



Dark Matter on the Lattice

SIMPs in an $Sp(4)$ dark sector

FAKT Workshop 2024

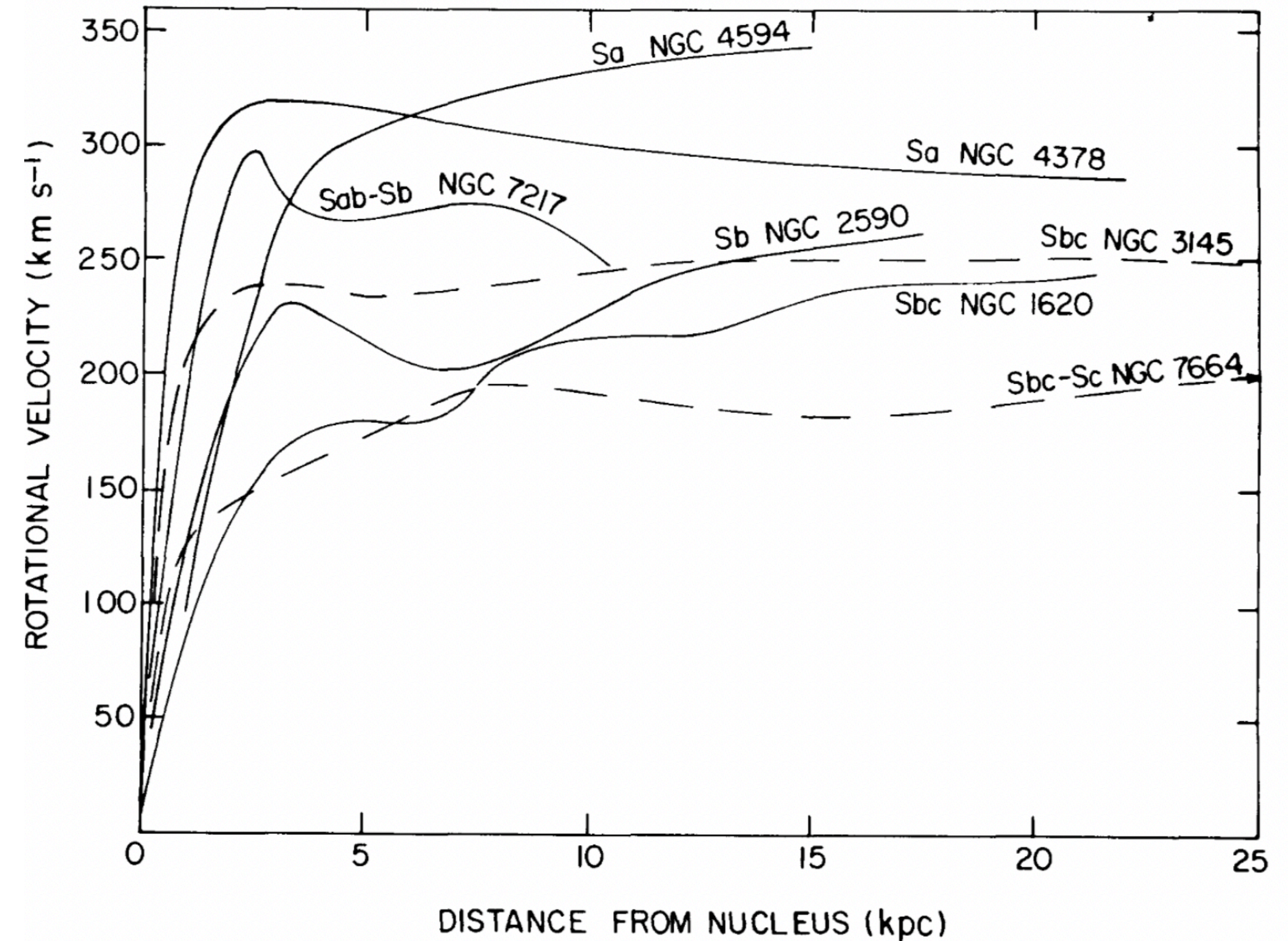
Particle Physics Retreat

Yannick Dengler, 23.2.23

With Axel Maas und Fabian Zierler

Dark Matter

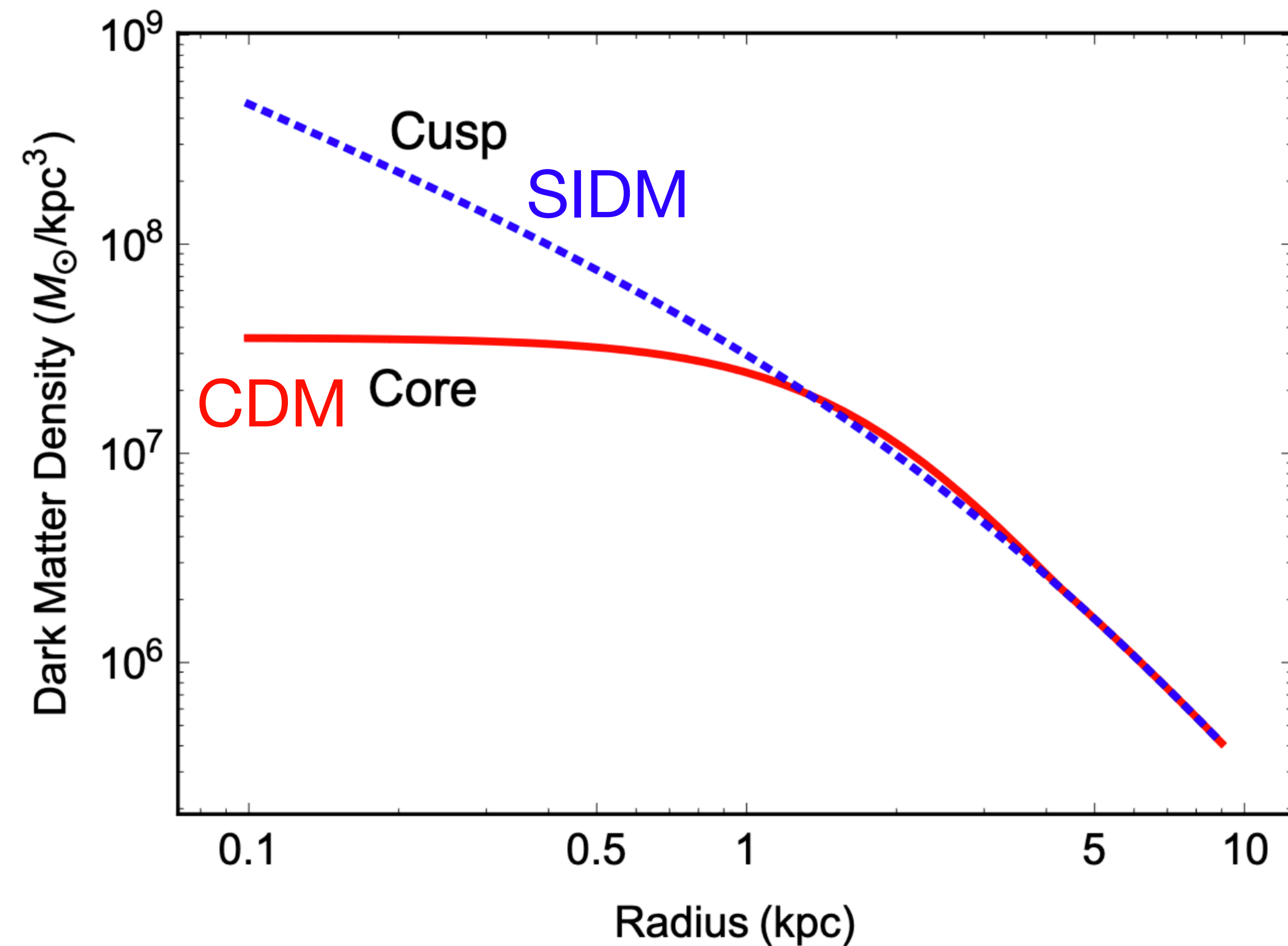
- Collection of phenomena beyond the standard model
 - Rotation curves, structure formation, etc.
- Possible explanations:
 - Modified gravity
 - Non observable form of matter
 - Particle beyond the SM



Rubin et al.: Ap.J.L. 225 (1978)

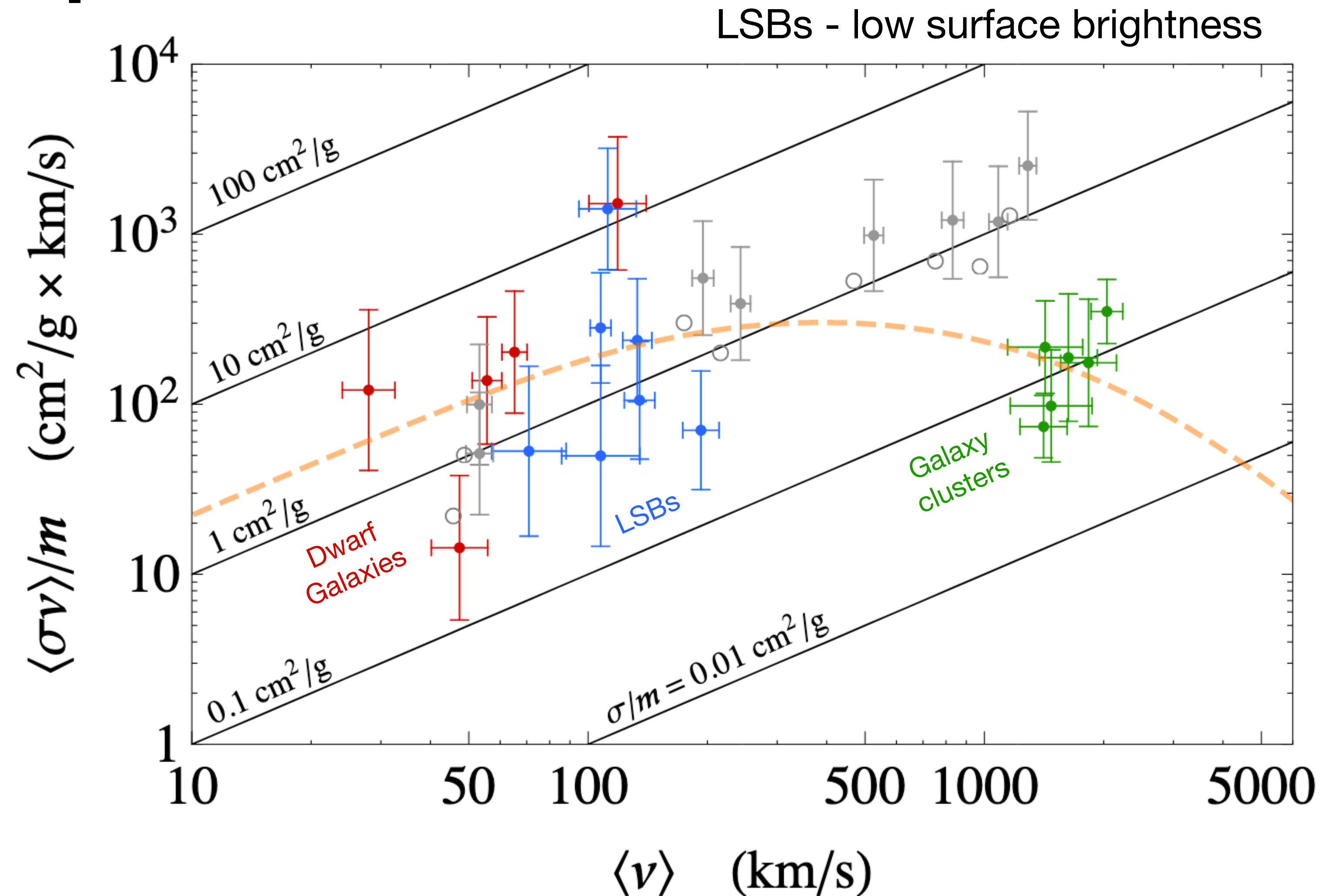
Dark Matter - Self-interaction

- Observations are in conflict with cold dark matter (CDM) models
 - "cusp vs. core", "too big to fail", etc.
- Possible solution:
 - Self-interacting dark matter (SIDM)
- Constraint by "bullet cluster"



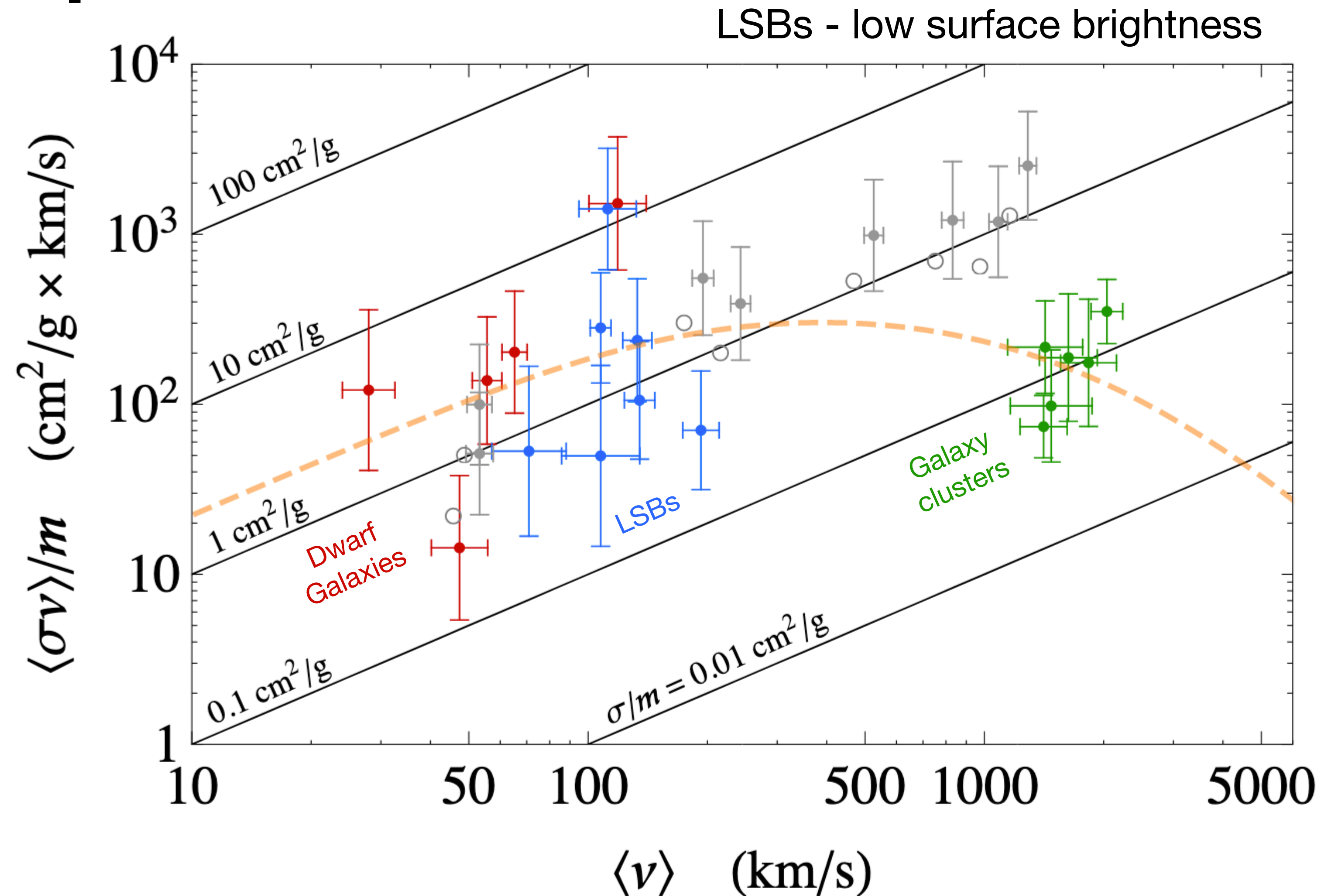
Dark Matter - Velocity-dependence

- "Dark halos as particle colliders"
- Cross-section from shape of halos
- Results prefer velocity-dependent cross section



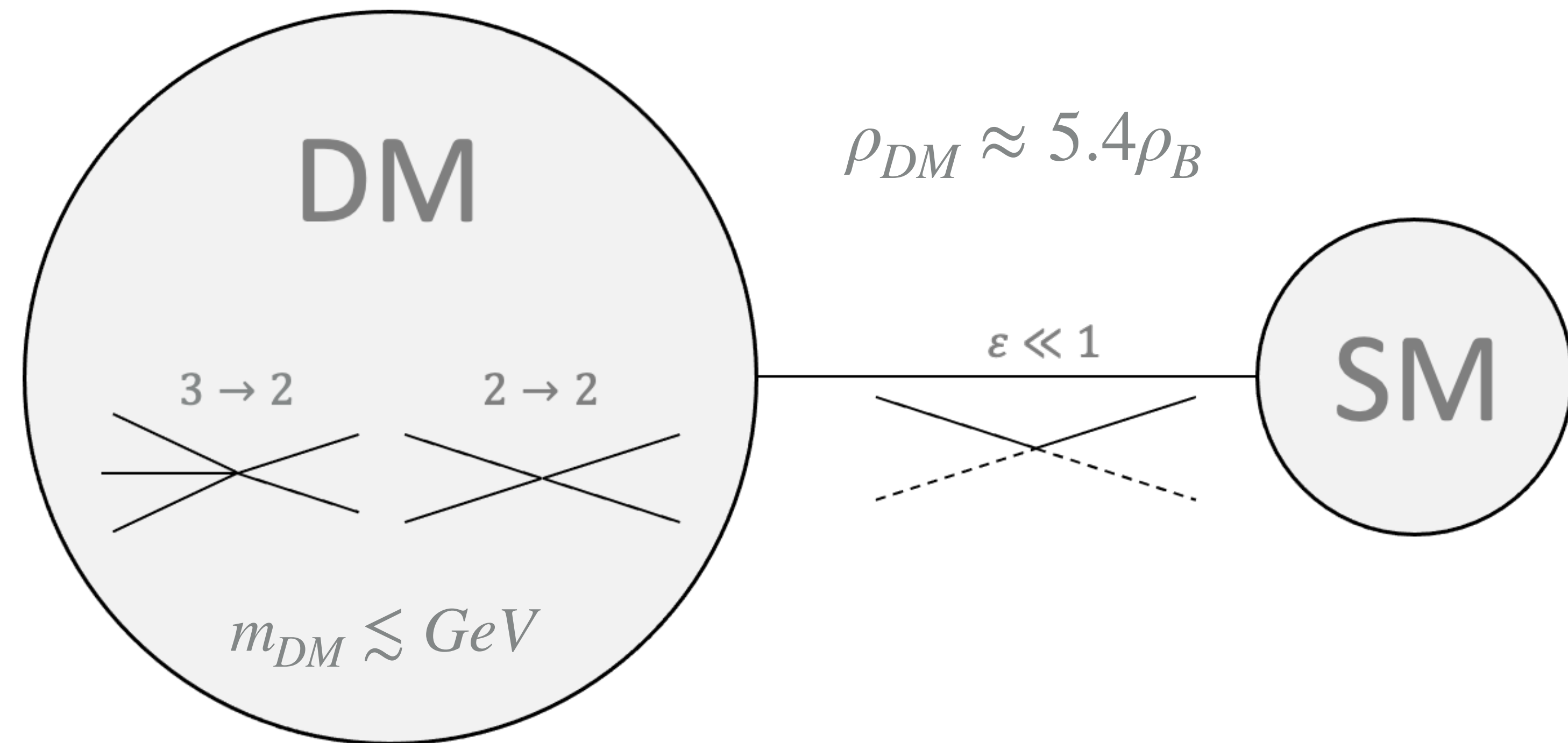
Dark Matter - Velocity-dependence

- "Dark halos as particle colliders"
- Cross-section from shape of halos
- Results prefer velocity-dependent cross section
- This work:
 - Blueprint: How to compare lattice results to this



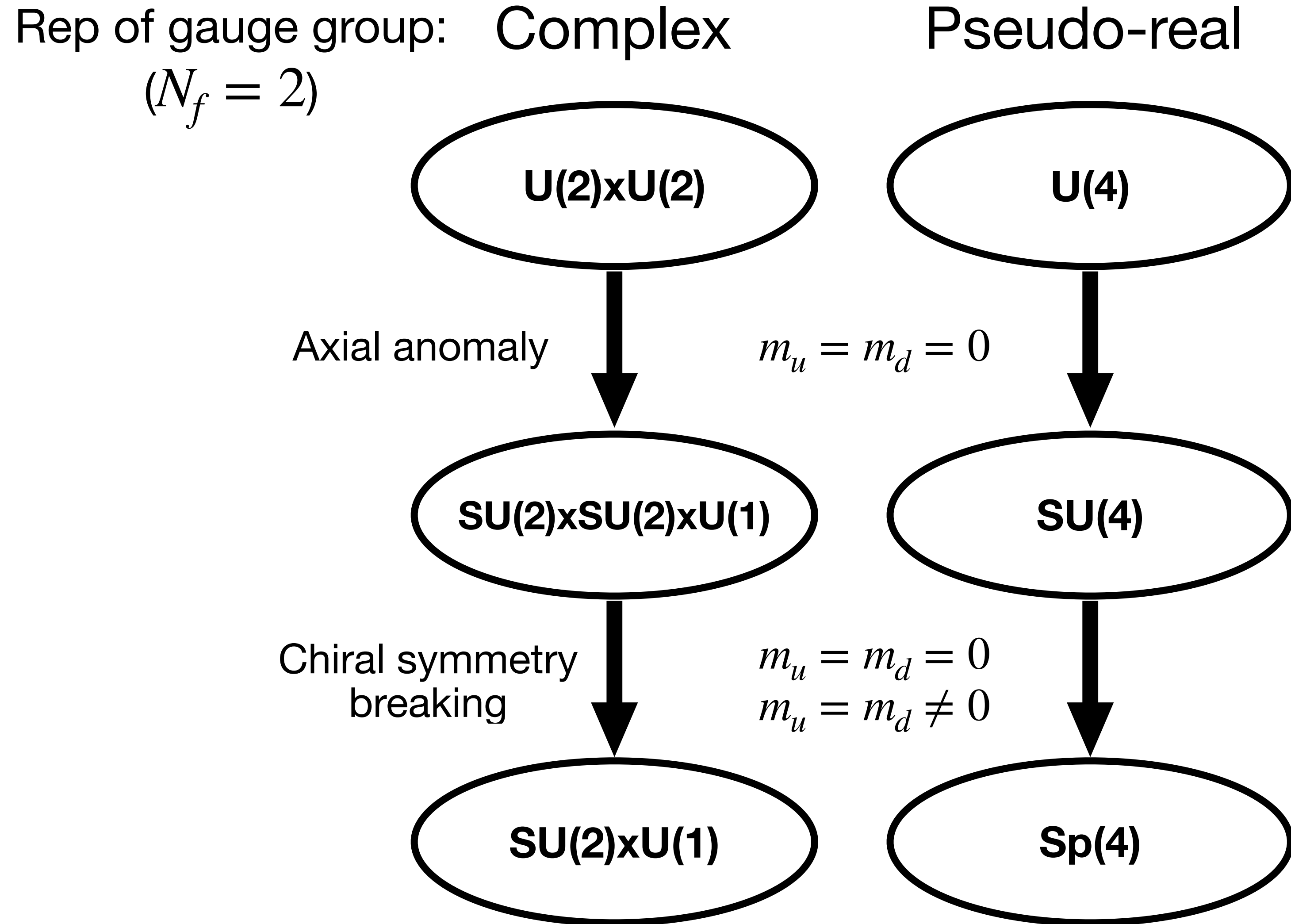
Dark Matter - SIMP

- One possible realization of SIDM
 - DM as a thermal relic of the early universe via freeze-out
- Number lowering process in the dark sector
 - Heat up of DM
- Heat flow from DM to SM via coupling
 - Mediator enables direct detection



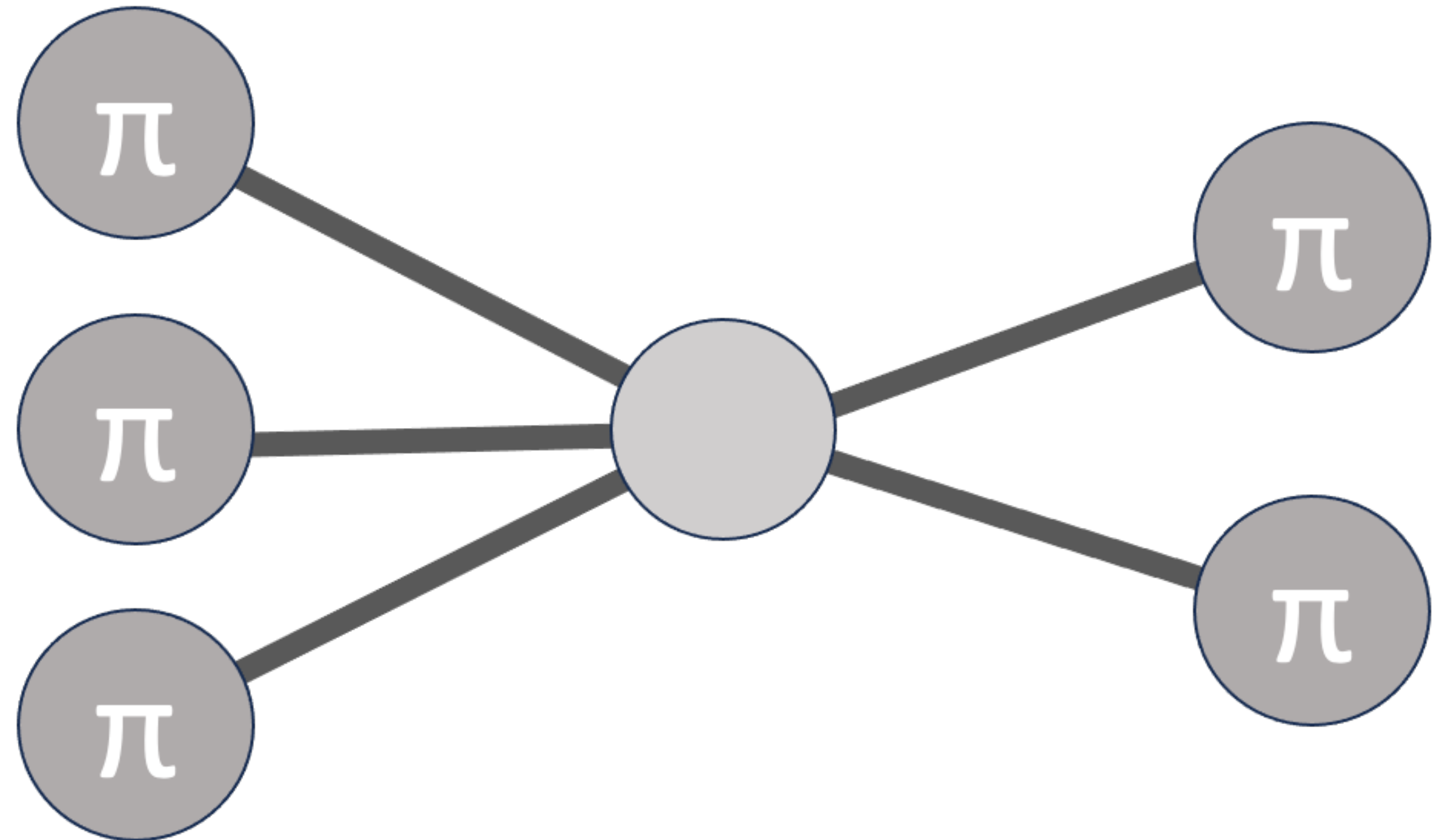
Minimal realisation

- $N_f = 2$ fundamental fermions in pseudo-real representation of gauge group
- Enlarged flavour symmetry
- Result: 5 pNGBs
- 3 → 2 process possible



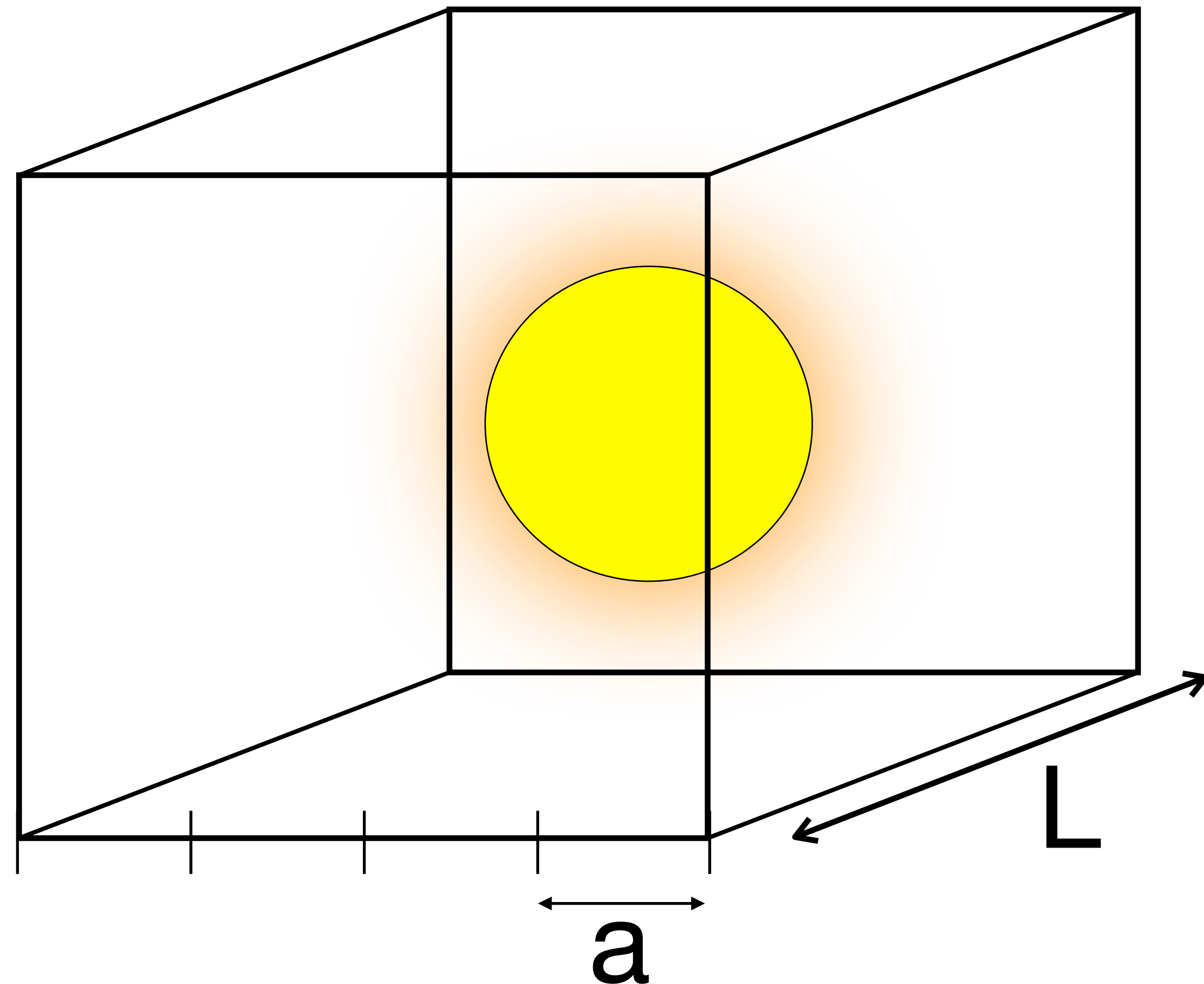
$Sp(4)$ gauge with $N_f = 2$

- "Zoo" of dark particles:
 - 5 "dark" Pions
 - 10 "dark" Rhos
 - and more
- Even number of colours:
 - No fermionic bound states



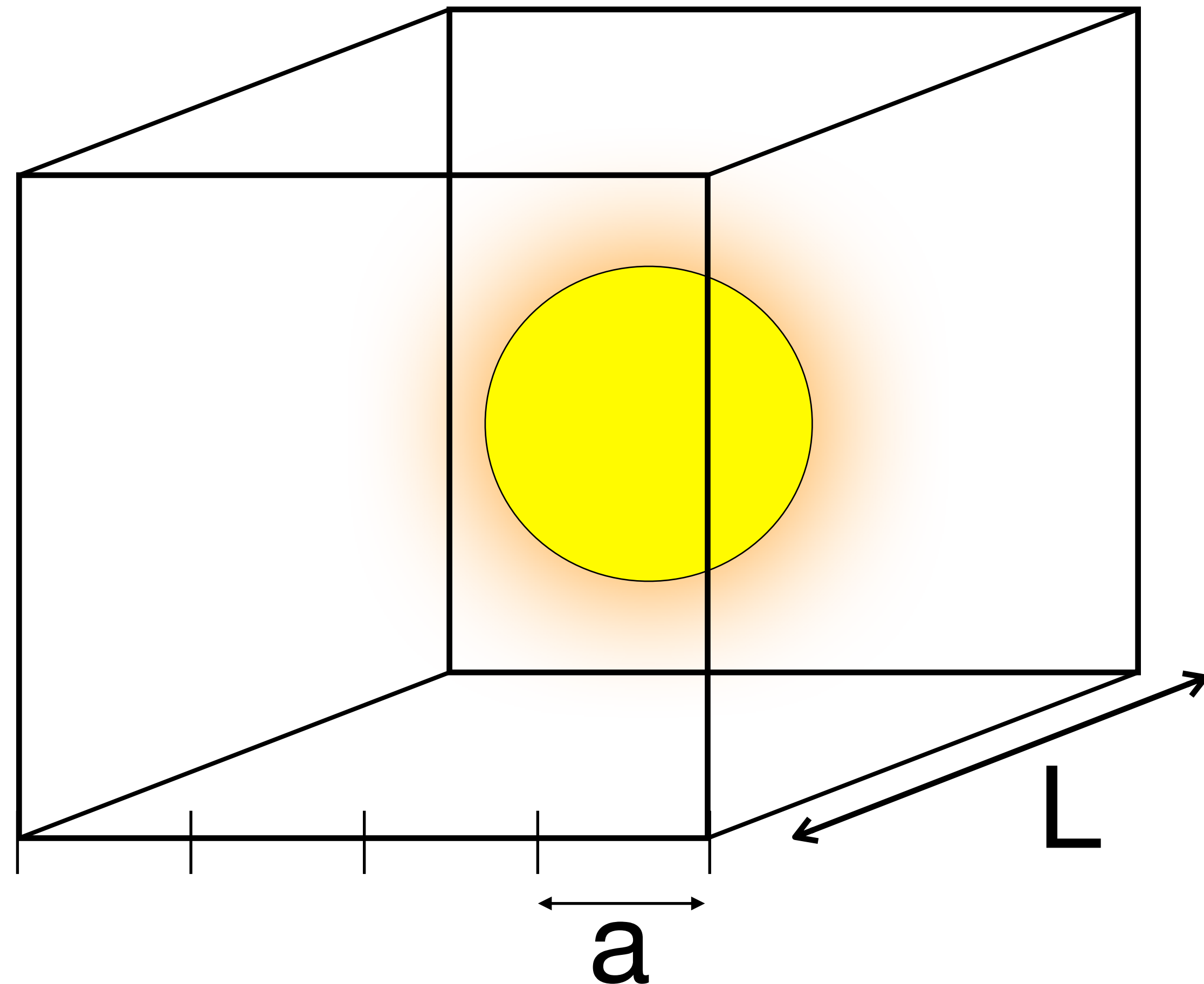
Lattice

- Sample gauge configurations in a discretized space-time
- Challenges:
 - IR and UV cutoff because of a and L
 - Discretization artifacts
 - Finite volume effects



Lattice

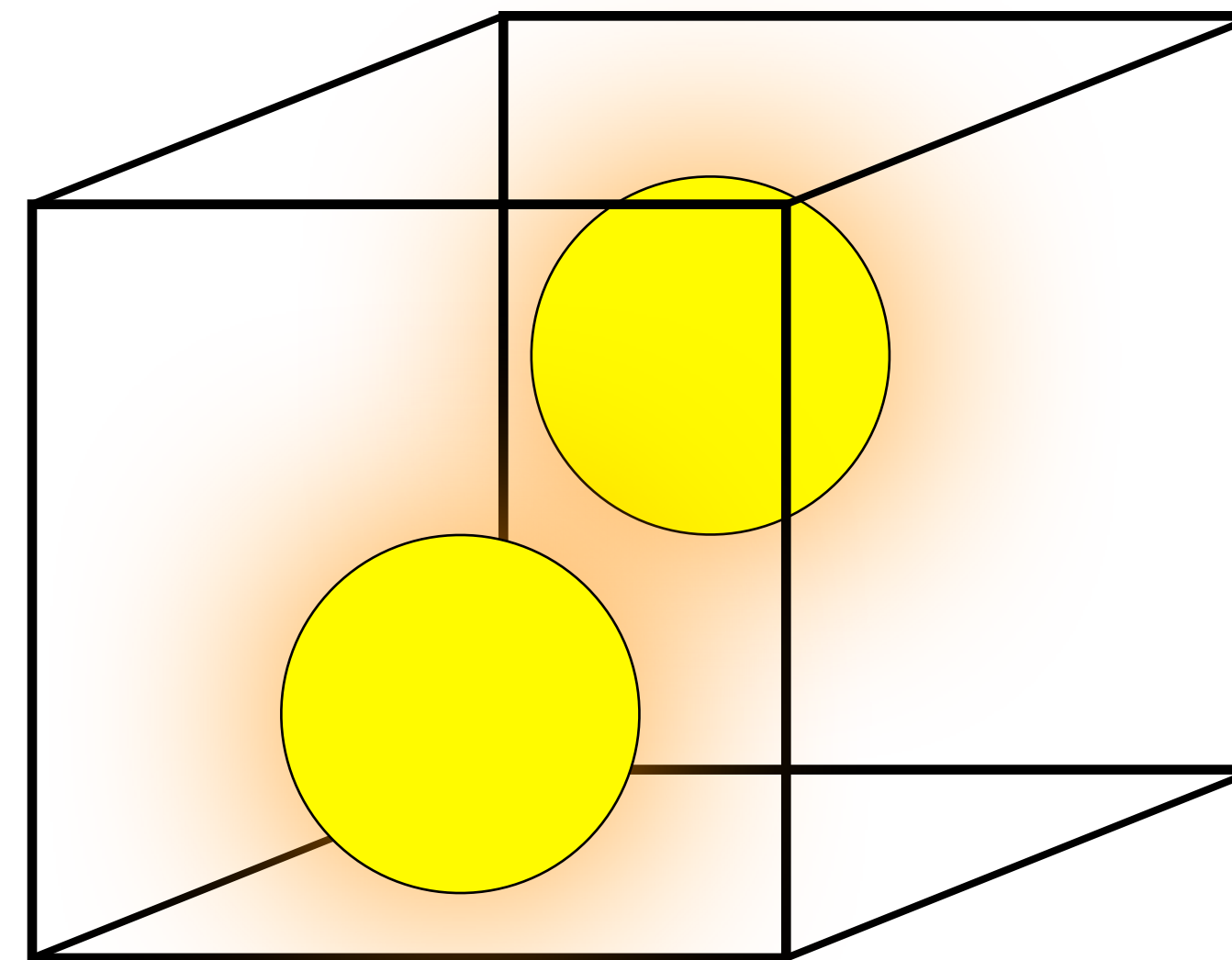
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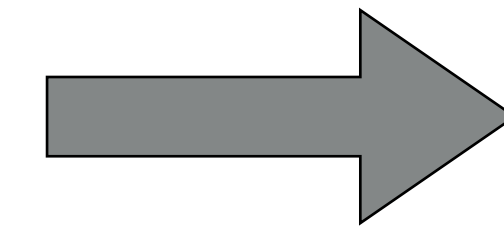
Lattice - Scattering

- Particles enclosed in a box
 - Energy levels are shifted in finite volume due to scattering effects
- Energy shift \leftrightarrow scattering properties

- $$\tan(\delta) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{L}_{00}^{\vec{0}}(1, q^2)}$$



Finite size


 \mathcal{M}

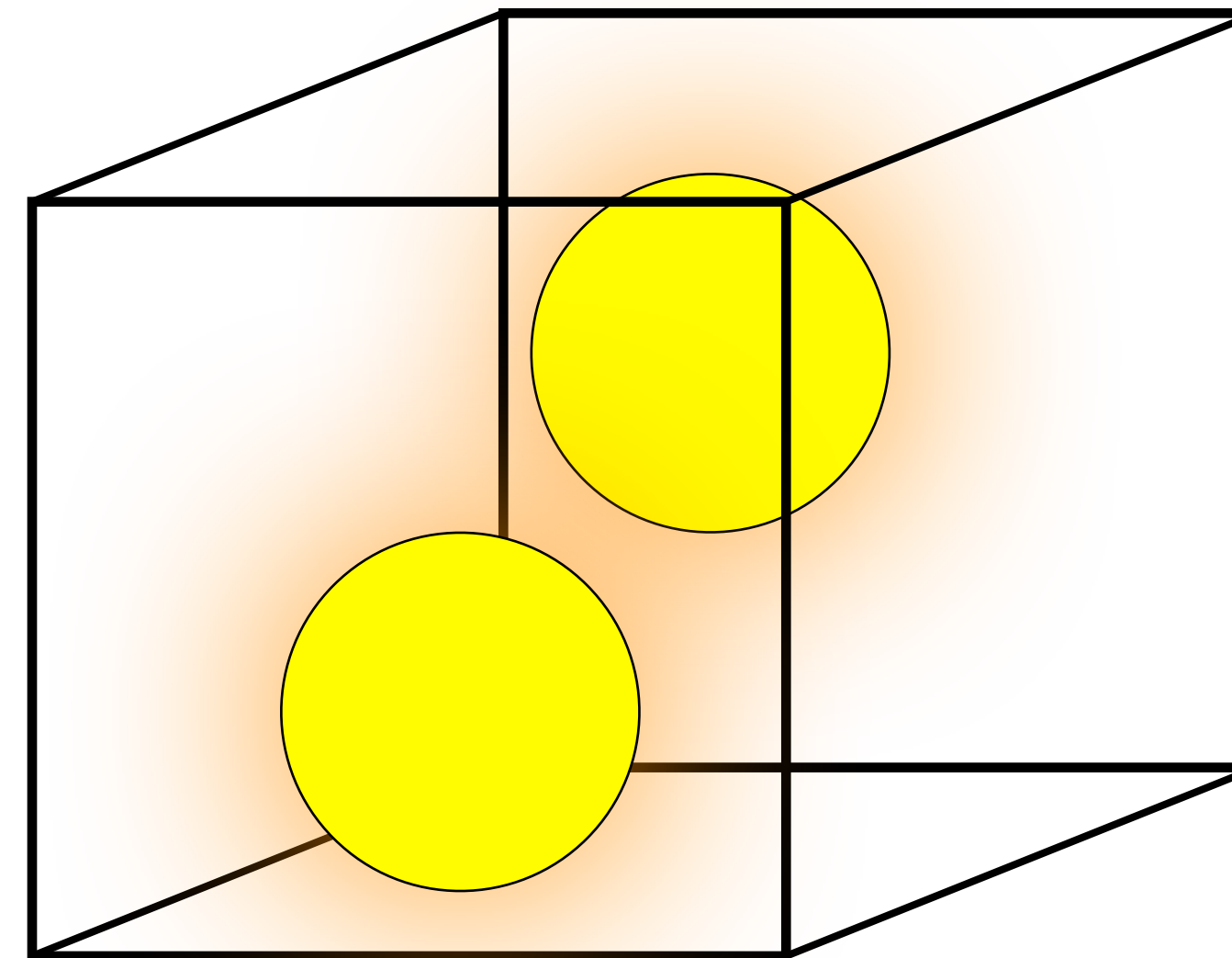
Infinite volume

Lattice - Scattering

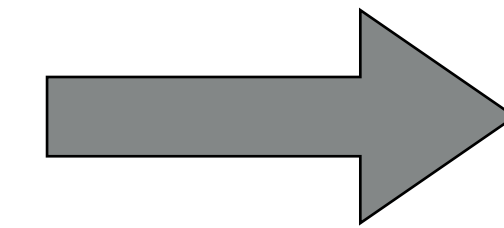
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← "Lüscher Zeta function"



Finite size



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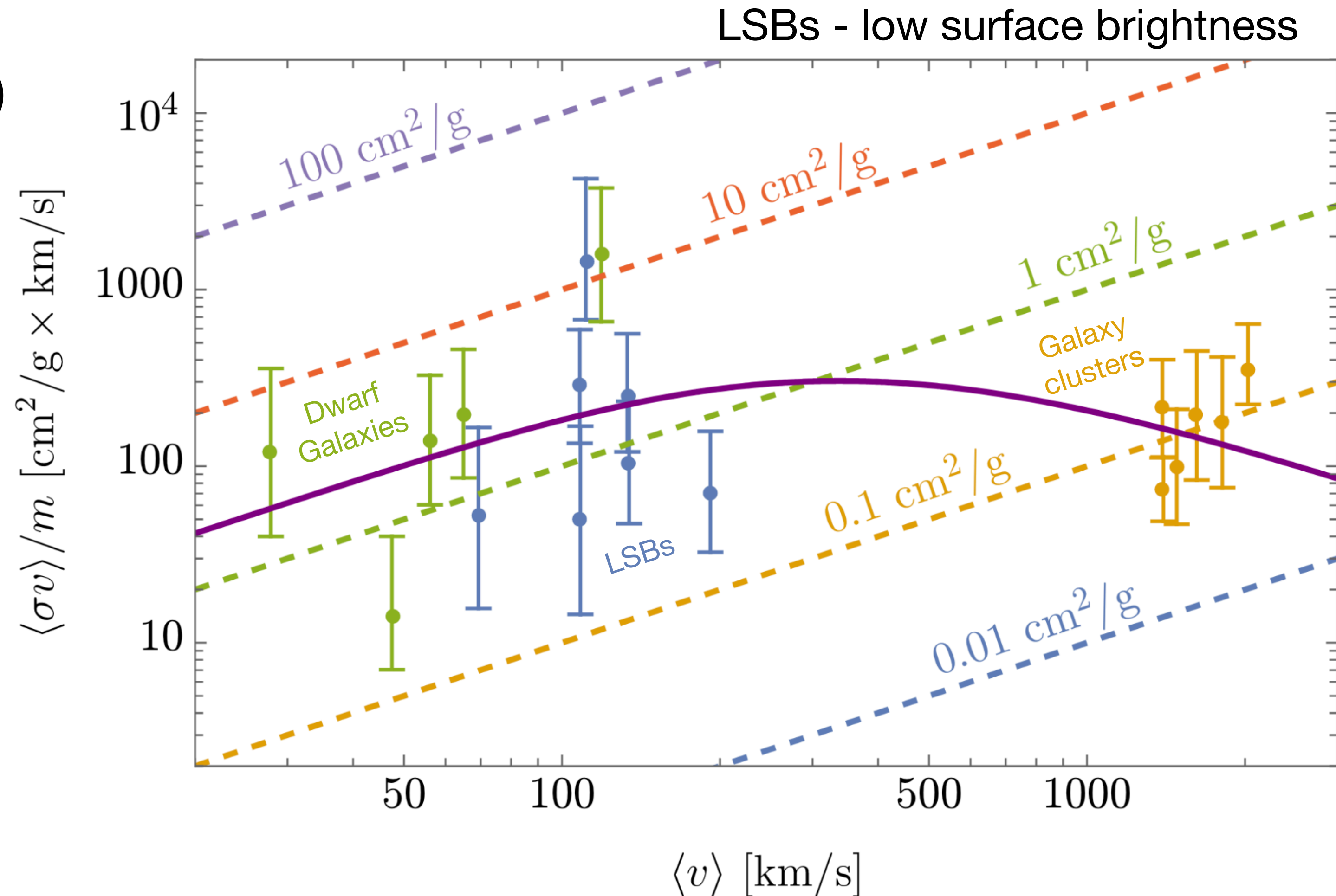
Infinite volume

Comparison to halo data

- Effective range-expansion (s-wave)

$$P \cot(\delta) = -\frac{1}{a} + \frac{P^2}{2r_e} + \mathcal{O}(P^4)$$

$$\frac{\langle \sigma v \rangle}{m} = \int_0^\infty v \sigma f_{MB}(v) dv$$



Comparison to halo data

- Effective range-expansion (s-wave)

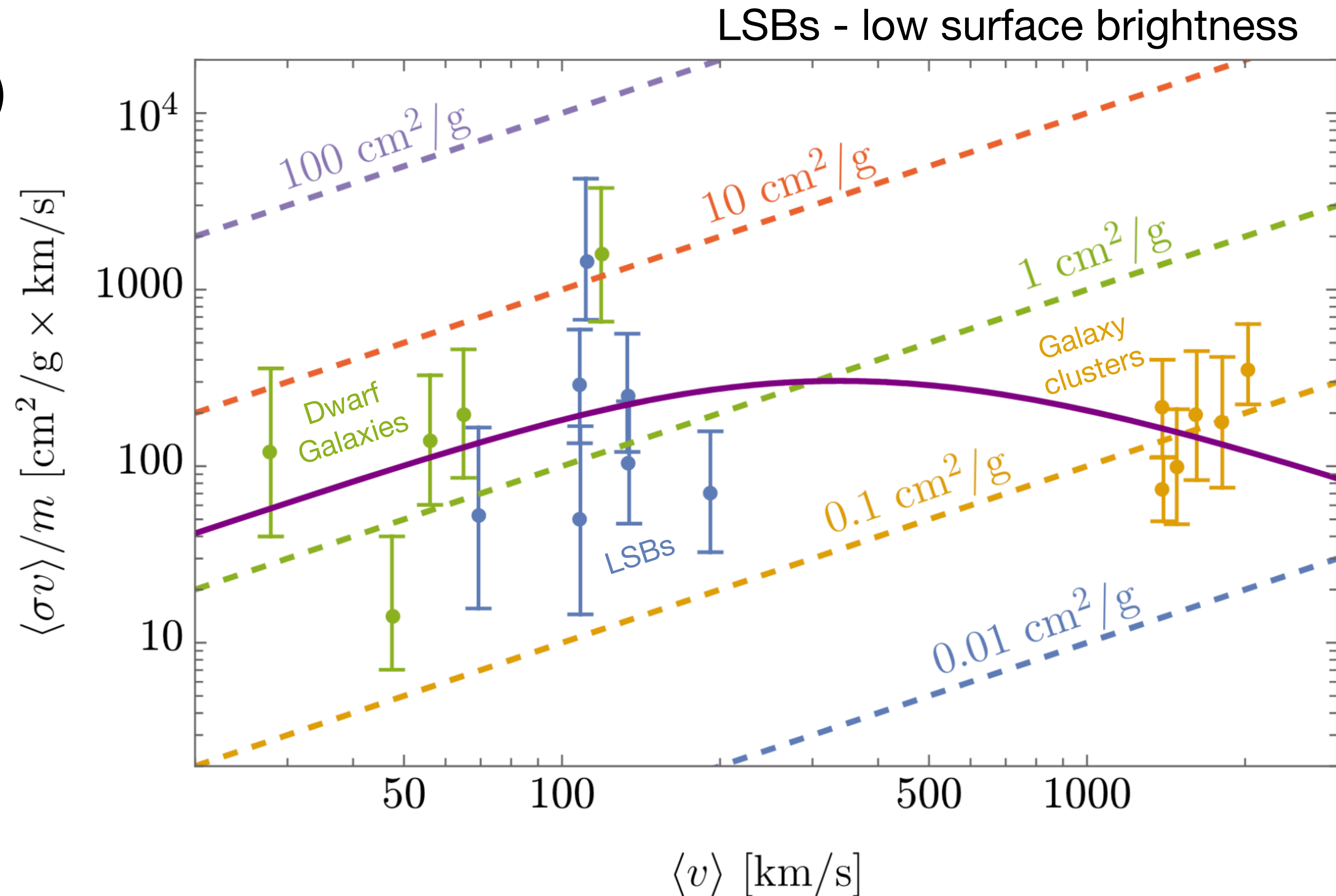
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$$\frac{\langle \sigma v \rangle}{m} = \int_0^\infty v \sigma f_{MB}(v) dv$$

$$\Rightarrow a = 22.2 \text{ fm}$$

$$\Rightarrow r_e = -2.59 \times 10^{-3} \text{ fm}$$

$$\Rightarrow m_{DM} = 16.7 \text{ GeV}$$

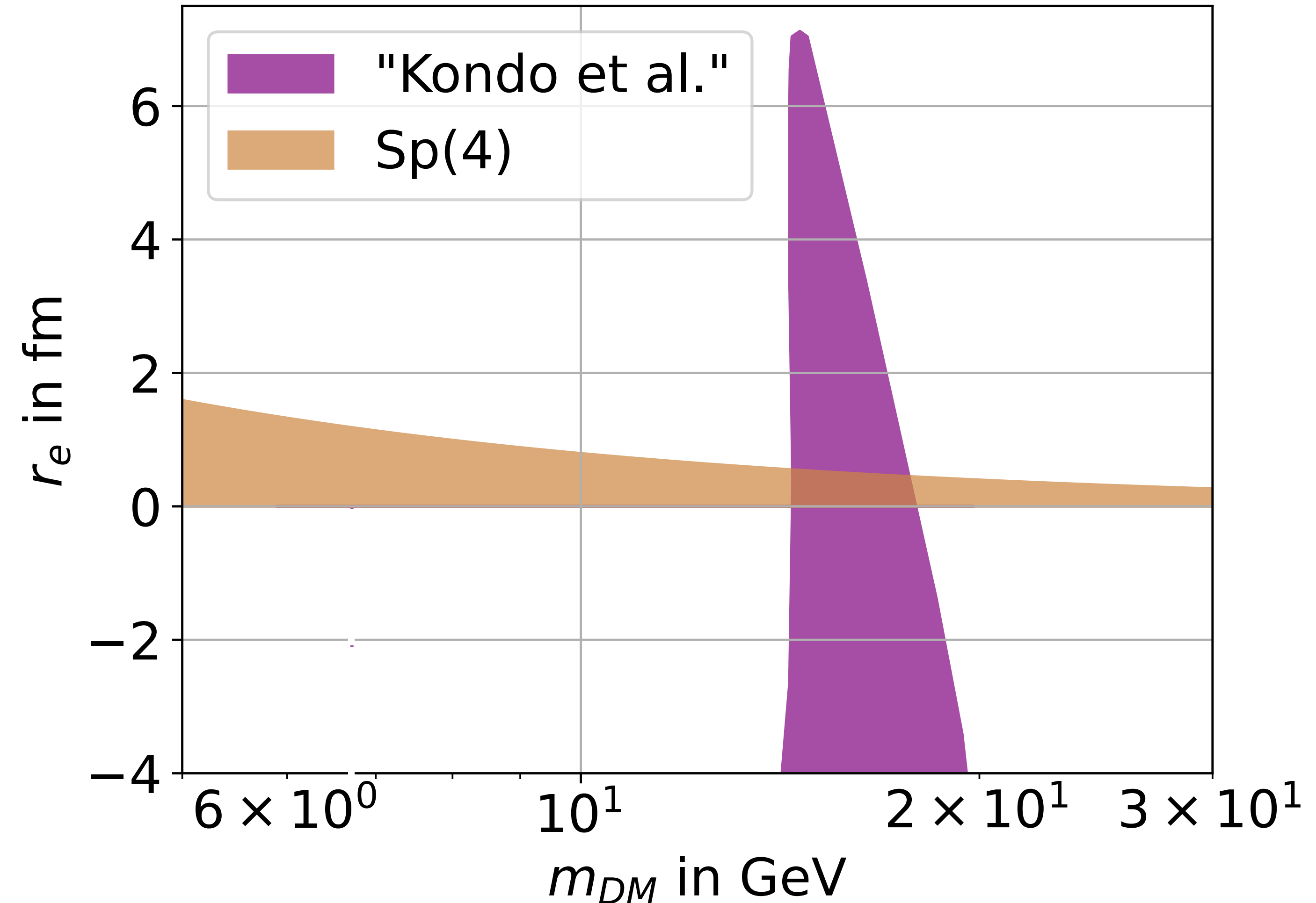


Comparison to halo data

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- Parameters do not agree

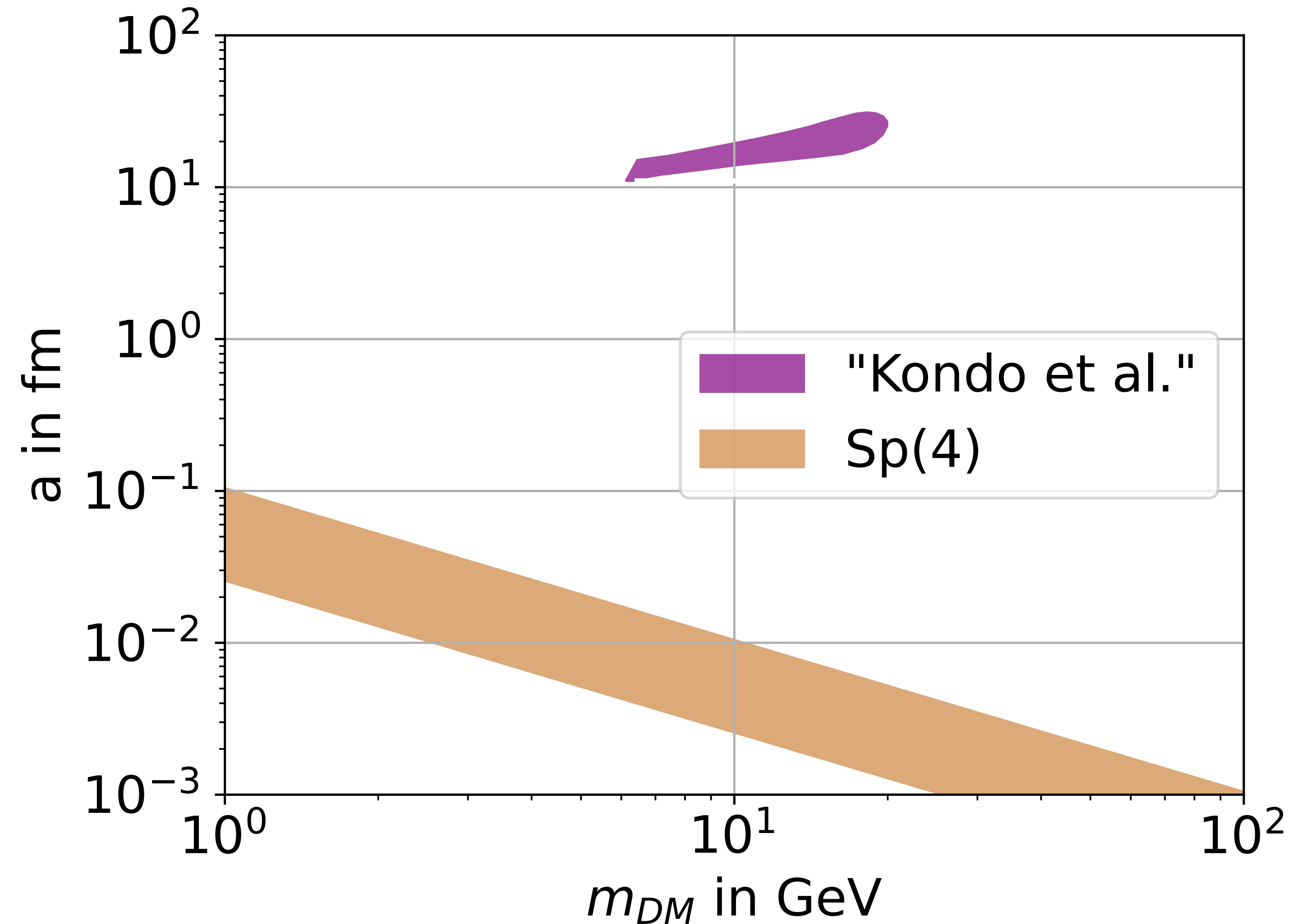


Comparison to halo data

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Kondo et al: J. High Energ. Phys. (2022)

Comparison to halo data

- Effective range-expansion (s-wave)

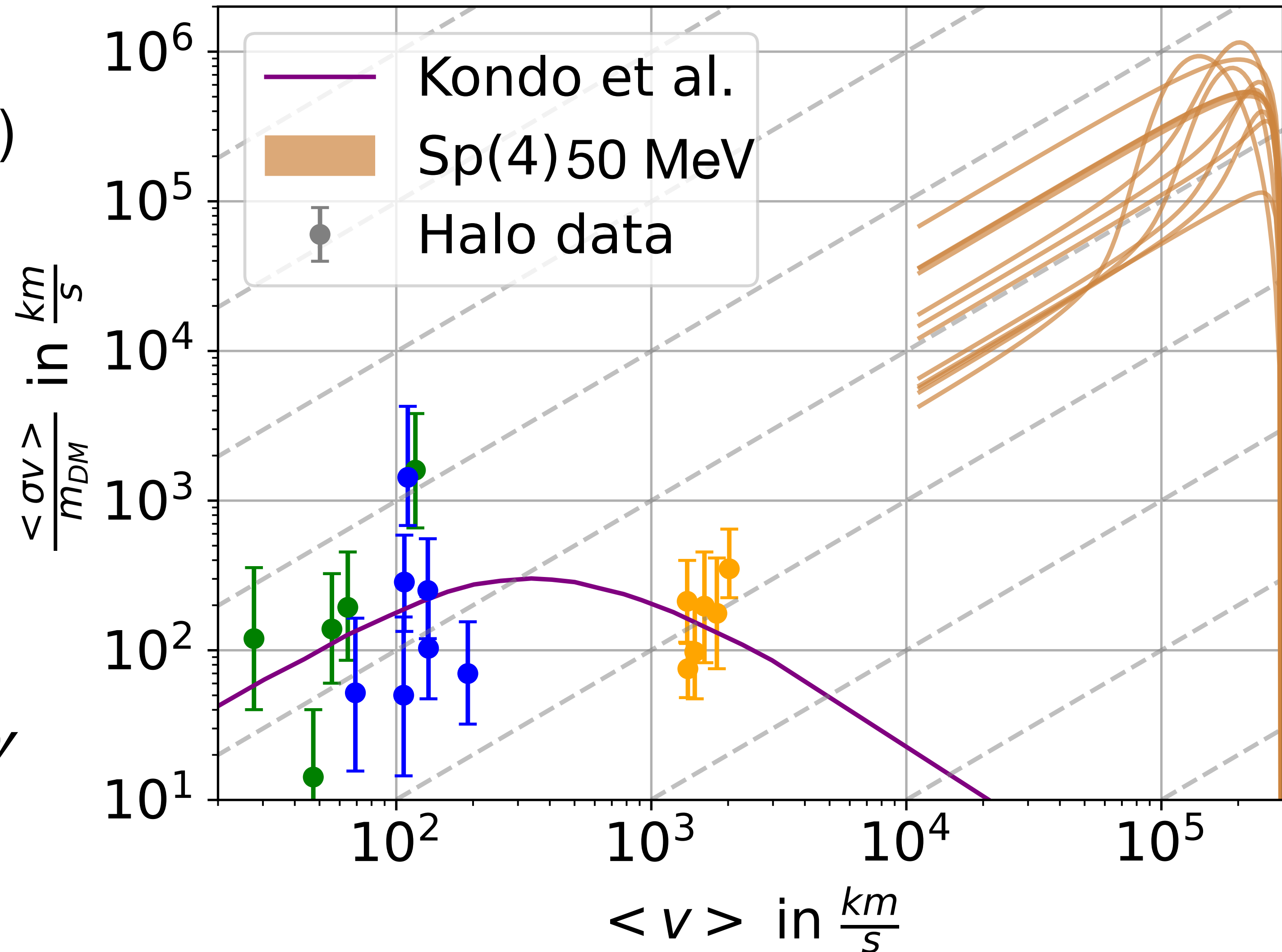
$$P \cot(\delta) = -\frac{1}{a} + \frac{P^2}{2r_e} + \mathcal{O}(P^4)$$

- Parameters do not agree

- Relativistic speeds:

$$\frac{\langle \sigma v \rangle}{m} = \int_1^\infty v(\gamma) \sigma f_{MJ}(\gamma) d\gamma$$

- $m_{DM} \approx 50 \text{ MeV}$



Outlook

- Lattice technicalities
- Provide low energy constants for "dark" χ -pT
- Full $\pi\pi\pi \rightarrow \pi\pi$ scattering cross section from the lattice

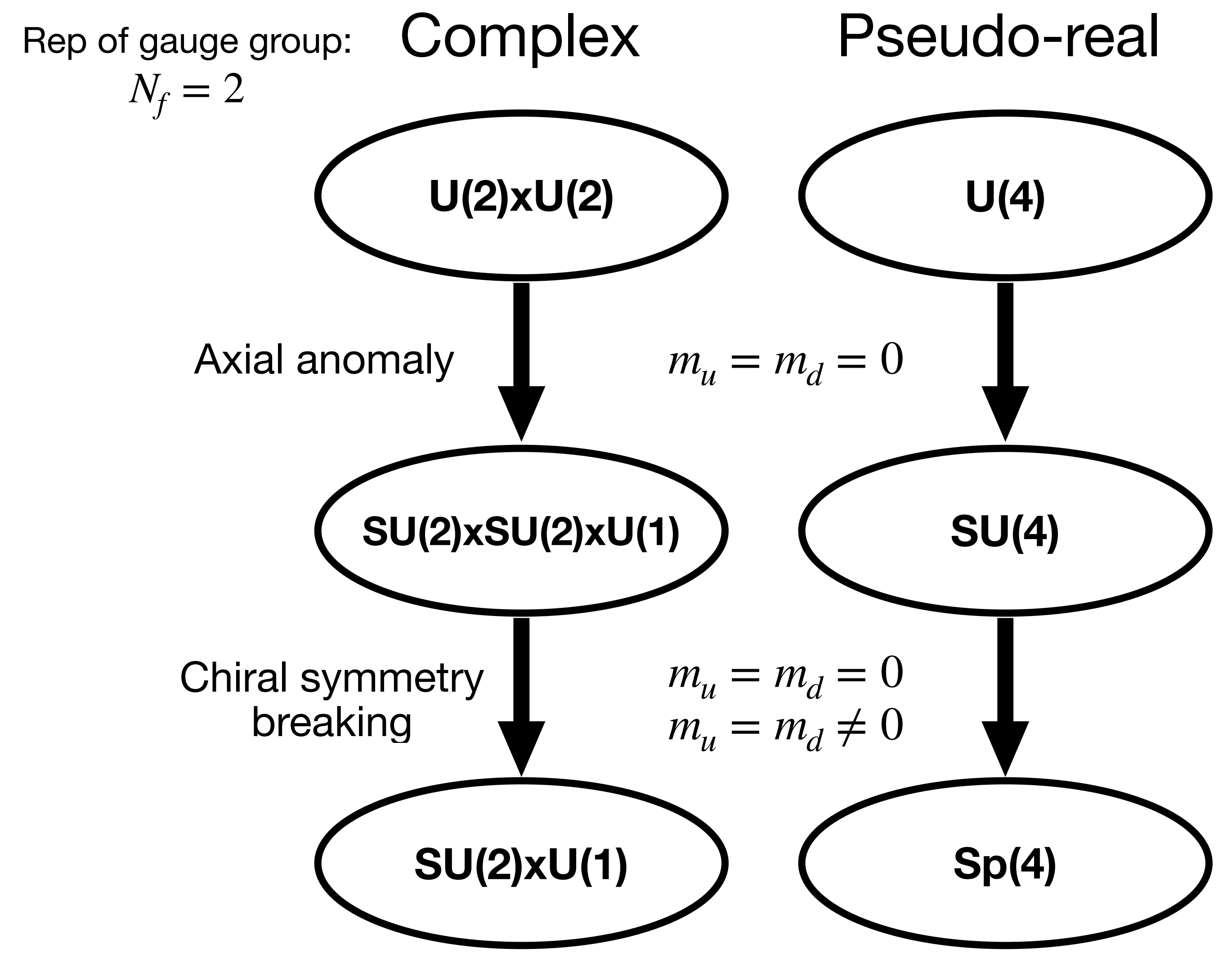
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Thank you!

Sp(4)_c vs. Sp(4)_f - clarification

- Symplectic groups always have an even dimension - Sp(2N)
- Flavour symmetry:
 - Needed for symmetry breaking pattern
- Gauge symmetry: Needed for the pseudo-real representation
 - Also SU(2) or Sp(6) for example possible



Comparison to halo data

- Effective range-expansion (s-wave)

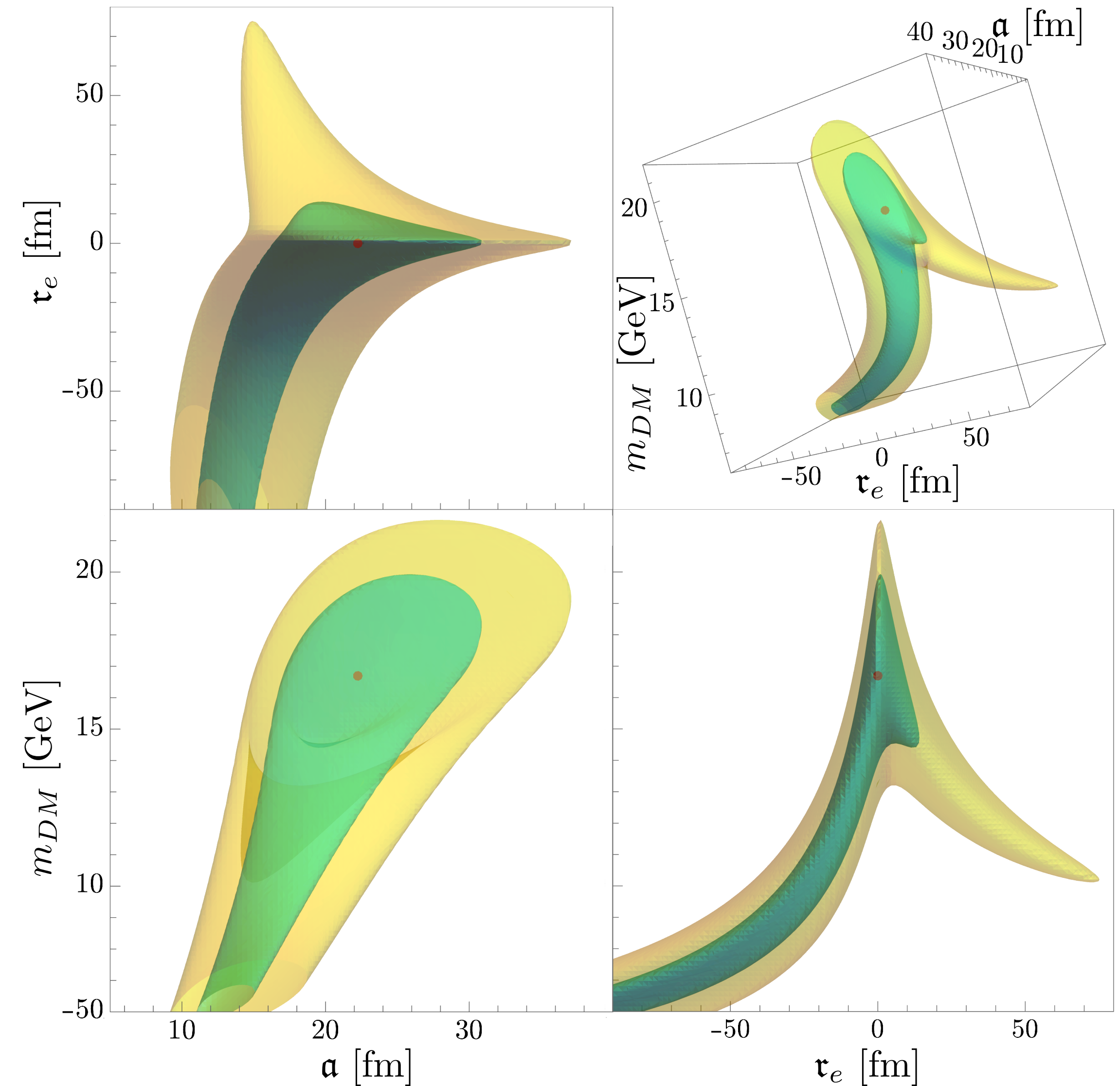
$$k \cot(\delta) = -\frac{1}{a} + \frac{k^2}{2r_e} + \mathcal{O}(k^4)$$

- Best fit:

- $a = 22.2 \text{ fm}$

- $r_e = -2.59 \times 10^{-3} \text{ fm}$

- $m_{DM} = 16.7 \text{ GeV}$

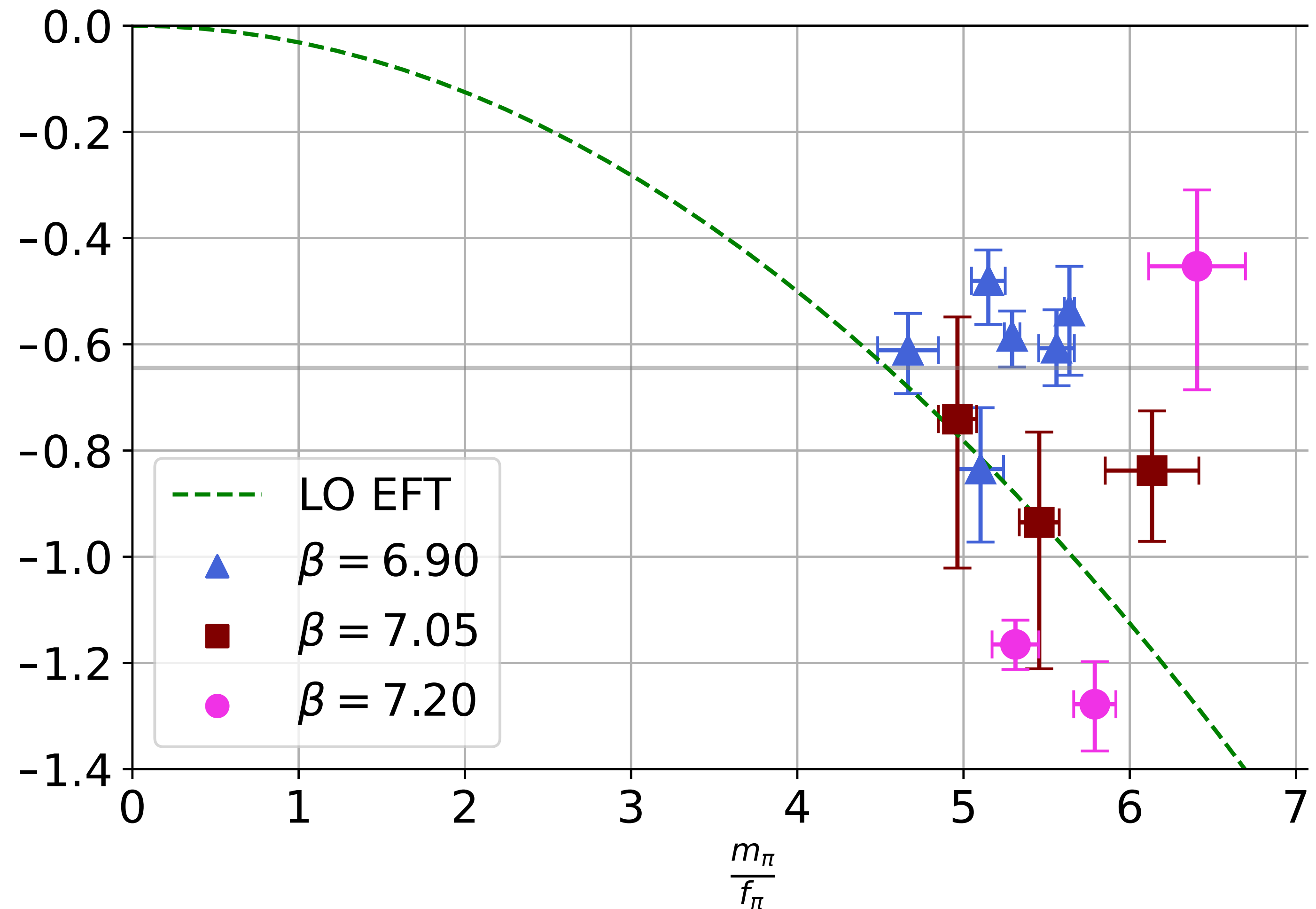


Comparison chiral perturbation theory

- $\chi p T$ prediction:

$$a_0 m_\pi = -\frac{1}{32} \left(\frac{m_\pi}{f_\pi} \right)^2$$

- Pion mass on edge or beyond validity



Comparison to astrophysical constraints

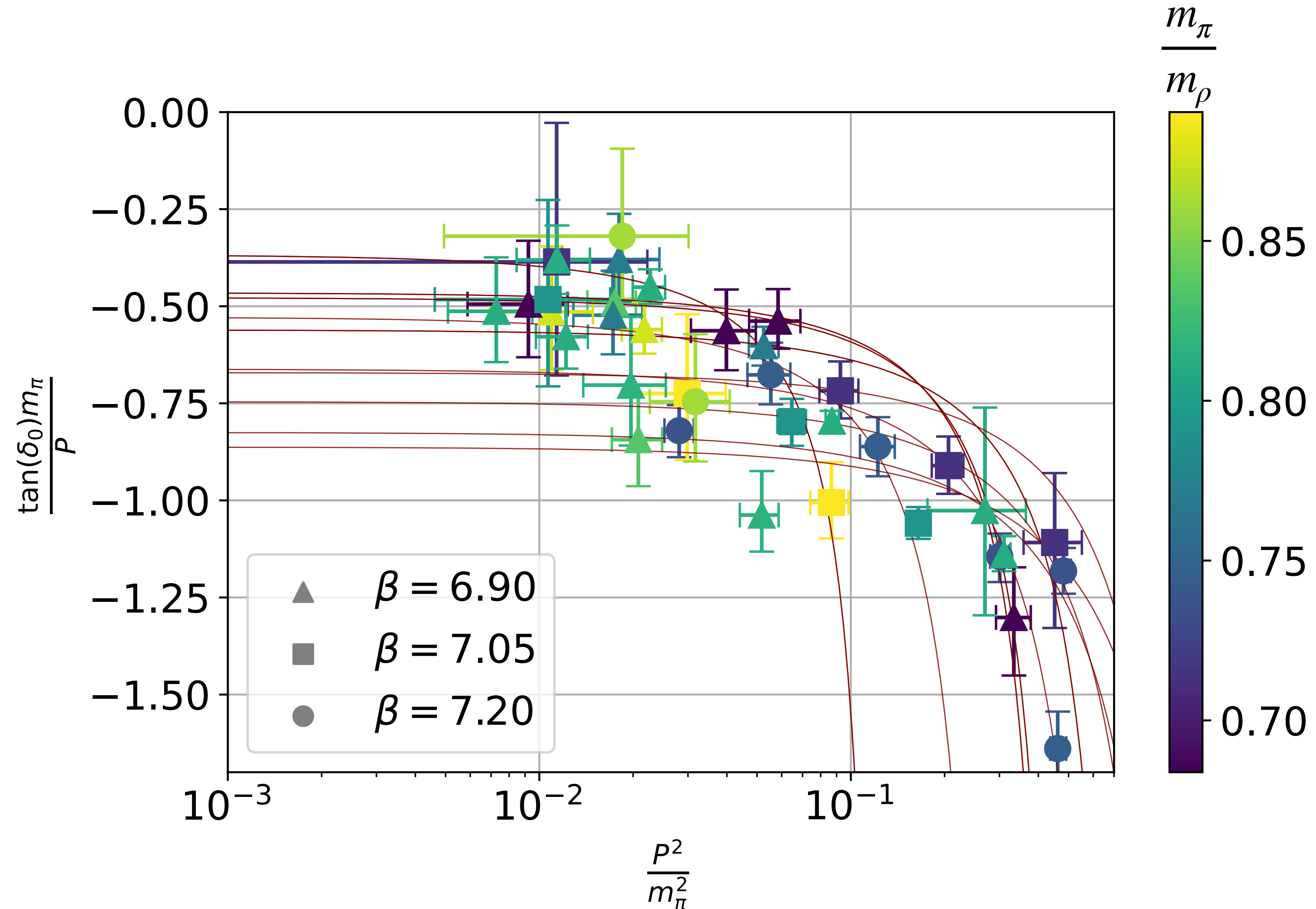
- Scattering length:

- $a_0 m_\pi = -0.65^{+0.2}_{-0.3}$

- $\frac{\sigma}{m} < 0.19 \frac{cm^2}{g}$

- Fixes the lattice constant

$\rightarrow m_{DM} > 115 MeV$



5 dark Pions

$$Sp(4)_f$$

- Pions form a 5-plet of the flavour symmetry
 - $\pi^+, \pi^0, \pi^-, \Pi_{ud}, \Pi_{\bar{u}\bar{d}}$
- What are the possible scattering channels?
- Tensor products of the corresponding representations
- 3 Isospin channels in $\pi\pi$:
- $l=0$ (1-dim), $l=1$ (10-dim), $l=2$ (14-dim)

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$

$$10 \otimes 5 = 5 \oplus 10 \oplus 35$$

$$5 \otimes 5 \otimes 5 = 3(5) \oplus 10 \oplus 30 \oplus 35$$

$$\pi\pi \rightarrow \pi\pi \quad (l=0,1,2)$$

$$\pi\pi \rightarrow \rho \quad (l=1)$$

$$\pi\pi \rightarrow \pi\pi\pi \quad (l=1)$$

$$\pi\pi \rightarrow \pi\pi\rho \quad (l=0,1,2)$$

etc.

Phenomenology of scattering channels

$$Sp(4)_f$$

- $l=2$ (14-dim):
 - (Probably) contributes most to $\pi\pi$ -scattering
 - 14 out of 25 possible combinations of Pions
- Considered in this talk

$$5 \otimes 5 = 1 \oplus 10 \oplus 14$$

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etc.

Phenomenology of scattering channels

- $l=0$ (1-dim):
 - (Probably) no large contribution to $\pi\pi$ -scattering
 - Mixing with the „singlet“
 - Numerically challenging („connected diagrams“)
- Not considered in this work

$$Sp(4)_f$$

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etc.

Phenomenology of scattering channels

$$Sp(4)_f$$

- $l=1$ (10-dim):
 - Mixing with the Rho
 - $\pi\pi\pi \rightarrow \pi\pi$
 - No contribution to $\pi\pi$ -s-wave scattering
- Tackled in the future

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etc.

Phenomenology of scattering channels

$$Sp(4)_f$$

- $l=2$:
 - Makes up most 2 π scattering (14/25)
 - Easiest on the lattice
- $l=1$:
 - No s-wave scattering
 - Mixing with dark Rho
 - $\pi\pi\pi \rightarrow \pi\pi$
- $l=0$:
 - Mixing with the flavour singlet

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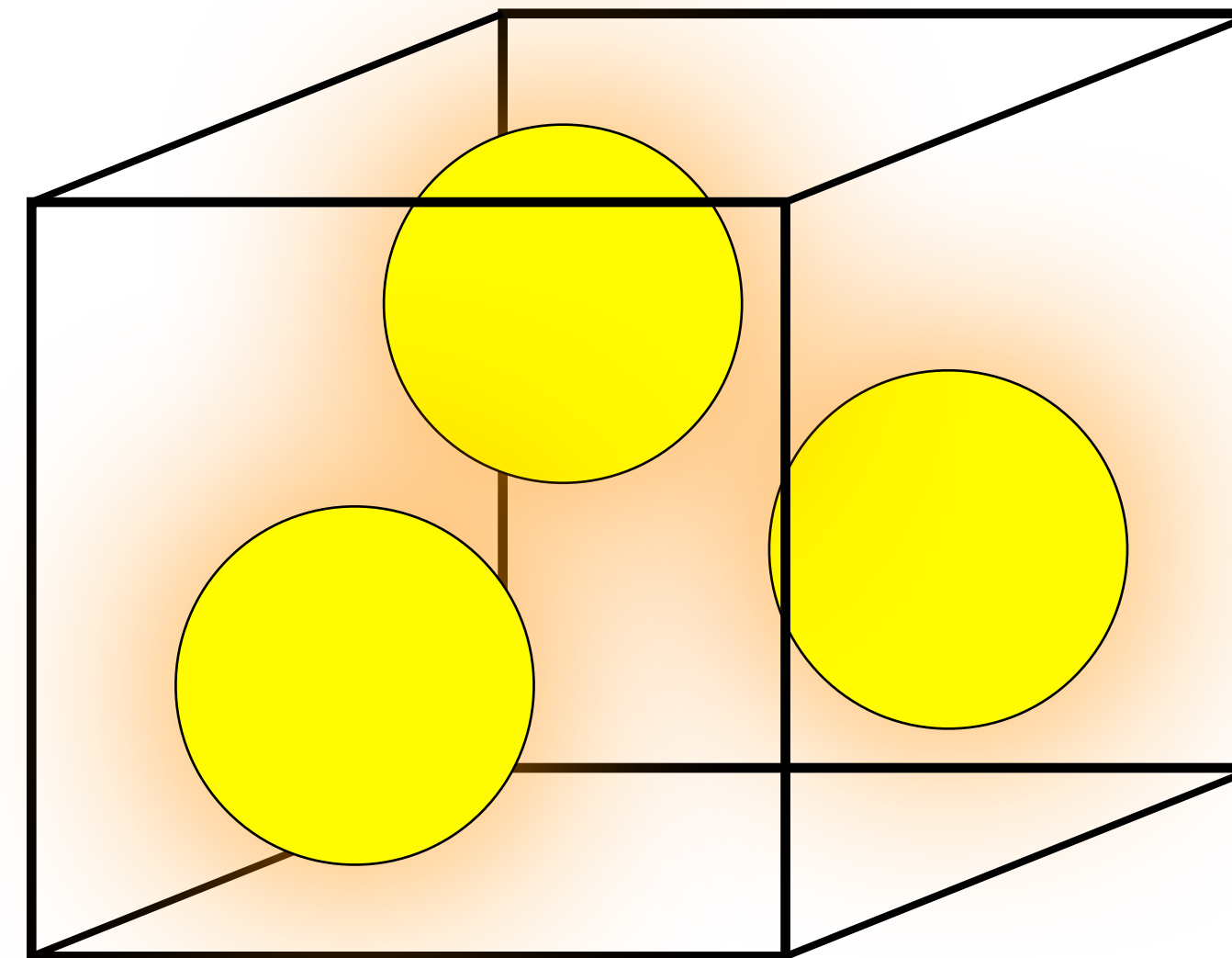
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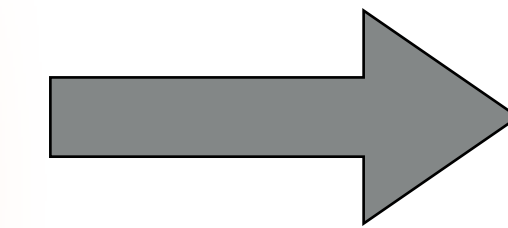
etc.

Lattice - Scattering

- Can be extended to the 3 particle case
- „3 particle quantization condition“
- $\det[F_3^{-1} + \mathcal{K}_3] = 0$
 - Full 2 \rightarrow 2 information needed



Finite size

 \mathcal{M}

Infinite volume