Factorization of Inclusive Semileptonic Off-Shell and Boosted Top Quark Decays in the Endpoint region

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with

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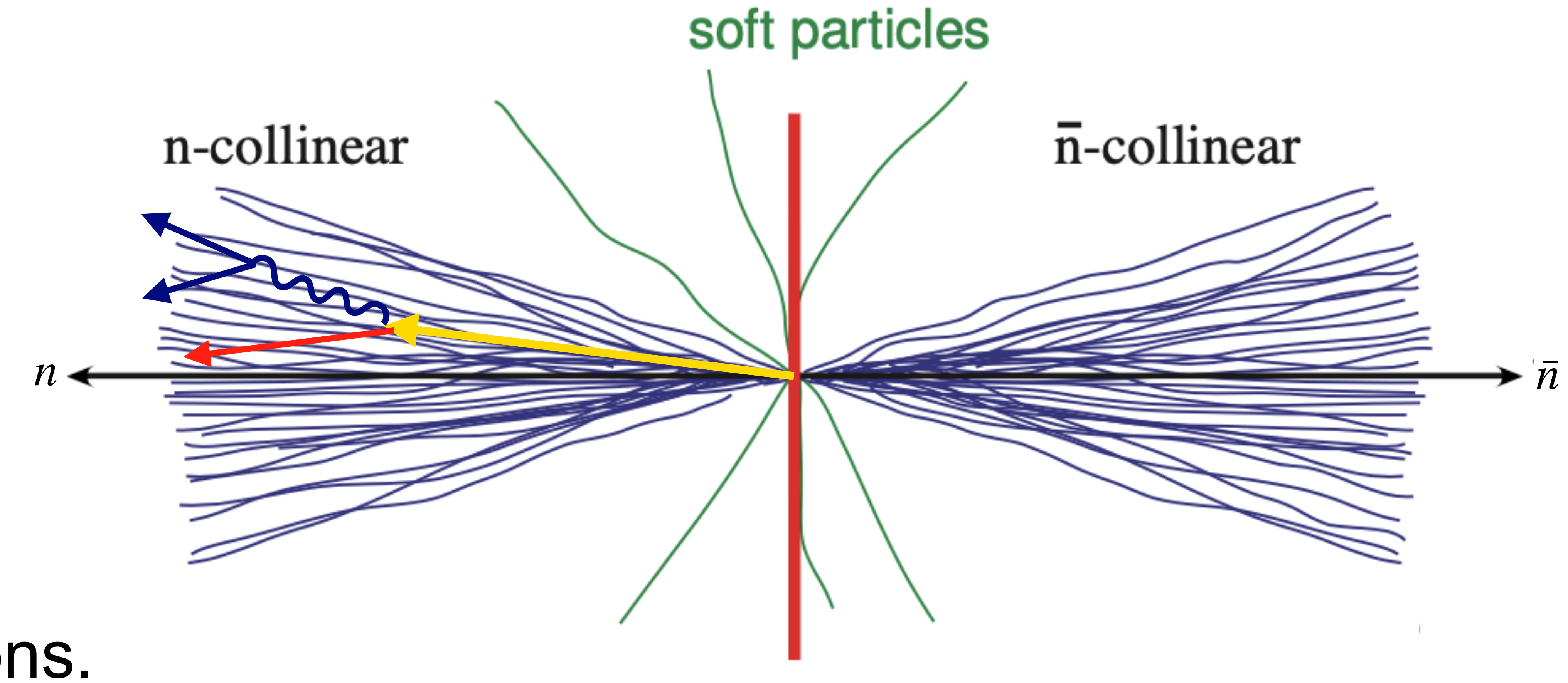
Motivation — Top Quark Physics

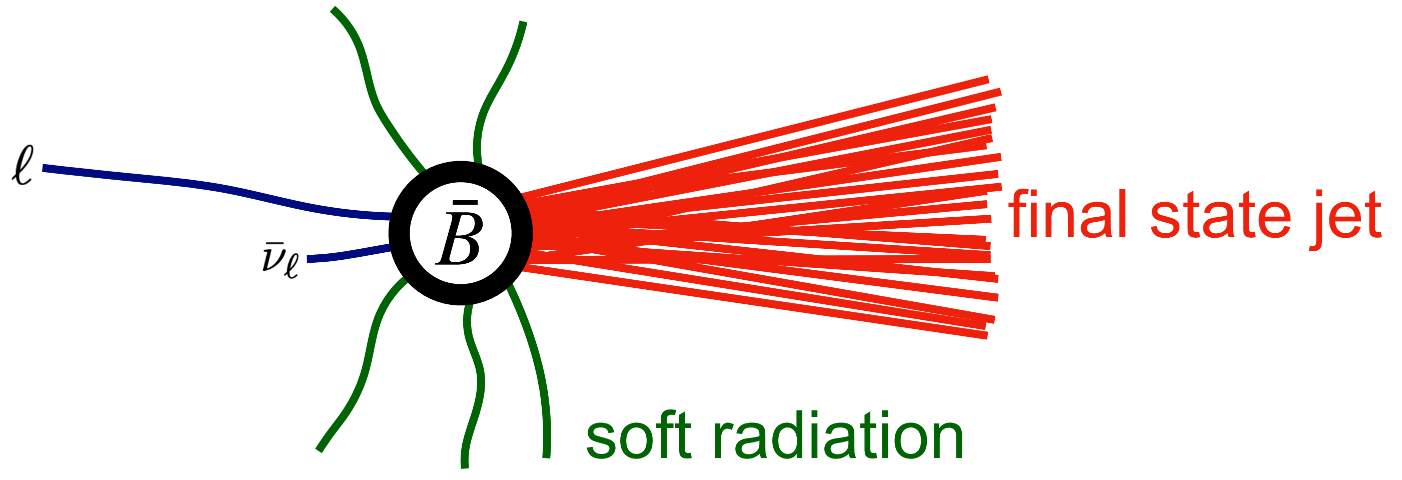
- The **top quark** is the heaviest known elementary particle.
- Due to its large mass the top quark plays a key role in consistency checks of the Standard Model and new-physics searches.
- Studies of top quark production and its decay commonly based on two approaches:
  - Narrow-width (NW) limit:
    - Top quark treated as on-shell particle.
    - Factorization of top production and decay dynamics.
  - Off-shell fixed-order computation:
    - Accounts for non-resonant, non-factorizable and finite lifetime effects.
    - Start-of-the-art: fixed-order NLO QCD.



- New approach: Combine properties of NW limit and off-shell computations.
  - **QCD factorization theorem** for off-shell boosted top quarks ($\longrightarrow$ SCET, bHQET)
    - Merge factorization approaches for boosted top production and semileptonic B decays.
  - Top quark state based on a measurement (and not on the concept of a “top particle”).
  - Incorporate resummed QCD corrections for differential top decay observables.
- Aim: Analytic control of top mass dependent decay observables.

Soft-Collinear Effective Theory (SCET)

- SCET: Used to describe energetic QCD processes where the final state particles have large energies compared to their invariant mass.
  - B meson decays to light particles in kinematic endpoint regions, e.g. $\bar{B} \to X_u \, \ell \, \bar{\nu}_\ell \, (m_X \ll E_X \sim \frac{m_B}{2})$
  - Jet production in pp collisions and e^+e^- collisions. $(m_J \ll E_J)$
  - …



- Momentum modes:

$$
\begin{aligned}
\text{collinear dofs :} \quad & p_n \sim Q(\lambda^2, 1, \lambda) \\
& p_{\bar{n}} \sim Q(1, \lambda^2, \lambda) \\
\text{soft dofs :} \quad & p_s \sim Q(\lambda^2, \lambda^2, \lambda^2) \\
\text{hard dofs :} \quad & p_h \sim Q(1, 1, 1)
\end{aligned}
$$

$$p^\mu = p^+ \frac{\bar{n}^\mu}{2} + p^- \frac{n^\mu}{2} + p_\perp^\mu \equiv (p^+, p^-, p_\perp)$$

$$p^2 = p^+ p^- + p_\perp^2$$

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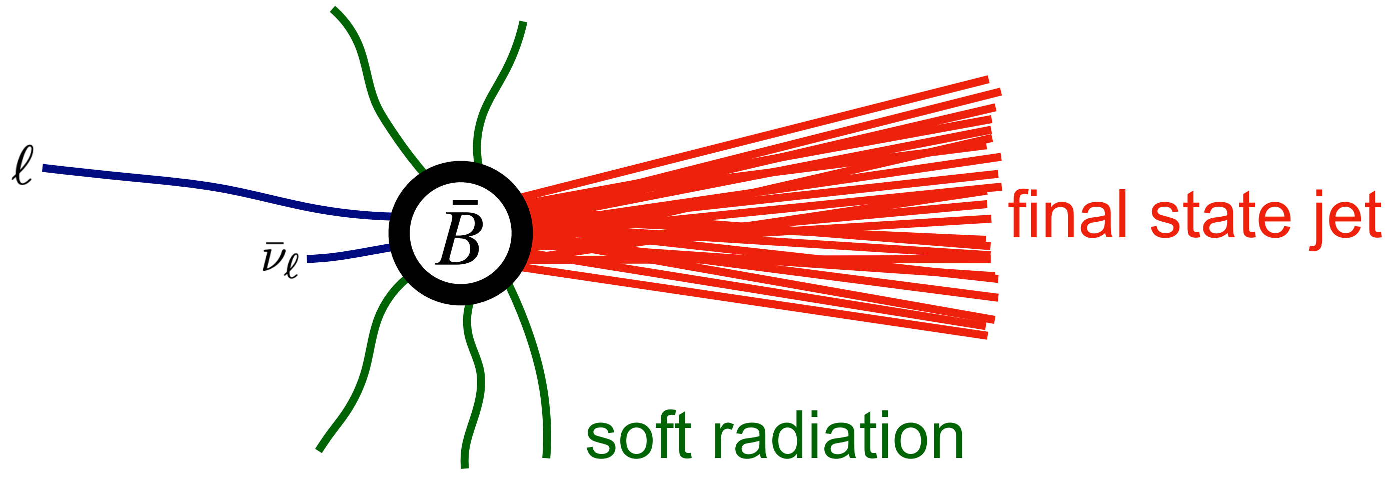
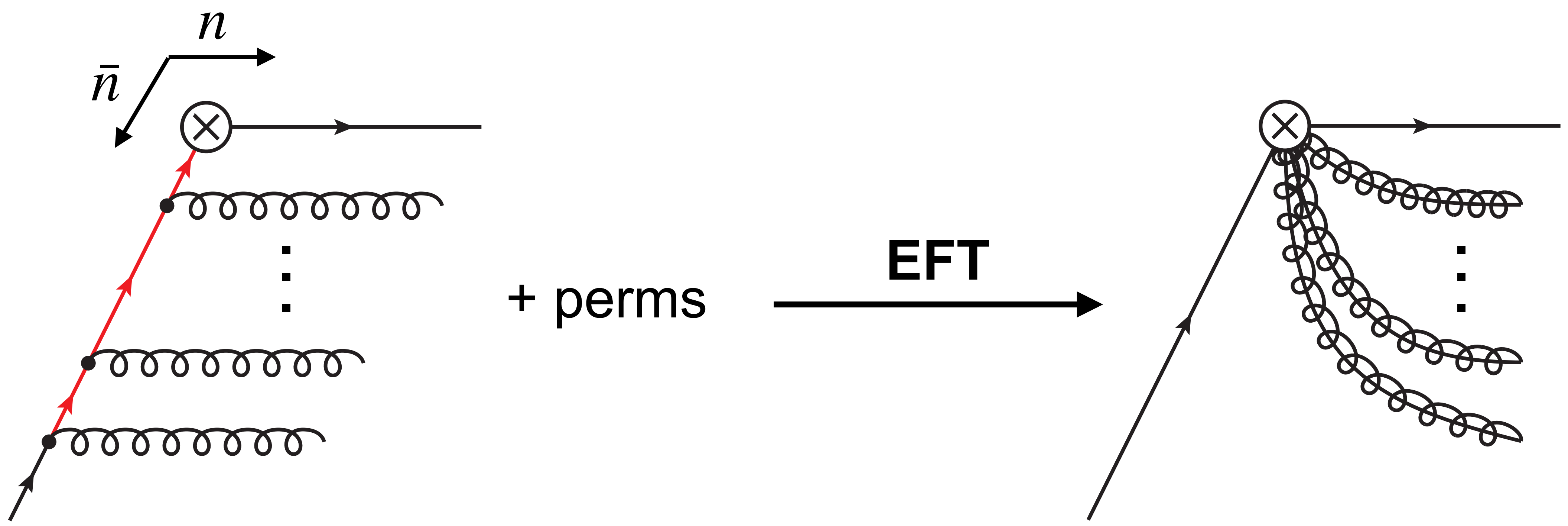


- Leading order collinear quark Lagrangian:

$$\mathcal{L}_n = \bar{\xi}_n \left[in \cdot D_s + gn \cdot A_n + gn \cdot A_s + i\not{D}_n^{\perp} W_n^{\dagger} \frac{1}{\bar{n} \cdot \mathcal{P}} W_n \, i\not{D}_n^{\perp} \right] \frac{\not{\bar{n}}}{2} \xi_n$$

collinear Wilson line



$$W_n(x) = P \exp \left(-ig_s \int_{-\infty}^{0} \mathrm{d}s \; \bar{n} \cdot A_n(s\bar{n} + x) \right)$$

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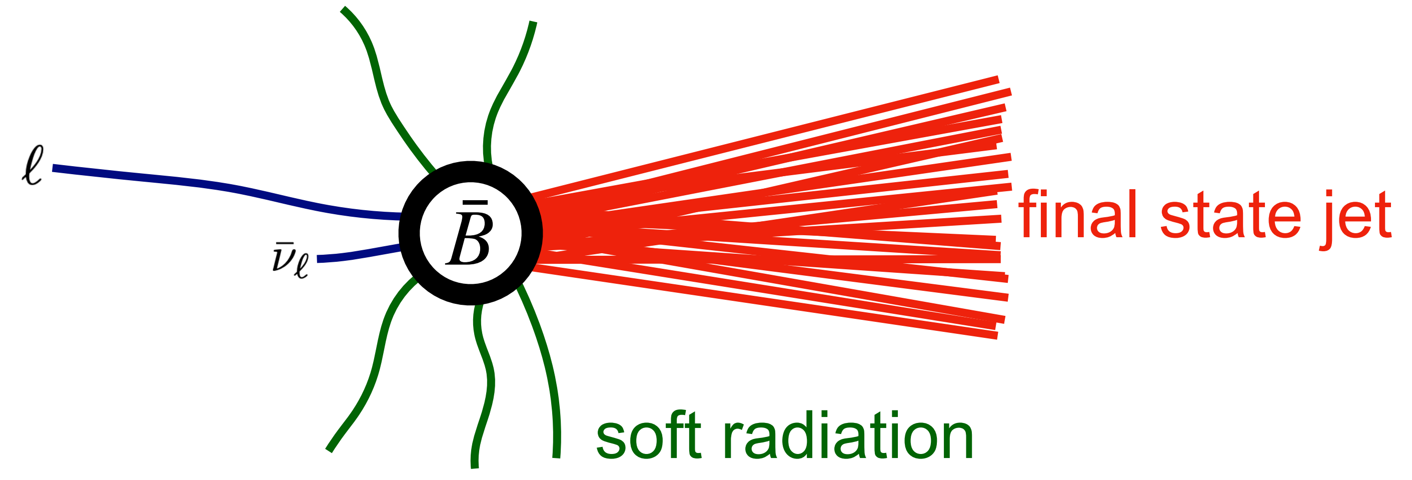

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collinear-soft coupling $iD_s^\mu = i\partial^\mu + gA_s^\mu$

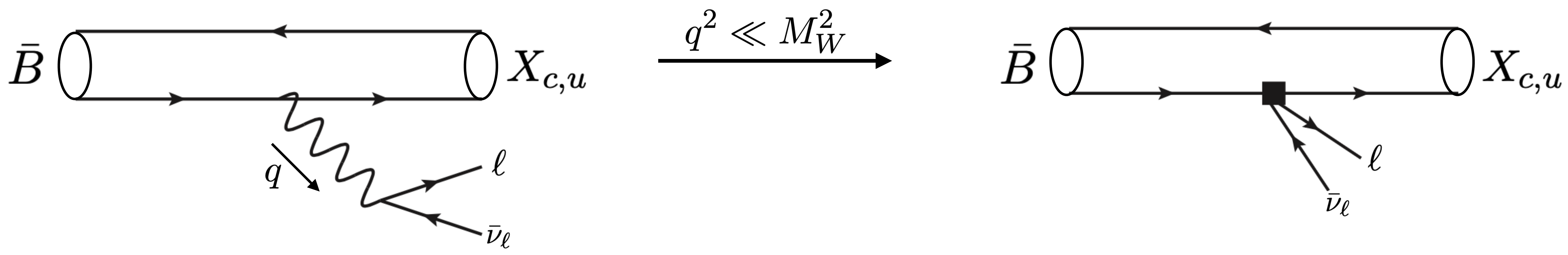
$$\xi_n \to Y_n \, \xi_n, \qquad W_n \to Y_n \, W_n \, Y_n^\dagger$$

$$Y_n(x) = \bar{\mathrm{P}} \exp\left(- ig_s \int_0^\infty \mathrm{d}s \; n \cdot A_s(ns + x) \right)$$

$$\mathcal{L}_n = \bar{\xi}_n \, in \cdot \partial_s \, \frac{\not{\bar{n}}}{2} \, \xi_n + \ldots$$

Recap: Semileptonic B decays

- Inclusive semileptonic decays $\bar{B} \to X_c\,\ell\,\bar{\nu}_\ell$ and $\bar{B} \to X_u\,\ell\,\bar{\nu}_\ell$ allow extraction of $|V_{cb}|$ and $|V_{ub}|$ from measurements of the decay spectra.



$$\frac{\mathrm{d}^3\Gamma}{\mathrm{d}E_\ell\,\mathrm{d}q^2\,\mathrm{d}h^2} \sim G_F^2\,L_{\mu\nu}\,W^{\mu\nu}$$

$$W^{\mu\nu} = \frac{1}{2m_B}\frac{1}{\pi}\,\mathrm{Im}\Big[\langle\bar{B}|\;i\!\int \mathrm{d}^4x\,\mathrm{e}^{-iq\cdot x}\,T\{J^{\dagger\mu}(0)J^\nu(x)\}\,|\bar{B}\rangle\Big], \qquad J^\nu = (\bar{c}\,\gamma^\nu P_L\,b)$$

- $\bar{B} \to X_c\,\ell\,\bar{\nu}_\ell$: $W^{\mu\nu}$ can be studied by using a local OPE within HQET.
  - Non-pert. physics encoded in matrix elements of local operators.
- $\bar{B} \to X_u\,\ell\,\bar{\nu}_\ell$: Cuts on E_ℓ or h^2 needed to eliminate $b \to c$ background events.
  - Restriction to phase space region of energetic jets with small invariant mass ⟶ **SCET**
  - OPE not applicable in this region ⟶ need to rely on factorization tools.

Factorization for Semileptonic B decays

- Factorized form of differential decay rate in the endpoint region:

Bauer, Fleming, Pirjol, Stewart '01
Bosch, Lange, Neubert, Paz '04

$$\frac{d^3\Gamma}{dE_\ell\, dq^2\, dh^2} \sim \Gamma_0\, H(E_\ell,\mu) \int dk^+\, J(k^+,\mu)\, S_{\rm shape}(m_B - m_b - k^+,\mu)$$

hard function

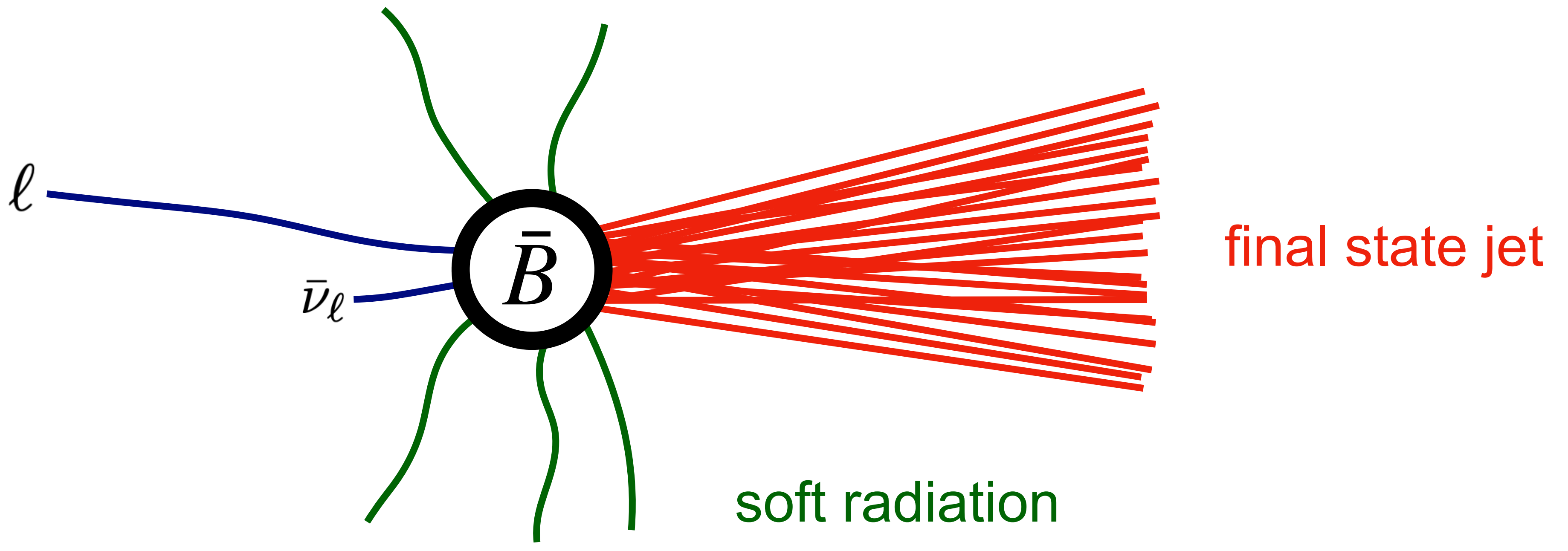
$$H \sim C^2$$

jet function

$$J(k) \sim {\rm Im}\left[\int d^4x\, e^{ik\cdot x}\, \langle 0|\, T[\bar\xi_n W_n(0)\, \bar{\not n}\, W_n^\dagger \xi_n(x)]\, |0\rangle\right]$$

soft function

$$S_{\rm shape}(k^+) = \langle \bar B|\, \bar h_v\, Y_n\, \delta(k^+ - in\cdot\partial)\, Y_n^\dagger\, h_v\, |\bar B\rangle$$



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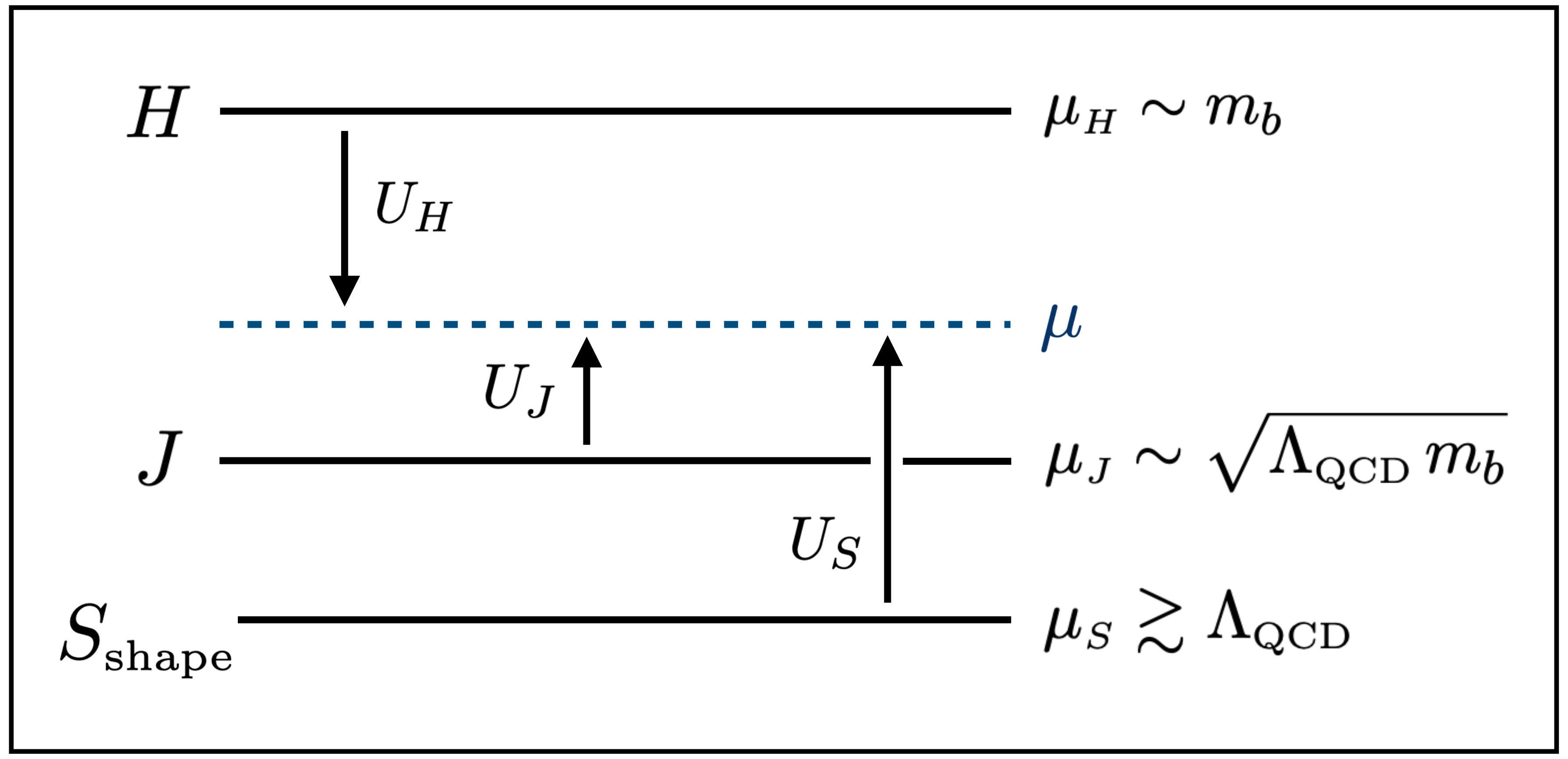
hard function: $\mu_H \sim m_b$

jet function: $\mu_J \sim \sqrt{\Lambda_{\rm QCD}\, m_b}$

soft function: $\mu_S \gtrsim \Lambda_{\rm QCD}$

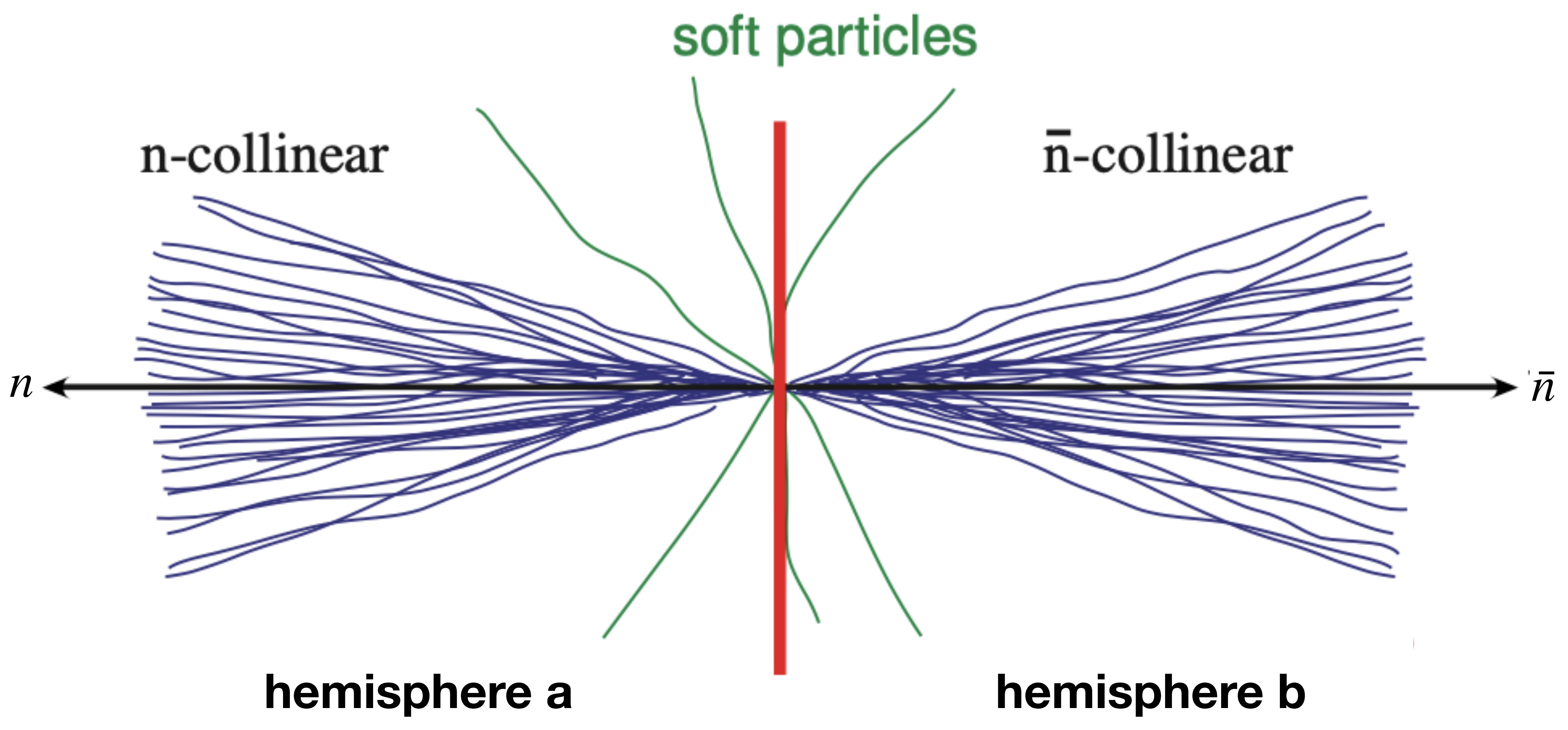


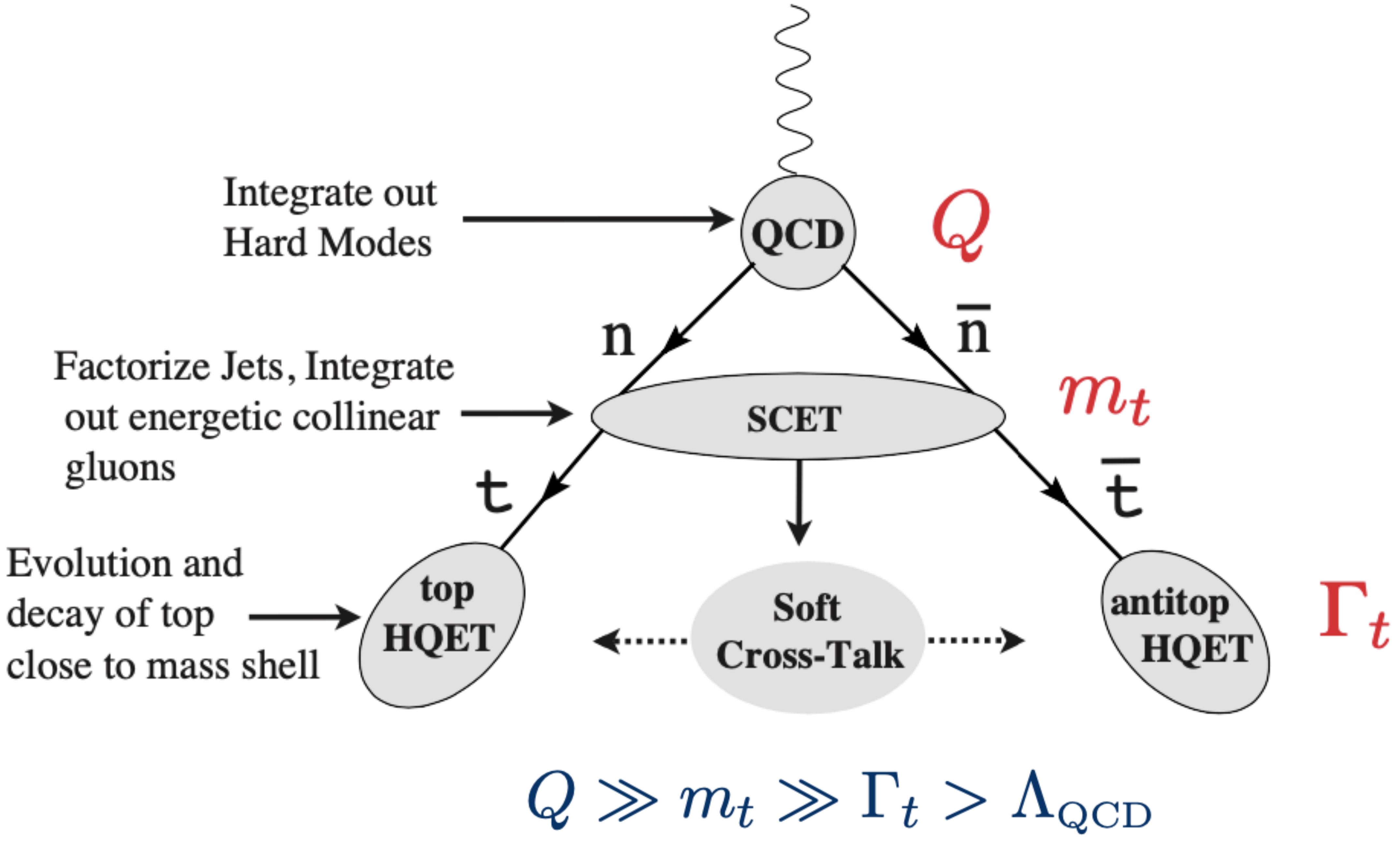
- Each sector depends only on a single physical scale → Large logarithms can be avoided.
- RGEs can be used to evolve the distinct sectors to a common scale.

Boosted top pair production in e^+e^- collisions

- Factorization approach for boosted top jet production in $e^+e^- \to t\bar{t}$ at c.m. energies $Q \gg m_t$.
  - Dijet region for factorization characterized by $s_{a,b} = M_{a,b}^2 - m_t^2 \ll m_t^2$.
  - Top state defined by measurements of $M_{a,b}$.

$$\frac{\mathrm{d}^2\sigma}{\mathrm{d}M_a^2\,\mathrm{d}M_b^2} \sim \sigma_0\, H_Q(Q,\mu) \int \mathrm{d}l\,\mathrm{d}l'\, J_t(s_a - Ql,\mu)\, J_{\bar{t}}(s_b - Ql',\mu)\, S(l,l',\mu)$$





[Fleming, Hoang, Mantry, Stewart: Phys.Rev.D77:074010 (2008)]

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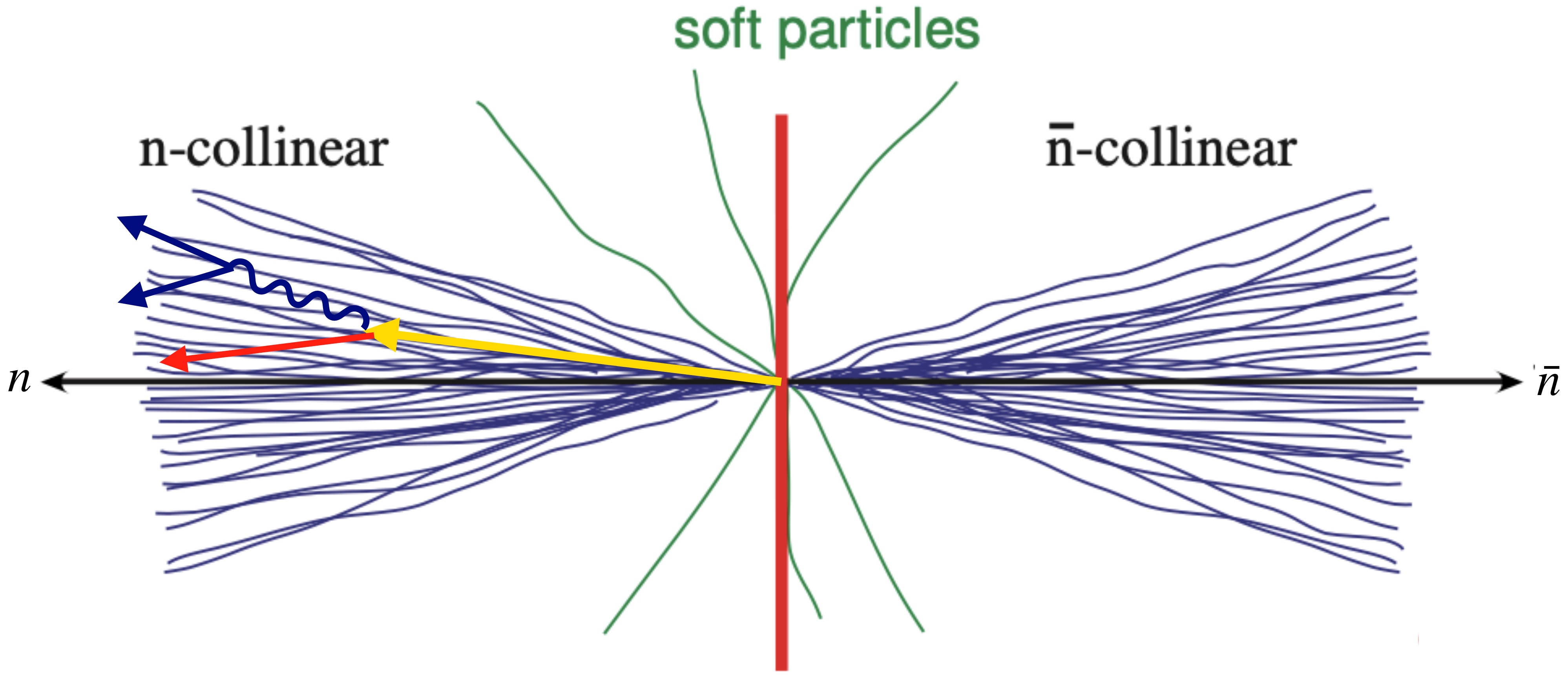
$$\frac{d^2\sigma}{dM_a^2\, dM_b^2} \sim \sigma_0\, H_Q(Q,\mu) \int dl\, dl'\, J_t(s_a - Ql, \mu)\, J_{\bar{t}}(s_b - Ql', \mu)\, S(l, l', \mu)$$



$$\frac{d^2\sigma}{dM_a^2\, dM_b^2} \sim \sigma_0\, H_Q \left[J_t \otimes J_{\bar{t}} \otimes S\right]$$

$$J_t \sim H_m \left[J_b \otimes S_{ucs}\right]$$

inclusive → differential

- Combination of factorization approaches for top quark production in e^+e^- collision with factorization methods known from heavy quark decays.
  - Examination of decay sensitive observables.
  - Treatment of finite lifetime effects and of the dynamics of the top quark decay products.
  - Study of effects at kinematic endpoint regions.

- Important application: Gauge invariant off-shell top quark decay.

Factorization approach for top jet function

- Starting point: $$J_t \propto \frac{1}{N_c\, m_t} \sum_X (2\pi)^3\, \delta^{(4)}(p - P_X)\, \mathrm{Tr}\Big[\langle 0 |\, \frac{\bar{n}\!\!\!/}{4}\, (W_n^\dagger h_v)(0)\, |X\rangle \langle X|\, (\bar{h}_v W_n)(0)\, |0\rangle \Big]$$

Insert decay operator into top jet function. → Match QCD current onto heavy-to-light EFT current. → Factorize momentum modes into distinct sectors.

$$\mathcal{O} = \big(\bar{b}\gamma^\mu P_L t\big)\big(\bar{\nu}_\ell \gamma_\mu P_L \ell\big)$$

$$\mathcal{J}^\mu \sim \sum_i C_i \left(\bar{\chi}_{n'} \Gamma_i^\mu Y_{n'}^\dagger h_v\right)$$

$\hat{s}_a \equiv s_a/m_t$

$$(s_{a,b} = M_{a,b}^2 - m_t^2 \ll m_t^2) \qquad \frac{\mathrm{d}^3 J_t(\hat{s}_a)}{\mathrm{d}E_\ell\, \mathrm{d}q^2\, \mathrm{d}h^2} \sim H(E_\ell, \mu) \int \mathrm{d}r^+\, J_b(r^+, \mu)\, S_{ucs}(\hat{s}_a - r^+, \mu)$$

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Known from semi-leptonic B decays

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“bHQET ultracollinear-soft function”

- New ingredient
- Can be computed perturbatively.
- Top width acts as infrared cut-off.
- Describes Fermi motion of the decaying top in the measured state with mass M_a.

$$S_{ucs}(r^+) \propto \int \mathrm{d}^4 y_1 \int \mathrm{d}^4 y_2 \sum_{X_{uc}} \delta^{(4)}(k_{uc}^+ - r^+)$$
$$\times \, \mathrm{Tr}\Big[\langle 0| (\bar{h}_v Y_{n'})(0) (W_n^\dagger h_v)(y_2) |X_{uc}\rangle \langle X_{uc}| (\bar{h}_v W_n)(y_1) (Y_{n'}^\dagger h_v)(0) |0\rangle \Big]$$

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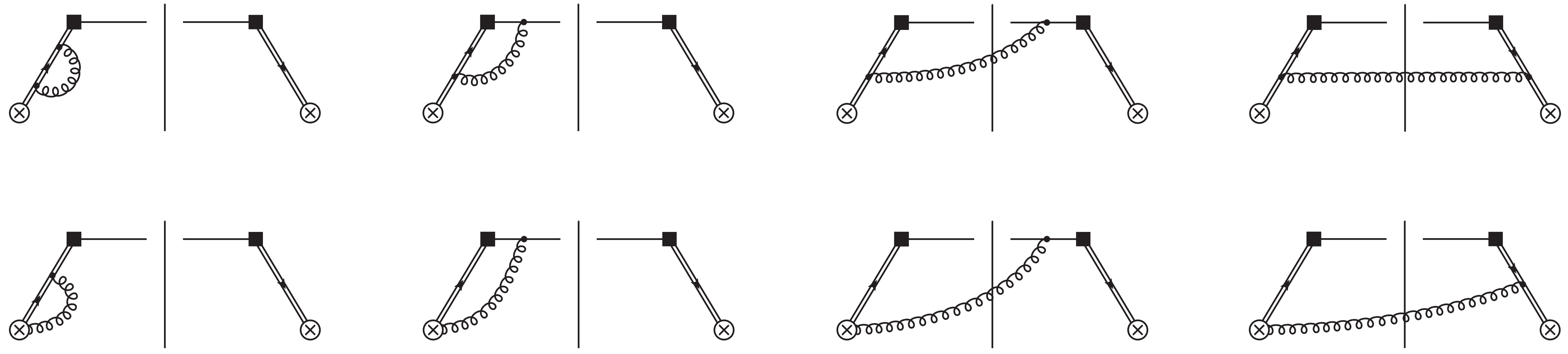

Summary & Outlook

Summary:

- Factorization is an important theoretical tool for calculations of processes involving hadrons.
  - Allows to factorize cross sections and decay rates into different parts that can either be calculated perturbatively or determined from data.
  - Sums large logarithms arising in perturbative calculations by means of RG equations.
- We aim to combine properties of the NW limit and off-shell computations for decaying top quark studies.
  - Merge existing factorization theorems for off-shell top production in e^+e^- collisions and for heavy quark decays.
  - Top state defined by measurements (and not by NW limit).
  - Leads to a gauge-invariant jet function for boosted top quarks including off-shell effects (up to leading order in m_t/Q).
  - Allows to study decay sensitive observables beyond the commonly employed NW limit.

Outlook:

- We want to study the NLO QCD corrections for off-shell and boosted top decays in the endpoint region.
  $\longrightarrow$ Analysis of our factorized approach with NLL resummation.
- Comparison with predictions from Monte-Carlo event generators.