Wilson line correllators in CGC and jets

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Outline

1 JIMWLK evolution: properties of the CGC
- Gluons field in high energy scattering
- The evolution equation for Wilson line correlators

2 Intelligent truncations
- Group constraints – coincidence limits a.k.a infrared & collinear safety
- Block hierarchies
- Gauge invariant truncations

3 Perspectives

4 Backup slides

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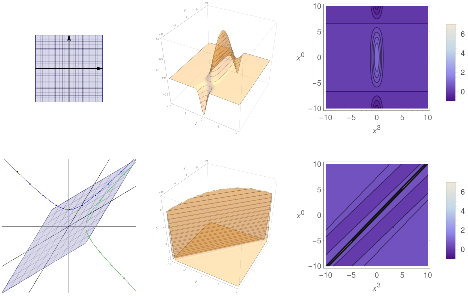
1. JIMWLK evolution: properties of the CGC
   - Gluons field in high energy scattering
   - The evolution equation for Wilson line correlators
2. Intelligent truncations
   - Group constraints – coincidence limits a.k.a infrared & collinear safety
   - Block hierarchies
   - Gauge invariant truncations
3. Perspectives
4. Backup slides

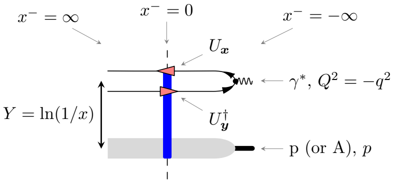
$\gamma^* A$ scattering at high energy – fields and interactions



$$A^\mu(x)$$

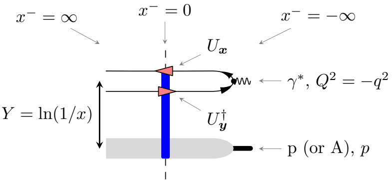
$$\downarrow \quad A'^\mu(x') = (\Lambda A)^\mu(\Lambda^{-1}x')$$

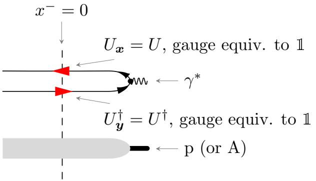
$$A'^\mu(x') = g^\mu{}_+ \underbrace{\delta(x^-) b^+(\boldsymbol{x})}_{\sim e^\eta A^+} (1 + O(e^{-\eta}))$$



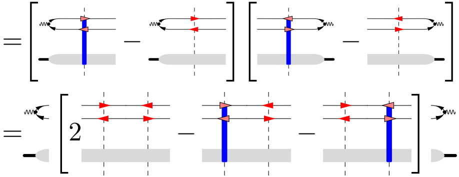
$\gamma^* A$ scattering at high energy – total X-section

Nikolaev, Zakharov, Frankfurt, Strikman, Levin, Mueller,... (dipole picture)



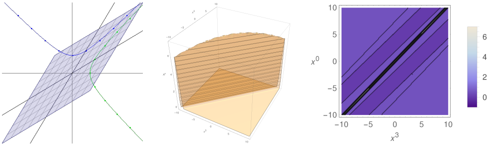


$\sigma_{\text{tot}}^{\gamma^* A}(Y, Q^2) =$

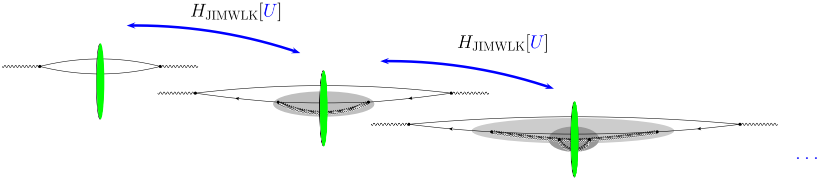


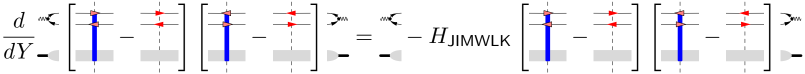
$$= \int d^2r \int d\alpha |\psi(\alpha, \boldsymbol{r}^2, Q^2)|^2 \;\; 2\text{Re} \int d^2b \frac{1}{N_c} \left\langle \text{tr}(\mathbb{1} - U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}}) \right\rangle (Y)$$

The need for evolution



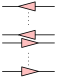
$$A'^{\mu}(x') = g^{\mu}{}_{+}\delta(x^-)b^+(\boldsymbol{x})(1+O(e^{-\eta}))$$

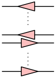
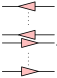


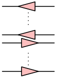
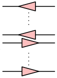


The JIMWLK evolution equation – Fokker-Planck form

Generalizes to arbitrary correlators:

$U_{x_1} \otimes \cdots \otimes U^\dagger_{y_{\bar n}} =:$ 

$\Big\langle$  $\Big\rangle(Y) := \int \hat{D}[U]$  $Z_Y[U]$

$\frac{d}{dY}\Big\langle$  $\Big\rangle(Y) = -\Big\langle H_{\text{JIMWLK}}$  $\Big\rangle(Y)$

The JIMWLK evolution equation – Fokker-Planck form

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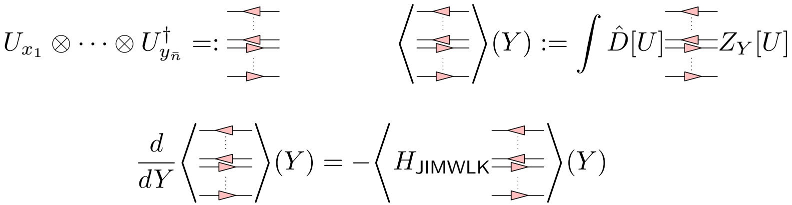


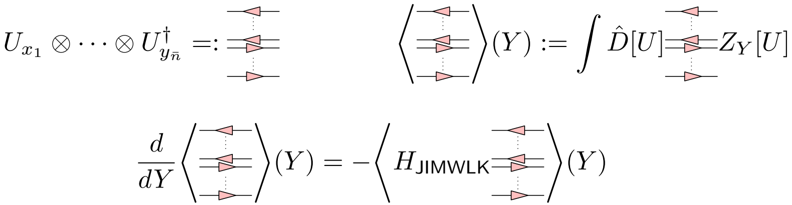
- $$\frac{d}{dY} Z_Y[U] = -H_{\text{JIMWLK}}[U] \; Z_Y[U]$$

Heribert Weigert **Nucl. Phys. A703**, 2002, 823

▸ explicit form

The JIMWLK evolution equation – Fokker-Planck form

Generalizes to arbitrary correlators:



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explicit form

- resums all $\sim [\alpha_s \ln(1/x)]^n$ (at LO)

The JIMWLK evolution equation – Fokker-Planck form

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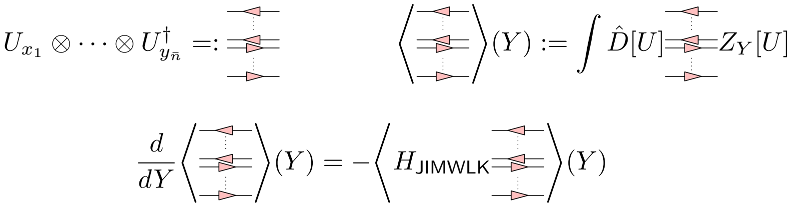

- $$\frac{d}{dY} Z_Y[U] = -H_{\text{JIMWLK}}[U] \;\; Z_Y[U]$$

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explicit form

- resums all $\sim [\alpha_s \ln(1/x)]^n$ (at LO)
- ⟶ energy dependence of $\langle \ldots \rangle(Y)$

The JIMWLK Hamiltonian

back

$$H_{\text{JIMWLK}} = -\frac{1}{2}\frac{\alpha_s}{\pi^2}\int_{\boldsymbol{xzy}} \mathcal{K}_{\boldsymbol{xzy}} \left[i\nabla^a_x i\nabla^a_y + i\bar{\nabla}^a_x i\bar{\nabla}^a_y + \tilde{U}^{ab}_z (i\bar{\nabla}^a_x i\nabla^b_y + i\nabla^a_x i\bar{\nabla}^b_y)\right]$$

$$\mathcal{K}_{\boldsymbol{xzy}} = \frac{(\boldsymbol{x}-\boldsymbol{z})\cdot(\boldsymbol{z}-\boldsymbol{y})}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}$$

[integration convention for x, z, y]

- $i\nabla^a_{\boldsymbol{x}}$ and $i\bar{\nabla}^a_{\boldsymbol{x}}$ are functional derivatives:

$$i\nabla^a_{\boldsymbol{x}} := -[U_{\boldsymbol{x}} t^a]_{ji}\frac{\delta}{\delta U_{\boldsymbol{x},ij}} \qquad i\bar{\nabla}^a_{\boldsymbol{x}} := [t^a U_{\boldsymbol{x}}]_{ji}\frac{\delta}{\delta U_{\boldsymbol{x},ij}}$$

- l. & r. inv vector fields, generate r & l rotations, e.g.:

$$\exp\left\{-i\int_{\boldsymbol{y}} \omega^a_{\boldsymbol{y}} i\nabla^a_{\boldsymbol{y}}\right\} U_{\boldsymbol{x}} = U_{\boldsymbol{x}} e^{i\omega^a_{\boldsymbol{x}} t^a}$$

- reps of the algebra:

$$[i\nabla^a_{\boldsymbol{x}}, i\nabla^b_{\boldsymbol{y}}] = \delta^{(2)}(\boldsymbol{x}-\boldsymbol{y}) i f^{abc} i\nabla^c_{\boldsymbol{x}} \quad [i\bar{\nabla}^a_{\boldsymbol{x}}, i\bar{\nabla}^b_{\boldsymbol{y}}] = \delta^{(2)}(\boldsymbol{x}-\boldsymbol{y}) i f^{abc} i\bar{\nabla}^c_{\boldsymbol{x}} \quad [i\bar{\nabla}^a_{\boldsymbol{x}}, i\nabla^b_{\boldsymbol{y}}] = 0$$

The JIMWLK Hamiltonian

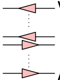
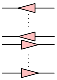
back

$$H_{\text{JIMWLK}} = -\frac{1}{2}\frac{\alpha_s}{\pi^2}\int_{\boldsymbol{xzy}} \mathcal{K}_{\boldsymbol{xzy}} \left[i\nabla^a_x i\nabla^a_y + i\bar{\nabla}^a_x i\bar{\nabla}^a_y + \tilde{U}^{ab}_z (i\bar{\nabla}^a_x i\nabla^b_y + i\nabla^a_x i\bar{\nabla}^b_y) \right]$$

$$\mathcal{K}_{\boldsymbol{xzy}} = \frac{(\boldsymbol{x}-\boldsymbol{z})\cdot(\boldsymbol{z}-\boldsymbol{y})}{(\boldsymbol{x}-\boldsymbol{z})^2(\boldsymbol{z}-\boldsymbol{y})^2}$$

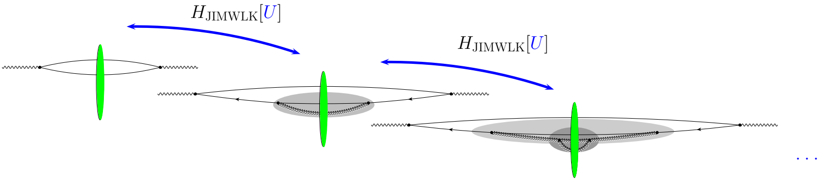
[integration convention for x, z, y]

- $i\nabla$ & $i\bar{\nabla}$ $\leftrightarrow$ left & right inv vector fields $\Rightarrow$

$\Big\langle$  $\Big\rangle(Y) = \Big\langle \exp\left[-H_{\text{JIMWLK}}(Y-Y_0)\right]$  $\Big\rangle(Y_0)$

remain Wilson line correlators for all Y

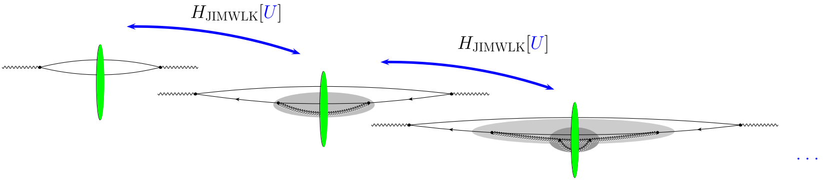
JIMWLK - Balitsky hierarchies



- Dipole evolution

$$\frac{d}{dY}\langle \operatorname{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle(Y) = \frac{\alpha_s}{\pi^2}\int d^2z\, \mathcal{K}_{\boldsymbol{xzy}} \Big(\langle [\tilde U_{\boldsymbol{z}}]^{ab} \operatorname{tr}(t^a U_{\boldsymbol{x}} t^b U^\dagger_{\boldsymbol{y}})\rangle(Y) - C_{\mathsf{f}}\langle \operatorname{tr}(U_{\boldsymbol{x}}U^\dagger_{\boldsymbol{y}})\rangle(Y)\Big)$$

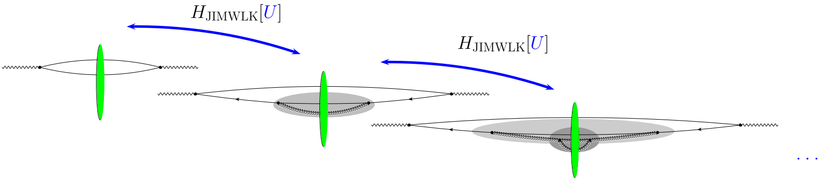
JIMWLK - Balitsky hierarchies



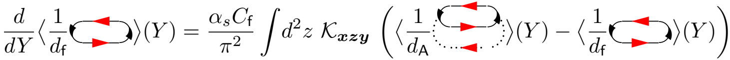
- Dipole evolution

$$\frac{d}{dY}\langle \frac{1}{d_{\mathsf{f}}} \bigcirc \rangle(Y) = \frac{\alpha_s C_{\mathsf{f}}}{\pi^2} \int d^2z\, \mathcal{K}_{\boldsymbol{xzy}} \left(\langle \frac{1}{d_{\mathsf{A}}} \bigcirc \rangle(Y) - \langle \frac{1}{d_{\mathsf{f}}} \bigcirc \rangle(Y) \right)$$

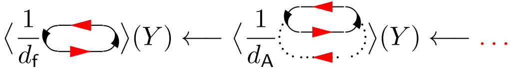
JIMWLK - Balitsky hierarchies



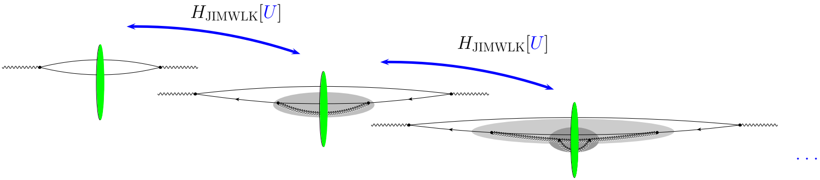
- Dipole evolution



- Inifinte hierarchy of dependent correlators



JIMWLK - Balitsky hierarchies



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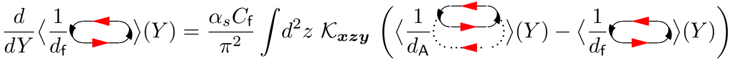

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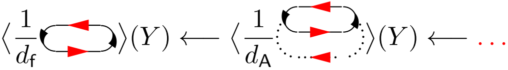
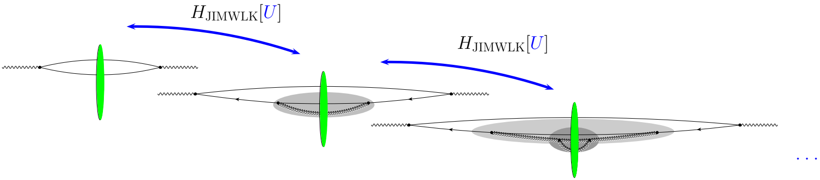


- Truncate to solve! Last resort? Informative!

JIMWLK - Balitsky hierarchies



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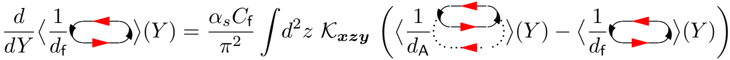

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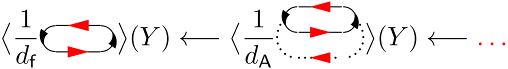
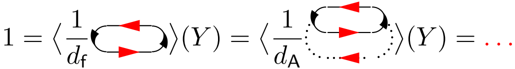


- Truncate to solve! Last resort? Informative!
- No target: $U_{\boldsymbol{x}} \to U$ – consistency from gauge invariance:



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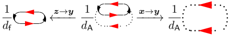
Group constraints and gauge invariance – examples

- Operator relations – gauge invariance: 

$$\frac{1}{d_A}\tilde{U}_{\boldsymbol{z}}^{ab} 2\mathrm{tr}(t^a U_{\boldsymbol{x}} t^b U_{\boldsymbol{y}}^\dagger) = \frac{1}{d_A} 2\mathrm{tr}(U_{\boldsymbol{z}} t^a U_{\boldsymbol{z}}^\dagger U_{\boldsymbol{x}} t^a U_{\boldsymbol{y}}^\dagger)$$

$\boldsymbol{z} \mapsto \boldsymbol{x}$ or $\boldsymbol{y}$: $\frac{1}{d_f}\mathrm{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)$ $\xrightarrow{\boldsymbol{y} \mapsto \boldsymbol{x}}$ 1

$\boldsymbol{y} \mapsto \boldsymbol{x}$: $\frac{1}{d_A}\tilde{\mathrm{tr}}(\tilde{U}_{\boldsymbol{z}} \tilde{U}_{\boldsymbol{x}}^\dagger)$ $\xrightarrow{\boldsymbol{z} \mapsto \boldsymbol{x}}$ 1

$$\frac{1}{d_A}\tilde{\mathrm{tr}}(\tilde{U}_{\boldsymbol{z}} \tilde{U}_{\boldsymbol{x}}^\dagger) = \frac{1}{d_A}\left(|\mathrm{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{z}}^\dagger)|^2 - 1\right)$$

Fierz:

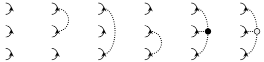
$$\frac{1}{d_A}\left(\mathrm{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{z}}^\dagger)\mathrm{tr}(U_{\boldsymbol{z}} U_{\boldsymbol{y}}^\dagger) - \frac{1}{N_c}\mathrm{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{y}}^\dagger)\right) \xrightarrow{\boldsymbol{y} \mapsto \boldsymbol{x}} \frac{1}{d_A}\left(|\mathrm{tr}(U_{\boldsymbol{x}} U_{\boldsymbol{z}}^\dagger)|^2 - 1\right)$$

- $\langle \ldots \rangle$ ➡ more correlators than degrees of freedom

- 2 gluon correlator $\in \mathbb{R}$
- all others possibly $\in \mathbb{C}$

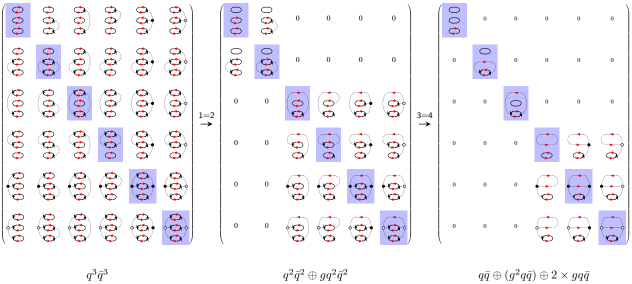
Correlators – singlet hierarchies

- Higher order correlator evolution *contains* that of lower correlators
  Example: coincidence limits of $q^3\bar{q}^3$: H.W., J. Alcock-Zeilinger
  - $3!$ singlet states (in almost direct corresp to the elements of S_3):



trace basis

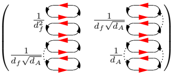
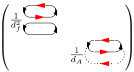
  - propagation through soft gluon field mixes *all*: links correlators



qq and $\bar{q}\bar{q}$ limits

Gauge invariant truncations of JIMWLK – the idea

- Indepdendent degrees of freedom? Example: $n = 2$ $q\bar{q}$ pairs!

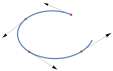
$\langle \mathcal{A} \rangle(Y) := \Big\langle$  $\Big\rangle(Y) \xrightarrow{y_2 \to x_2} \Big\langle$  $\Big\rangle(Y) \xrightarrow[\text{no target}]{U_{\boldsymbol{x}} \to \mathbb{1}} \mathbb{1}$

- $\langle \mathcal{A} \rangle(Y) \in \mathbb{R}^{n! \leftarrow n!}$ a curve in a vector space. (JIMWLK!) Vectorially

tangent

$$\frac{d}{dY}\langle \mathcal{A} \rangle(Y) = \mathcal{M}(Y) \langle \mathcal{A} \rangle(Y)$$

curve



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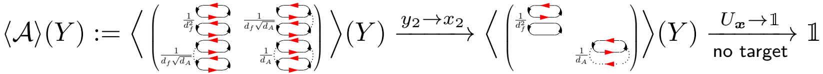

- $\mathcal{M}(Y)$ inherits conicidence limits – lives in a vector space (not a curved 1pG) Parametrize $\mathcal{M}(Y)$ (*not* $\langle \mathcal{A} \rangle(Y)$)

$=: \mathcal{E}(Y, Y_0) \in$ curved 1pG

$$\langle \mathcal{A} \rangle(Y) = P_Y \exp\left[\int_{Y_0}^{Y} dY' \; \mathcal{M}(Y) \right] \langle \mathcal{A} \rangle(Y_0)$$

$\in$ some vector space

Gauge invariant truncations: parametrize $\mathcal{M}$

- **Rule 1:** Wilson line correlators must map onto Wilson line correlators: *must respect coincidence limits.*
- **Rule 2:** JIMWLK respects matrix symmetry of $\langle\mathcal{A}\rangle(Y)$!
  Adapted basis expansion:

$$\mathcal{A} = b_{(i)} A^i = b^{\text{sy}}_{(i)} \mathcal{A}^i_{\text{sy}} + b^{\text{asy}}_{(i)} \mathcal{A}^i_{\text{asy}}$$

  $\mathcal{E}(Y, Y_0)$ and $\mathcal{M}(Y)$ **must** take block form:

$$\frac{d}{dY} b_{(i)} \langle\mathcal{A}^i\rangle(Y) = b_{(i)} \mathcal{M}^i{}_j(Y) \langle\mathcal{A}^j\rangle(Y)$$

- **Rule 3:** All entries of $\mathcal{A}$ invariant under independent global color rot V_L and V_R. Example:

$\forall V_L, V_R \in \mathrm{SU}(n)$

- **Rule 4:** Cross sections remain real! (Subtle consequences)

Gauge invariant truncations: parametrize $\mathcal{M}$

- The most general $\mathcal{M}(Y)$? **Parametrization equation:**

$$b_{(i)}\mathcal{M}^i{}_j(Y)\langle A^j\rangle(Y)$$
$$= \Big\langle \int_{Y_0}^{Y} dy \Big[\frac{1}{2} \int_{\boldsymbol{u}_1\boldsymbol{u}_2} G_{\boldsymbol{u_1}\boldsymbol{u_2}}(y)\delta^{a_1a_2}\left(i\bar\nabla^{\boldsymbol{u}_1}_{a_1} i\bar\nabla^{\boldsymbol{u}_2}_{a_2} + i\nabla^{\boldsymbol{u}_1}_{a_1} i\nabla^{\boldsymbol{u}_2}_{a_2}\right)$$
$$+ \sum_{m=3}^{\infty}\sum_{l=0}^{m}\sum_{\alpha,\beta}\frac{1}{m!}\int_{u_1\ldots v_{m-l}} G^{(\alpha,\beta)}_{\boldsymbol{u}_1\ldots\boldsymbol{v}_{m-l}} C^{a_1\cdots a_l}_{\alpha} C^{b_1\cdots b_{m-l}}_{\beta}$$
$$\times \left(\nabla^{\boldsymbol{u}_1}_{a_1}\cdots\nabla^{\boldsymbol{u}_l}_{a_l}\bar\nabla^{\boldsymbol{v}_1}_{b_1}\cdots\bar\nabla^{\boldsymbol{v}_{m-l}}_{b_{m-l}} + (\nabla\leftrightarrow\bar\nabla)\right)\Big]\mathcal{A}\Big\rangle(Y)$$

- Using ∇ and $\bar\nabla$ + coincidence constraints on $G_{\_,\ldots,\_}$: **Rule 1**
- Symmetry $\nabla\leftrightarrow\bar\nabla$: **Rule 2**
- Color structure of mixed terms *must factorize*: **Rule 3**

$$C^{a_1\cdots a_k}C^{b_1\cdots b_l}\nabla^{\boldsymbol{u}_1}_{a_1}\ldots\nabla^{\boldsymbol{u}_1}_{a_k}\bar\nabla^{\boldsymbol{v}_1}_{b_1}\ldots\bar\nabla^{\boldsymbol{v}_l}_{b_l}$$

- No $m=3$ term for total cross section! **Rule 4**

Gauge invariant truncations: parametrize $\mathcal{M}$

- $G_{\boldsymbol{u}_1\ldots\boldsymbol{u}_n}$? Think structure functions for Wilson lines
- one for each independent n-gluon singlet $C^{a_1\ldots a_n}$
  $\Rightarrow$ need workable algorithm for all of these HW, J A-Z, sketch below
- parametrization up to an initial condition at Y_0
- coincidence limits constrain $G_{\boldsymbol{u}_1\ldots\boldsymbol{u}_n}$ HW, R Moerman, sketch below
- gauge invariant at any order
- Result:
  - $$\langle\ldots\rangle(Y) \mapsto \langle\ldots\rangle[G^{(\alpha,\beta)}_{\boldsymbol{u}_1\boldsymbol{u}_2}, G^{(\alpha,\beta)}_{\boldsymbol{u}_1\boldsymbol{u}_2\boldsymbol{u}_3}, \text{etc}](Y)$$
  - Tuncate at a given number of coodinates n: ignore higher $n+k$ particle correlations
  - N_c exact within that truncation. NO large N_c limit!

Gauge invariant truncations: parametrize $\mathcal{M}$

- Example: truncate at $G_{\_,\_}$ (Gaussian truncation): Color dipoles only!
- Evolution is imposed by JIMWLK via Balitsky hierarchy:

$$\frac{d}{dY}\langle \frac{1}{d_{\mathsf{f}}} \bigcirc \rangle(Y) = \frac{\alpha_s C_{\mathsf{f}}}{\pi^2}\int d^2z\, \mathcal{K}_{\boldsymbol{xzy}} \left(\langle \frac{1}{d_{\mathsf{A}}} \ominus \rangle(Y) - \langle \frac{1}{d_{\mathsf{f}}} \bigcirc \rangle(Y) \right)$$

$$\frac{d}{dY}\langle \frac{1}{d_{\mathsf{A}}} \ominus \rangle(Y) = \ldots \qquad \text{must ignore}$$

$$\vdots$$

- From parametrization equation

$$\frac{1}{d_{\mathsf{f}}}\langle \bigcirc \rangle = \frac{1}{N_c}\langle \mathrm{tr}(U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})\rangle = e^{-C_f G_{\boldsymbol{xy}}}$$

$$\frac{1}{d_{\mathsf{A}}}\langle \ominus \rangle = \frac{1}{d_A}\langle U^{ab}_{\boldsymbol{z}} 2\mathrm{tr}(t^a U_{\boldsymbol{x}} t^b U^\dagger_{\boldsymbol{y}})\rangle = e^{-\left\{\left[\frac{N_c}{2}((G_{\boldsymbol{xz}}+G_{\boldsymbol{zy}})-G_{\boldsymbol{xy}}\right)-C_{\mathsf{f}} G_{\boldsymbol{xy}}\right\}(Y)}$$

- manifestly gauge invariant – just enough degrees of freedom

$\boldsymbol{z} \mapsto \boldsymbol{x}$ or $\boldsymbol{y}$ → $C_{\mathsf{f}}\mathrm{tr}(U_{\boldsymbol{x}} U^\dagger_{\boldsymbol{y}})$ — $\boldsymbol{y} \mapsto \boldsymbol{x}$ → $2N_c C_{\mathsf{f}}$

$$U^{ab}_{\boldsymbol{z}} tr(t^a U_{\boldsymbol{x}} t^b U^\dagger_{\boldsymbol{y}}) \quad = \mathrm{tr}(U_{\boldsymbol{z}} t^a U^\dagger_{\boldsymbol{z}} U_{\boldsymbol{x}} t^a U^\dagger_{\boldsymbol{y}})$$

$\boldsymbol{y} \mapsto \boldsymbol{x}$ → $\frac{1}{2}\tilde{\mathrm{tr}}(\tilde{U}_{\boldsymbol{z}} \tilde{U}^\dagger_{\boldsymbol{x}})$ — $\boldsymbol{z} \mapsto \boldsymbol{x}$ → $2N_c C_{\mathsf{f}}$

Outline

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- Gluons field in high energy scattering
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4 Backup slides

Three points, four points, etc?

- Space of gluon singlets from trace basis
- Choose color structures with symmetries:

$$\int\limits_{\boldsymbol{u}_1\ldots\boldsymbol{u}_n} G^{(K)}_{\boldsymbol{u}_1\ldots\boldsymbol{u}_n} C^{a_1\ldots a_n}_{(K)} i^n \bar{\nabla}^{\boldsymbol{u}_1\cdots\boldsymbol{u}_n}_{a_1\cdots a_n} = \int\limits_{\boldsymbol{u}_1\ldots\boldsymbol{u}_n} \frac{1}{n!}\sum_{\sigma\in S_n} G^{(K)}_{\boldsymbol{u}_{\sigma(1)}\ldots\boldsymbol{u}_{\sigma(n)}} C^{a_{\sigma(1)}\ldots a_{\sigma(n)}}_{(K)} i^n \bar{\nabla}^{\boldsymbol{u}_1\cdots\boldsymbol{u}_n}_{a_1\cdots a_n}$$

  Simultaneous symmetry in color and coords.
- Or use a "generic one":

$M^{-1}(M^{-1})^t = (M^tM)^{-1}$ is dagonal with α_I on diagonals

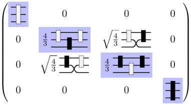
$$\frac{1}{m!}\sum_{i=1}^{m!}(\sigma_i\circ G)_{\boldsymbol{u}_1\ldots\boldsymbol{u}_m}(\sigma_i\circ C)_{a_1\ldots a_m} = \frac{1}{m!}\sum_{(Iij)}\alpha_I (T_{(Iij)}\circ G)_{\boldsymbol{u}_1\ldots\boldsymbol{u}_m}(T_{(Iij)}\circ C)_{a_1\ldots a_m}$$

$$= \sum_{(Iij)}\frac{1}{|I|}(T_{(Iij)}\circ G)_{\boldsymbol{u}_1\ldots\boldsymbol{u}_m}(T_{(Iij)}\circ C)_{a_1\ldots a_m}$$

$|I| = \frac{m!}{\alpha_I}$

where $\sigma_k = T_{(Iij)} {M^{(Iij)}}_k$

Three points:

$T_{(Iij)} \leftrightarrow$ 

with $T_{(Iij)}T_{(Klm)} = \delta_{IK}\delta_{jl}T_{(Iim)}$

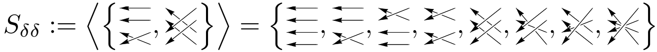
details

Work in progress

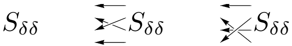
- Program completed for three point truncation
- Four point truncation involves mixed contributions:

$$\int\limits_{u_1\ldots u_4} G_{u_1\ldots u_4}\delta^{a_1a_2}\delta^{a_3a_4}\Big(i\nabla^{u_1}_{a_1}i\nabla^{u_2}_{a_2}i\bar\nabla^{u_3}_{a_3}i\bar\nabla^{u_4}_{a_4}+(\nabla\leftrightarrow\bar\nabla)\Big)$$

Attack via invariance subgroup

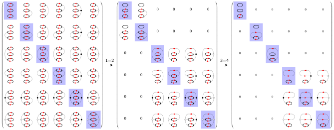


and associated cosets

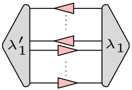


- Subtract iterated Gaussian truncation ... ("connected 4-points")

Context



- This applies to whereever QCD triggers factorized singlet transitions mediated by Wilson lines

$\langle \mathcal{A} \rangle^{\lambda_1'}{}_{\lambda_1} = \Big\langle$  $\Big\rangle$

JIMWLK, Jet evolution (SCET)

- JIMWLK coincidence limits = collinear & infraread safety in jets
- Truncation in term of n-point correlations. At this level N_c-exact.
- Stay tuned for 4-point results....

Thank you for your attention

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Singlet states with $q\bar{q}$-pairing: trace singlet bases

- An algorithm by example: $(q\bar{q})^3$

$$\mathrm{id} = (1)(2)(3) \longrightarrow \delta_{\bar{q}_1 q_1} \otimes \delta_{\bar{q}_2 q_2} \otimes \delta_{\bar{q}_3 q_3} =$$

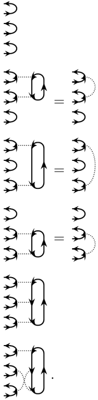
$$(12)(3) \longrightarrow \mathrm{tr}\,(t^{a_1} t^{a_2})\,[t^{a_1}]_{\bar{q}_1 q_1} \otimes [t^{a_2}]_{\bar{q}_2 q_2} \otimes \delta_{\bar{q}_3 q_3} = \; = $$

$$(13)(2) \longrightarrow \mathrm{tr}\,(t^{a_1} t^{a_3})\,[t^{a_1}]_{\bar{q}_1 q_1} \otimes \delta_{\bar{q}_2 q_2} \otimes [t^{a_3}]_{\bar{q}_3 q_3} = \; = $$

$$(23)(1) \longrightarrow \mathrm{tr}\,(t^{a_2} t^{a_3})\,\delta_{\bar{q}_1 q_1} \otimes [t^{a_2}]_{\bar{q}_2 q_2} \otimes [t^{a_3}]_{\bar{q}_3 q_3} = \; = $$

$$(123) \longrightarrow \mathrm{tr}\,(t^{a_1} t^{a_2} t^{a_3})\,[t^{a_1}]_{\bar{q}_1 q_1} \otimes [t^{a_2}]_{\bar{q}_2 q_2} \otimes [t^{a_3}]_{\bar{q}_3 q_3} =$$

$$(132) \longrightarrow \mathrm{tr}\,(t^{a_1} t^{a_3} t^{a_2})\,[t^{a_1}]_{\bar{q}_1 q_1} \otimes [t^{a_2}]_{\bar{q}_2 q_2} \otimes [t^{a_3}]_{\bar{q}_3 q_3} = \; .$$



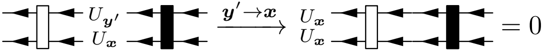
- orthogonalize (not unique in general)

- don't forget to normalize!

back

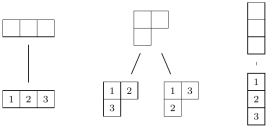
States for qq or $\bar{q}\bar{q}$ limits

- Zeroes from symmetry:

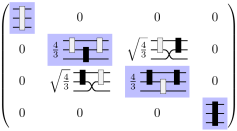


correct but mentally misleading

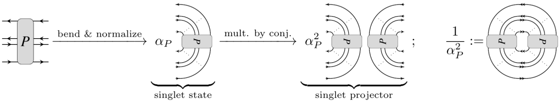
- Form states from hermitian Young projectors. Example $q^3\bar{q}^3$



leads to



- Bend and normalize to form states

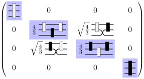


- $\langle \mathcal{A} \rangle$ shows block structure after like limits

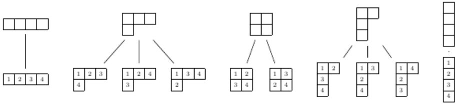
back

Gluon-singlets from trace bases: !n states in general

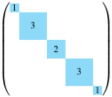
- Three: $!3 = 2$ states. Projected by symmetry recall



- Four: $!4 = 9$ states, project by symmetry via Young projectors, viz.



leads to



Dimension patterns: 2, 0, 0, 0, 2, 2, 1, 1, 1, 0

Example state:

$$P_{\begin{smallmatrix}1&2\\3&4\end{smallmatrix}}\, v = \frac{c'_3}{12}\Big(\operatorname{tr}(t^{a_1}t^{a_2}t^{a_3}t^{a_4}) + \operatorname{tr}(t^{a_1}t^{a_2}t^{a_4}t^{a_3}) - 2\operatorname{tr}(t^{a_1}t^{a_3}t^{a_2}t^{a_4})$$
$$+ \operatorname{tr}(t^{a_1}t^{a_3}t^{a_4}t^{a_2}) - 2\operatorname{tr}(t^{a_1}t^{a_4}t^{a_2}t^{a_3}) + \operatorname{tr}(t^{a_1}t^{a_4}t^{a_3}t^{a_2})\Big)$$
$$- \frac{c'_4}{6}\Big(\operatorname{tr}(t^{a_1}t^{a_4})\operatorname{tr}(t^{a_2}t^{a_3}) + \operatorname{tr}(t^{a_1}t^{a_3})\operatorname{tr}(t^{a_2}t^{a_4}) - 2\operatorname{tr}(t^{a_1}t^{a_2})\operatorname{tr}(t^{a_3}t^{a_4})\Big)$$

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