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# Defining the Monte Carlo Top Mass Parameter

This talk reports on new work Oliver Jin,  
Simon Plätzer and Daniel Samitz  
which is being published soon and  
arXiv:1807.06617

**André H. Hoang**

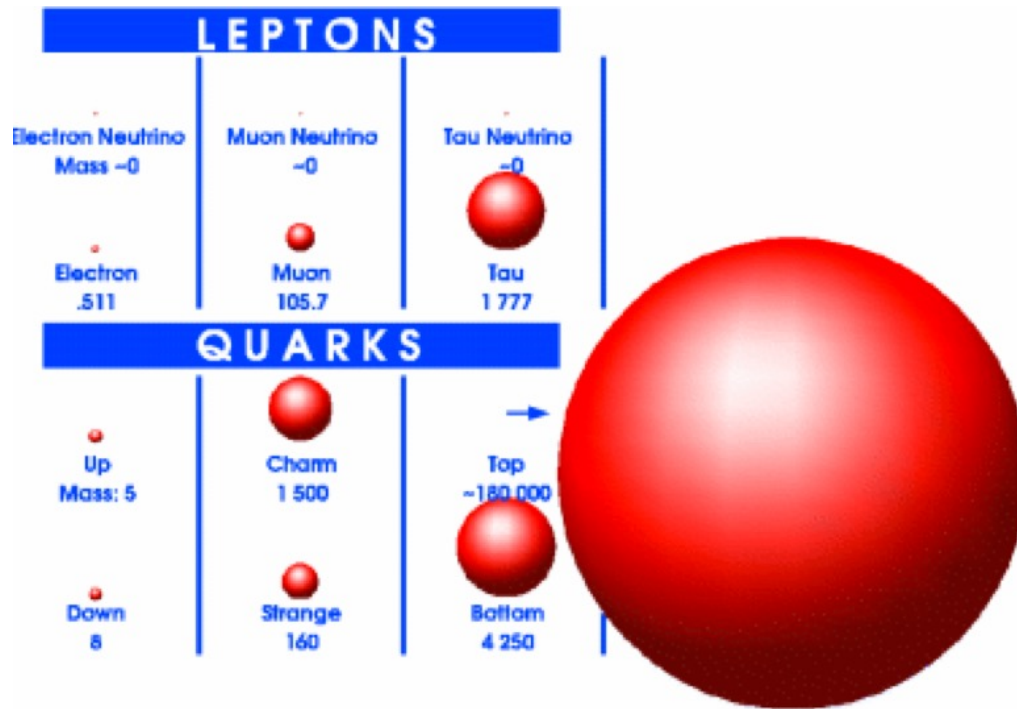
University of Vienna

*∫dk* **Π** Doktoratskolleg  
Particles and Interactions



**FWF**  
Der Wissenschaftsfonds.

# .. not just the heaviest SM particle



- Top quark: heaviest known particle
  - Most sensitive to the mechanism of mass generation
  - Peculiar role in the generation of flavor.
  - Top might not be the SM-Top, but have a non-SM component.
  - Top as calibration tool for new physics particles (SUSY and other exotics)
  - Top production major background in new physics searches
  - One of crucial motivations for New Physics
- Very special physics laboratory:  $\Gamma_t \gg \Lambda_{\text{QCD}}$ 
    - Top treated a particle:  $p_T$ , spin,  $\sigma_{\text{tot}}$ ,  $\sigma(\text{single top})$ ,  $\sigma(\text{tt+X})$ ,...  $\rightarrow q \gg \Gamma_t$
    - Quantum state sensitive low-E QCD and unstable particle effects:  $m_t$ , endpoint regions  $\rightarrow q \sim \Gamma_t$
    - Multiscale problem:  $p_T$ ,  $m_t$ ,  $\Gamma_t$ ,  $\Lambda_{\text{QCD}}$ , . . . (depends on resolution of observable)

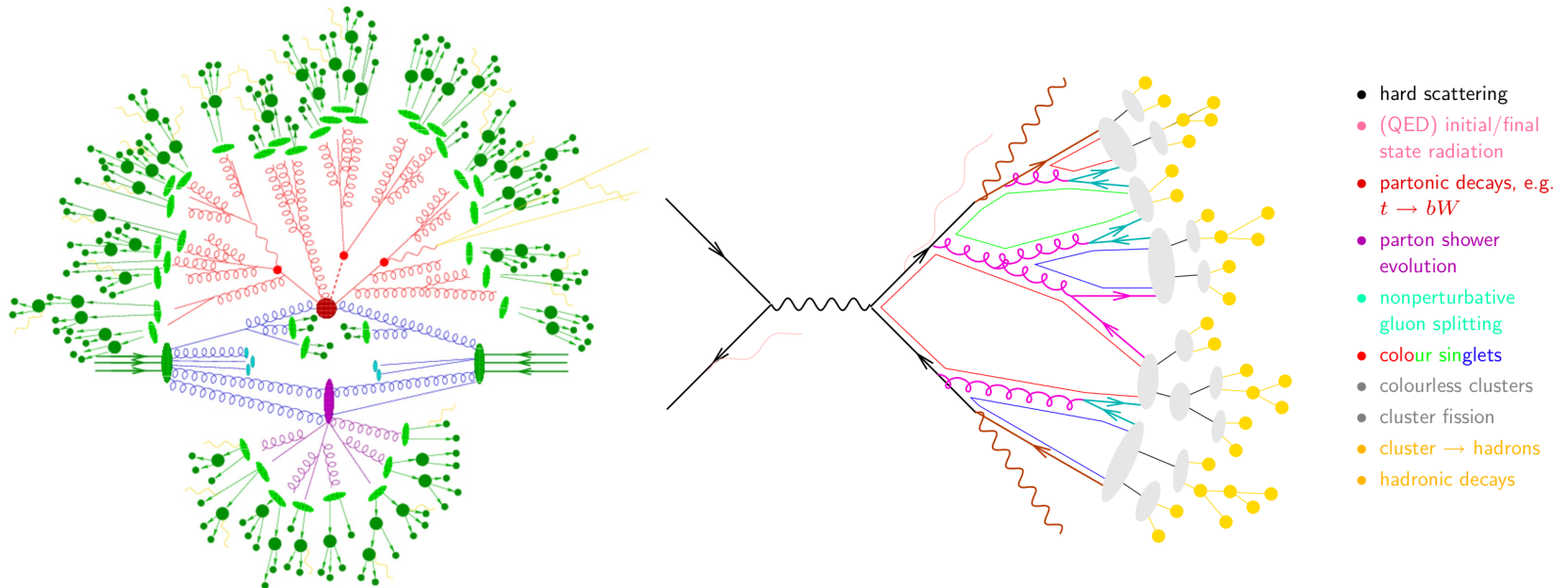


# What is $m_t^{\text{MC}}$ ?

What does the question mean in the first place?

→ It means that we can provide the relation 
$$m_t^{\text{MC}} = m_t^{\text{scheme}}(\mu) + \frac{\alpha_s(\mu)}{\pi} \delta m^{\text{scheme}} + \dots$$

How can we address this messy question in the first place?



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There are 4 essential ingredients to address the problem from first principles:

Understanding of top  
as a quantum state

NLL precise MC  
parton shower

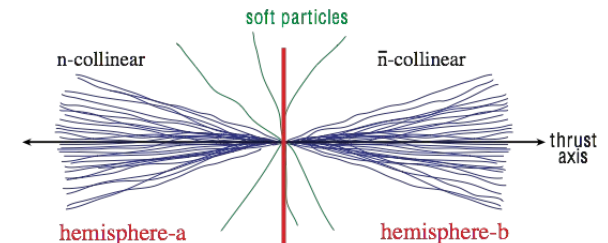
Factorization  
compatible MC  
hadronization model

Hadron level analytic  
QCD predictions

Is there an observable where all 4 ingredients are known ?

Yes: Event-shape observables in  $e^+e^-$  collisions  
for boosted top pair production:  
2-jettiness, thrust, ...

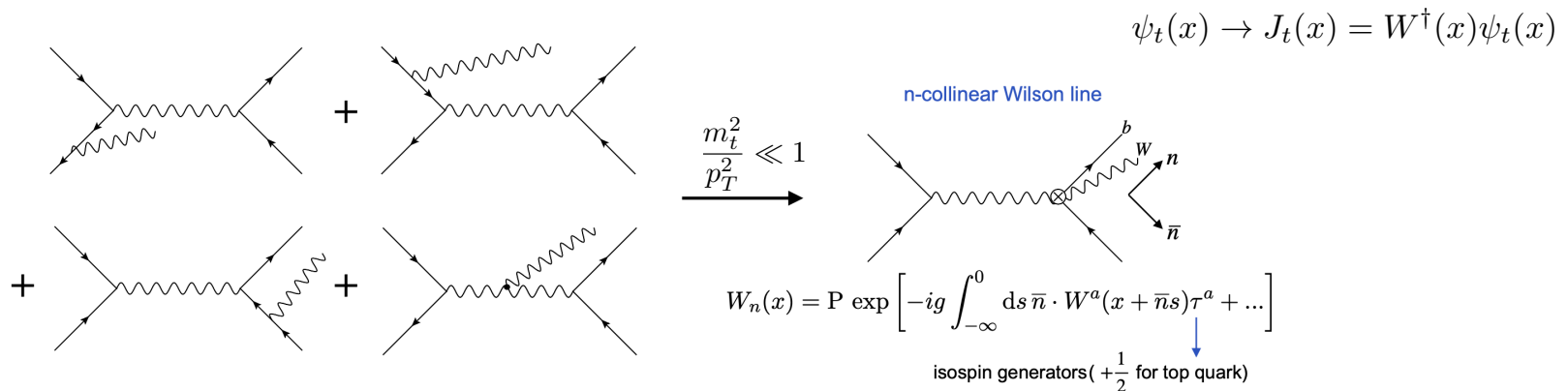
Fleming, AHH, Mantry, Stewart 2007



# (A) Beyond the picture of a top particle

If we stick to the picture of a top particle the only mass that is ever relevant is the pole mass = pole of the top propagator.

For boosted top quarks the effects of QCD and electroweak radiation (incl. top decay) in the top direction can be described by a QCD+electroweak Wilson line, which generalizes the top particle picture.

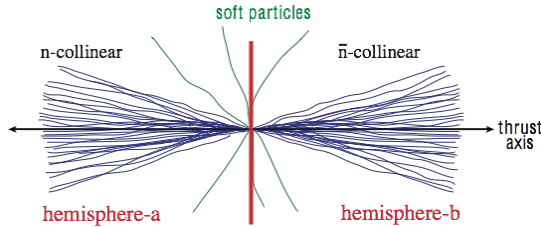


- Wilson line also contains gluons collinear with the top quark
- Describes coherent (collinear) gauge-invariant coherent off-shell (top decay products)+gluon states → "top state" defined by measurement
- Process universality (in analogy to PDFs) → factorization
- Only soft gluon radiation radiation is process dependent

This is not a new concept in QCD, but rather new for top decays.

AHH, Plätzer, Regner, Ruffa w.i.p

# (B) Boosted top eventshapes



$$\tau = 1 - \max_{\vec{n}} \frac{\sum_i |\vec{n} \cdot \vec{p}_i|}{Q} \quad \tau \rightarrow 0 \quad \frac{M_1^2 + M_2^2}{Q^2}$$

$$Q = E_{\text{c.m.}}$$

Insensitive to details of top decay

Hadron level:

$$\frac{d\sigma}{d\tau}(\tau, Q, m, \delta m) = \int_0^{Q\tau} d\ell \underbrace{\frac{d\hat{\sigma}_s}{d\tau}\left(\tau - \frac{\ell}{Q}, Q, m, \delta m\right)}_{\text{Partonic cross section}} S_{\text{mod}}(\ell)$$

Shape function

Partonic cross section

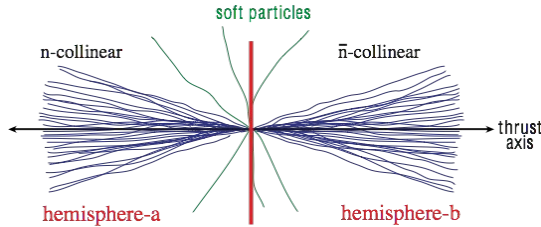
Partonic cross section (uses effective theories SCET, bHQET):

$$\left( \frac{d^2\sigma}{dM_t^2 dM_{\bar{t}}^2} \right)_{\text{hemi}} = \sigma_0 H_Q(Q, \mu_m) H_m\left(m, \frac{Q}{m}, \mu_m, \mu\right) \times \int_{-\infty}^{\infty} dl^+ dl^- B_+\left(\hat{s}_t - \frac{Ql^+}{m}, \Gamma, \mu\right) B_-\left(\hat{s}_{\bar{t}} - \frac{Ql^-}{m}, \Gamma, \mu\right) S_{\text{hemi}}(l^+, l^-, \mu)$$

$$B_+(\hat{s}, \Gamma_t, \mu) = \text{Im} \left[ \frac{-i}{12\pi m_J} \int d^4x e^{ir \cdot x} \langle 0 | T \{ \bar{h}_{v_+}(0) W_n(0) W_n^\dagger(x) h_{v_+}(x) \} | 0 \rangle \right]$$

# (B) Boosted top eventshapes

AHH, Plätzer, Samitz (2018)



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Shape function

Parton cross section

- $S_{\text{mod}}$  leading nonperturbative corrections only from large-angle soft radiation: linear sensitive to  $\Lambda_{\text{QCD}}$
- Any top mass renormalization scheme can be implemented  $m_t^{\text{pole}} = m + \delta m$
- Can be calculated with a finite IR cut  $\lambda$  for the parton cross section:
  - ▶ Redefinition of  $S_{\text{mod}}$  (large-angle soft radiation)
  - ▶ Change of mass scheme: (ultra-collinear radiation)
 
$$m_t^{\text{pole}} \rightarrow m_t(\lambda) = m_t^{\text{pole}} - \delta m(\lambda), \quad \delta m(\lambda) \sim \alpha_s(\lambda) \lambda$$

$m_t(\lambda)$  is the pole of the top propagator in the presence of the IR cutoff  $\lambda$

# (C) Angular ordered parton shower (Herwig)

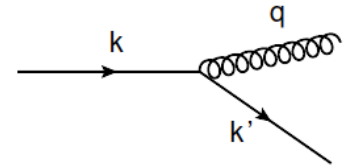
## Partonic level cross section

Catani, Trentadue, Turnock Webber (1993)

$$\frac{d\hat{\sigma}}{d\tau} = \int dk^2 dk'^2 \delta\left(\tau - \frac{k^2 + k'^2}{Q^2}\right) J(Q^2, k^2) J(Q^2, k'^2)$$

AHH, Plätzer, Samitz (2018)

- Agrees exactly with partonic cross section obtained from analytic factorized calculations at NLL!
- CB is NLL precise for inclusive event shapes.
- For vanishing IR cutoff: CB mass parameter  $m_t^{\text{CB}} = m_t^{\text{pole}}$



**BUT: Parton showers in MC generators have an IR cutoff to prevent infinite multiplicities**

$$q_{\perp} > Q_0$$

- Linear dependence on  $Q_0$  from large-angle soft and ultracollinear radiation
- Matches analogous calculations for analytic calculations
- Realized accurately by Herwig's shower

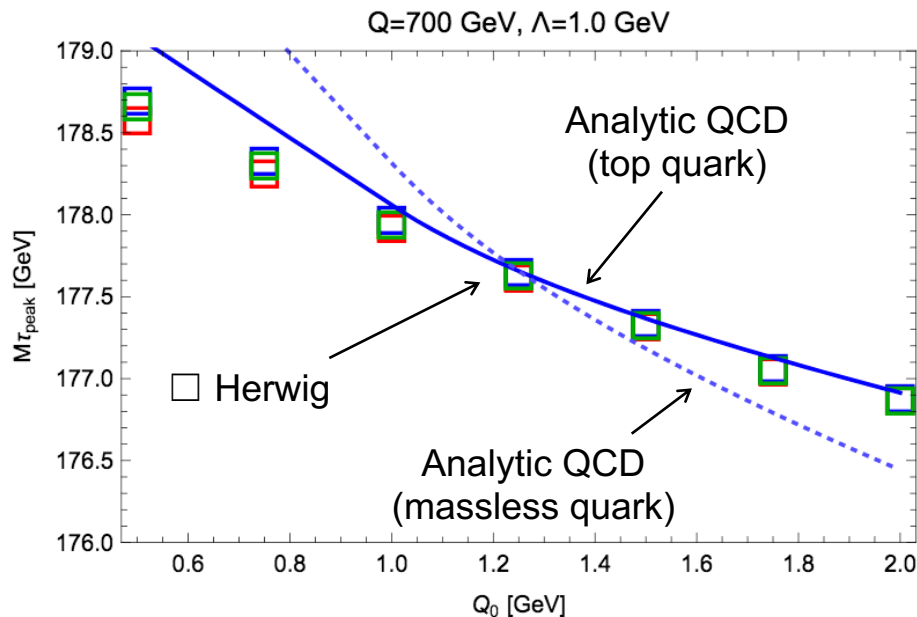
$$m_t^{\text{CB}}(Q_0) = m_t^{\text{pole}} - \frac{2}{3} Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s(Q_0)^2)$$

$$m_t^{\text{CB}}(Q_0) = m_t^{\text{MSR}}(Q_0) - \frac{2}{3} \left(1 - \frac{2}{\pi}\right) Q_0 \alpha_s(Q_0) + \mathcal{O}(\alpha_s^2(Q_0))$$

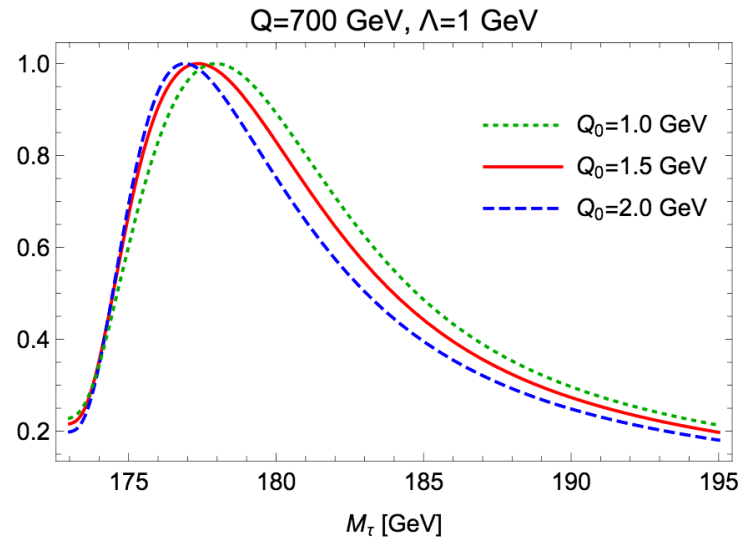
# (C) Angular ordered parton shower (Herwig)

Peak position of  $M_\tau = \frac{Q^2 \tau_2}{2m_t}$  ( $Q = E_{\text{cm}}$ )

Parton level analysis (no hadronization corrections)



How well does a hadronization model satisfy this criterium?



- Herwig parton level simulations in full agreement with analytic calculations
- **Change of  $Q_0$ :**  
Physical predictions **should remain** unchanged when

- ▶ hadronization model is retuned ( $Q_0$  dependent tune)
- ▶ generator mass is interpreted as  $m_t^{\text{CB}}(Q_0)$

# (D) Factorization compatible hadronization model

AHH, Jin, Plätzer, Samitz to appear

Standard shower cut treatment for all MC generators:

- Shower-cutoff scale  $Q_0$  = one of many hadronization model parameters

BUT: To gain control over the shower's top mass parameter:

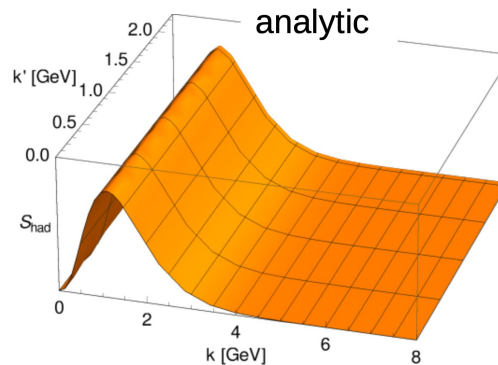
- The shower-cutoff scale  $Q_0$  must be promoted to a factorization scale, such that hadron level descriptions are shower-cut independent. [Plätzer arXiv:2204.06956](#)
- The parton-level to hadron-level transfer matrix must behave like a shape function!

$$\frac{d\sigma}{d\tau}(\tau, Q) = \int d\hat{\tau} \frac{d\hat{\sigma}}{d\hat{\tau}}(\hat{\tau}, Q) \underbrace{T(\tau, \hat{\tau}, \{Q, Q_0\})}_{\text{Transfer matrix}}$$

Transfer matrix should have the property  $T(\tau, \hat{\tau}, \{Q, Q_0\}) = T(\tau - \hat{\tau})$

$$T\left(\frac{k}{Q} = \tau - \hat{\tau}, \frac{k'}{Q} = \hat{\tau}, \{Q, Q_0\}\right)$$

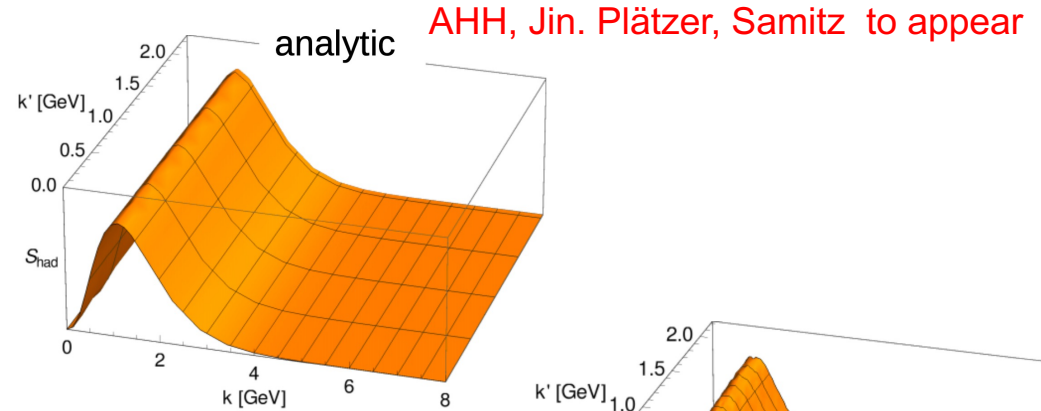
Transfer matrix should have this form:



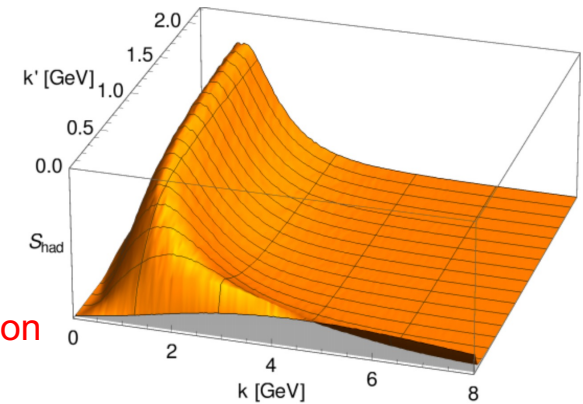
# (D) Factorization compatible hadronization model

$$T \left( \frac{k}{Q} = \tau - \hat{\tau}, \frac{k'}{Q} = \hat{\tau}, \{Q, Q_0\} \right)$$

transfer matrix should have this form:



But it actually looks like this:  
( $Q_0 = 1.00$  GeV)

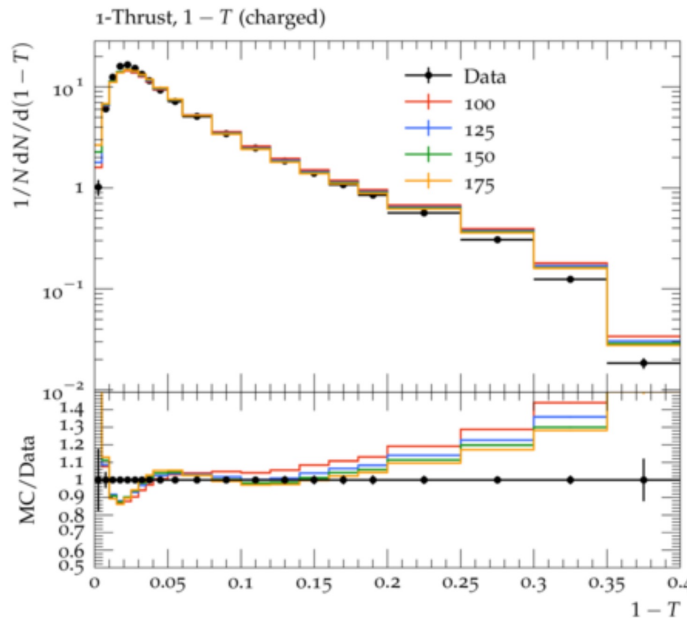


Peak region hadronization  
inconsistent with QCD  
factorization!

Description of observables not quite  
shower-cutoff independent (after tuning)

(Thrust at  $Q = M_Z$ )

$Q_0 = ( 1.00, 1.25, 1.50, 1.75 )$  GeV

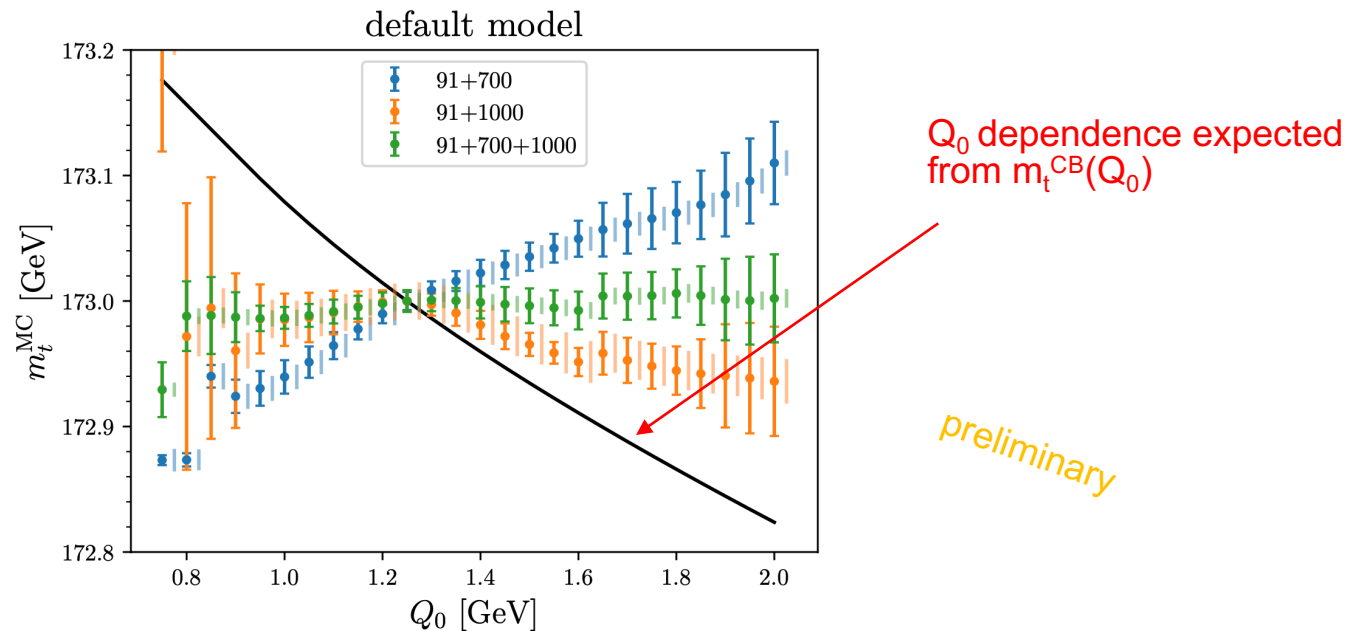


# (D) Factorization compatible hadronization model

AHH, Jin, Plätzer, Samitz to appear

Include top “pseudo” 2-jettiness data into the standard tuning data set:

- Also tune the top mass parameter  $m_t^{\text{MC}}$  for different  $Q_0$  values (to top pseudo data generated for  $Q_0=1.25$  GeV)



Default Herwig hadronization model modifies  $m_t^{\text{MC}}$  in an unphysical way in compatible with QCD factorization.

$$\rightarrow m_t^{\text{Herwig}}(Q_0) \neq m_t^{\text{CB}}(Q_0)$$

# Factorization compatible hadronization model

AHH, Jin, Plätzer, Samitz to appear

## Dynamic model: forced gluon splitting

- If the splitting had taken place in the parton shower it would have been generated from the splitting function

$$dP(g \rightarrow q\bar{q}) \sim \frac{dq^2}{q^2} \alpha_s(q^2) \left(1 - 2z(1-z) + \frac{2m_q^2}{q^2}\right) \Theta(q^2 z(1-z) - m_q^2)$$

giving the gluon a virtuality  $q^2$

- Use the probability distribution for this dynamically generated virtuality as „gluon constituent mass”  $m_g$
- Set a highest possible scale for the non-pert. gluon splitting  $\tilde{Q}_g$   
(new tuning parameter instead of fixed  $m_g$ )
- Need to IR regularize the splitting function (because evolve below cutoff):  $dP(g \rightarrow q\bar{q}) \rightarrow d\tilde{P}(g \rightarrow q\bar{q})$

freeze out strong coupling at some low scale to avoid Landau pole

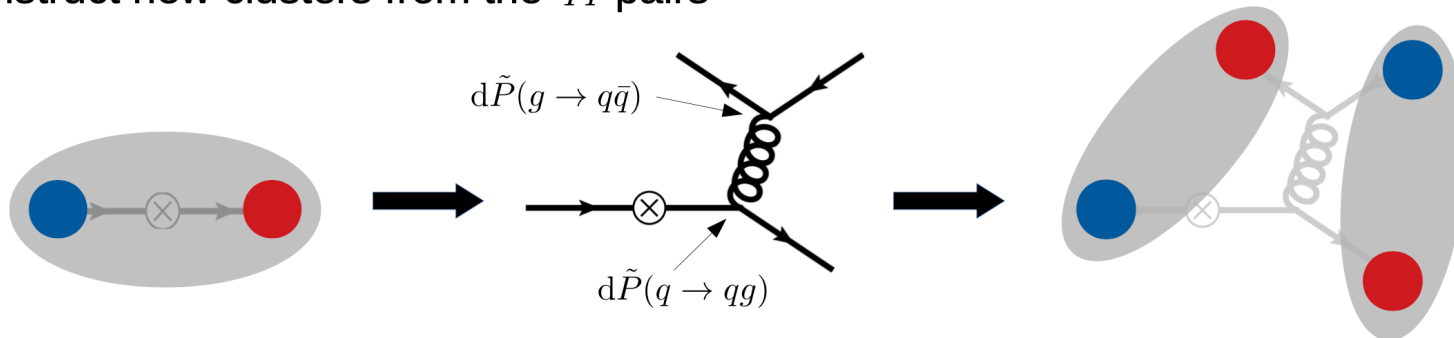
use constituent masses for quarks

# Factorization compatible hadronization model

AHH, Jin, Plätzer, Samitz to appear

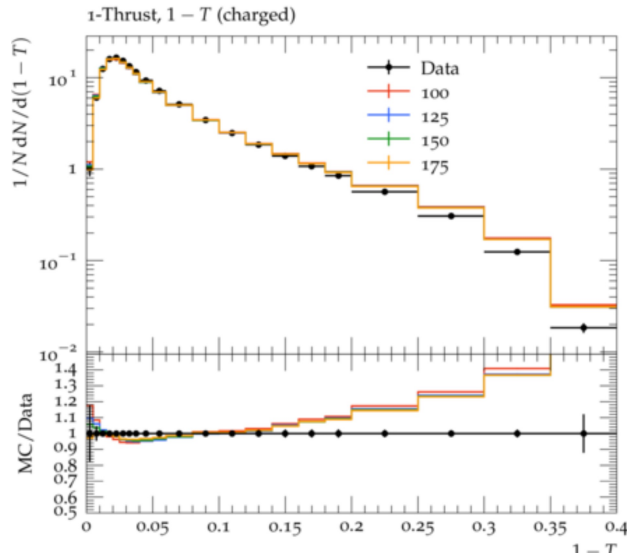
## Dynamic model: cluster fission

- Want a mass distribution of the daughter clusters that resembles as much as possible the mass distribution dynamically generated by low scale emissions of the parton shower
- Radiate a gluon from one of the cluster's constituents according to  $d\tilde{P}(q \rightarrow qg)$   
set a maximum scale  $\tilde{Q}_q$  of the splitting (new tuning parameter for fission instead of  $P_{\text{Split}}$ )
- Split the gluon according to  $d\tilde{P}(g \rightarrow q\bar{q})$
- Construct new clusters from the  $q\bar{q}$  pairs



# Factorization compatible hadronization model

Transfer function much better consistent with QCD factorization



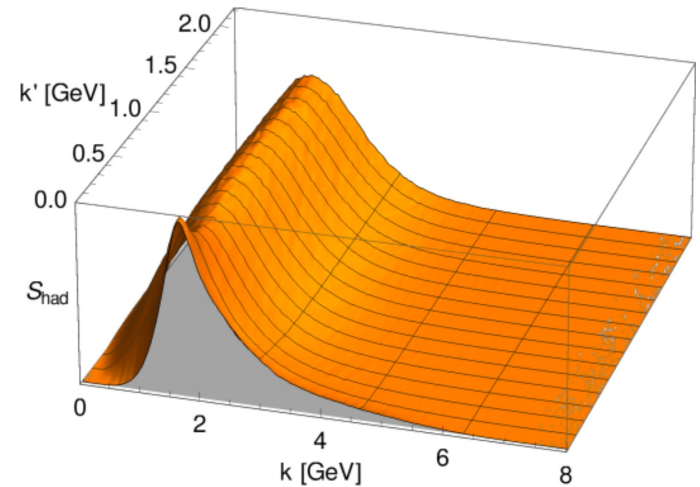
Observables much less dependent on  $Q_0$

Tunes  $m_t^{MC}$  fully consistent with expectations from analytic QCD calculation

(“pseudo data” generated for  $Q_0=1.25$  GeV)

$$\rightarrow m_t^{\text{Herwig}}(Q_0) = m_t^{\text{CB}}(Q_0)$$

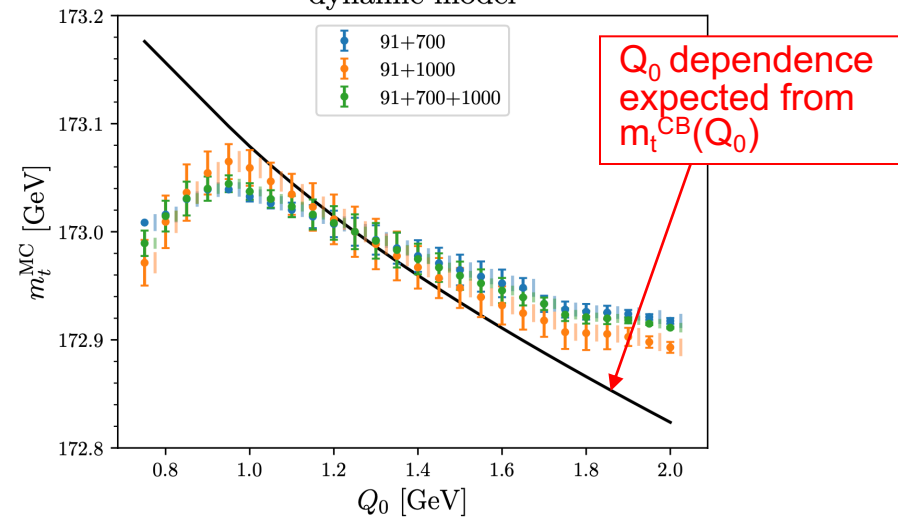
AHH, Jin, Plätzer, Samitz to appear



Thrust

preliminary

dynamic model



AHH, Jin, Plätzer, Samitz to appear

# What is $m_t^{\text{MC}}$ ?

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→ It means that we can provide the relation 
$$m_t^{\text{MC}} = m_t^{\text{scheme}}(\mu) + \frac{\alpha_s(\mu)}{\pi} \delta m^{\text{scheme}} + \dots$$

There are 4 essential ingredients to address the problem from first principles:

## Understanding of top as a quantum state

- Abandon top as on-shell asymptotic state (“top particle”)
- Beyond narrow-width limit (top = top decay final states)
- Top has color and needs additional gluons to become observable
- Observable top state = coherent top+gluon (color singlet) state

## Hadron level analytic QCD predictions

- NLL precise analytic QCD perturbative prediction
- First principles treatment of hadronization corrections
- NLO control over IR factorization scale for perturb. and non-pert. Effects
- Analytic description of shower cut  $Q_0$  dependence

## NLL precise MC parton shower

- NLL precision needed to achieve NLO accurate definition of  $m_t^{\text{MC}}$
- Analytic NLO precision concerning the IR cutoff  $Q_0$  or the parton shower evolution
- Shower cutoff  $Q_0$  = an IR factorization scale
- Parton shower determines the meaning of  $m_t^{\text{MC}}(Q_0)$

## Factorization compatible MC hadronization model

- Shower cutoff  $Q_0$  is not treated as a tuning variable, but as factorization scale
- Hadronization model has to respond to different choices of  $Q_0$  to compensate for changes of  $Q_0$  in the parton shower.
- Should not provide nonperturbative corrections to  $m_t^{\text{MC}}$

→ The MC top mass parameter can be promoted to a renormalization scheme and its NLO relation to any other top mass renormalization scheme can be calculated.

# Final remarks and Outlook

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- Universality of the current insights
  - So far we have all theoretical ingredients to interpret  $m_t^{\text{MC}}$  only for  $e^+e^-$  eventshape distributions and the Herwig MC generator (coherent branching + cluster hadronization)
  - MC generators do not have the same precision for all observables
  - It could be that the parton shower cutoff  $Q_0$  may lead to observable-dependent interpretations of  $m_t^{\text{MC}}$
- Progress to generalize the current results will take many years of work because many new theory tools need to be developed (→ coherent top state)
- Upcoming: factorization of differential coherent (off-shell) top decay  
→ observables such as  $E_{\text{lepton}}$ ,  $M_{\text{b-jet lepton}}$  for fiducial top states
- Future plans: dipole shower, string hadronization (Pythia)
- LHC: MPI and UE hadronization models still need to be better understood from the QCD perspective (→ theoretical brickwall for LHC hadronization models )

# Old Default Model

# New Dynamical Model

Shower cutoff dependence of predictions with tuned hadronization model

