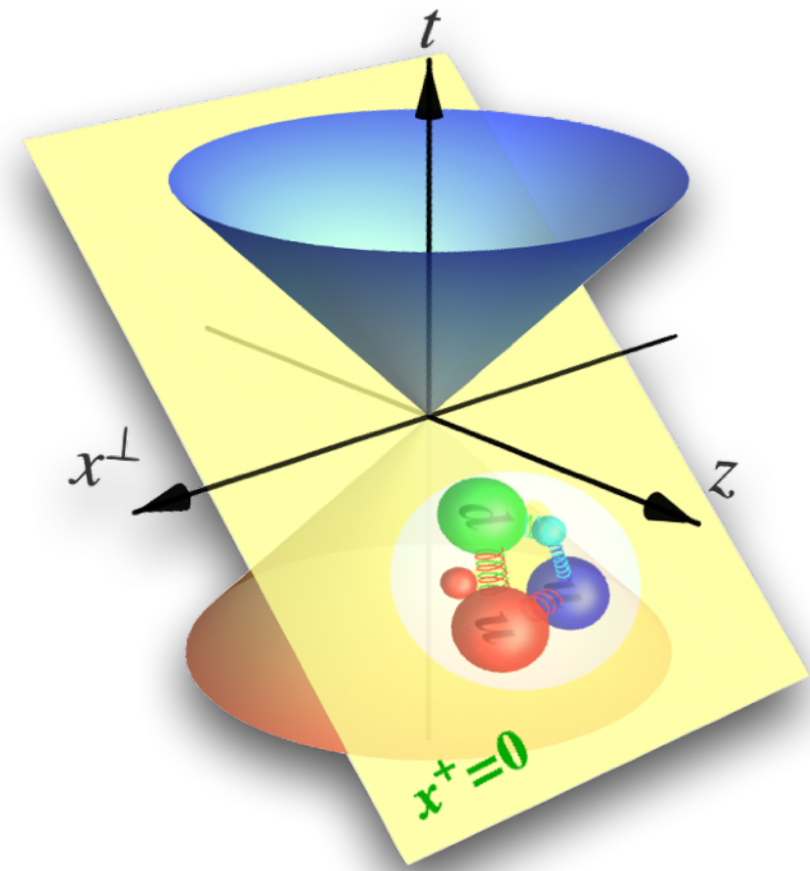
- Results for masses and decay constant of pseudoscalar mesons.

J^P	Meson	$m^{\rm CI}$	$m^{\rm e/l}$	E	F	$f^{\rm CI}$	$f^{\rm e/l}$
0^-	$\pi(u\bar{d})$	0.14	0.14	3.59	0.47	0.10	0.092
	$K(u\bar{s})$	0.50	0.50	3.70	0.55	0.11	0.11
	$D(u\bar{c})$	1.87	1.87	3.25	0.39	0.15	0.15(1)
	$D_s(s\bar{c})$	1.96	1.97	3.45	0.54	0.16	0.18
	$\eta_c(c\bar{c})$	2.90	2.98	3.74	0.90	0.20	0.24(1)
	$B(u\bar{b})$	5.30	5.30	2.98	0.18	0.14	0.13
	$B_s(s\bar{b})$	5.38	5.37	3.26	0.27	0.16	0.16
	$B_c(c\bar{b})$	6.16	6.28	4.25	0.79	0.21	0.35

Solution BSE

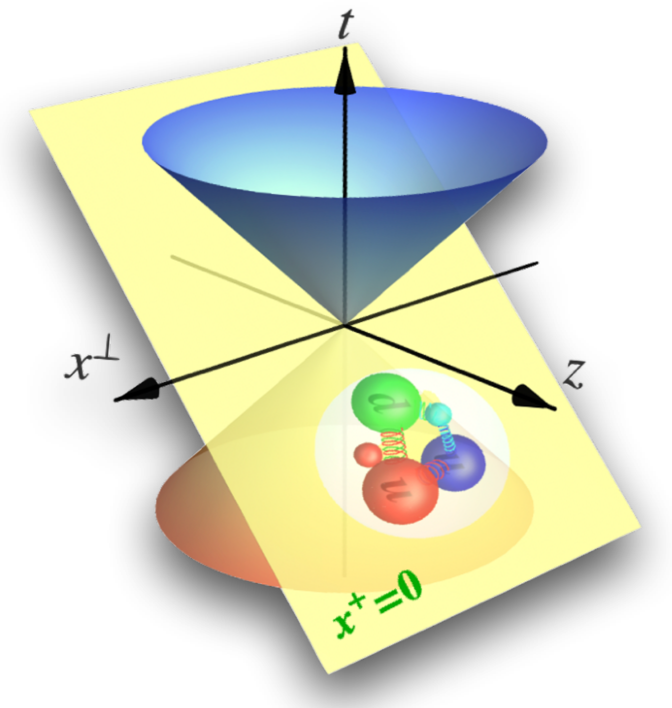






Meson Distribution Amplitudes on the Light Cone

Light-Cone Distribution Amplitudes (LCDAs)



$$\phi_M(x, \mu)$$

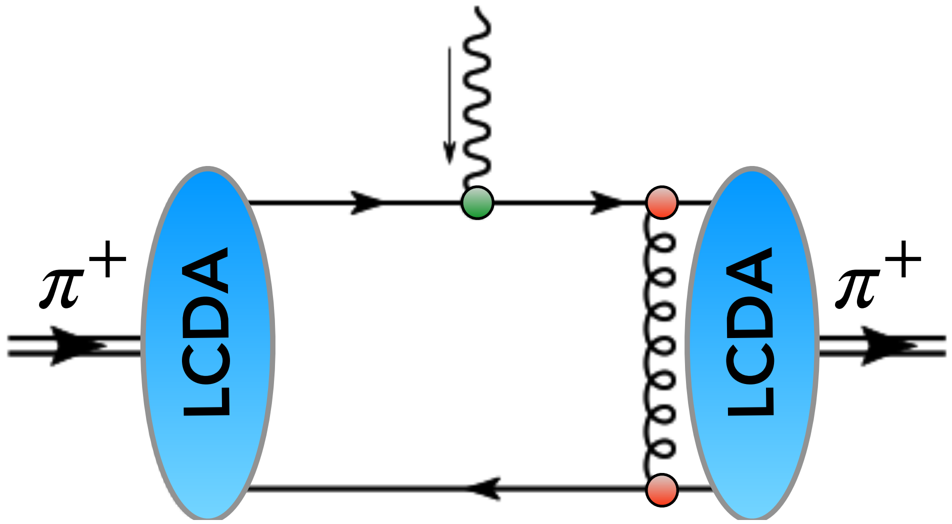
- The LCDAs are scale-dependent functions can be understood as the closest relative of quantum mechanical wave functions in quantum field theory

- $\phi_M(x, \mu)$ expresses the light-front fraction of the hadron's momentum carried by a valence quark. Allows for a probability interpretation of partons.

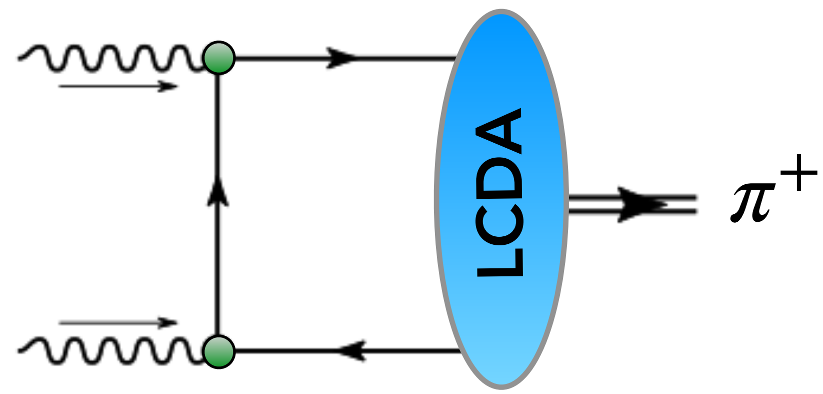
- x is the light-front momentum fraction: $x = k^+/P^+$ and μ the renormalization scale.

Light-Cone Distribution Amplitudes (LCDAs)

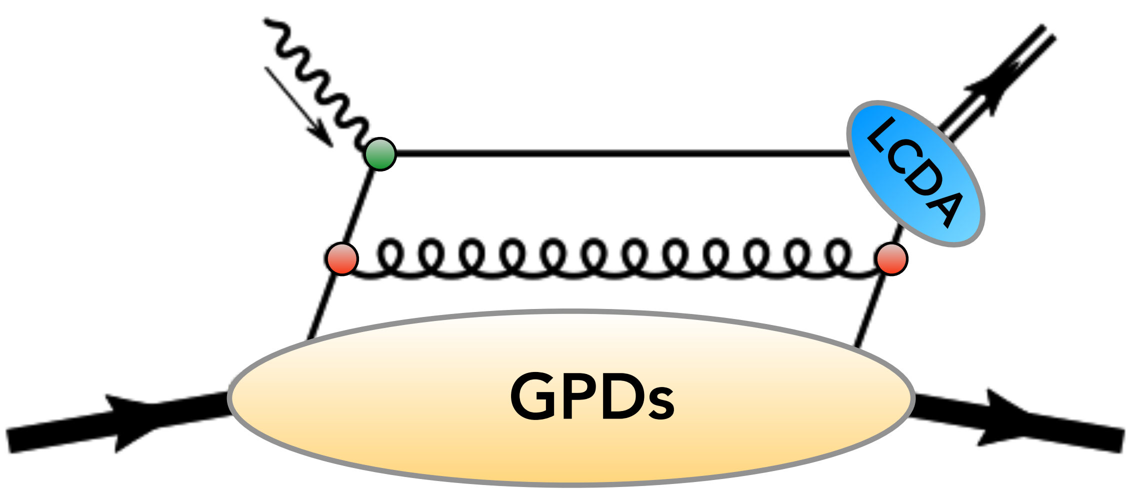
- Hard exclusive scattering processes.

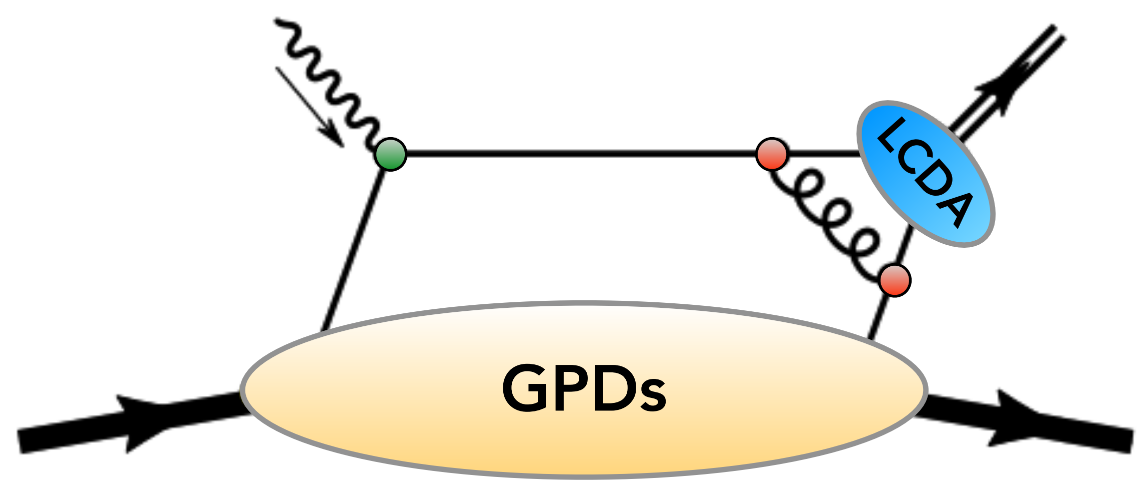


$$Q^2 F_\pi(Q^2) \to 16\pi f_\pi^2 \alpha_s(Q^2)$$



$$Q^2 F_{\gamma^*\gamma\pi}(Q^2) \to 2 f_\pi$$

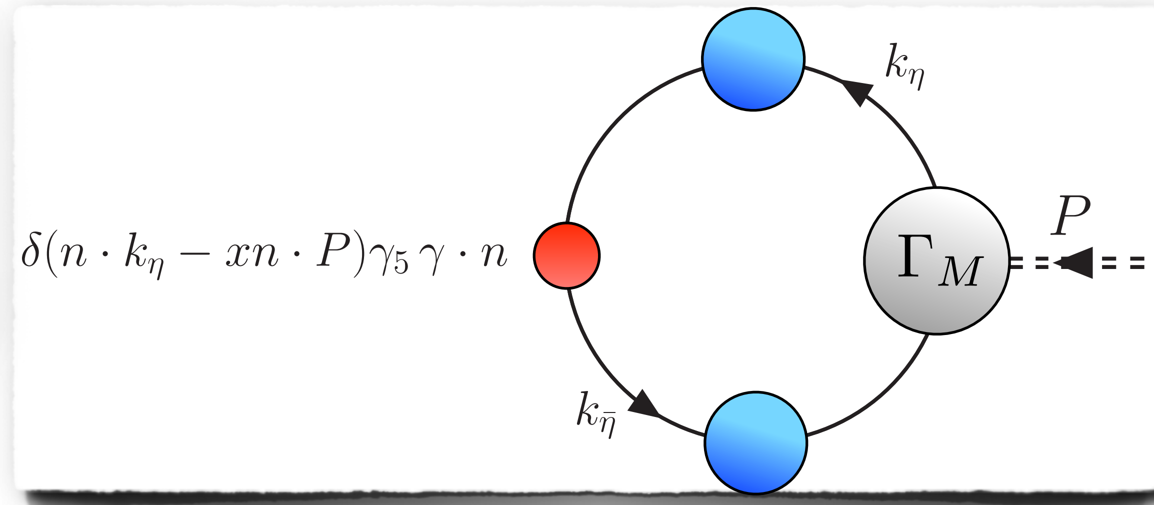




Light-Cone Distribution Amplitudes

- **Light-cone projection of the Bethe-Salpeter wave function $\chi_M(k,P)$**

$$\langle 0|\bar{q}_f(y_2 n)\mathcal{W}[y_2 n, y_1 n]\,\gamma\cdot n\,\gamma_5\, q_g(y_1 n)|M(P)\rangle$$
$$= i f_M\, n\cdot P \int_0^1 dx\, e^{-in\cdot P(y_1 x + y_2 \bar{x})}\phi_M(x,\mu)$$



f_M = weak decay constant

$n^2 = 0$

$n\cdot P = -m_M$

$$f_M\phi_M(x,\mu) = \frac{\mathcal{Z}_2 N_c}{\sqrt{2}}\,\mathrm{Tr}_D\int^\Lambda \frac{d^4k}{(2\pi)^4}\,\delta(n\cdot k_\eta - xn\cdot P)\gamma_5\,\gamma\cdot n\;\chi_M(k_\eta,k_{\bar{\eta}})$$

$\chi_M(k_\eta,k_{\bar{\eta}}) := S(k_\eta)\Gamma_M(k,P)S(k_{\bar{\eta}})$ ➡ **Meson's Bethe-Salpeter wave function (BSWF)**

$\Gamma_M(k,P)$ ➡ **Meson's Bethe-Salpeter Amplitude**

$S(k_\eta)$ ➡ **Quark propagator**

L. Chang, I.C. Cloët, J.J. Cobos-Martínez, C.D. Roberts, S.M.Schmidt, P. C. Tandy, Phys. Rev. Lett. 110 (2013)
J. Segovia, L. Chang, I. C. Cloët, C. D. Roberts, S. M. Schmidt, Phys. Lett. B 731 (2014)

Meson LCDAs

$$\phi_M(x,\mu)$$

- Projection of BS wave-function onto the light front :

$$f_M \phi_M(x,\mu) = \frac{\mathcal{Z}_2 N_c}{\sqrt{2}} \mathrm{Tr}_D \int^\Lambda \frac{d^4k}{(2\pi)^4} \delta(n\cdot k_\eta - x n\cdot P)\gamma_5\, \gamma\cdot n\; \chi_M(k_\eta, k_{\bar\eta})$$

$$\chi_M(k_\eta, k_{\bar\eta}) := S(k_\eta)\Gamma_M(k,P)S(k_{\bar\eta})$$

- Computing Mellin moments

$$\langle x^m \rangle = \int_0^1 dx\, x^m \phi_M(x,\mu)$$

$$\left\langle x^0 \right\rangle = \int_0^1 dx \phi_M(x,\mu) = 1$$

$$\rightarrow \quad \langle x^m \rangle = \frac{\mathcal{Z}_2 N_c}{\sqrt{2} f_M} \mathrm{Tr}_D \int^\Lambda \frac{d^4k}{(2\pi)^4} \frac{(n\cdot k_\eta)^m}{(n\cdot P)^{m+1}} \gamma_5\, \gamma\cdot n\; \chi_M(k_\eta, k_{\bar\eta})$$

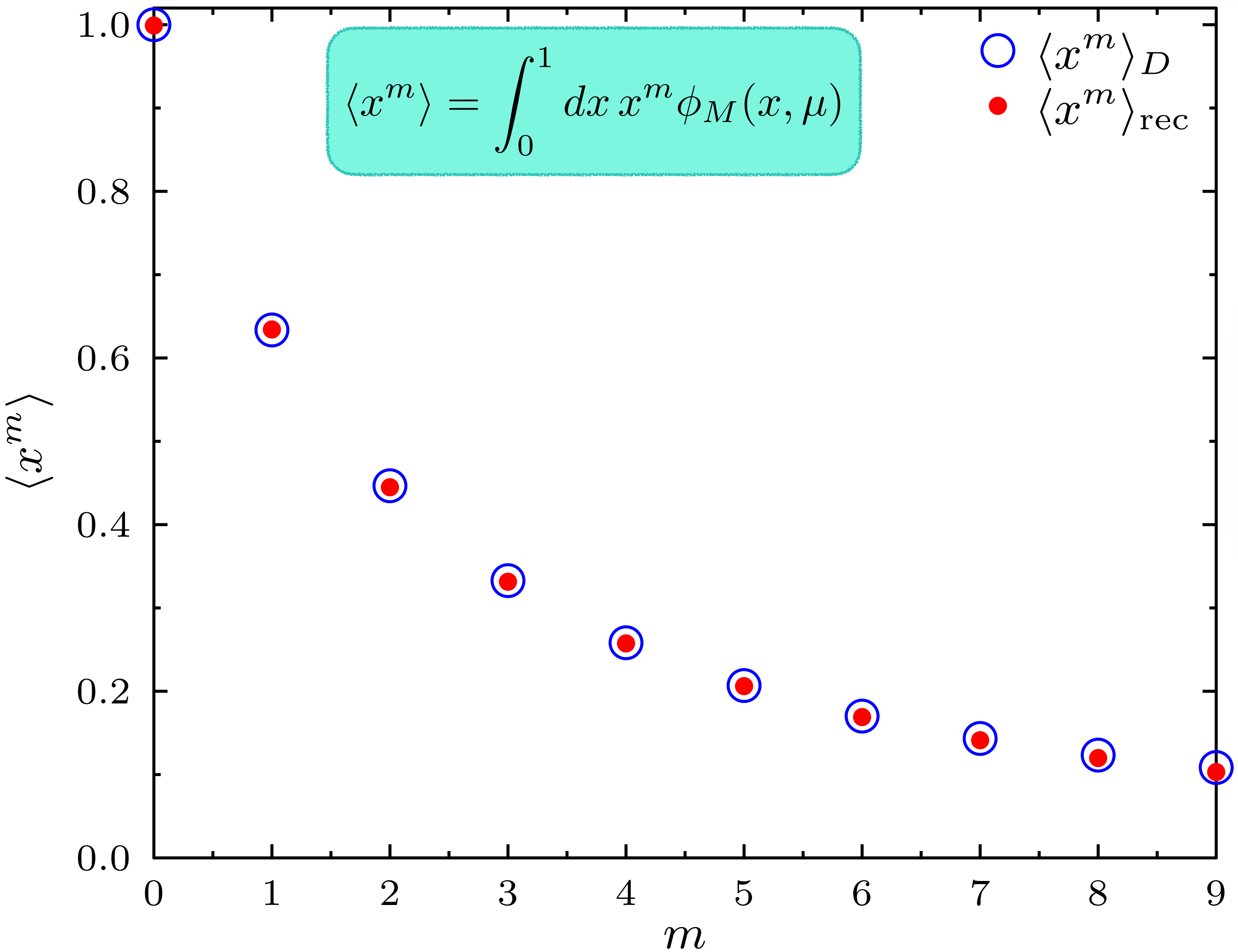
Meson LCDAs

$$\phi_M(x,\mu)$$

- Reconstructing LCDAs

$$\phi_\pi^{\text{rec.}}(x,\mu) = \mathcal{N}(\alpha)\,[x\bar{x}]^{\alpha-1/2}\,[1+a_2 C_2^\alpha(2x-1)]$$

$$\phi_H^{\text{rec.}}(x,\mu) = \mathcal{N}(\alpha,\beta)\,4x\bar{x}\,e^{4\,\alpha x\bar{x}+\beta(x-\bar{x})}$$



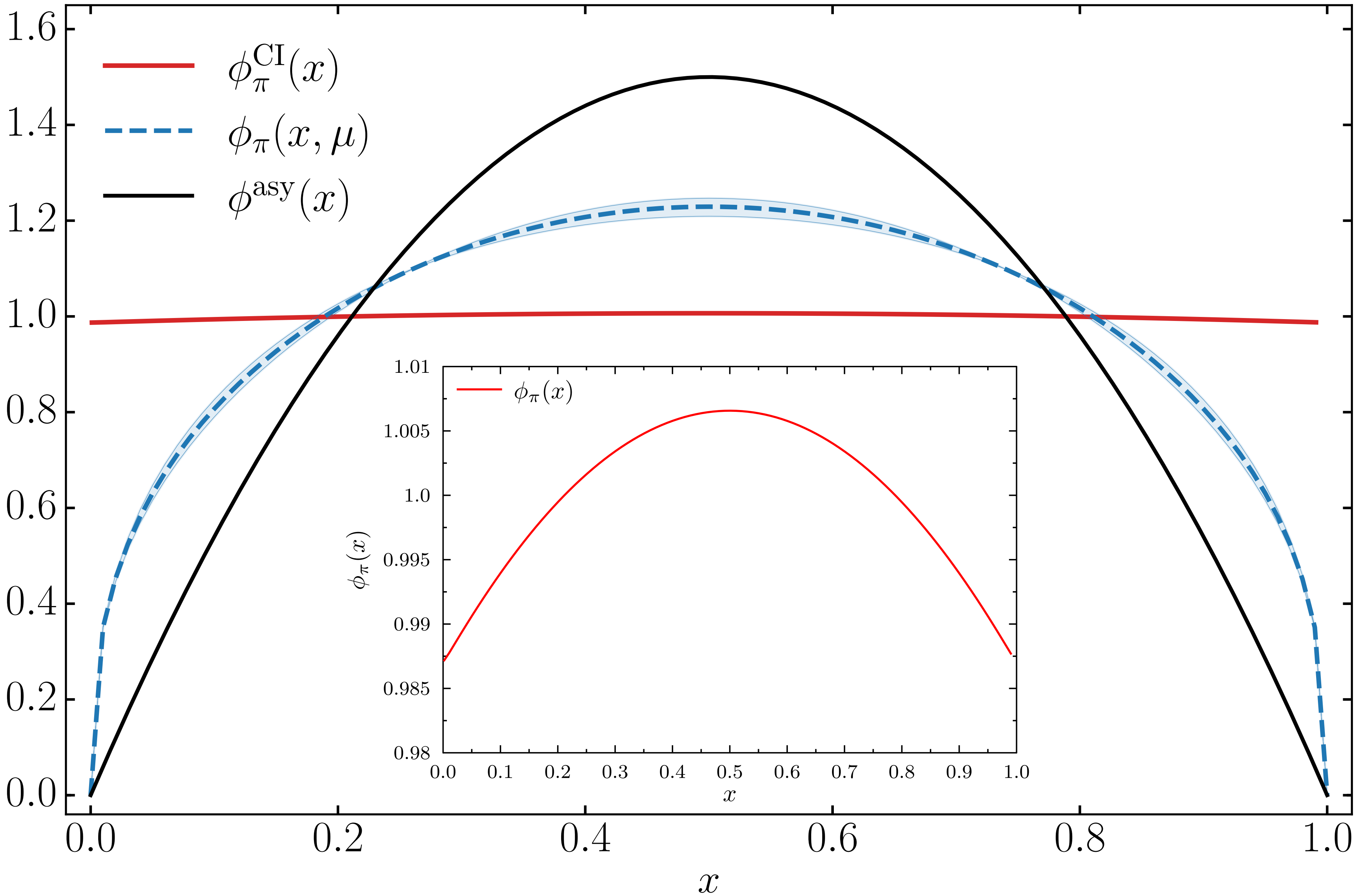
$$\epsilon(\alpha,\beta) = \sum_{m=1}^{m_{\max}} \left| \frac{\langle x^m \rangle_{\text{rec.}}}{\langle x^m \rangle_H} - 1 \right|$$

Meson LCDAs

$$\phi_M(x,\mu)$$

$$\phi^{\text{asy}}(x) = 6x(1-x)$$

$$\frac{\mathcal{G}_f(q^2)}{q^2} = \mathcal{G}_f^{\text{IR}}(q^2) + 4\pi\tilde{\alpha}_{\text{PT}}(q^2)$$



F. E. S, R. C. da Silveira, J. Cobos-Martínez, B. El-Bennich and E. Rojas, Eur. Phys. J. C 80, 955 (2020)

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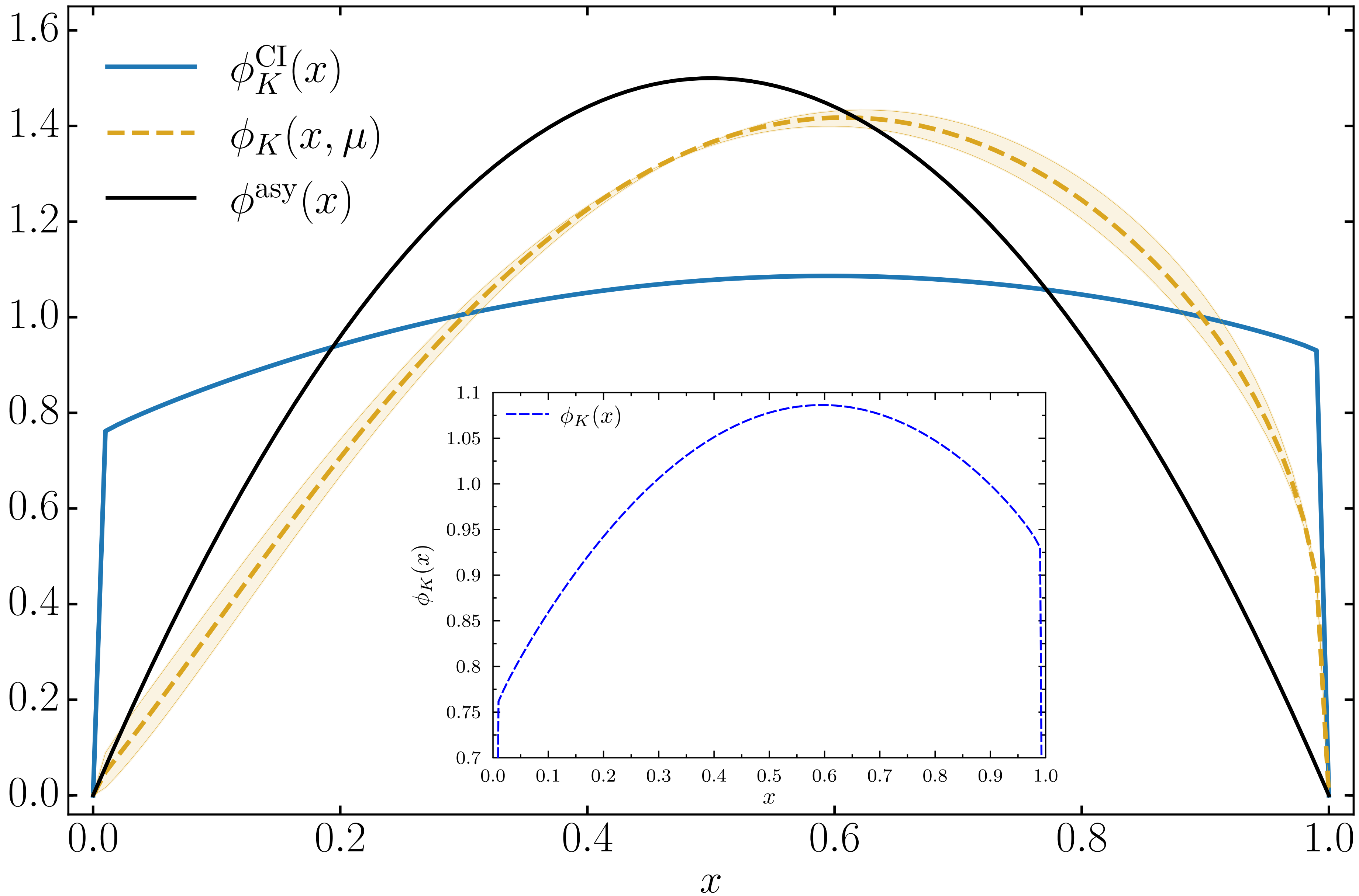

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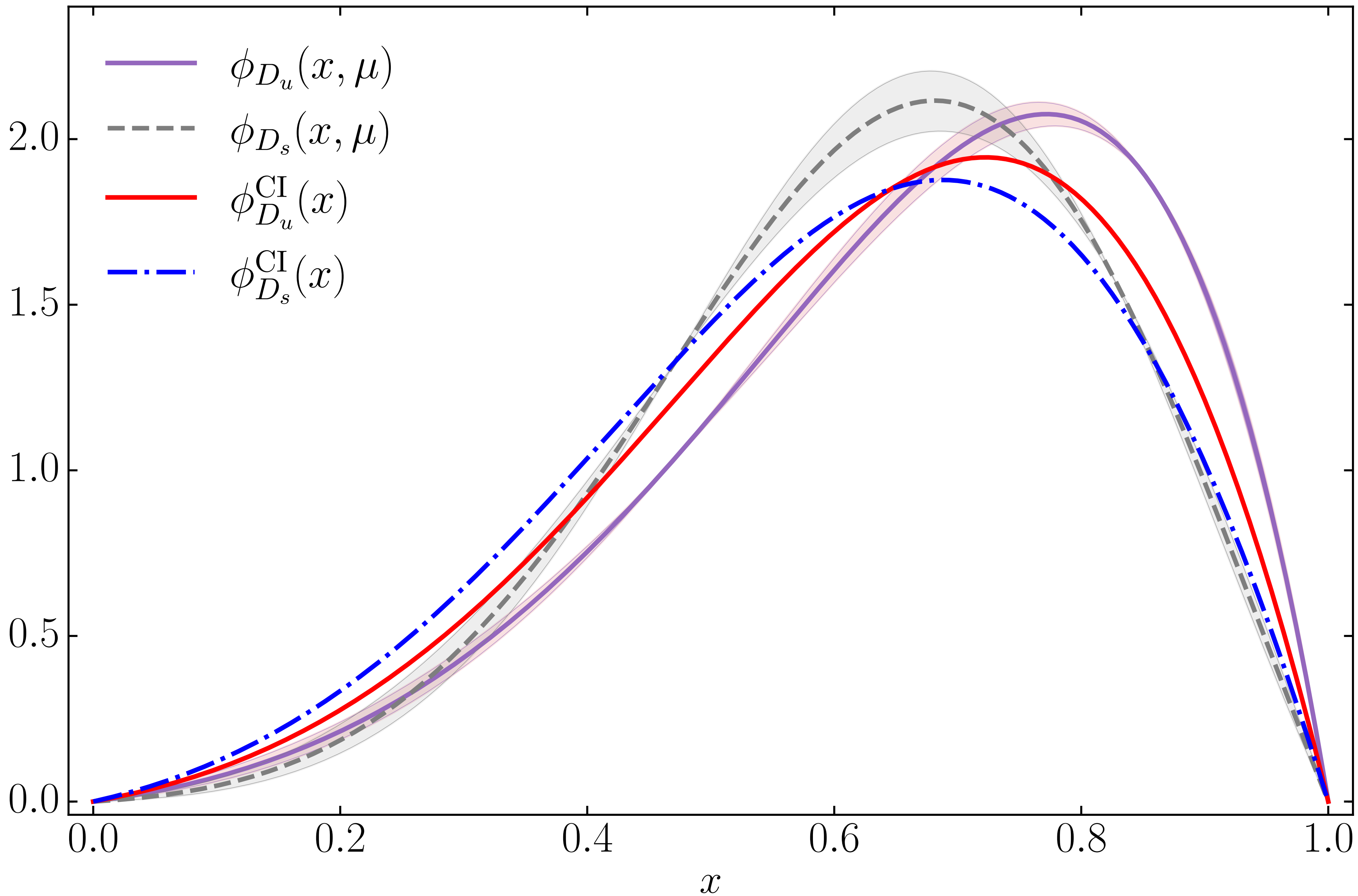

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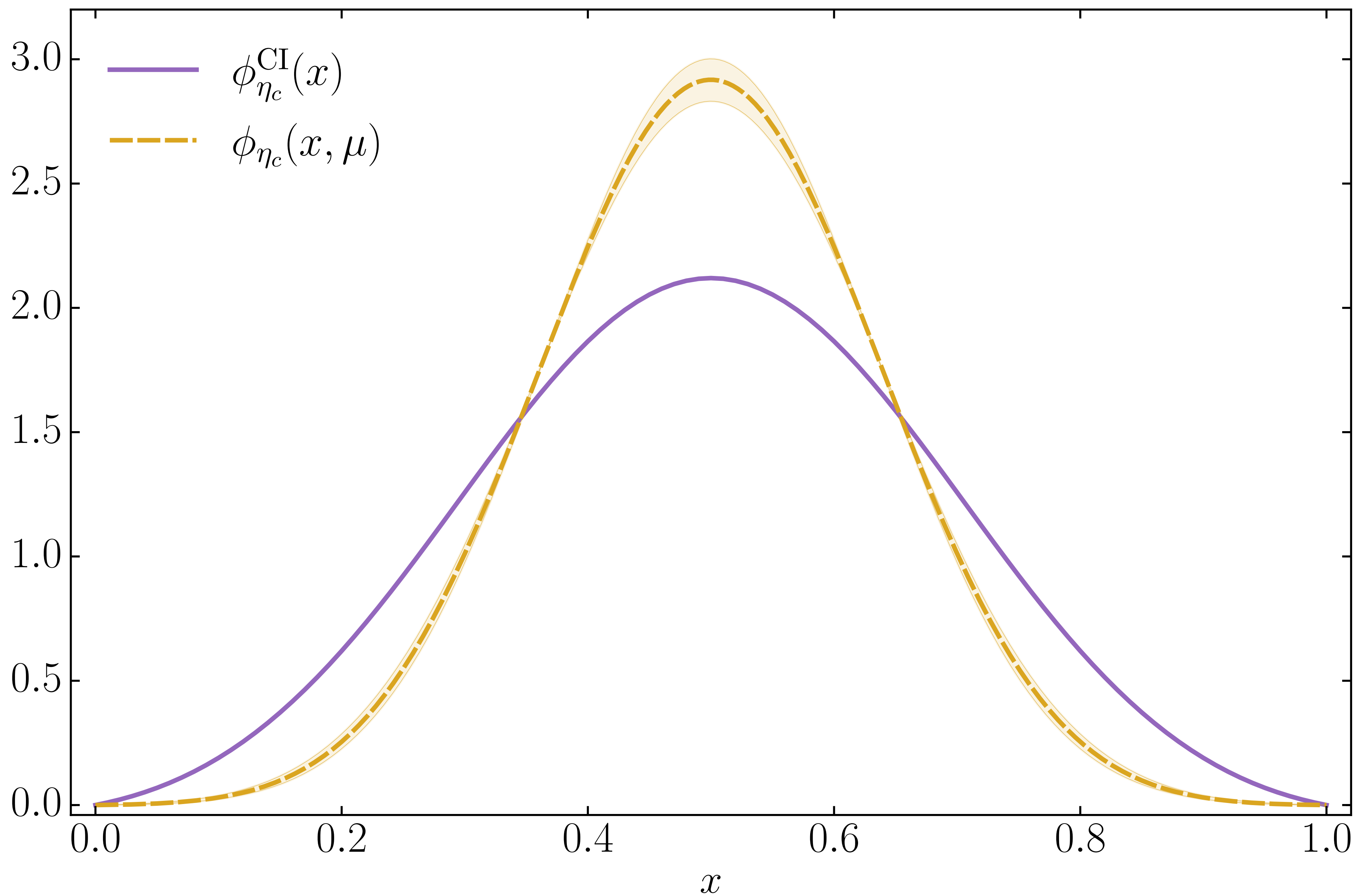


F. E. S, R. C. da Silveira, J. Cobos-Martínez, B. El-Bennich and E. Rojas, Eur. Phys. J. C 80, 955 (2020)

Conclusions

- The Contact Interaction model, when applied within the framework of DSE and BSE, provides a valuable tool for investigating non-perturbative QCD phenomena.

- Despite its simplifications, the CI model offers a qualitative understanding of hadronic physics and aids in the computation of LCDAs, though with limitations in accuracy for high-momentum and detailed dynamical processes.

- It serves as a useful complement to more sophisticated approaches in the study of QCD.

Thank you!

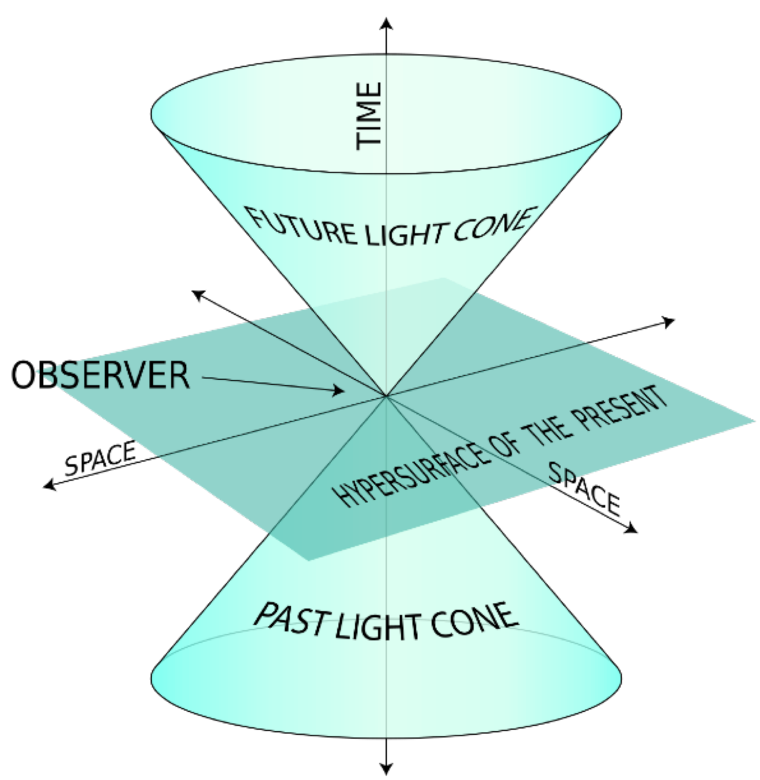
Light-Cone Distribution Amplitudes

- In relativistic **QFT** the infinite degrees of freedom do not allow for a straightforward definition of a particle's **WF** as in quantum mechanics.

  ✳Particles interact and their number is not conserved

- An alternative is formulate the theory on the light-front because the eigenfunctions of the light-front Hamiltonian are independent of the system's four-momentum.

- Coordenadas del cono de luz:



$$x_\mu = (x^+, x^-, x_\perp)$$

$$x^\pm = \frac{1}{\sqrt{2}}(x^0 \pm x3) \qquad x_\perp = (x^1, x^2)$$