Birdtracks & Deconfinement

Consider pure SU(N) gauge theory, no quarks

Deconfining phase transition

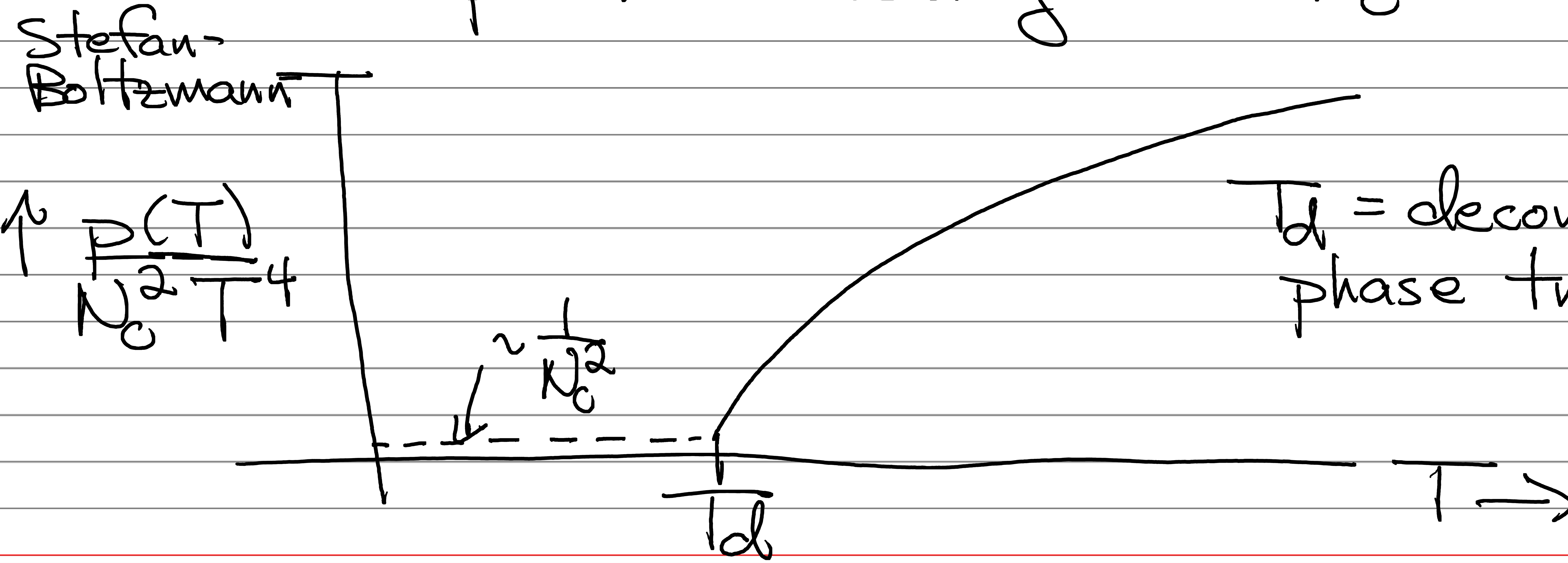
@ temperature $T \neq 0$

$T=0$: confinement $\Rightarrow$ only color singlets

pressure $p(T) \sim N^0 \sim 1$

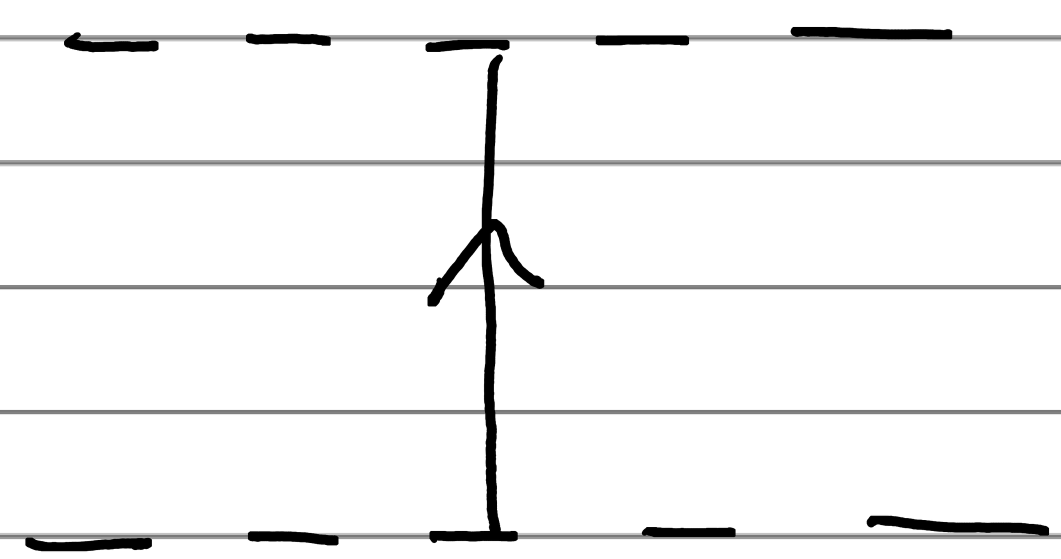
$T \to \infty$: asymp. freedom $\Rightarrow$

$p(T) \sim$ ideal gas $\sim N_c^2 T^4$



T_d = deconfining phase trans.

Order parameters



Thermal Wilson line @ $T \neq 0$

$$\mathbb{L}(\bar{x}) = \mathbb{P}\, e^{ig\int_0^{1/T} A_0(\bar{x},x)\,dx}$$

Under aperiodic gauge transf.

$$\mathbb{L}(\bar{x}) \to \Omega^{+}(\bar{x}, 1/T)\, \mathbb{L}(\bar{x})\, \Omega(\bar{x})$$

Choose $\Omega(\bar{x}, 1/T) = \underbrace{e^{2\pi i j/N}}_{\in Z(N)} \Omega(\bar{x}, 0)$ $\quad j = 0, \ldots (N-1)$

Allowed gauge transf. for adjoint fields

$$A_\mu \to \Omega^{+} D_\mu \Omega = A_\mu \quad \text{for } \in Z(N)$$

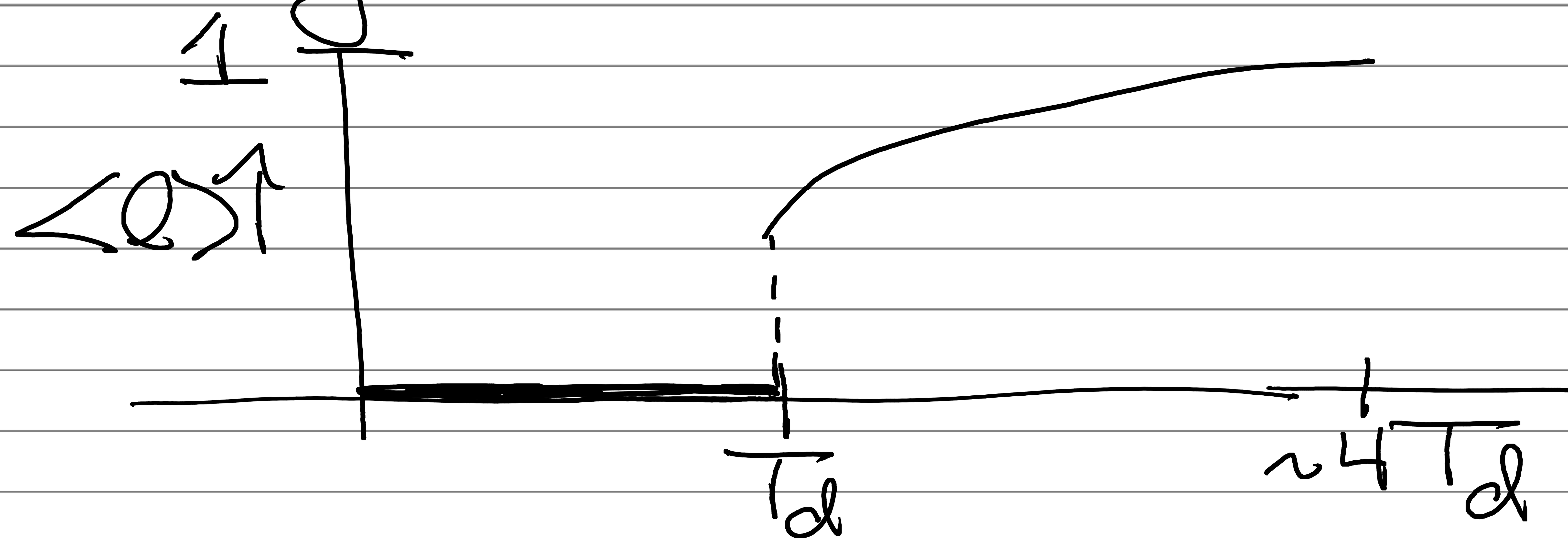
Not for fundamental fields

$$\bar{q}(\bar{x}, 1/T) = e^{2\pi i j/N}\, \bar{q}(\bar{x}, 0) \neq q(\bar{x}, 0)$$

Deconfinement

Order parameters: $\ell_k = \mathrm{tr}\, \mathbb{L}^k \qquad k = 0, \ldots, (N-1)$

Lattice: usually measure $\ell_1 \equiv \ell$



Also ~ independent of N if $N \geq 3$
(2nd order for $N = 2$)

$T < T_d$: $Z(N)$ symmetric phase

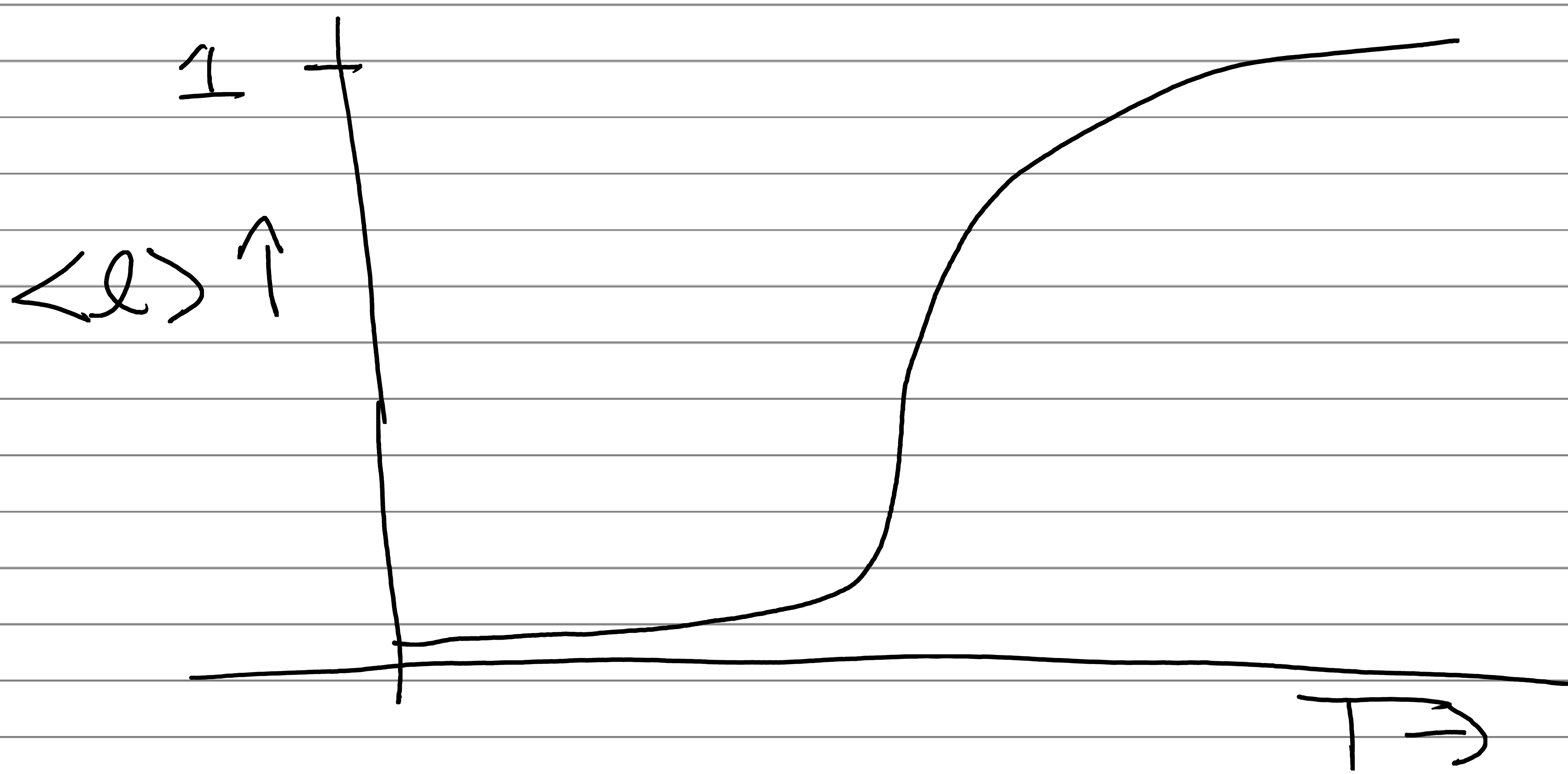
$T \geq T_d$: $Z(N)$ spon'y broken

With quarks

$Z(N)$ no longer valid symmetry; add terms

$$+ \; h_{qk} \left(\operatorname{tr} \mathbb{L} + c.c. \right) + \ldots$$

$\Rightarrow \langle \ell \rangle \neq 0$ at all T



Breaking of $Z(N)$ depends on # qks, etc.

"Semi"- QGP

How to characterize $0 < \ell < 1$?

Semi- Quark Gluon Plasma. Under gauge transf.

$$\mathbb{L}(x) \to \Omega^{\dagger}(x)\, \mathbb{L}(x)\, \Omega(x)$$

$\Rightarrow$ eigenvalues of $\mathbb{L}$ gauge invariant.

Simplest possible ansatz

$$(A_0^{cl})^{ab} = \frac{2\pi T}{g} q_a \delta^{ab} \qquad a,b = 1..N$$

$$\mathbb{L}^{cl} = e^{2\pi i q}$$

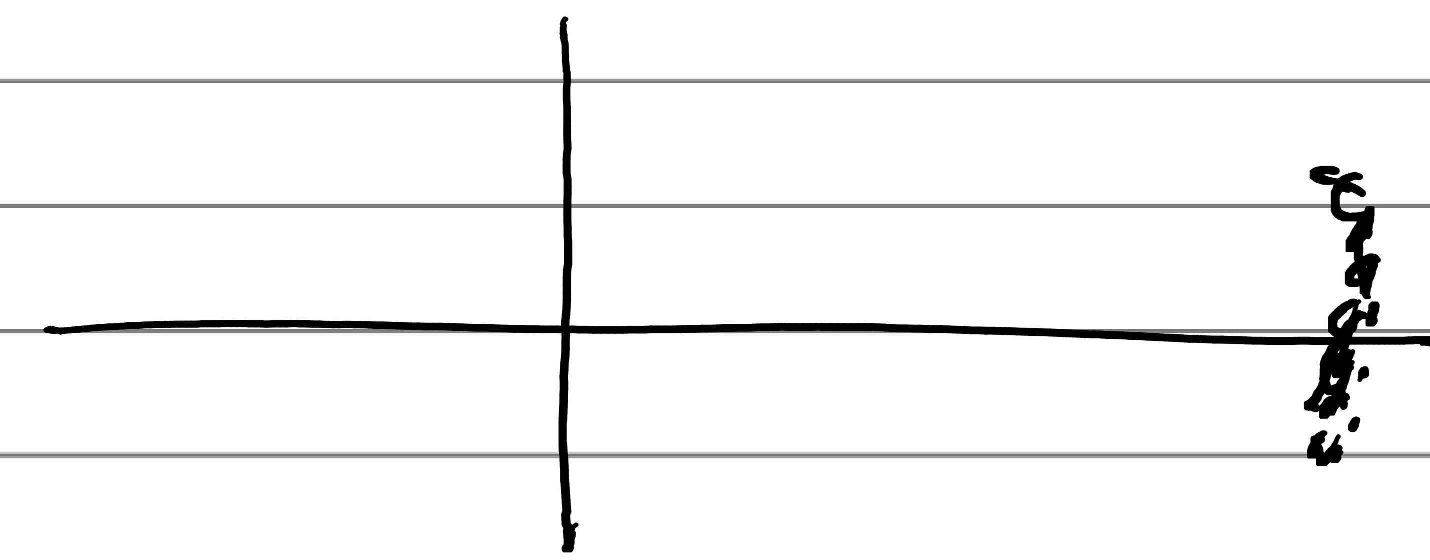
$N-1$ ind. q ; $\sum_1^N q_a = 0$.

Just expand in constant background field

q_a- distributions

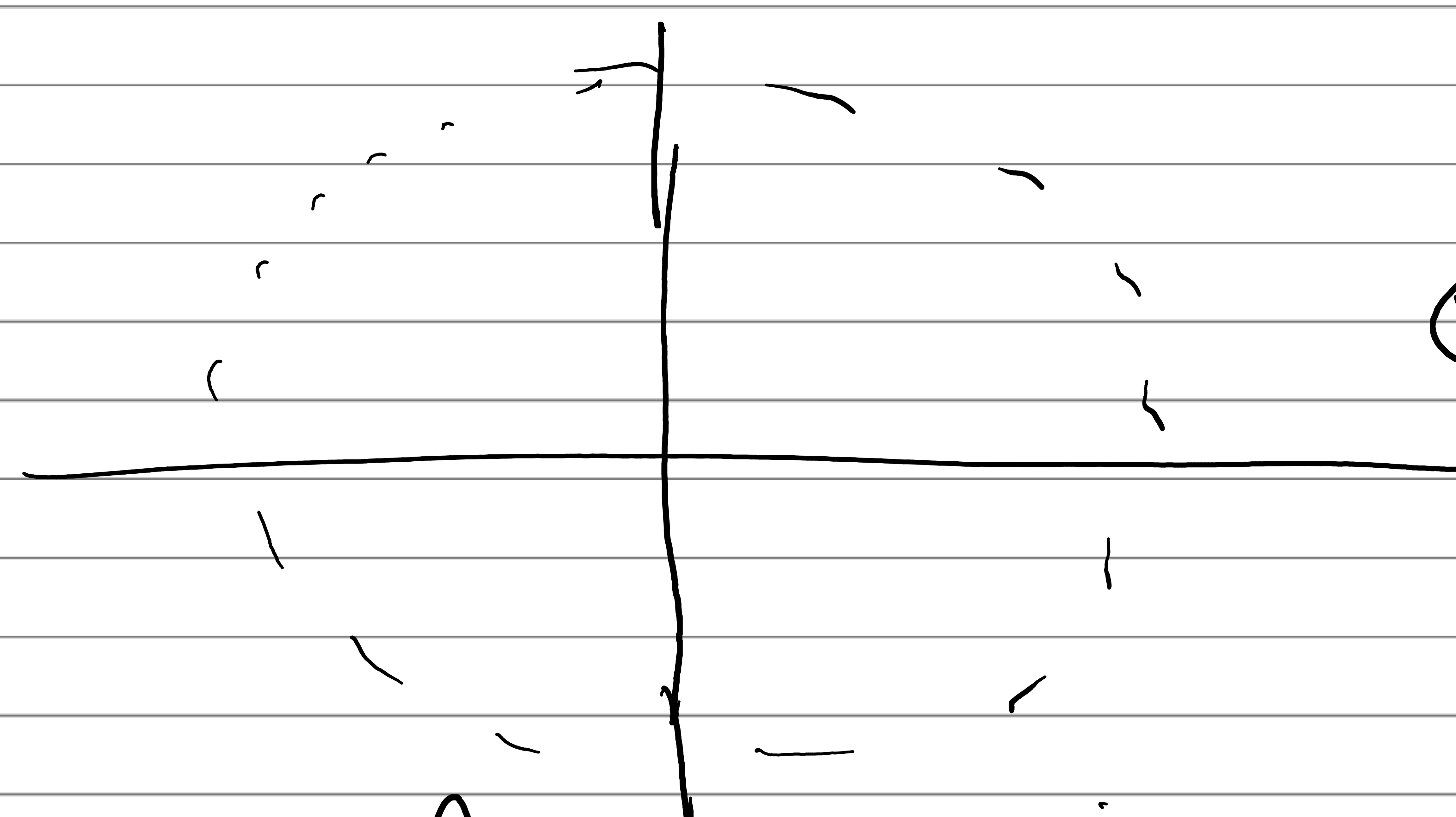
Look at dist. in q_a. Typically at large N ~ many simplifications

$T \to \infty$: $q_a \sim 0$



$e^{2\pi i q_a}$

$T < T_d$:



$e^{2\pi i q_a}$

Confinement $\Rightarrow$ uniform eigenvalue distribution $= \sum_a e^{2\pi i q_a} = 0$

Birdtracks

Don't care which q_a point in SU(N) space

Only physics in distribution in q_a.

$T \neq 0$:

$$q(1/T) = -q(0) \qquad \text{Fermi stat.'s}$$

$$A_\mu(1/T) = +A_\mu(0) \qquad \text{Bose-Einstein stat.'s}$$

q: $p_0 = 2\pi T(n + \frac{1}{2}) \qquad n = 0, \pm 1, \pm 2, \ldots$

g: $p_0 = 2\pi T n$

Now compute propagators in A_0^{cl} background

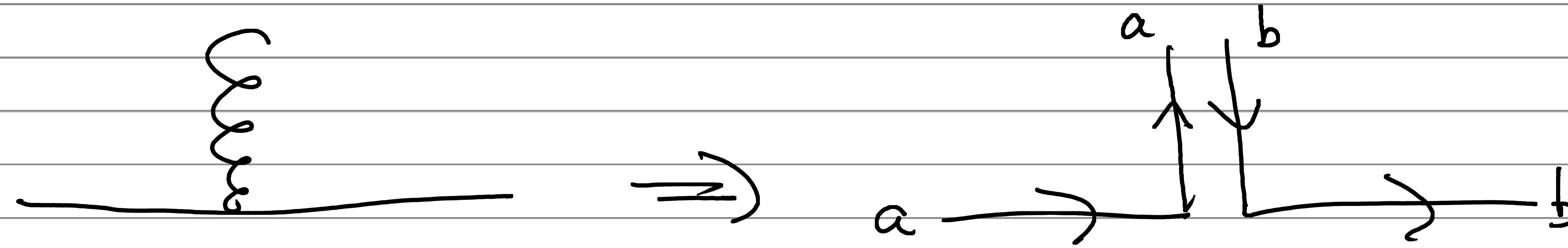
$$D_0 = \partial_0 - 2\pi T i q_a \qquad \text{in fund. rep.}$$

$$= \partial_0 - 2\pi T i (q_a - q_b) \qquad \text{in adj. rep.}$$

color off-diagonal. Dominate @ $N \to \infty$

Qk: $a \longrightarrow \; = \dfrac{1}{(p_0 + q_a)\gamma_0 + \vec{p}\cdot\vec{\gamma}}$

Gl: $\begin{matrix} a \longrightarrow \\ b \longleftarrow \end{matrix} \; = \dfrac{1}{(p_0 + q_a - q_b)^2 + \vec{p}^{\,2}}$

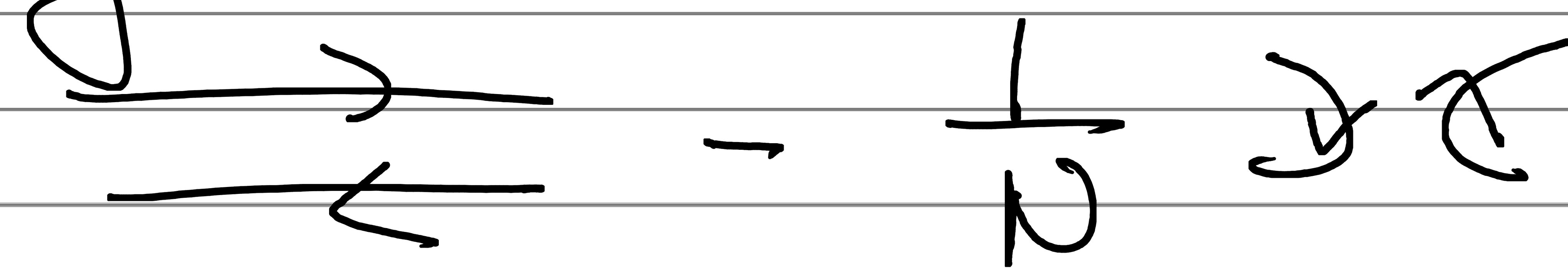


ugh

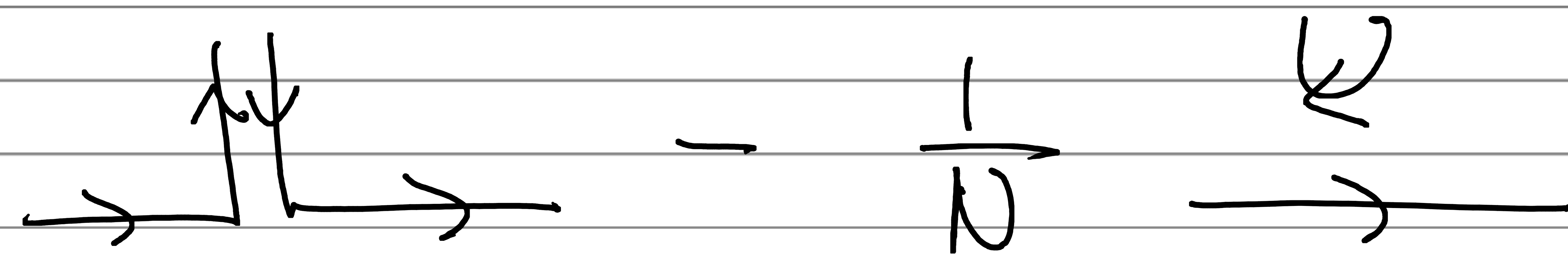
Feynman diagrams =
same BUT keep
track of q_a!

Gluon propagator

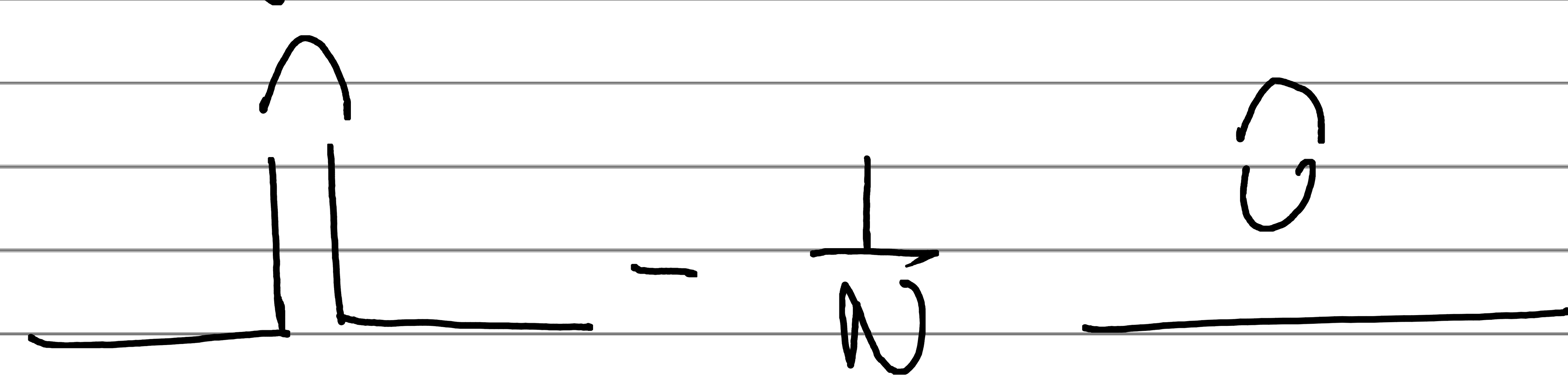
Correctly:



Corresponds to basis



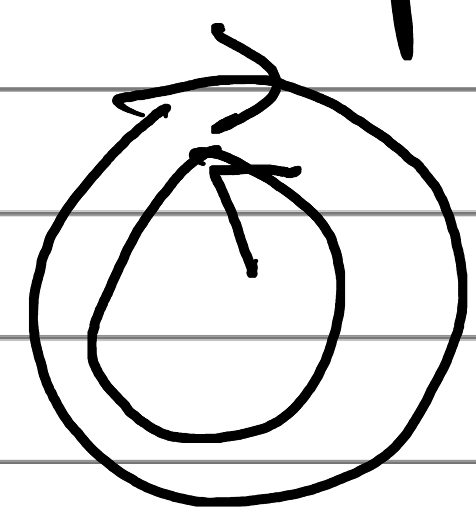
Off-diagonal generators same, but +1 diagonal generators. Overcomplete set:

$\sum_A t^{AA} =$  $= 0$

Very convenient for computation!

<u>Free energy ~ 1</u>

Compute for weak coupling, 3+1 dim's

 $= \sum_{a,b} \operatorname{tr} \ln\left(\left(p_0 + (q_a - q_b)\right)^2 + \vec{p}^{\,2} \right)$

$$\sim T^4 \sum_{n=1}^{\infty} \frac{|\operatorname{tr} \mathbb{L}^n|^2}{n^4} \sim \sum_{a,b} V_4(q_a - q_b)$$

$$V_4(q) = \# \, T^4 \, q^2 (1-q)^2$$

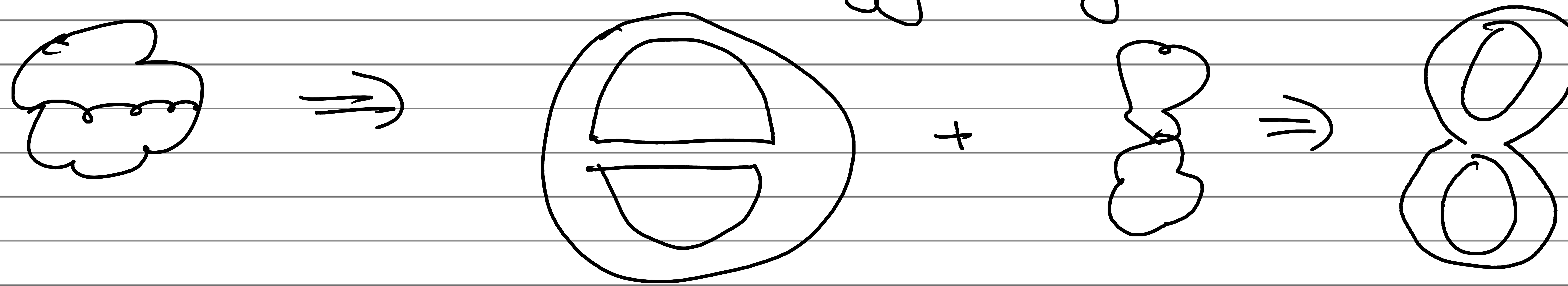
$$q = |q| \text{ modulo } 1$$

q_a <u>angular</u> variables

$$\mathbb{L} = e^{2\pi i q_a}$$

Simple analytic form for free energy ~ 1.

Free energy $\sim g^2$



+ . . .

Sum all diagrams still find simple result $\sim g^2 *$ 1 loop result.

But - need to incorporate shift in $g \sim g^2 *$ finite func. (g).

Eigenvalues are gauge invariant, "bare" g is not.

Simplicity lost at NNLO, $\sim g^3$

Confinement

Need ef. Lag. where $\langle qa \rangle$: perturbative

$\Rightarrow$ confined phase.

Lattice: for pure gauge,

$$p(T) \approx \# \left(T^4 - T_d^2 \, T^2 \right)$$

Leading correction to pressure $\sim T^2$

Manifestly non-perturbative

Need to decrease pressure, so must be a "ghost" field! Real ghost.

Manifestly effective model. Why T^2?!

(Hidden strings?)

2-D ghosts

Embed 2-D field in 4-D isotropically

$$\bigcirc \sim \underline{\underline{T_d^2}}\, T \sum_n \int dp_{\parallel}\ \mathrm{tr}\ \ln\left((p_0 + q_a - q_b)^2 + p_{\parallel}^2\right)$$

$$\sim T_d^2 T^2 \sum_{n=1}^{\infty} \frac{|\mathrm{tr}\, \mathbb{L}^n|^2}{n^2} \sim \sum_{a,b} V_2(q_a - q_b)$$

Now $\quad V_2 \sim -q(1-q) \longrightarrow q = |q| \bmod 1$

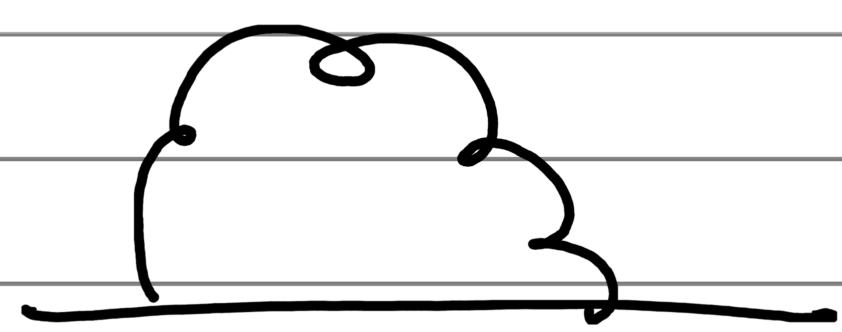
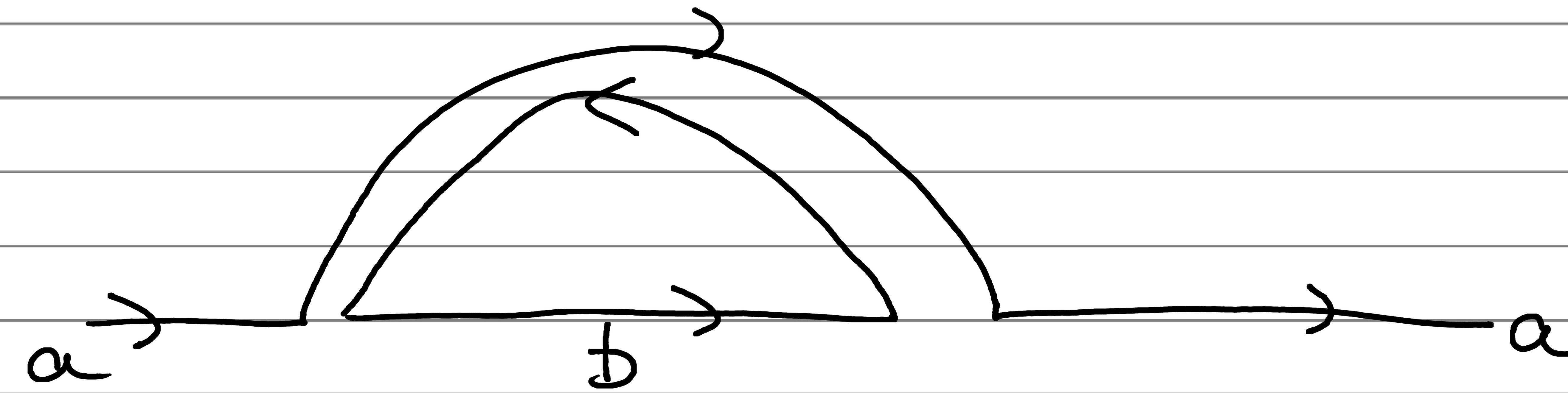
q_a periodic variable!

$V_2 \sim -q$ at small $q \Rightarrow$ <u>smooth</u> transition from pert. phase, $\langle q_a \rangle = 0$ to semi-QGP, $\langle q_a \rangle \sim T_d^2/T^2 \neq 0$.

Eff. model

Gluons + quarks and 2-D ghost fields

Backtracks essential!

 = 

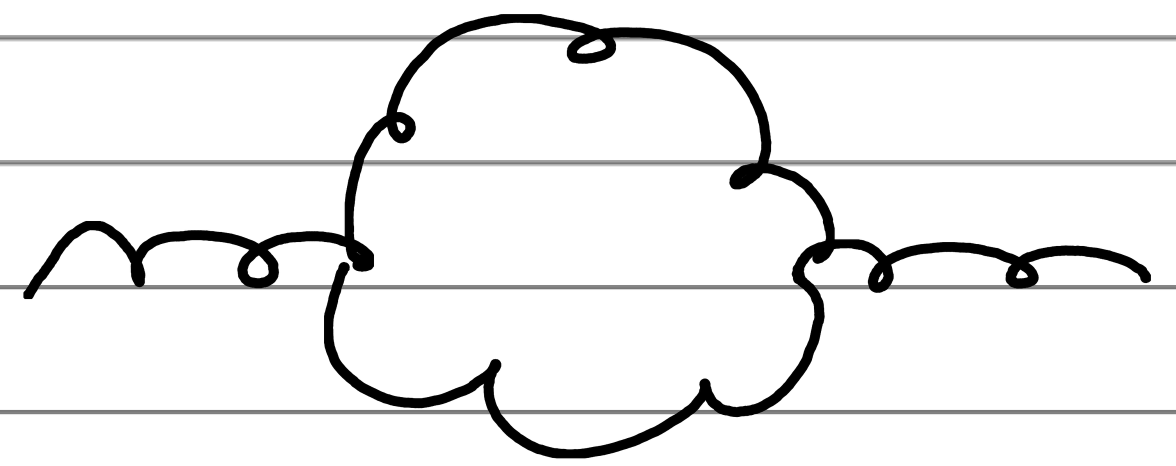
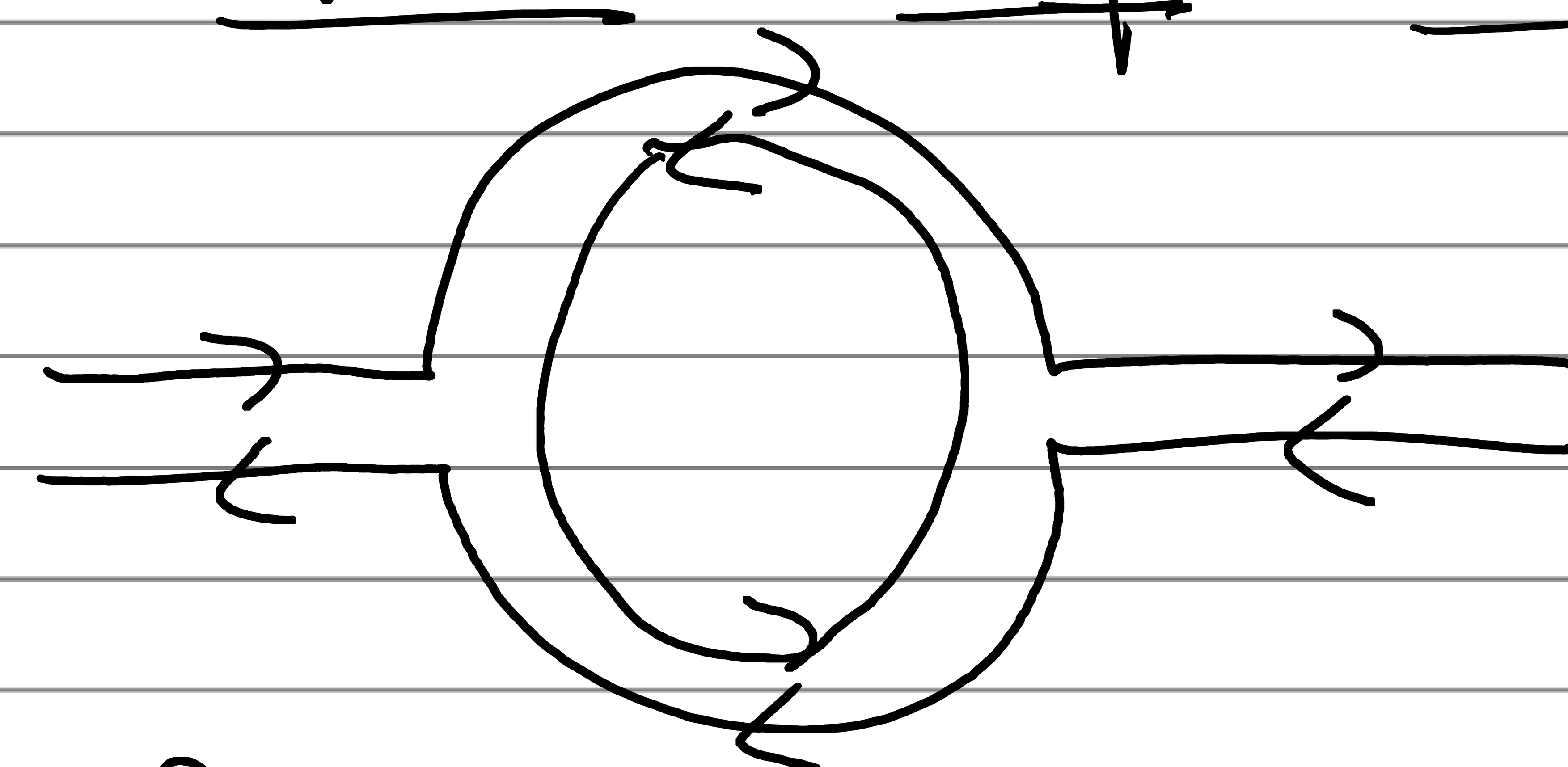
$\sim (m^2_{gk})_{ab}\, \delta\Sigma_{HTL}(\omega = i p_0, \bar{p})$ Hard Thermal Loop fnc.

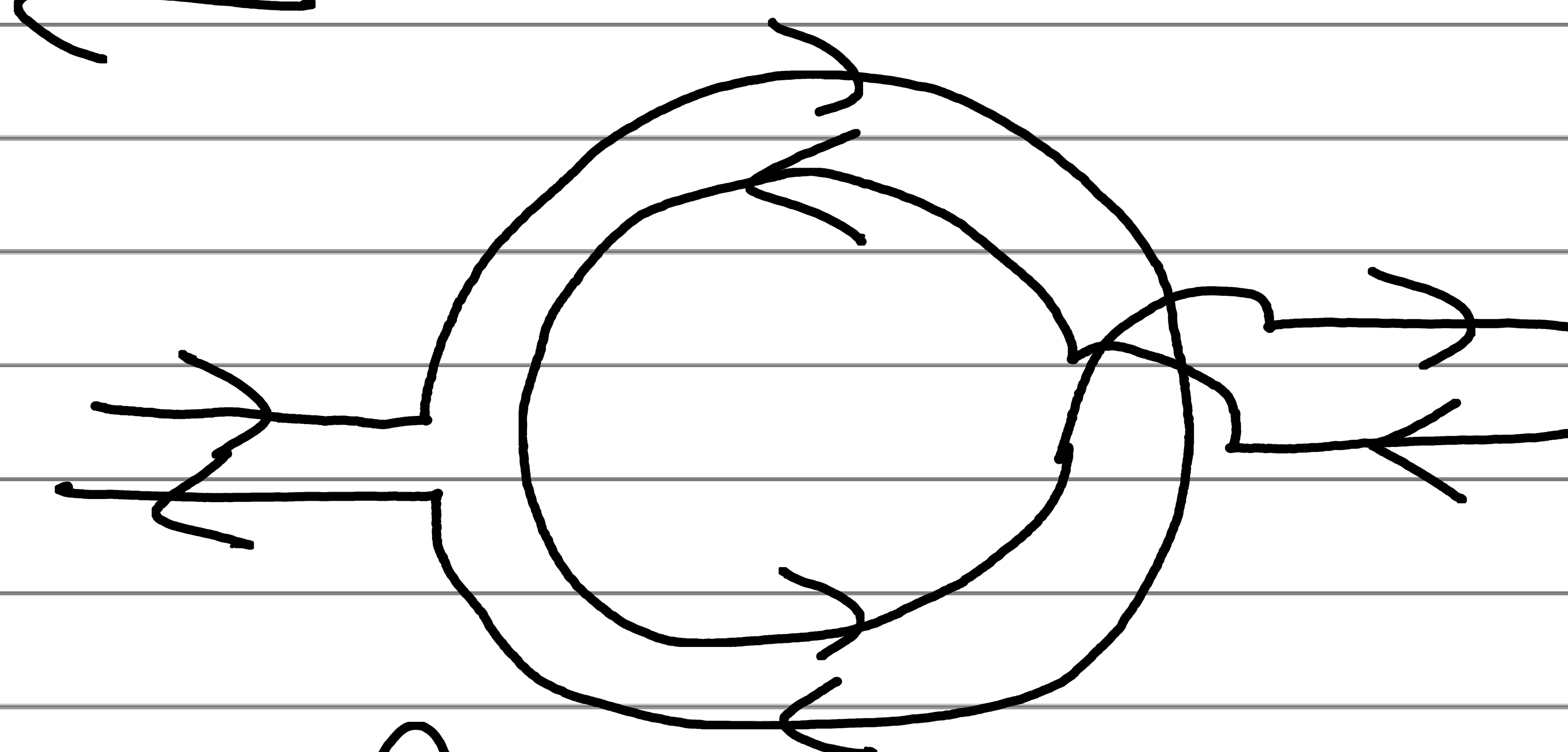
$\to \delta_{ab}\, g^2 T^2 \left(\sum_{c=1}^{N} \mathcal{A}(q_a - q_c) - \mathcal{A}(q_c + \tfrac{1}{2}) + \dots \right)$

$\mathcal{A}(q) = 1 - 6q(1-q)$ $\quad q \to |q| \bmod 1$

Generalizes usual pert. result

Gluon self energy

 $=$ 

Include terms from gluons and 2-D ghosts. $-$ 

Essential for cancellation of "tadpole" terms $\sim T^3$; soln. Eq. of Motion!

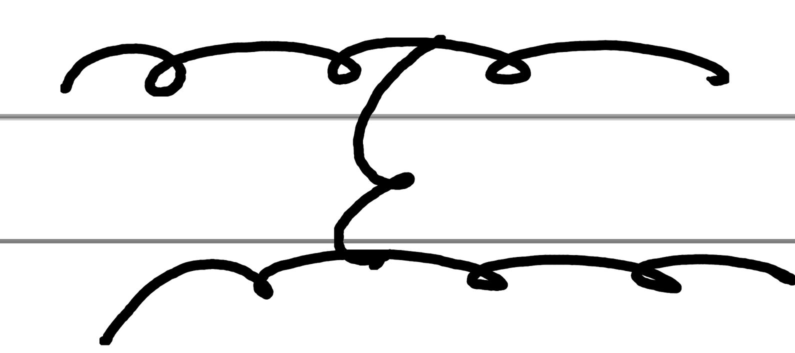
$$\Pi_{\mu\nu}^{ab,cd} \sim (m^2_{g\ell})^{ab,cd} \quad \delta\Pi^{HTL}_{\mu\nu}(\omega = ip_0, \vec{p})$$

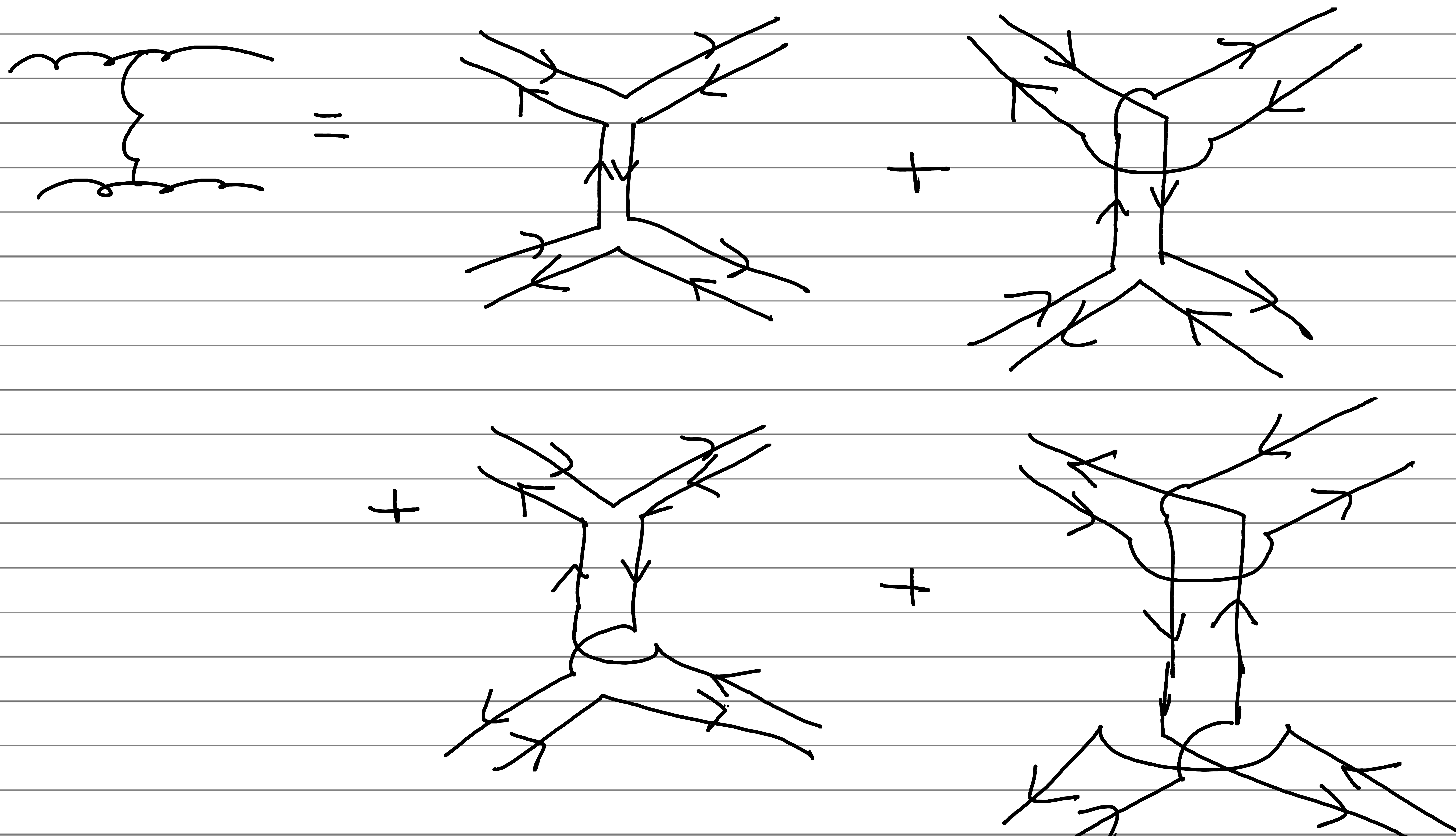
$$\sim g^2 T^2 \left(\delta^{ad}\delta^{bc}\left(\sum_{e=1}^{N} \mathcal{A}(q_a - q_e) + \mathcal{A}(q_e - q_b)\right) + N_f \underset{\text{qks}}{\mathcal{Q}} \right)$$

$$+\, g^2 T_d^2 \, \delta^{ad}\delta^{bc}$$

2D ghosts

Transport coeff's

Shear viscosity $\eta \sim 1/\sigma$, $\sigma \sim \left| \right.$  $\left. \right|^2$



No simple result

Have to keep track of all the qq's.
Do Boltzmann expansion in powers of l's

Actually, $\eta \sim \dfrac{\text{density}}{\text{cross-section}}$.

For both gluons and quarks, as $\langle \ell \rangle \to 0$, cross-section $\to 0$, but density wins

$\ell \to 0$: $\eta \sim \dfrac{T^3}{g^4 \ln 1/g} |\ell|^2$ (not obvious...) (full exp. messy)

Namely: only way to obtain small η is from large g^2, Semi-QGP: g can be moderate, but medium dilute due to confinement

Summary

Develop effective theory for "semi"-QGP.
Need to add 2-D ghosts to decrease pressure. Then compute pert'y in g^2.
Find: do obtain $\eta \to 0$ as $l \to 0 \sim l^2$.
Need detailed fit to pressure @ $N \to \infty$.
Can add quarks directly.
Backtracks essential! Don't care about details of color basis, only change in eigenvalue distribution.

References

T^2 term in pressure: RDP ph/0608242

Birdtracks and Hard Thermal Loops:
Yoshimasa Hidaka & RDP 0906,1751

Shear viscosity $\bar{c}$ gluons: YH & RDP 0803,0453
0906,1751

Effective matrix models:
A. Dumitru, Y. Guo, YH, C. Korthals-Altes 1011,3820
(several other works) 1205,0137

Effective models $\bar{c}$ 2-D ghosts YH & RP 2009,03903

In progress: Manas Debnath, Ritesh Ghosh,
Najmul Haque, YH & RDP

Shear and bulk viscosity + ...