

Symplectic Singularities in Physics: Their Origins and Significance

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FAKT workshop 2024: particle physics retreat

— February 22-23, 2024 —

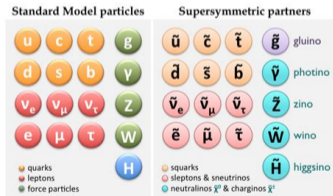


Quantum field theory: **successful** but **challenging**.

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Simplification: supersymmetry (SUSY) = extension of Poincare symmetry

- control over quantum corrections
→ “protected” quantities
- window into non-Lagrangian theories or non-trivial interacting theories in $d = 5, 6$



Vast landscape of SUSY theories with 8 supercharges

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- All local operators and correlation functions \longrightarrow too difficult
- Simpler, but robust quantities:
 - { symmetries,
 - { vacua,
 - { partition functions

Supersymmetric vacua

Hall-mark feature of supersymmetric (superconformal) theories:
→ **large continuous space \mathcal{M} of SUSY vacua**

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Facts:

- **Geometry** of \mathcal{M} depends on # SUSY and on space-time dimension
- **Singularities** of \mathcal{M} hint on non-trivial physics

Today: 8 supercharges in dim d (e.g. 4d $\mathcal{N} = 2$)

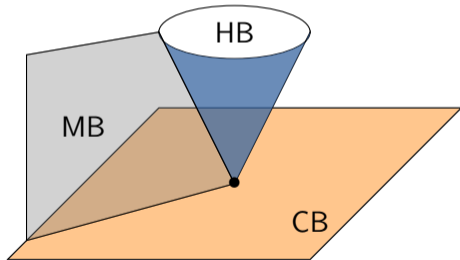
- **hypermultiplet:** e.g. electrons + SUSY partners \supset 4 real scalars
- **vector multiplet:** e.g. gauge field + SUSY partners \supset $6 - d$ real scalars

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space of SUSY vacua

$$\mathcal{M} = \underbrace{\left\{ \begin{array}{l} \langle \text{hypers} \rangle \neq 0 \\ \langle \text{vectors} \rangle = 0 \end{array} \right\}}_{\text{Higgs branch}} \cup \underbrace{\left\{ \begin{array}{l} \langle \text{hypers} \rangle = 0 \\ \langle \text{vectors} \rangle \neq 0 \end{array} \right\}}_{\text{Coulomb branch}} \cup \underbrace{\left\{ \begin{array}{l} \langle \text{hypers} \rangle \neq 0 \\ \langle \text{vectors} \rangle \neq 0 \end{array} \right\}}_{\text{mixed branch}}$$

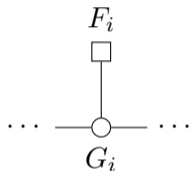


Quivers - a useful short-hand notation

A **quiver**: graph with vertices and edges

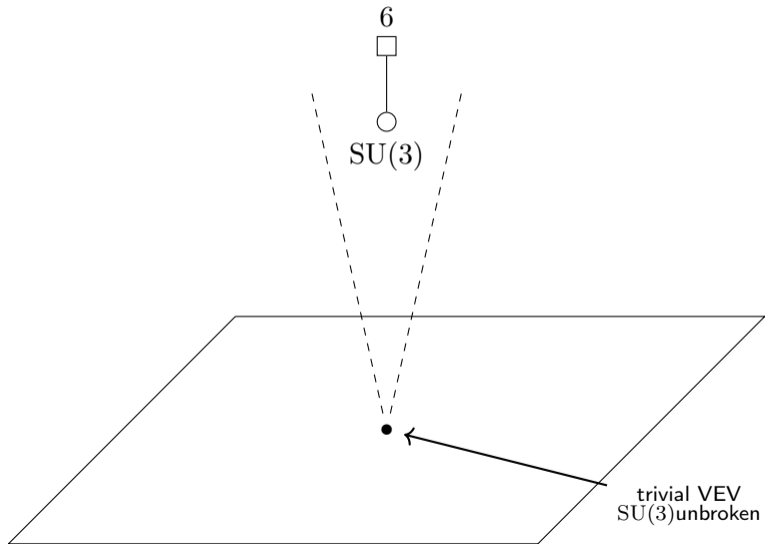
- \bigcirc vertices: **gauge group** factor G_i
- \square vertices: **flavour group** factor F_i
- — edges: bifundamental hypermultiplet

+ interaction terms fully determined by supersymmetry

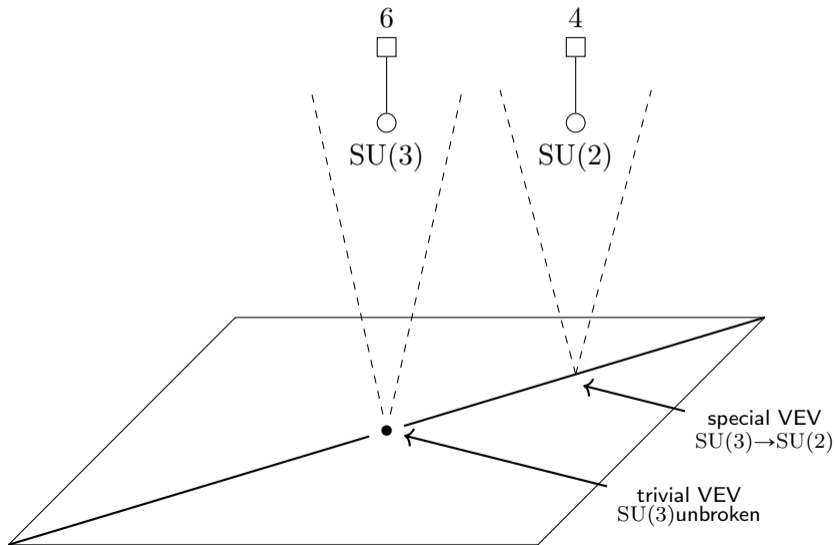


encodes an **entire**
Lagrangian SQFT

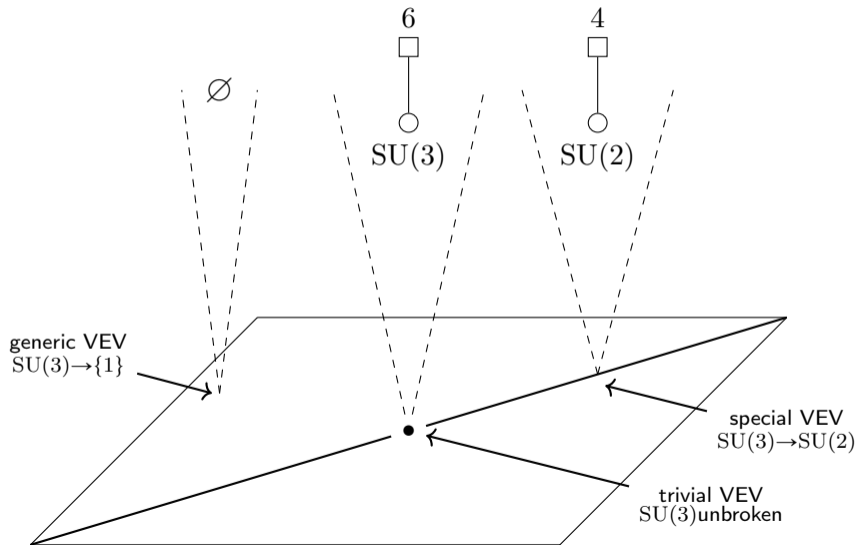
Phase diagram for Higgs mechanism



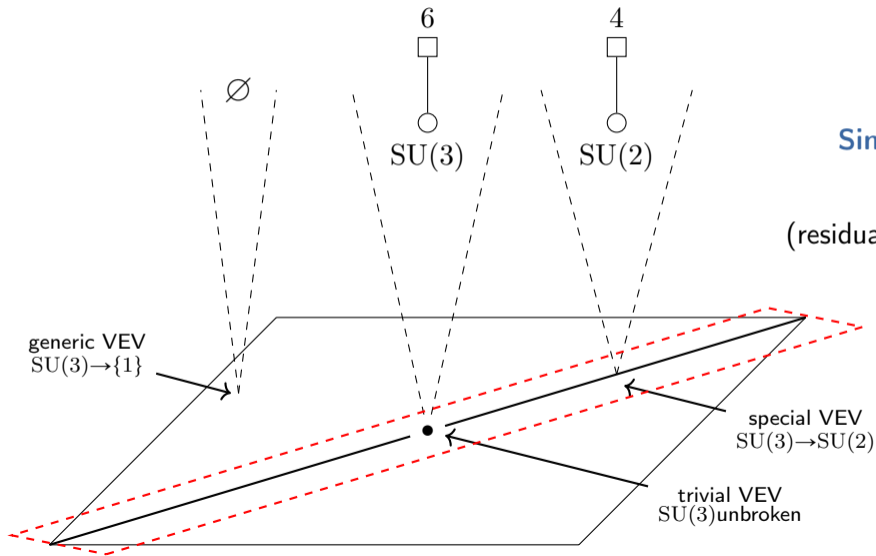
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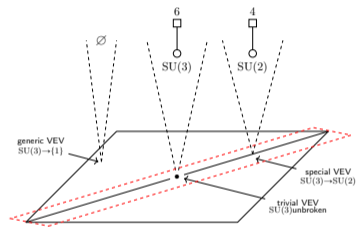


Takeaway:

Singularity on HB



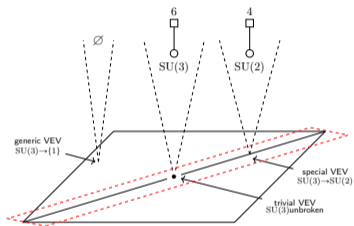
(residual) gauge symmetry



$$\left\{ \begin{array}{l} \text{VEVs} \\ \text{SU}(3) \rightarrow \{1\} \end{array} \right\}$$

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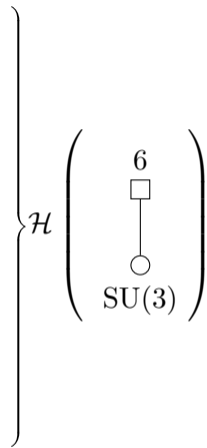
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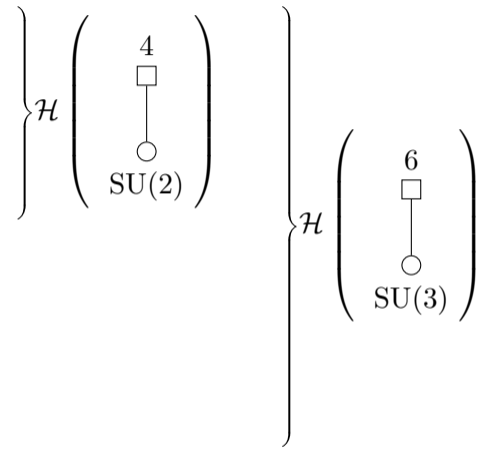
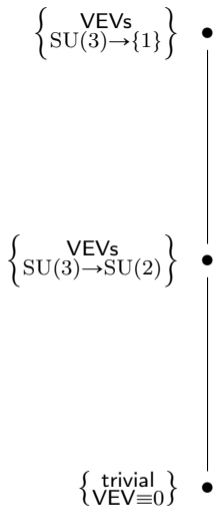
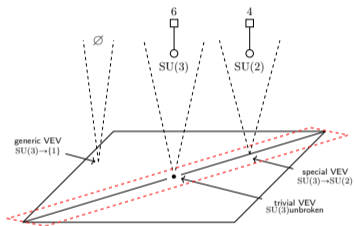


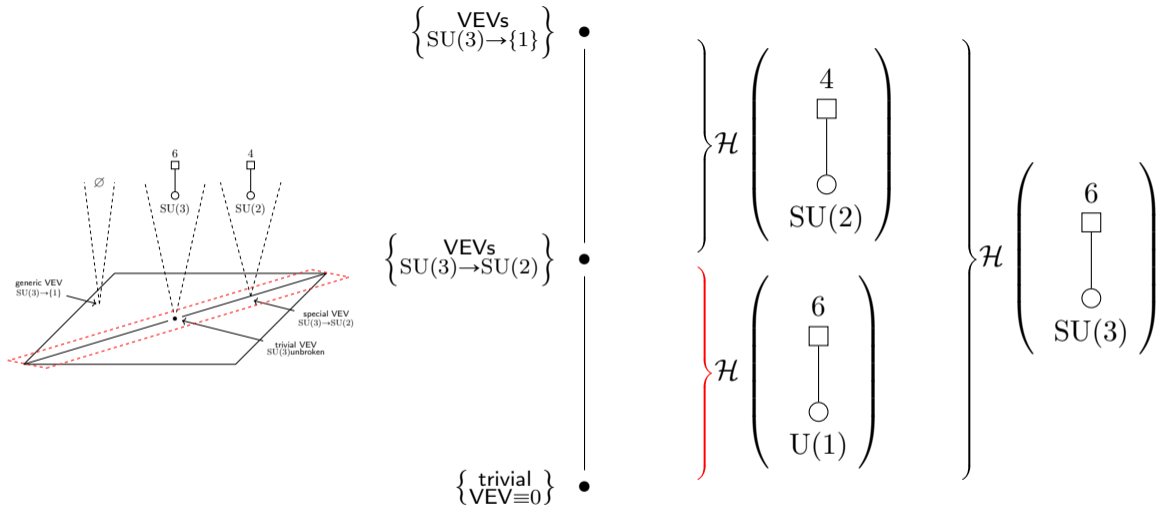
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Takeaway: **pattern of partial Higgsing** = **finite stratification of its Higgs branch**

Magnetic quivers

String theory: existence of non-trivial, interacting SQFTs with 8 supercharges

→ called 6d $\mathcal{N} = (1, 0)$, 5d $\mathcal{N} = 1$, and 4d $\mathcal{N} = 2$

Magnetic quivers

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Intrinsically strongly coupled and/or non-Lagrangian.

→ Higgs branches = symplectic singularities beyond “semi-classical physics”.

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Proposal: For a theory \mathcal{T} with 8 supercharges in $\dim = 3, 4, 5, 6$ in phase P :

$$\mathcal{H}(\mathcal{T}, P) \cong \mathcal{C}(\mathcal{Q}) \quad \text{[MS \& collaborators since 2018/19]}$$

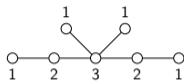
for an auxiliary combinatorial object \mathcal{Q} – called the **magnetic quiver**.

What can we do?

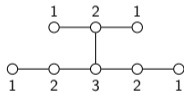
5d $\mathcal{N} = 1$
 $SU(3)_0$
 gauge theory
 with $N_f = 6$
 fund. flavours

at low energies:
 Higgs branch
 =
 hK quotient

at fixed point:
 Higgs branch
 \neq
 hK quotient



magnetic quiver



$$\mathfrak{su}(6) \oplus \mathfrak{u}(1)$$

symmetry

$$\mathfrak{su}(6) \oplus \mathfrak{su}(2) \oplus \mathfrak{su}(2)$$



Hasse diagram



Symplectic singularities appear in many contexts

- Higgs branches in $d = 3, 4, 5, 6$
- Coulomb branches in $d = 3$
- moduli space of instantons
- ...

Takehome messages

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Interplay: mathematical data \longleftrightarrow physical insights

- singularities of \mathcal{M} \longleftrightarrow massless degrees of freedom
- finite stratification \longleftrightarrow (generalised) partial Higgs mechanism

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Magnetic quivers – a new tool

- explore previously inaccessible **quantum Higgs branches**
- uncover new symplectic singularities

Many open questions that deserve exploration

- Higgs branch of SQFTs with 8 supercharges
- Network of Higgs branch RG-flows
- Classification of minimal singularities (minimal Higgs transitions)
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Thank you!