UNIVERSITÄT GRAZ
UNIVERSITY OF GRAZ

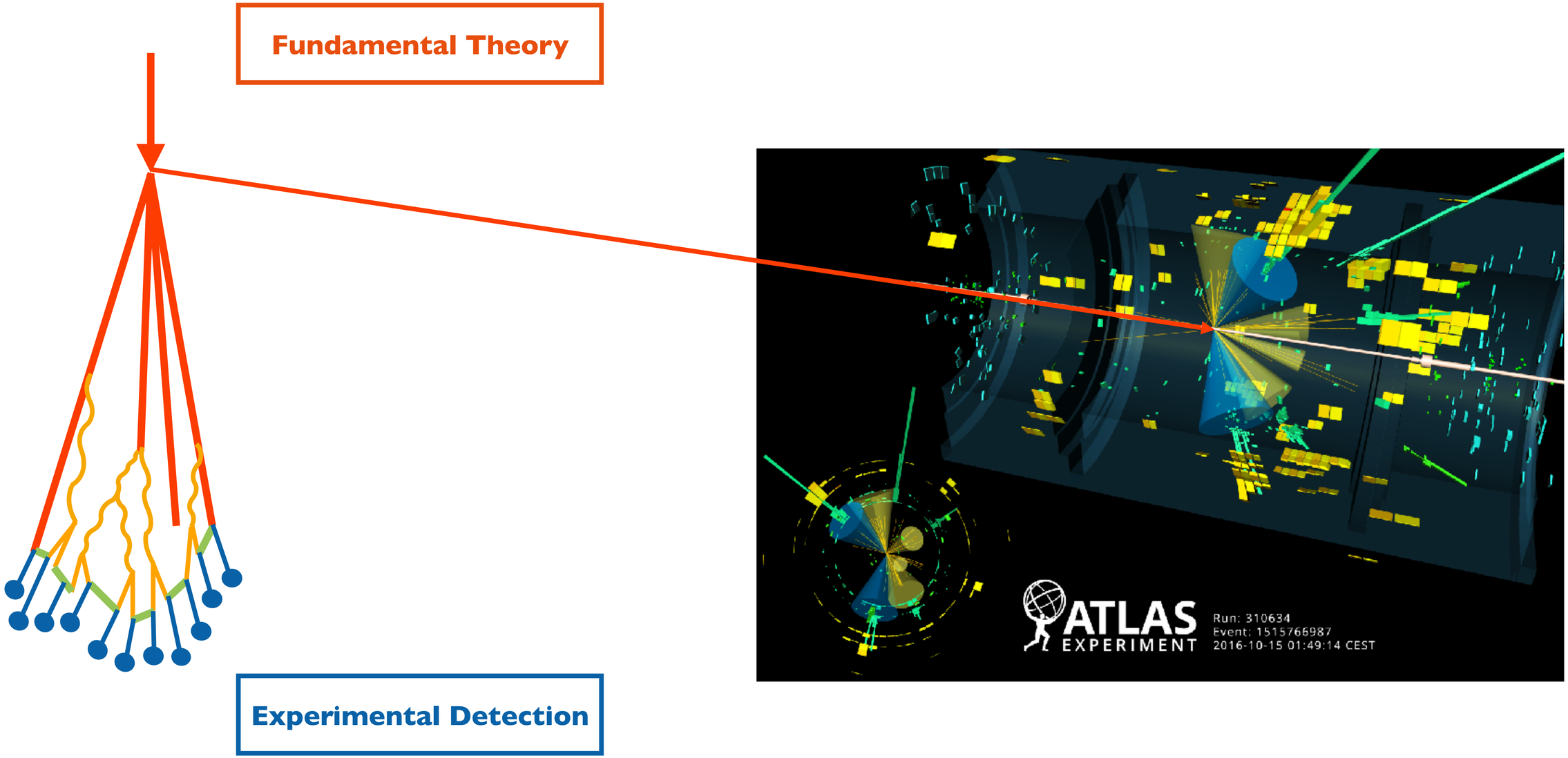
(Colour) evolution

Simon Plätzer
Institute of Physics — NAWI, University of Graz
Particle Physics — University of Vienna

At the
FAKT meeting
Bruck/Mur | 20 February 2023

NAWI Graz
Natural Sciences

The Complexity of Observations



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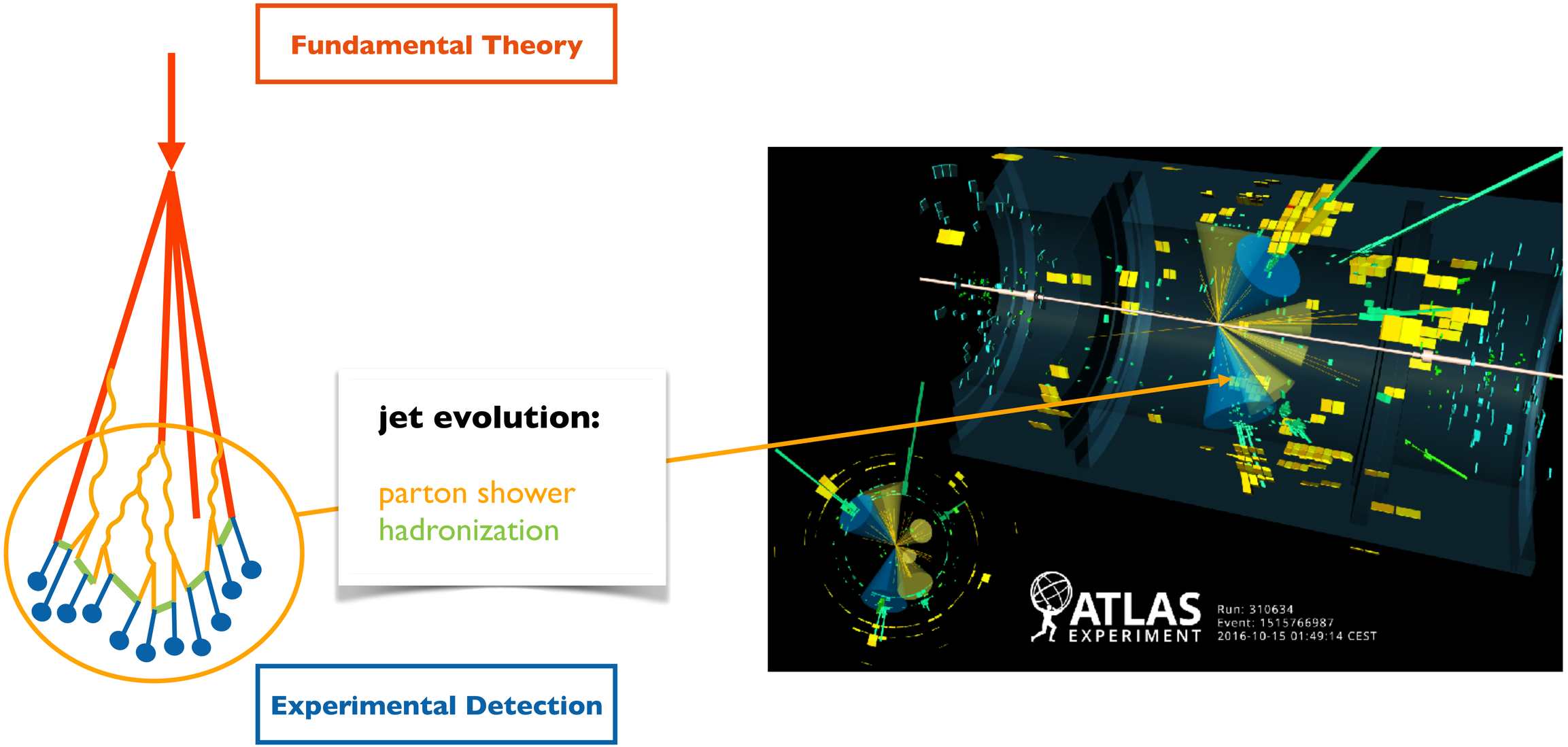
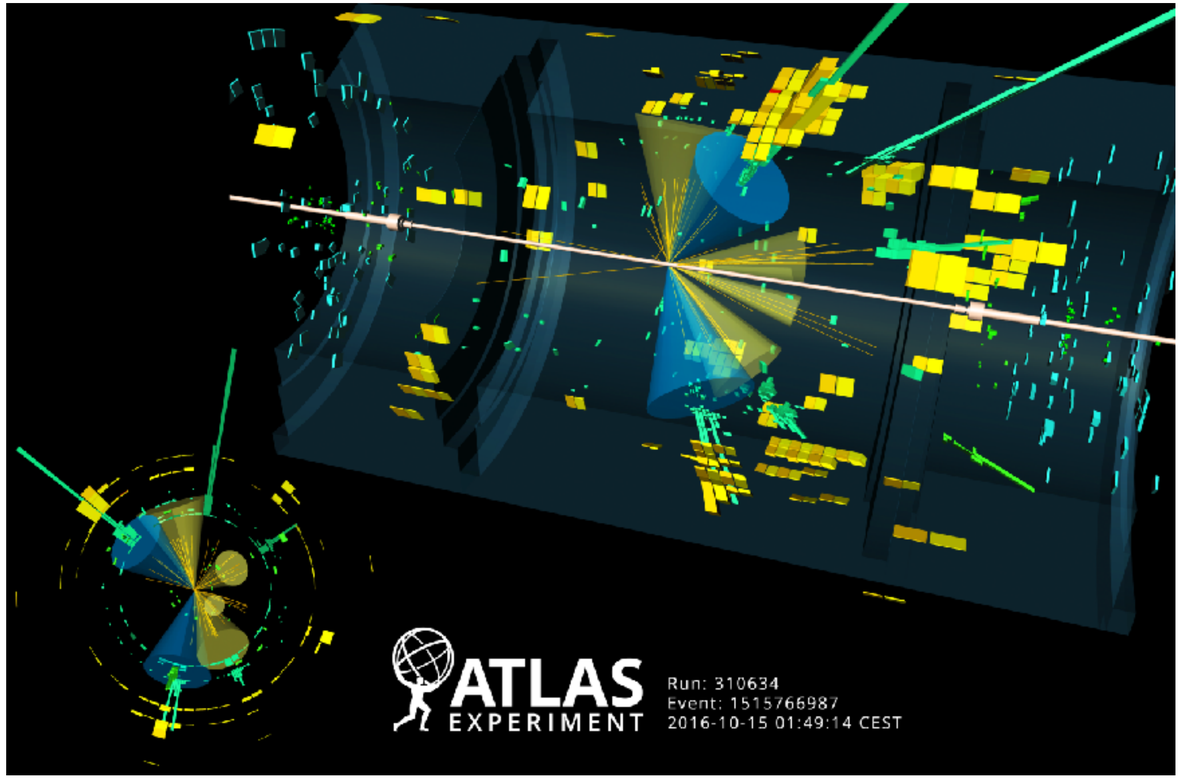


The Complexity of Observations

SCIENTIFIC AMERICAN. TECHNOLOGY | OPINION

use

Confirmed! We ~~Live in~~ a Simulation



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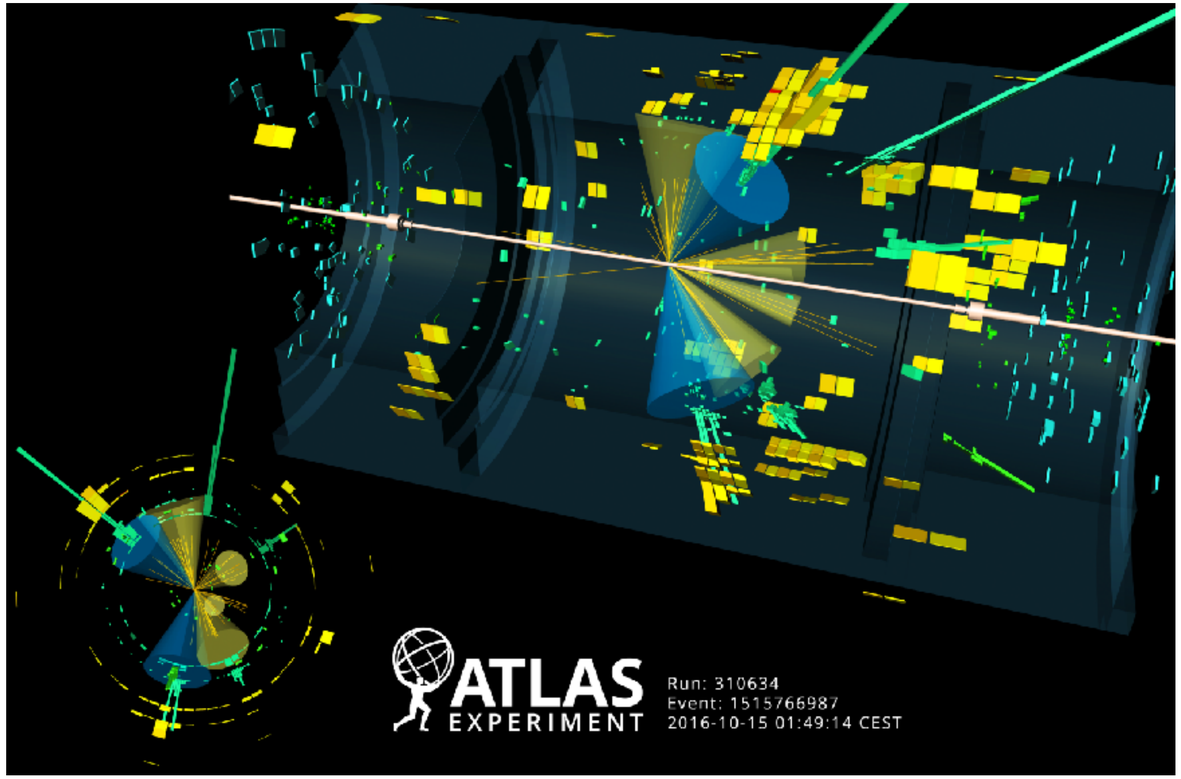
- as an event generator



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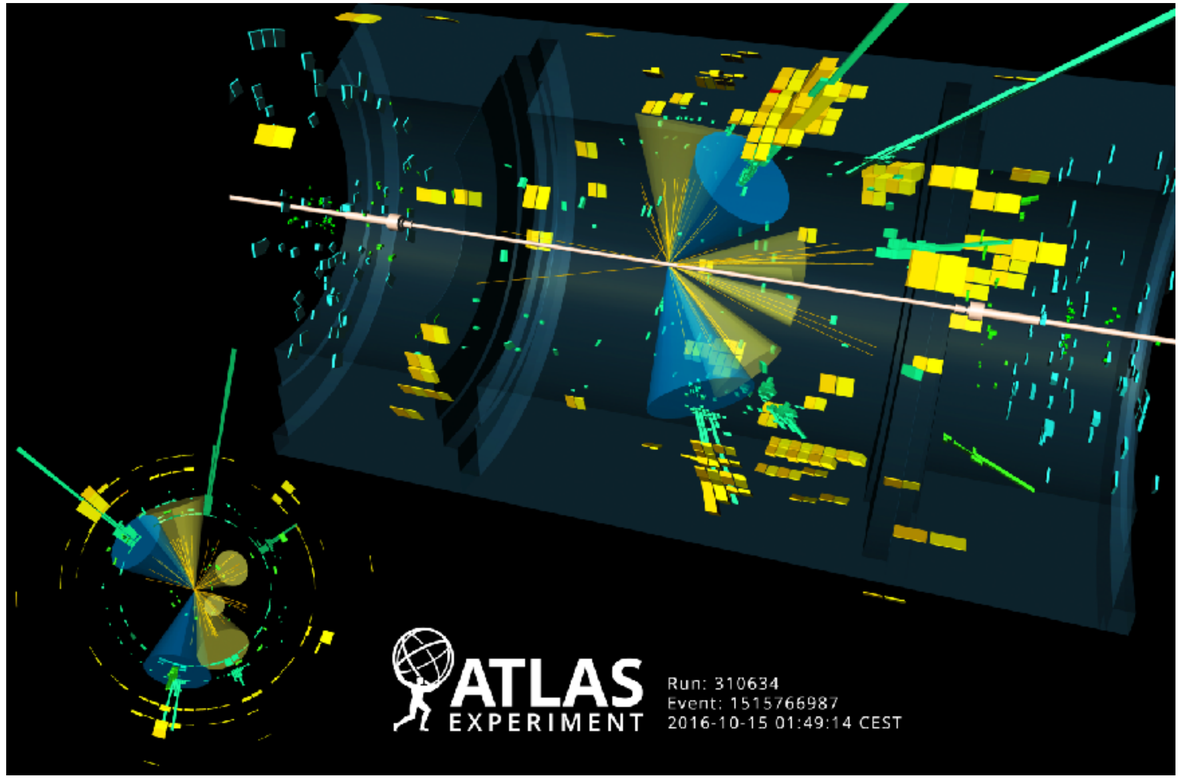

The Complexity of Observations

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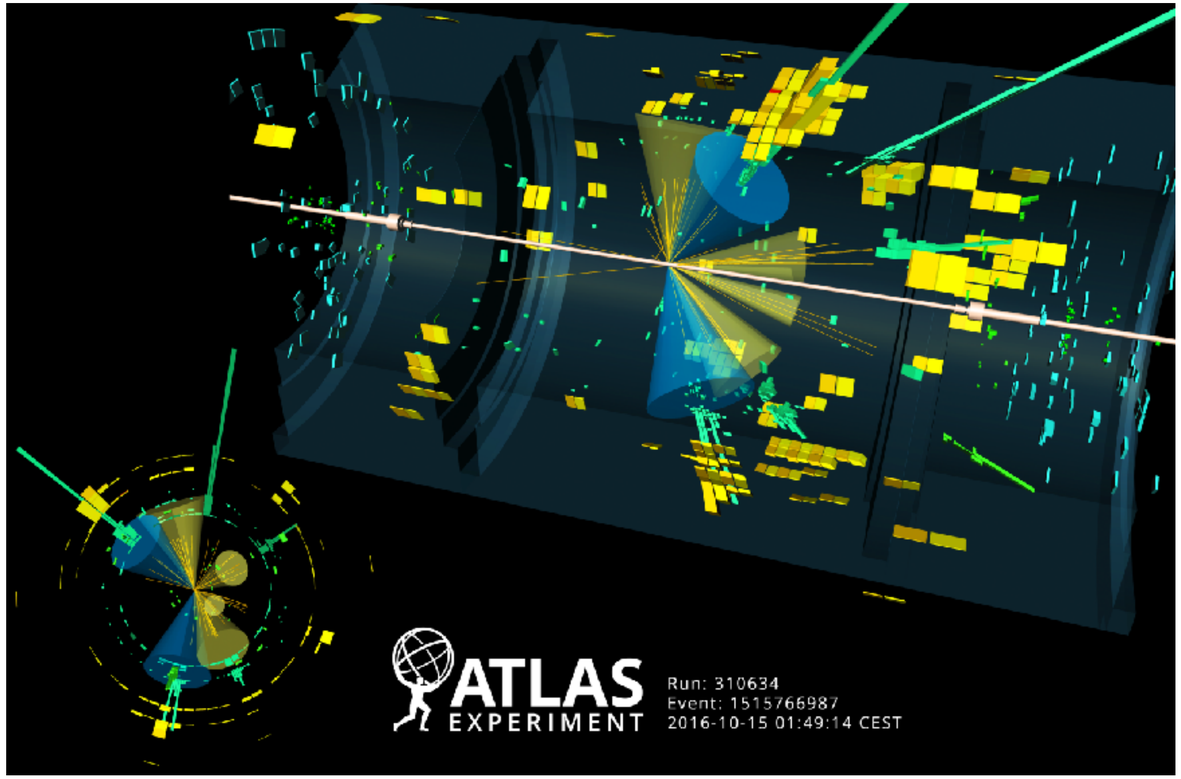
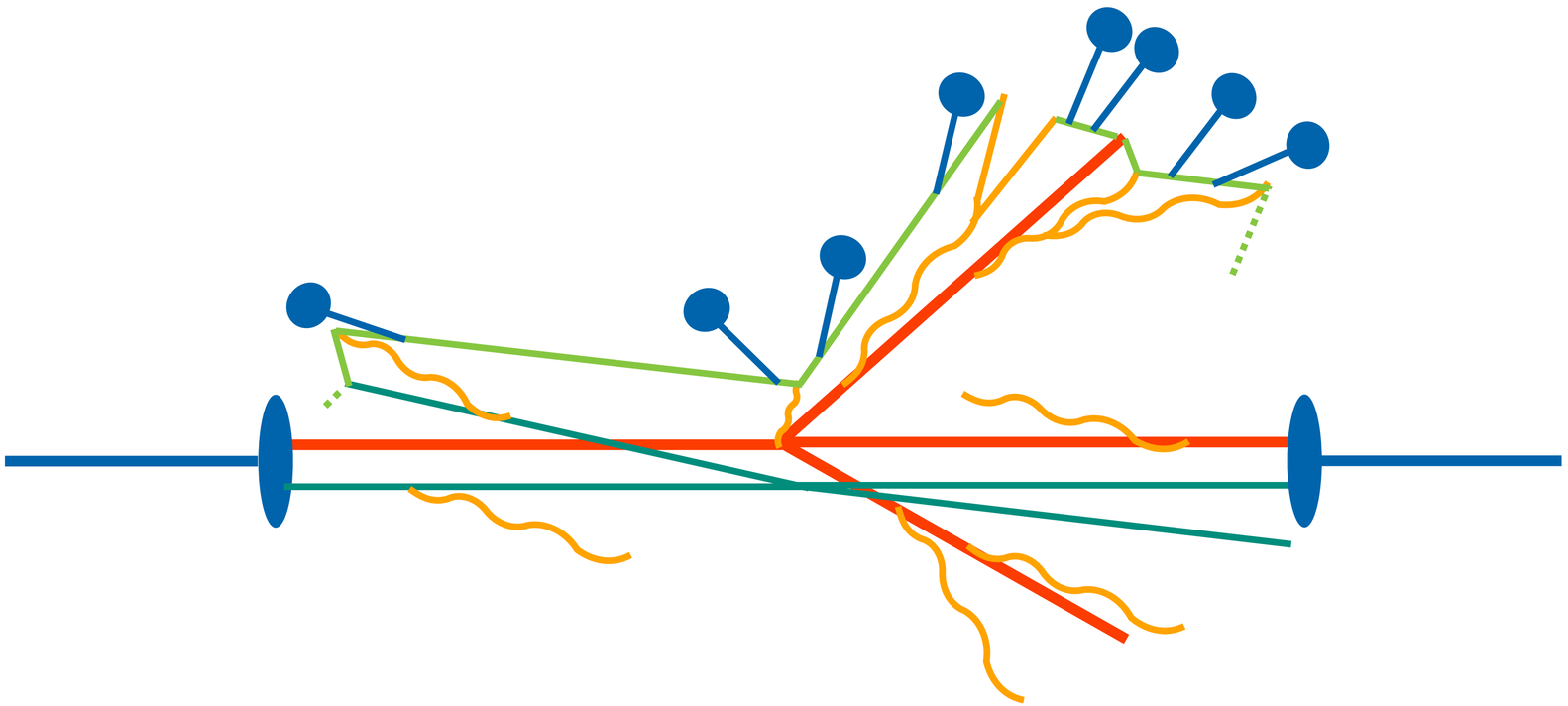
Confirmed! We ~~Live in~~ use a Simulation

- as an event generator
- as an exact tool for resummation
- as a means to explore amplitudes and structures in QFT



Event generators



$$\mathrm{d}\sigma \sim \mathrm{L} \times \mathrm{d}\sigma_H(Q) \times \mathrm{PS}(Q \to \mu) \times \mathrm{MPI} \times \mathrm{Had}(\mu \to \Lambda) \times \ldots$$

Amplitudes

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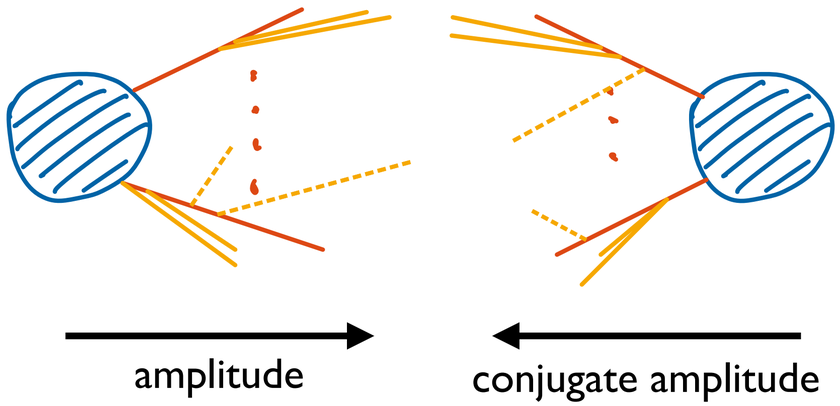
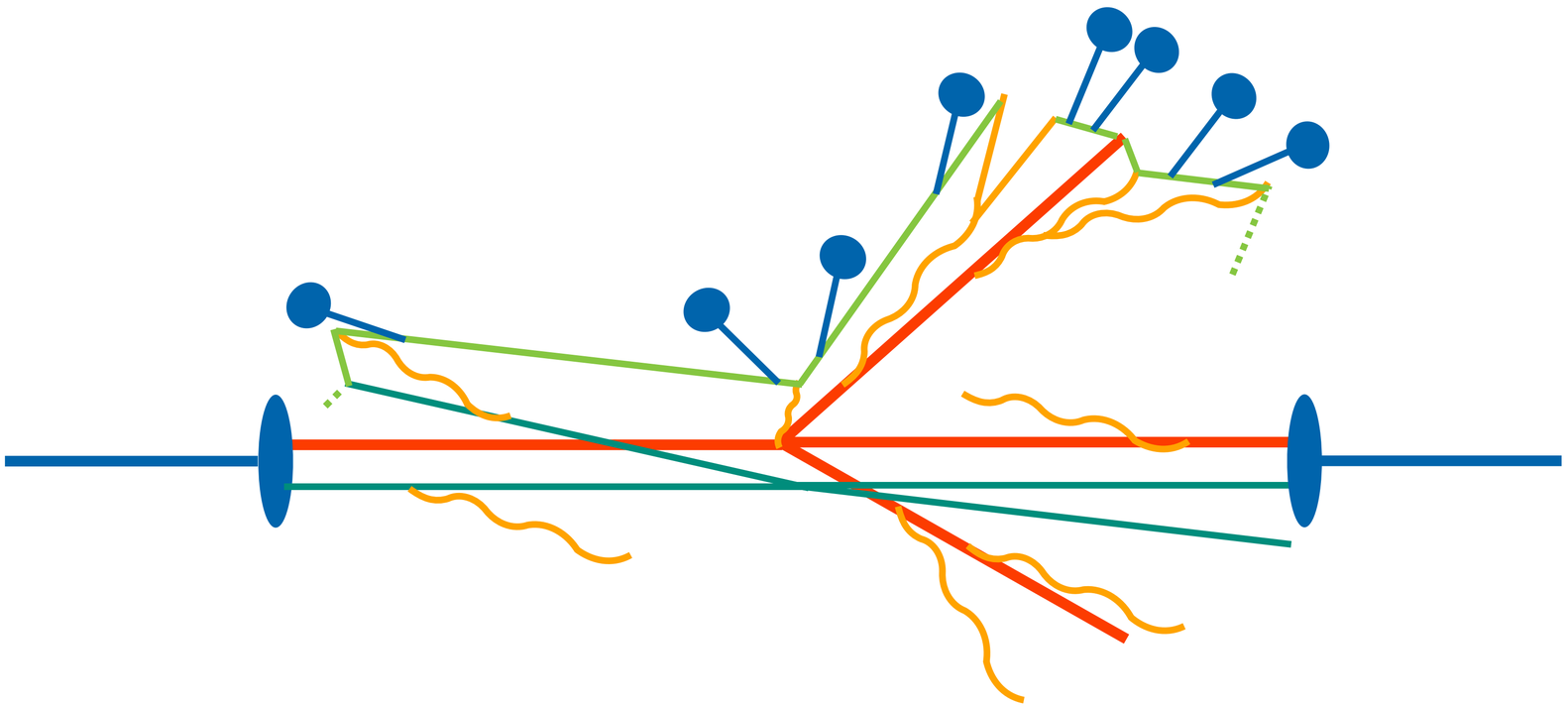


collinear
soft

$$\sim \quad \mathcal{M}_n^{+}(p_1, \ldots, p_n)\, T \cdots T \,\vdots\, T \cdots T\, \mathcal{M}_n(p_1, \ldots, p_n)$$

Suggests an iterative procedure to build amplitude and conjugate amplitude with many emissions.

Amplitudes



$$\mathrm{d}\sigma \sim \mathrm{Tr}\left[\mathbf{PS}(Q \to \mu)\mathrm{d}\mathbf{H}(Q)\mathbf{PS}^{\dagger}(Q \to \mu)\mathbf{Had}(\mu \to \Lambda)\right]$$

Factorisation and evolution

[Plätzer – JHEP 07 (2023) 126]

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm} \right] \mathrm{d}\phi_m u(\phi_m)$$

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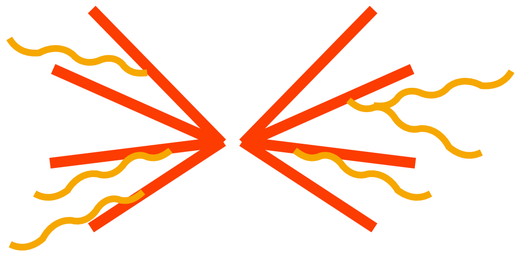

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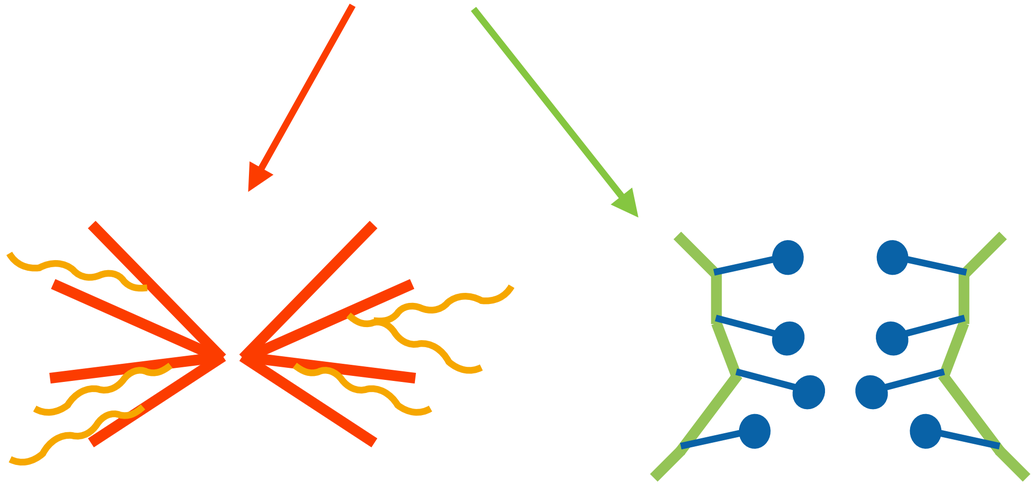

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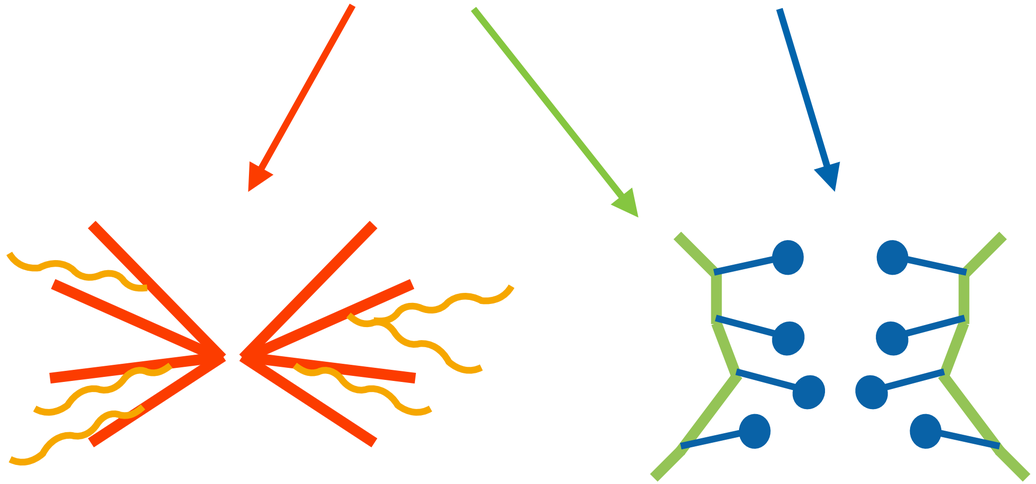

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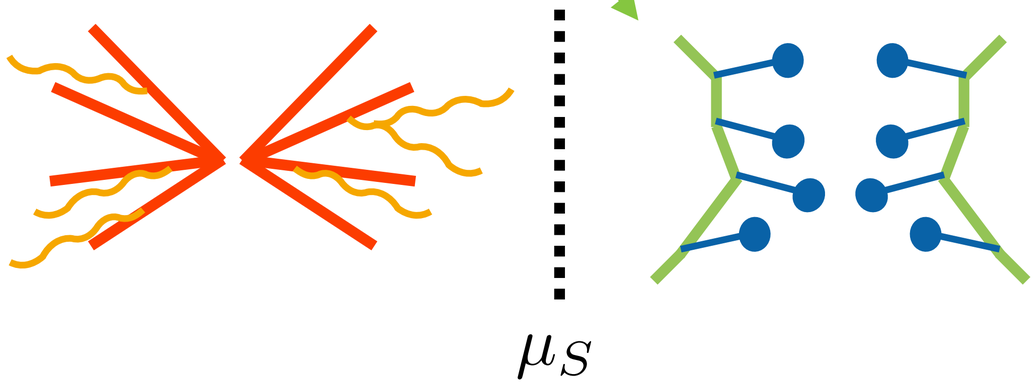

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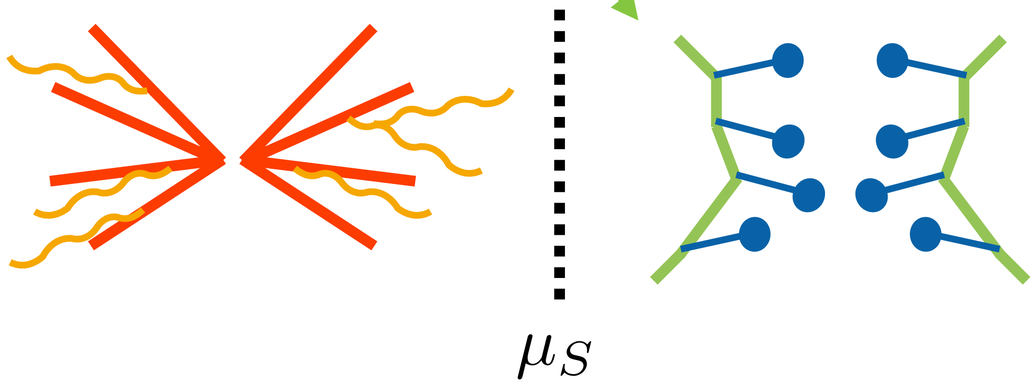

calculate building blocks

derive evolution

construct model response

constrain by data

Factorisation and evolution

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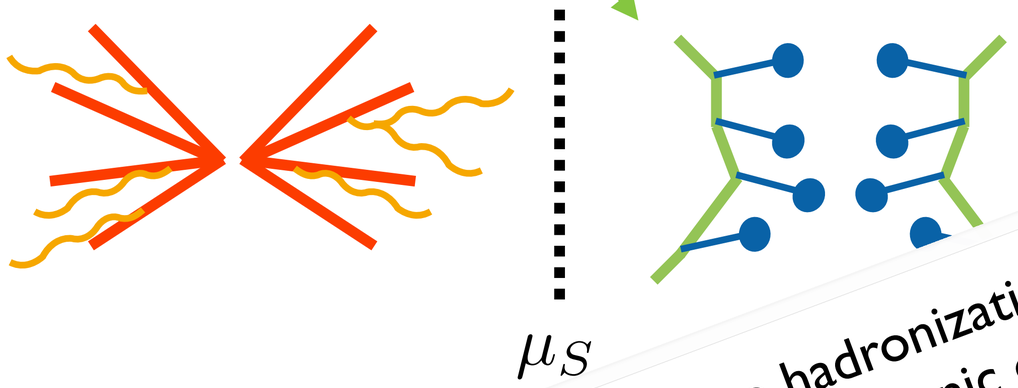


Not limited to hadronization — can also re-arrange factorisation of partonic observables in this way.

[e.g. resummation of NGL in SCET — Becher, Neubert et al.]

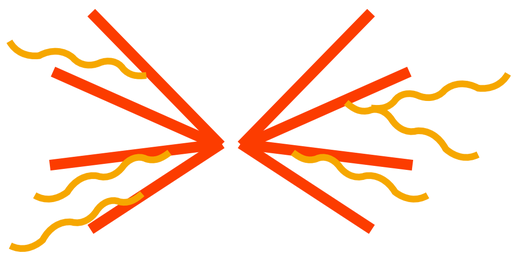
calculate building blocks

derive e

construct
respons

constrain by data

The role of the IR cutoff



Just a technical parameter?

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

- If a shower exploits unitarity, then implicit virtual corrections essentially use a cutoff-dependent renormalisation scheme. [Hoang, Plätzer, Samitz – JHEP 10 (2018) 200]

My approach: "renormalise" bare colour operators. [Plätzer – JHEP 07 (2023) 126]

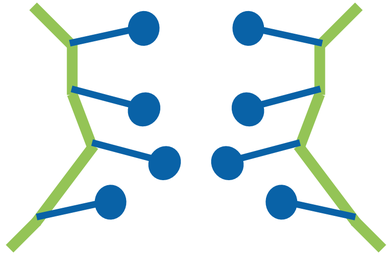
Subtract IR divergencies in unresolved regions

$$\mathbf{U}_n = \mathcal{X}_n \left[\mathbf{S}(\mu_S), \mu_S\right]$$

Re-arrange to resum IR enhancements

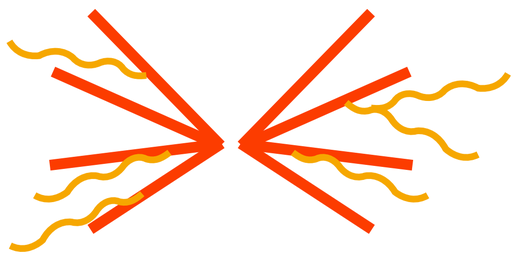
$$\mathbf{M}_n Z_g^n = \mathcal{Z}_n \left[\mathbf{A}(\mu_S), \mu_S\right]$$

$$\sigma = \sum_n \alpha_S^n \int \mathrm{Tr} \left[\mathbf{A}_n(\mu_S) \mathbf{S}_n(\mu_S)\right] \mathrm{d}\phi_n$$



- Even if we include explicit virtual contributions, a cutoff is present. This is independent of the UV renormalisation scheme, and impacts the structure of the resummation. [Plätzer – (slow) progress]

The role of the IR cutoff



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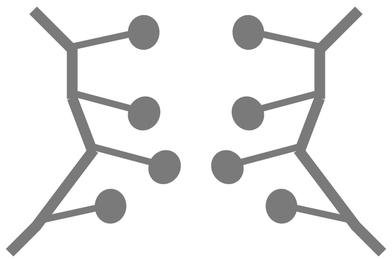
- Provides factorisation and subtractions for renormalised matrix elements in unresolved regions, consistency enforced by overall power counting

$$\mathbf{M}_n \to \alpha_s^n (\mathbf{M}_n^{(0)} + \alpha_s [\mathbf{M}_n^{(1)} - \mathbf{X}_n^{(1)} \mathbf{M}_n^{(0)} - \mathbf{M}_n^{(0)} \mathbf{X}_n^{(1)\dagger} - \mathbf{F}^{(1,0)} \mathbf{M}_{n-1}^{(0)} \mathbf{F}_n^{(1,0)\dagger}] + \mathcal{O}(\alpha_s^2))$$



- Virtual and real correction can use different cutoff forms and values even in different regions (soft, collinear, Glauber, …).
- At higher orders we find appropriate removal of iterated subtractions.

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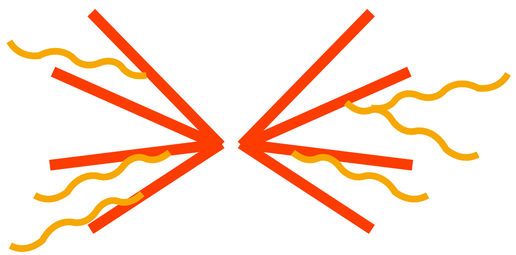

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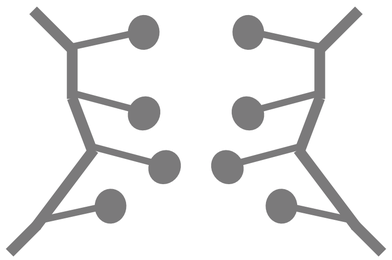
unresolved emission
at leading power



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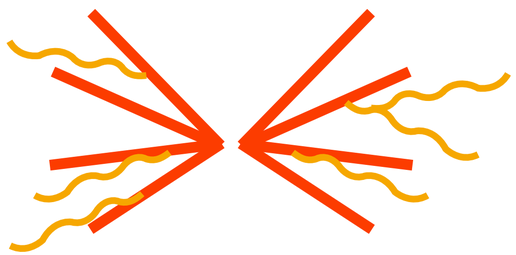

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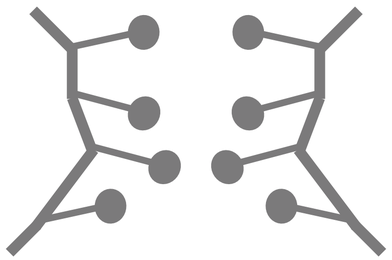
unresolved emission at leading power

loop divergence at leading power, no unresolved emission



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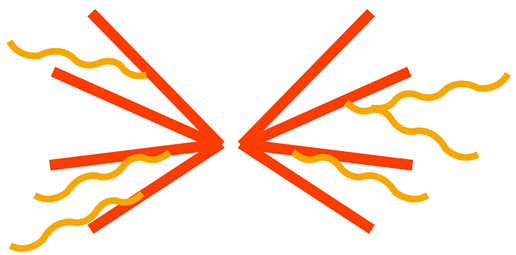

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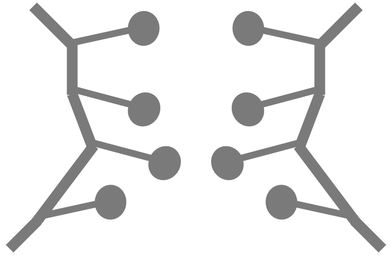
subtraction for loop divergence



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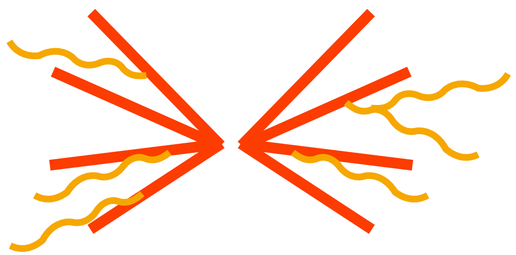

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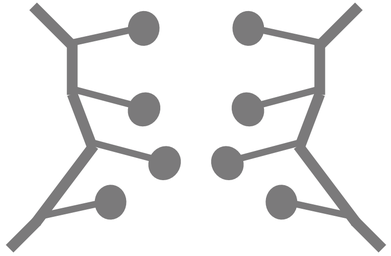

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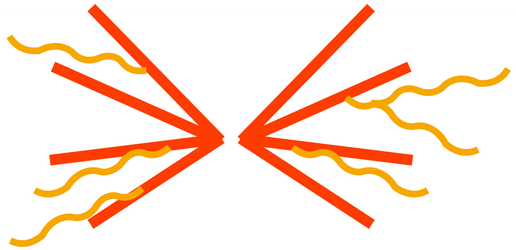
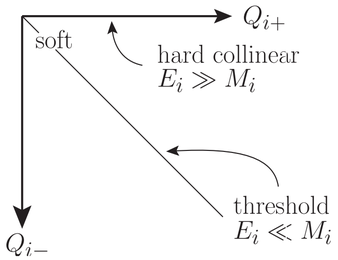
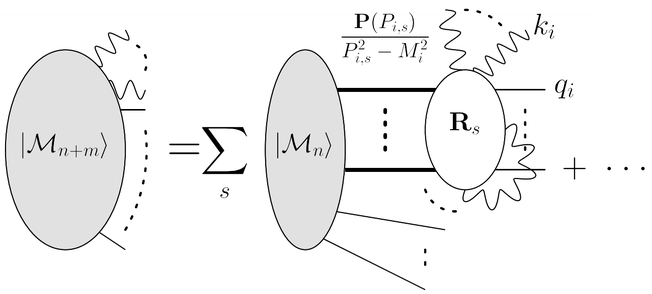
Building shower and resummation algorithms

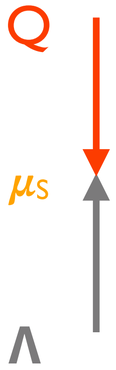


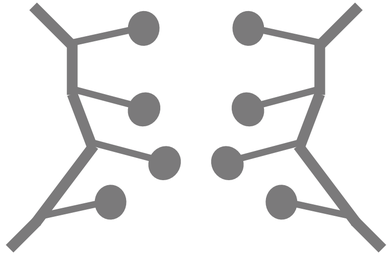
Factorisation of amplitudes and power expansions.





[Löschner, Plätzer, Ruffa, Sjödahl — '20+] [Plätzer & Weigert – wip]





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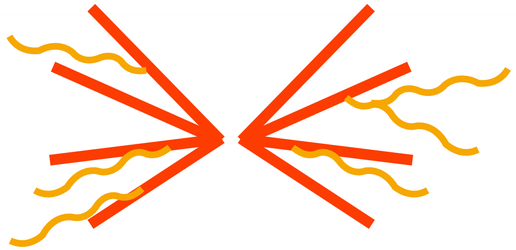

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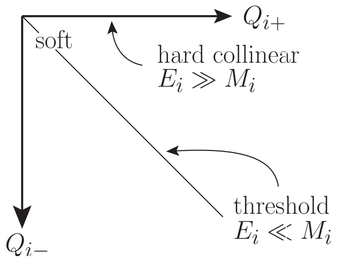

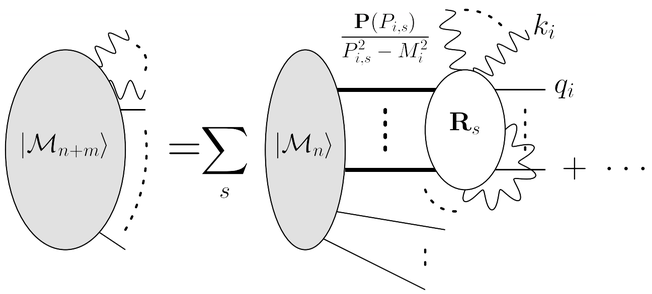

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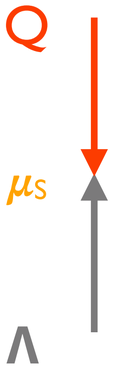
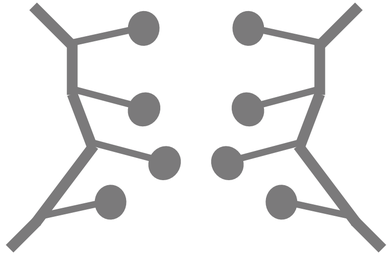


NLL parton showers — Herwig 7 dipole shower

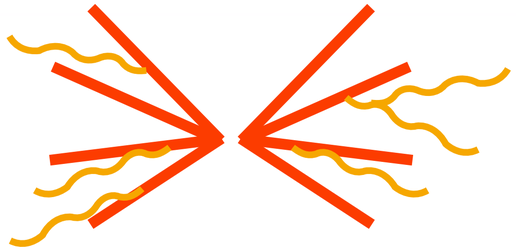
[Forshaw, Holguin, Plätzer — '20+] [Duncan, Holguin, Plätzer, Sule – wip]

$$H(\alpha_s) \times \exp\left(Lg_1(\alpha_s L) + g_2(\alpha_s L) + \alpha_s g_3(\alpha_s L) + \ldots\right)$$

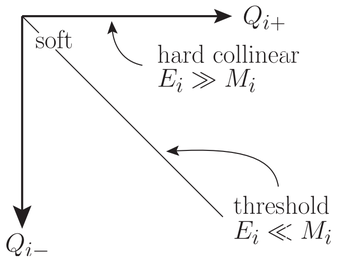
LL — qualitative NLL — quantitative NNLL — precision

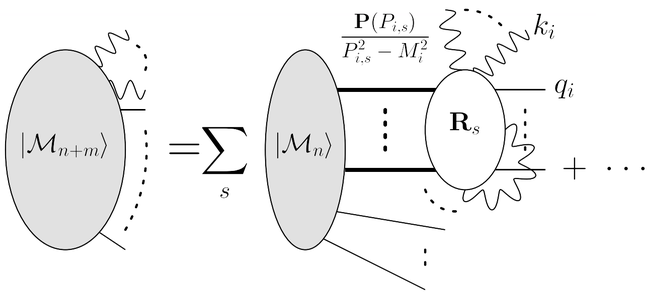


Building shower and resummation algorithms



Factorisation of amplitudes and power expansions.





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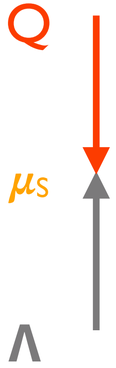

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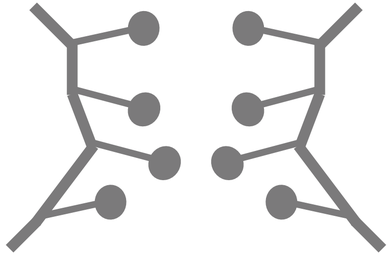
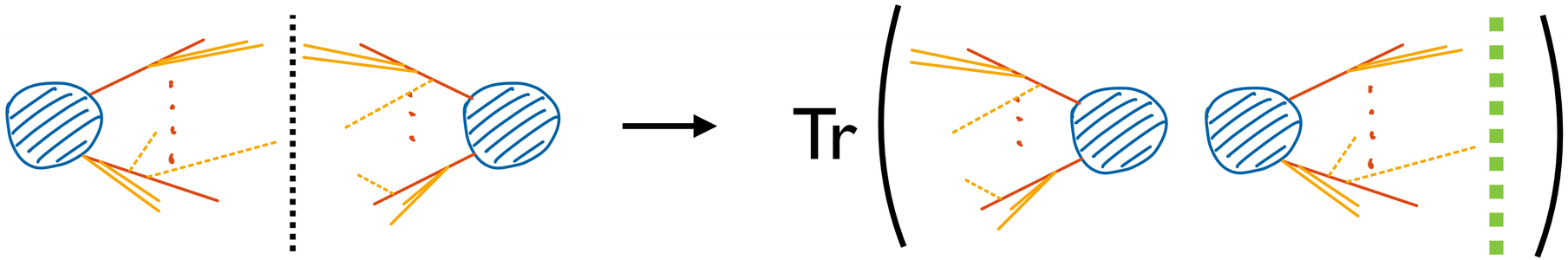


Amplitude evolution and resummation algorithms.

- Started with non-global logarithms. [Forshaw, Plätzer et al. — '18+]
- Establishing links to JIMWLK, EFT, direct QCD resummation.

[Forshaw & Plätzer – wip] [Plätzer & Weigert – wip]

Amplitude evolution



$$\mathrm{d}\sigma \sim \mathrm{Tr}\left[\mathbf{PS}(Q\to\mu)\mathrm{d}\mathbf{H}(Q)\mathbf{PS}^\dagger(Q\to\mu)\mathbf{Had}(\mu\to\Lambda)\right]$$

$$\mathbf{A}_n(q) = \int_q^Q \frac{\mathrm{d}k}{k}\, \mathrm{P}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}(k')}\, \mathbf{D}_n(k)\, \mathbf{A}_{n-1}(k)\, \mathbf{D}_n^\dagger(k)\, \overline{\mathrm{P}}e^{-\int_q^k \frac{\mathrm{d}k'}{k'}\mathbf{\Gamma}^\dagger(k')}$$

Markovian algorithm at the amplitude level: Iterate parton exchanges and emission.
Different histories in amplitude and conjugate amplitude needed to include interference.

CVolver solves evolution equations in colour flow space. Flexible for dedicated resummation and new parton showers.

One-loop structures … [Plätzer '13]
Soft evolution … [Angeles, De Angelis, Forshaw, Plätzer, Seymour – '18]
Soft + collinear evolution … [Forshaw, Holguin, Plätzer – '19]
Two-loop structures … [Plätzer, Ruffa — '21]
First Monte Carlo implementation … [De Angelis, Forshaw, Plätzer — '21]
Emissions beyond leading order … [Löschner, Plätzer, Simpson-Dore — '20]

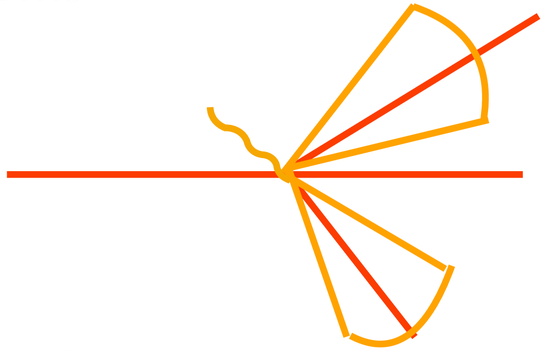
Amplitude evolution: new results

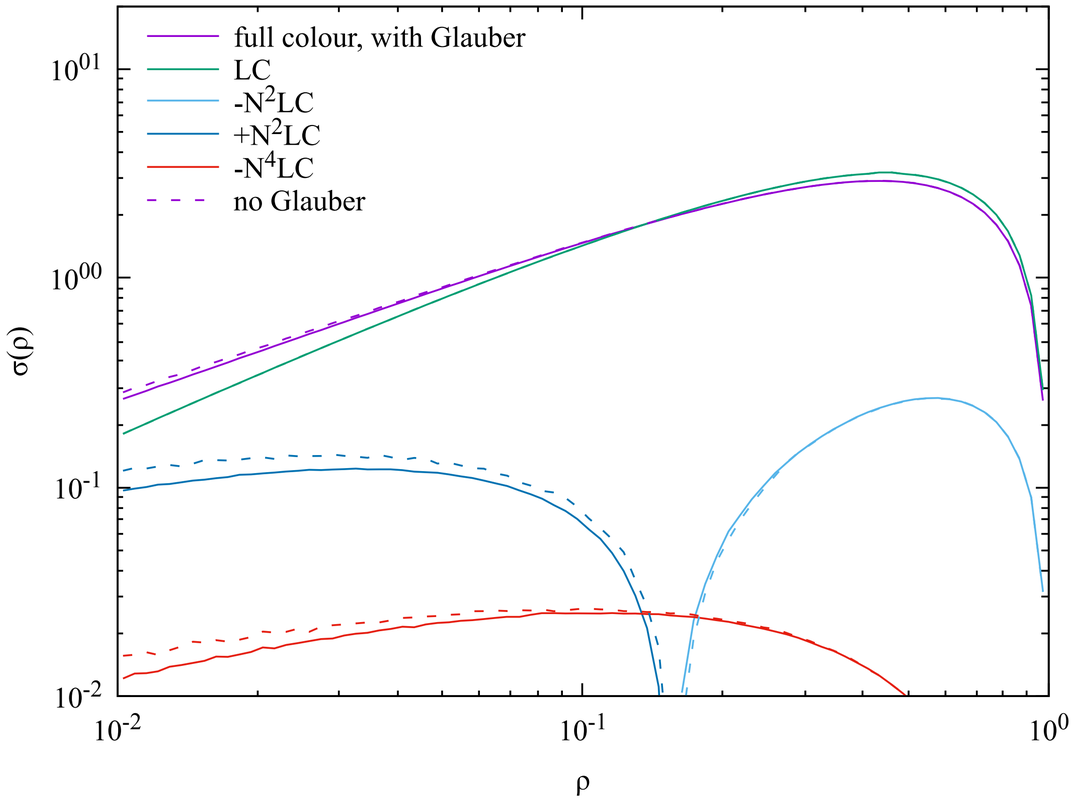
Full hadron collider and multi-jet campaign:

- QCD jet production and vetoes
- VBF including all interferences
- e+e- to hadronic WW — demand for FCC

Physics questions:

- Impact of Glauber exchanges
- Recoil to (inter-)jet radiation
- Impact of interference terms





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[QCD jets with additional emissions — also relevant to top]

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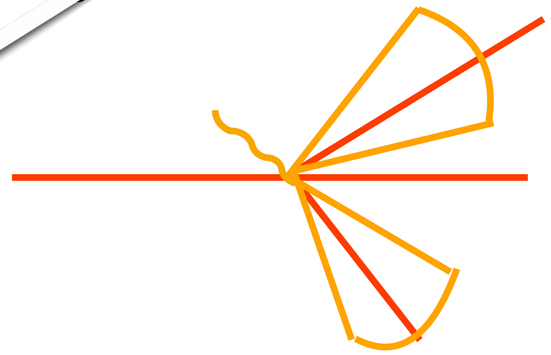
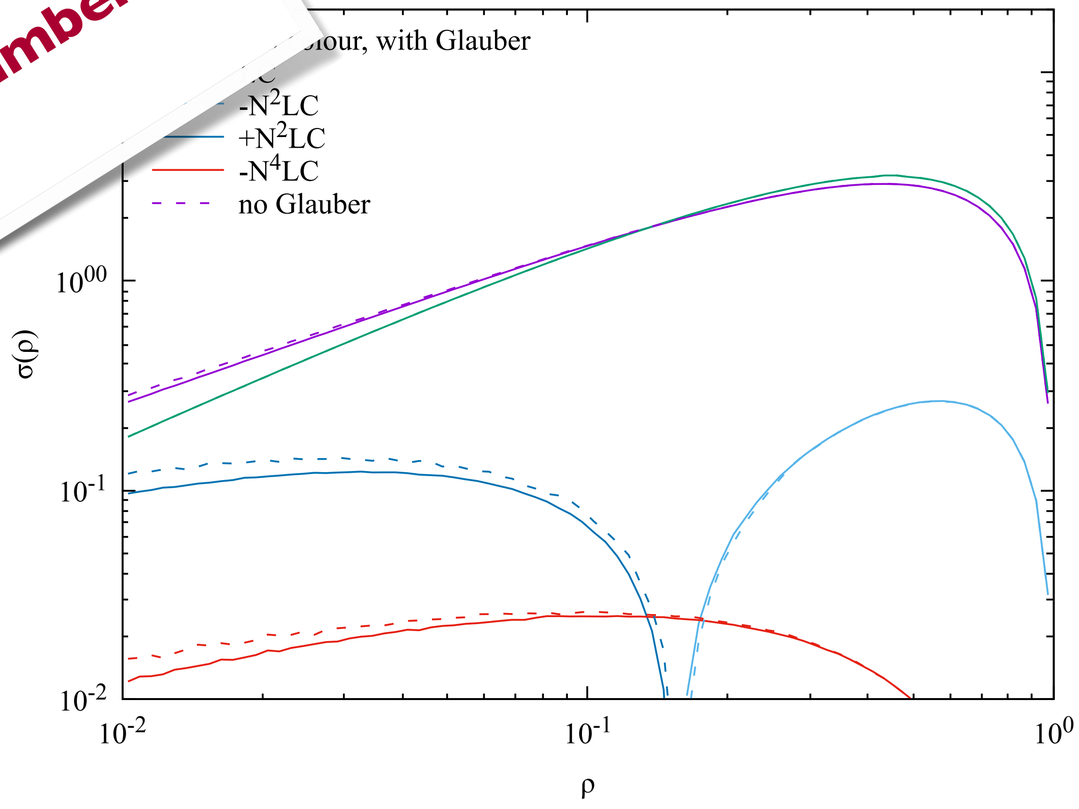
Inaccessible to inclusive approaches with fixed number of emissions as well as normal parton showers.





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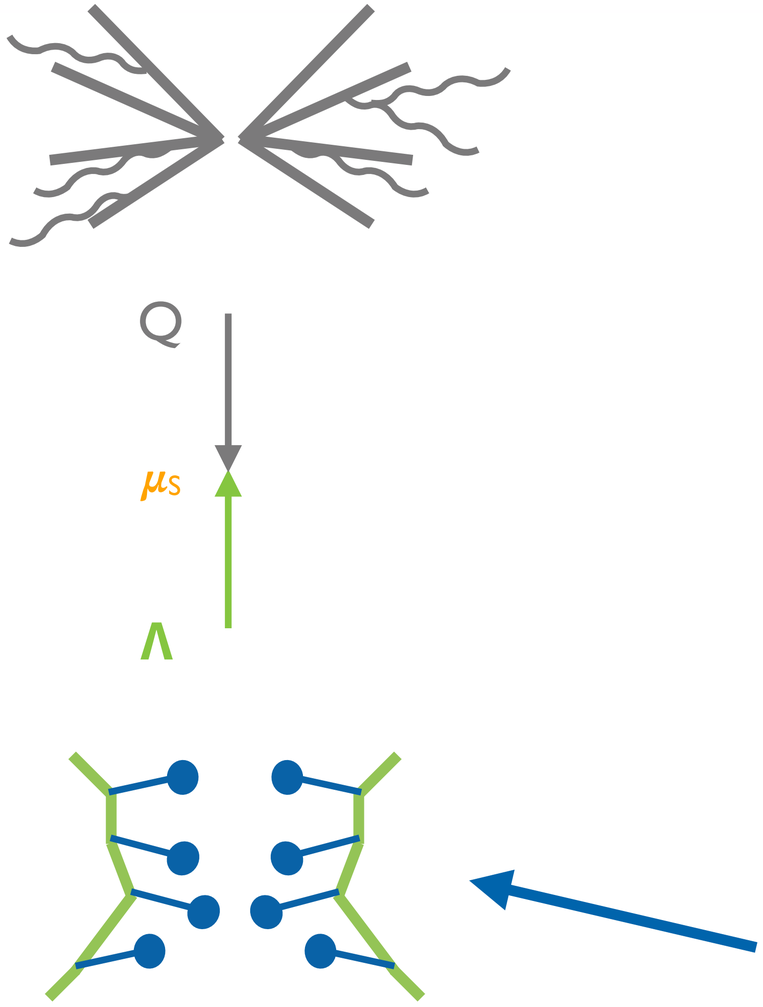
Building and constraining hadronization models



Hadron correlations not predicted or simplistic, baryon rates not right, cutoff dependence uncontrolled (fitted together with strong coupling), uncertainties not quantified … impact to ML more than worrying.

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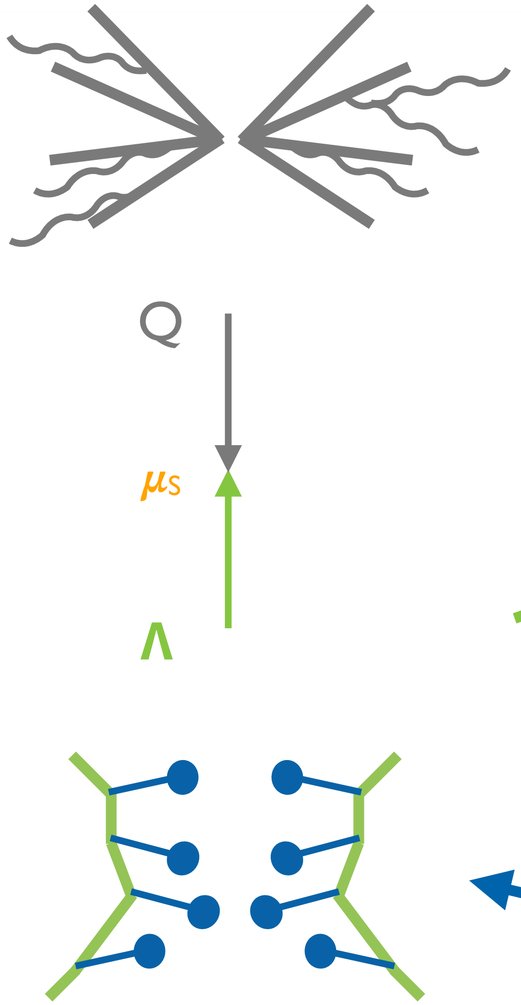
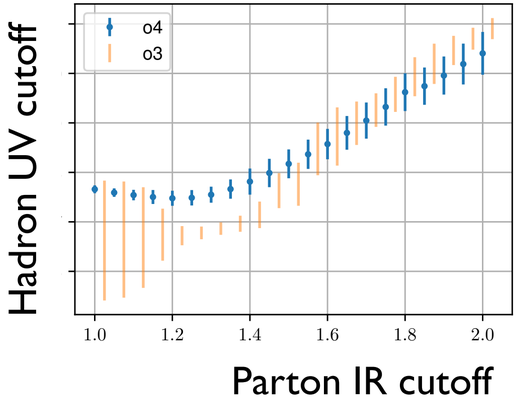


Towards a smooth matching of shower and hadronization at the infrared cutoff — inspired by coherent branching.

[Hoang, Jin, Plätzer, Samitz — in preparation]



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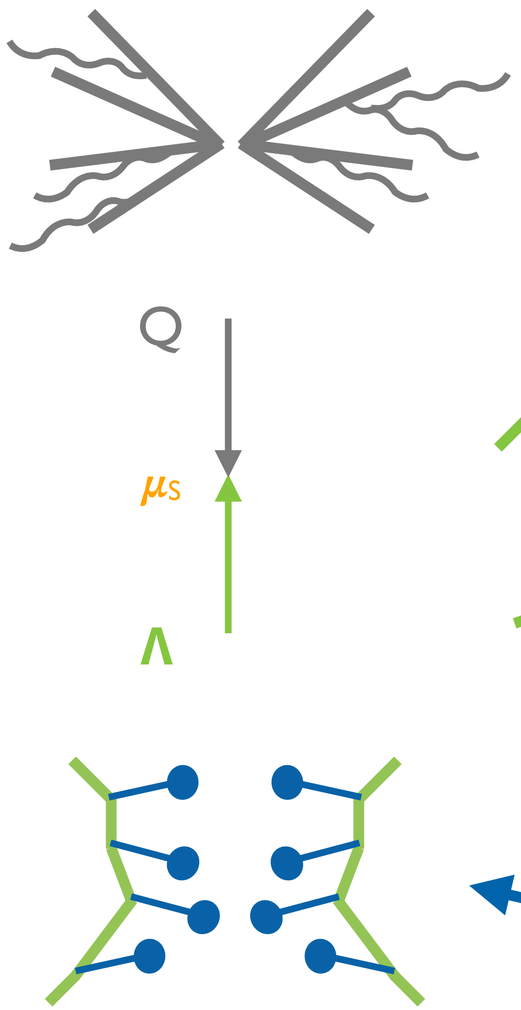
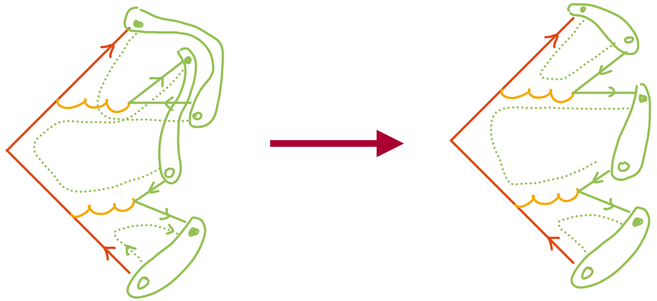


Towards a full model of cluster evolution with fission and colour reconnection informed by perturbative evolution.



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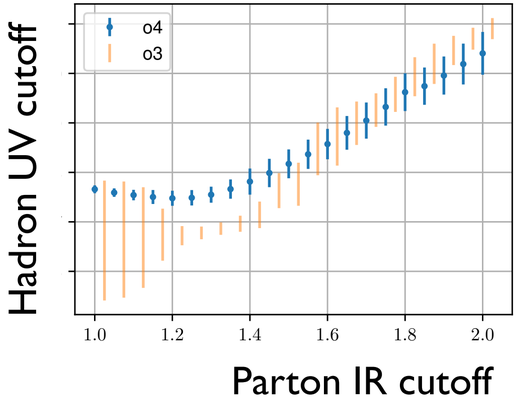

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Why?

Hadronization and the interface to showers lacks:

- comprehensive uncertainty estimates
- predictivity
- extrapolation across different energy regimes
- links to analytic models …

NB: Machine learned models will only continue to parametrize our ignorance.

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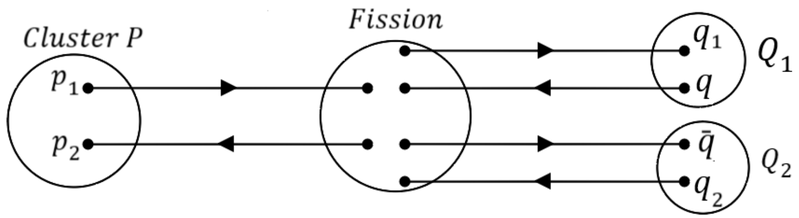
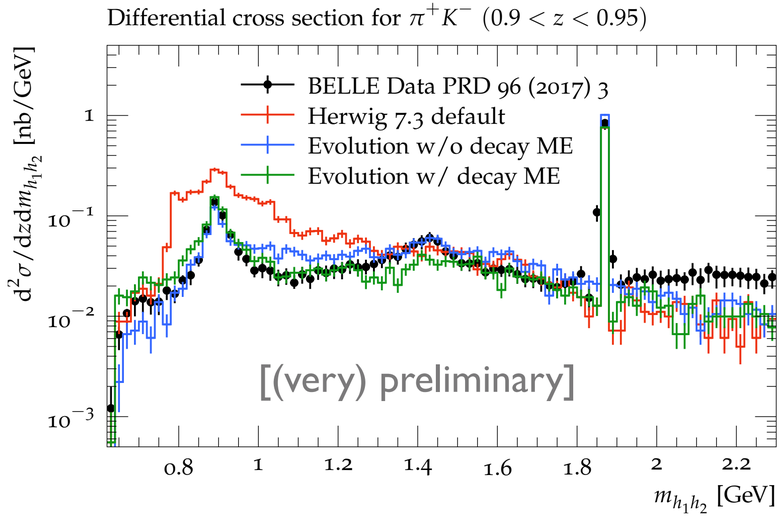


[figure by Jan Priedigkeit]



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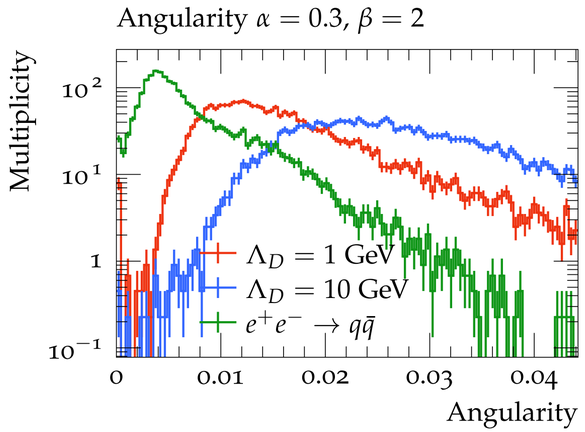
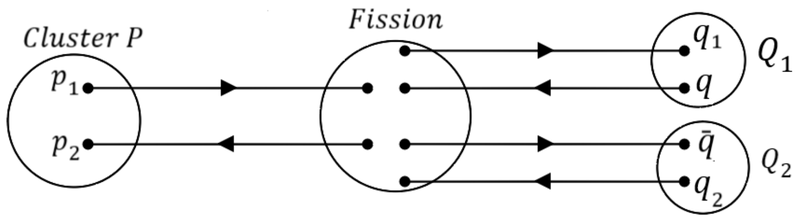
Most needed when hadronizing the unknown: strongly interacting dark matter.



[Kulkarni, Massouminia, Plätzer, Stafford — in preparation]



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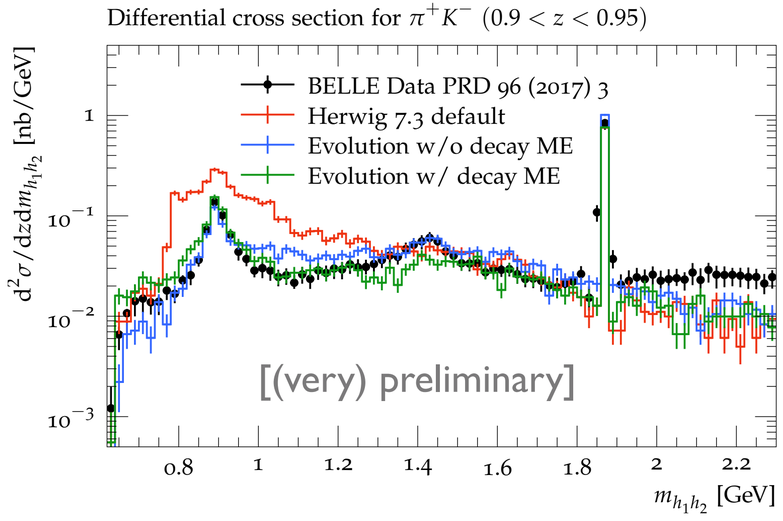

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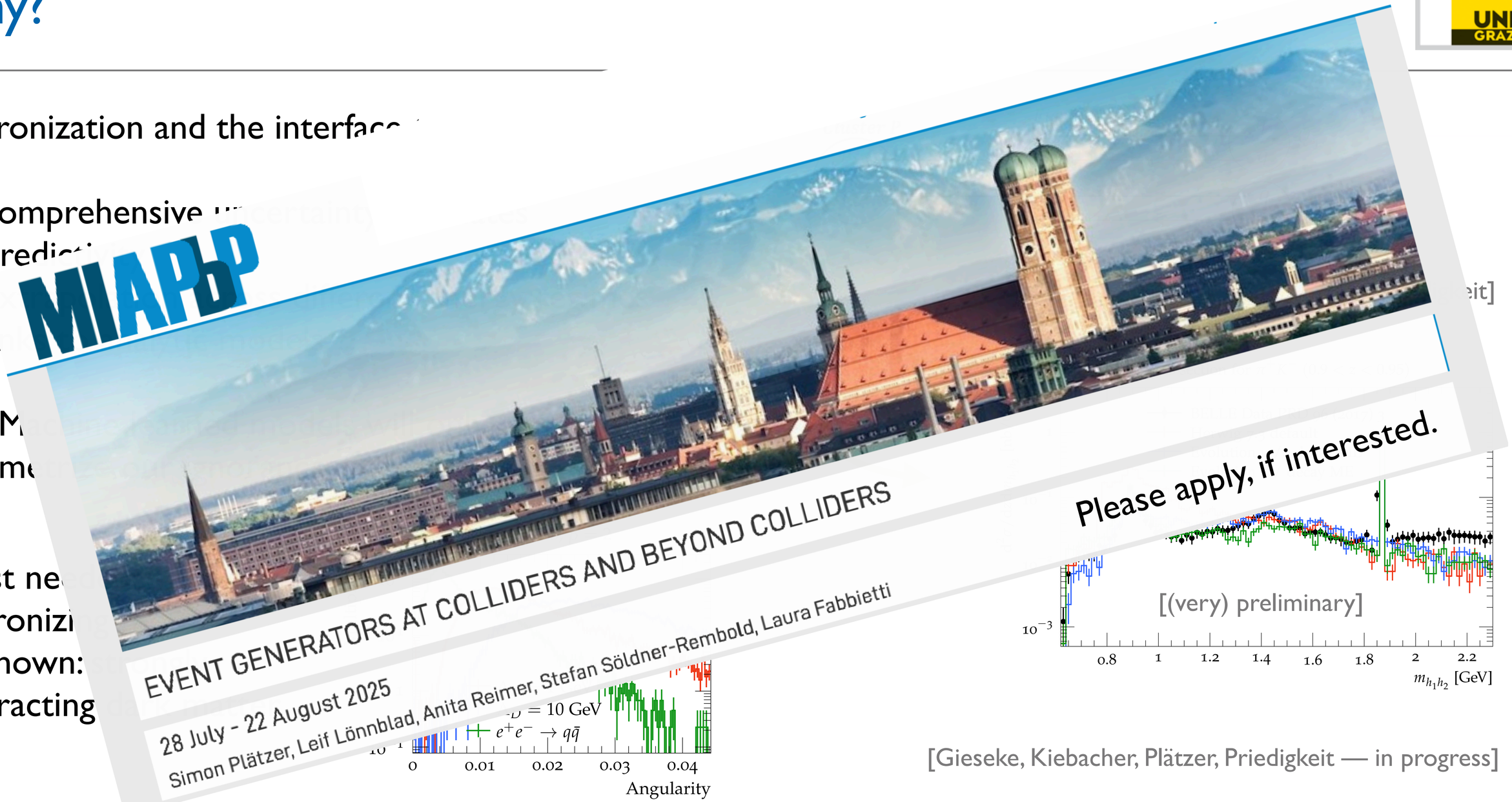

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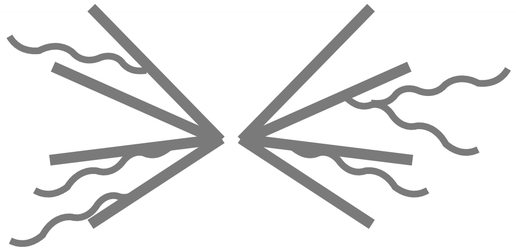
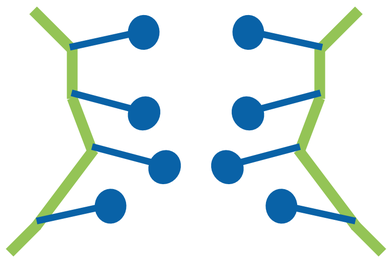
What structures are admissible?



Subtracted ("renormalised") observable defines a very general criterion of infrared safety: finiteness means the bare observable must admit **cancellations local in momentum and colour space.**

$$\mathbf{S}_n = \mathbf{Z}_n^\dagger \mathbf{U}_n \mathbf{Z}_n + \sum_{s=1}^{\infty} \alpha_S^s \int \mathbf{E}_{n+s}^{(s)\dagger} \mathbf{U}_{n+s} \mathbf{E}_{n+s}^{(s)} \prod_{i=n+1}^{n+s} \mu_R^{2\epsilon} [\mathrm{d}p_i] \tilde{\delta}(p_i)$$



This structure is ubiquitous if we talk about **electroweak final states** (in isospin space) and if we want to predict fully detailed and exclusive final states as needed for an event generator.

Observables singular at his level are genuine non-perturbative.

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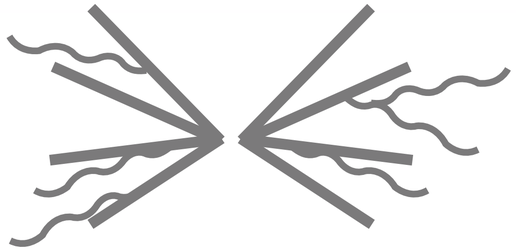

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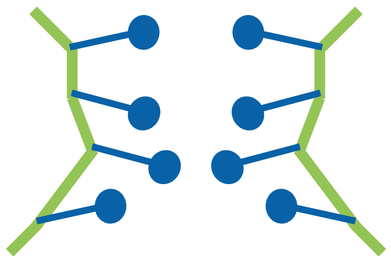
infrared resolution vs observable

$$\mathbf{S}_n = \mathbf{1}_n u(p_1, ..., p_n)$$
$$- \alpha_s \int \mu_R^{2\epsilon} [\mathrm{d}p_{n+1}] \tilde{\delta}(p_{n+1}) \hat{\mathbf{D}}_{n+1}^{(1,0)\dagger} \hat{\mathbf{D}}_{n+1}^{(1,0)} \Theta_{n,1} \left[u(p_1, ..., p_n, p_{n+1}) - u(p_1, ..., p_n) \right] + \mathcal{O}(\alpha_s^2)$$



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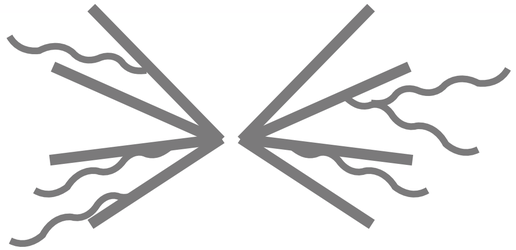
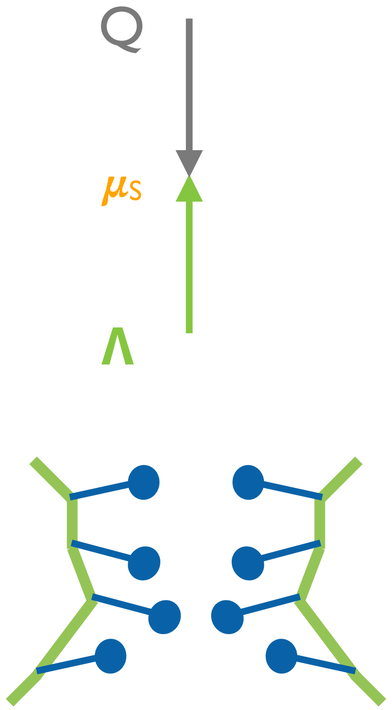
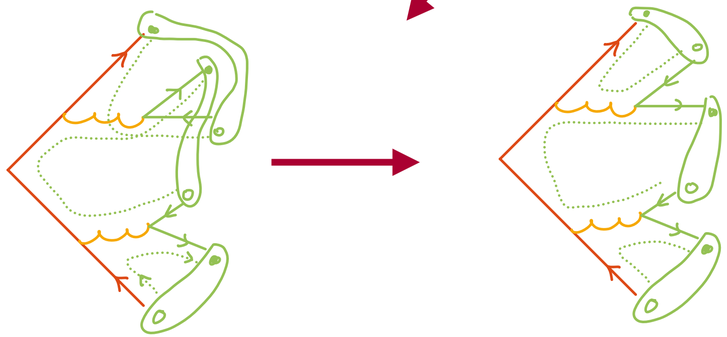
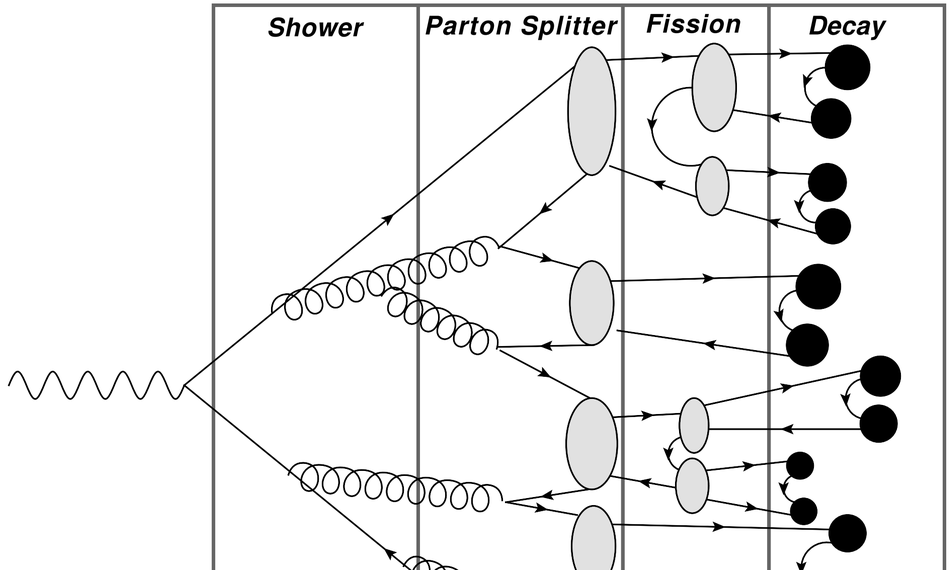


Hadronization models would start by studying clusters.
Possible relation to amplitudes from functional methods.



$$\partial_S \mathbf{S}_n = -\tilde{\mathbf{\Gamma}}^{\dagger}_{S,n}\mathbf{S}_n - \mathbf{S}_n\tilde{\mathbf{\Gamma}}_{S,n} + \sum_{s\geq 1}\alpha_S^s \int \tilde{\mathbf{R}}^{(s)\dagger}_{S,n+s}\mathbf{S}_{n+s}\tilde{\mathbf{R}}^{(s)}_{S,n+s}\prod_{i=n+1}^{n+s}[\mathrm{d}p_i]\tilde{\delta}(p_i)$$





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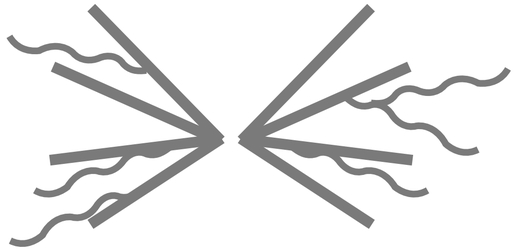
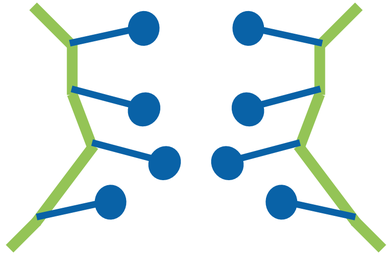


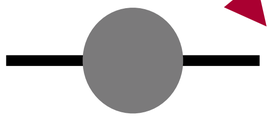


Generally we need to understand exclusive processes and factorisation, and (renormalised) LSZ and projections onto physical (singlet) final states.

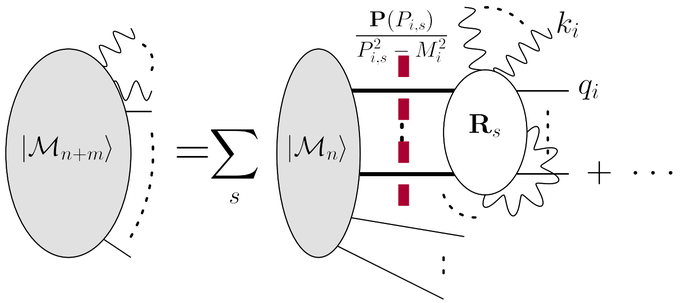
$$K^{\mu}_{i,s} = \Lambda^{\mu}{}_{\nu}\left(Q^{\nu}_{i,s} + \delta_{i,s}\, n^{\nu}_{i,s}\right)$$

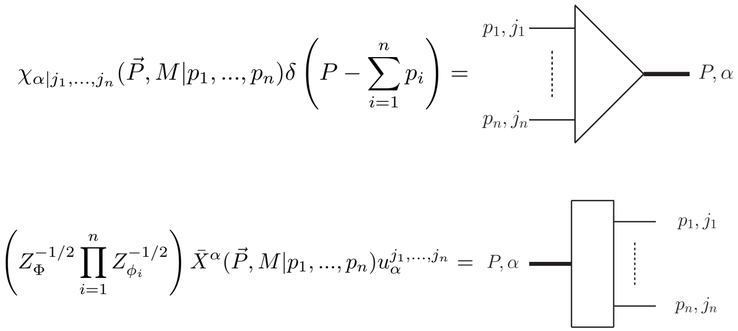
$$q^{\mu}_{i} = \Lambda^{\mu}{}_{\nu}\left(\alpha p^{\nu}_{i} + \frac{(1-\alpha^2)M_i^2 + p_i\cdot Q_{i,s}}{2\alpha\; n_{i,s}\cdot p_i} n^{\nu}_{i,s}\right) - K^{\mu}_{i,s}$$

Momentum mappings to systematically factor renormalised matrix elements.

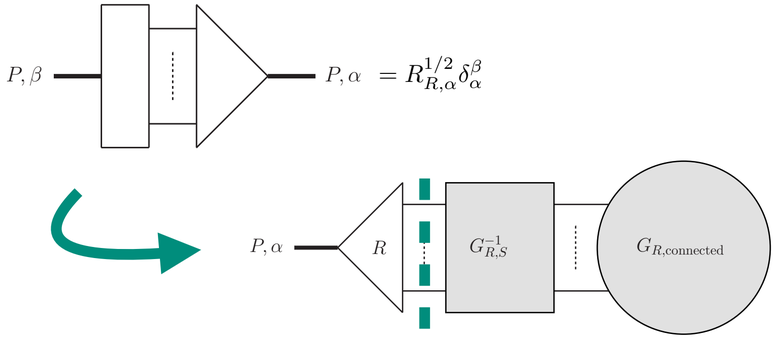


$$= \frac{1}{2p_i\cdot Q_{i,s}}\frac{\Psi(\Lambda p_i, M_i)\bar{\Psi}(\Lambda p_i, M_i)}{1-\Sigma'(M_i^2)} + \mathcal{O}(\lambda)$$





Composite particle scattering — for FMS as well as to study exclusive processes.



[Plätzer, Sjödahl — '22]

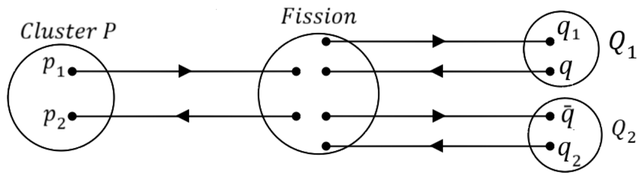
[Maas, Plätzer — in progress]

Summary

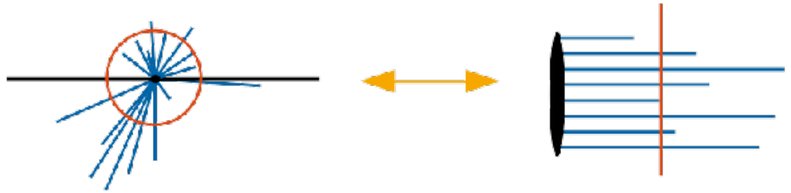
Colour space evolution equations:

- exiting theoretical tool to build parton shower and resummation algorithms,
- important subject in their own right to study structures in (QCD) amplitudes.

For event generators, parton shower accuracy is crucial, but hadronization is the elephant in the room. Get this back on top of the agenda, and link to non-perturbative methods.



Factorisation of infrared physics will teach us about the development of hadronization models, colour reconnection, initial states, JIMWLK and more.



The framework outlined here has significant room for extensions and further analytic work and simulation, either in or beyond the large-N limit, in QCD, electroweak physics, or even gravitational dynamics.

$$\sigma = \sum_{n,m} \int \int \mathrm{Tr}_n \left[\mathbf{M}_n \mathbf{U}_{nm}\right] \mathrm{d}\phi_m u(\phi_m)$$

Thank you!