

# Fits to $B \rightarrow D^* l \nu$ with novel techniques for $|V_{cb}|$ studies

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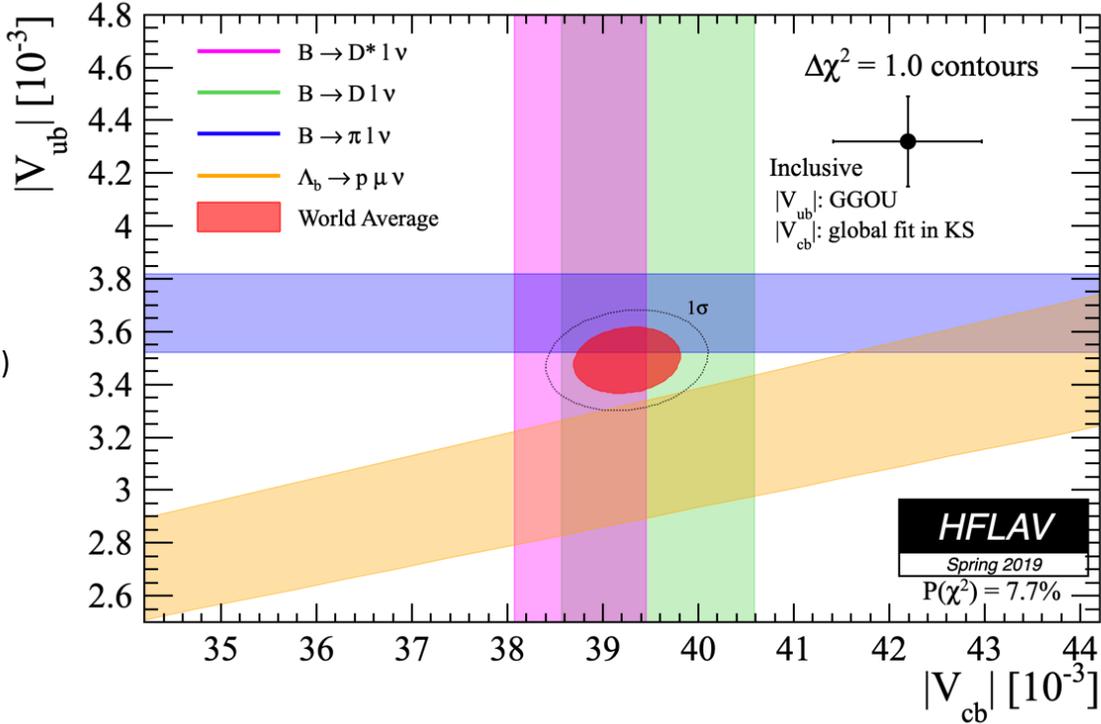
# Anomalies and Precision

- Measurement of CKM matrix elements is a powerful test of the SM
- Inclusive and exclusive measurements of  $|V_{cb}|$  (and  $|V_{ub}|$ ) are in tension
- $R(D)$  and  $R(D^*)$  depend on the same models used to determine  $|V_{cb}|$

Inclusive  $|V_{cb}| = (42.2 \pm 0.8) \times 10^{-3}$   
↗ Exp. + theor. error

$B^0 \rightarrow D^{*-} \ell^+ \nu_\ell$   $|V_{cb}| = (38.4 \pm 0.7 \pm 0.5 \pm 1.0) \times 10^{-3}$   
↙ Stat. error  
↙ Sys. error  
↘ Error from truncation (for PDG)

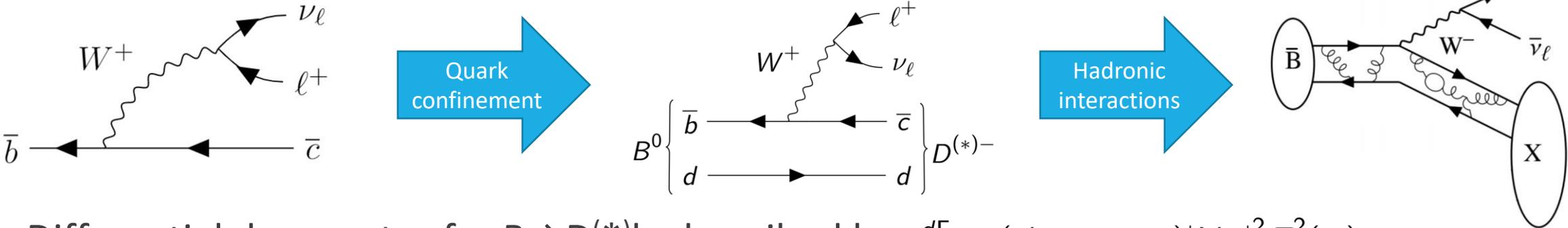
$B \rightarrow D \ell^+ \nu_\ell$   $|V_{cb}| = (40.1 \pm 1.0) \times 10^{-3}$   
↙ Combination error  
↘ [PDG \(2020\)](#)



- Developments in LQCD calculations may help resolve these anomalies

# A new era in LQCD calculations

- Cannot measure CKM elements from quark interactions  $\Rightarrow$  form factor description

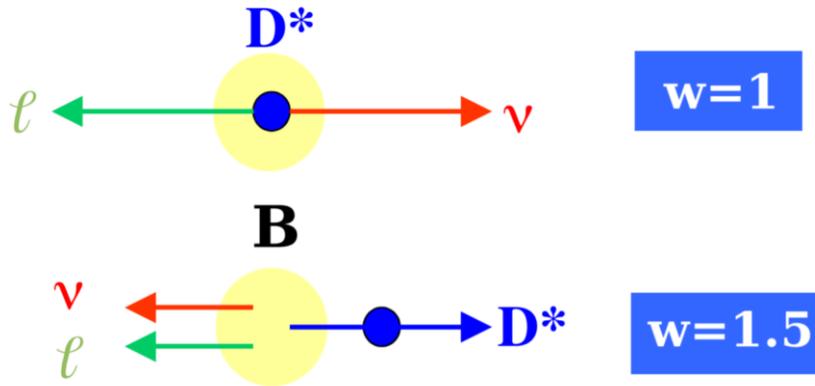


- Differential decay rates for  $B \rightarrow D^{(*)} \ell \nu$  described by  $\frac{d\Gamma}{dw} \propto (\text{phase space}) |V_{cb}|^2 \mathcal{F}^2(w)$
- Wave function overlap at zero hadronic recoil ( $w=1$ ) gave LQCD predictions with no constraints on shape

$$w = \frac{P_B \cdot P_{D^*}}{m_B m_{D^*}} = \frac{m_B^2 + m_{D^*}^2 - q^2}{2m_B m_{D^*}}$$

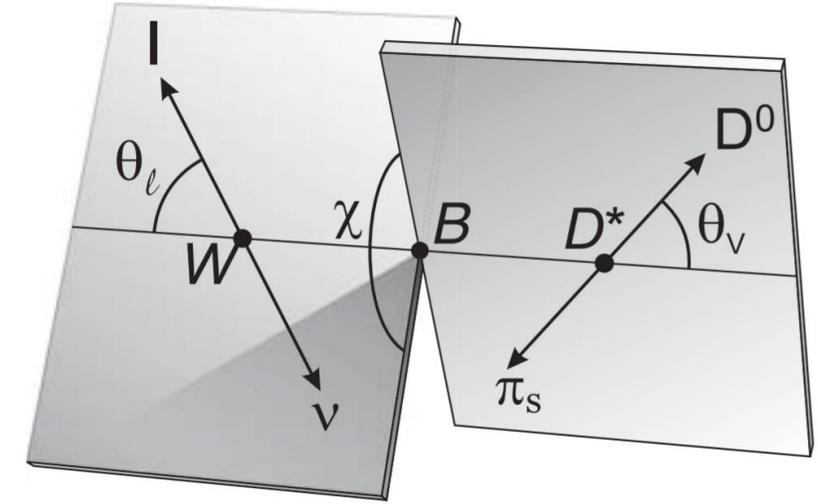
- New developments in LQCD provide calculations of form factors at non-zero hadronic recoil

JLQCD (2020): [B  \$\rightarrow\$  D<sup>\(\\*\)</sup>  \$\ell\$   \$\nu\$  form factors...](https://doi.org/10.22323/1.363.0139) doi.org/10.22323/1.363.0139  
 Fermilab-MILC (2021): [Semileptonic form factors...](https://arxiv.org/abs/2105.14019) arXiv:2105.14019



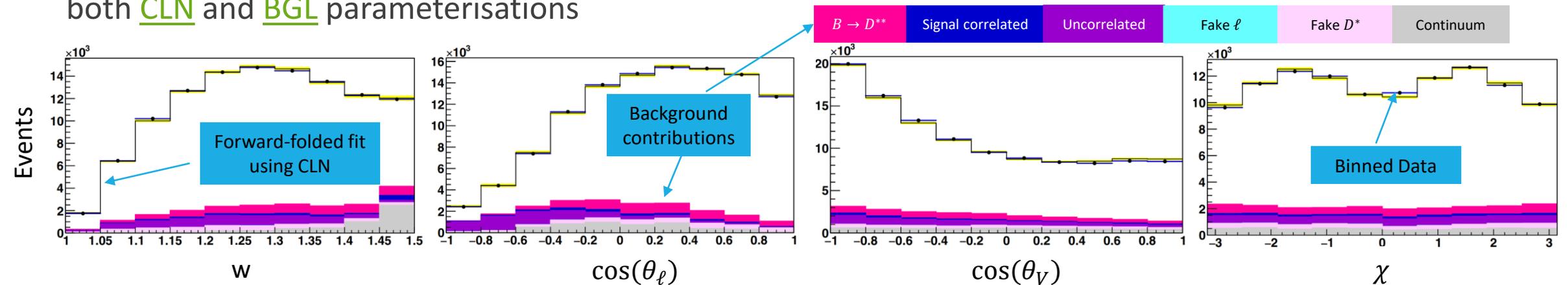
# B → D\* l ν Decay

- Exclusive  $|V_{cb}|$  measurements have historically had some model dependence
- Differential decay rate for  $l = e, \mu$ , is a function of four observables: hadronic recoil ( $w$ ) and three angles ( $\theta_\ell, \theta_V, \chi$ )



$$\frac{d\Gamma(B^0 \rightarrow D^{*\mp} \ell^+ \nu_\ell)}{dw d\cos\theta_\ell d\cos\theta_V d\chi} = \frac{\eta_{EW}^2 3m_{B^0} m_{D^{*\pm}}^2}{4(4\pi)^4} G_F^2 |V_{cb}|^2 \sqrt{w^2 - 1} (1 - 2wr + r^2) \times \{ (1 - \cos\theta_\ell)^2 \sin^2\theta_V H_+^2 + (1 + \cos\theta_\ell)^2 \sin^2\theta_V H_-^2 + 4\sin^2\theta_\ell \cos^2\theta_V H_0^2 - 2\sin^2\theta_\ell \sin^2\theta_V \cos 2\chi H_+ H_- - 4\sin\theta_\ell (1 - \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_+ H_0 + 4\sin\theta_\ell (1 + \cos\theta_\ell) \sin\theta_V \cos\theta_V \cos\chi H_- H_0 \},$$

- Belle 2019 untagged study ([doi.org/10.1103/PhysRevD.100.052007](https://doi.org/10.1103/PhysRevD.100.052007)) performed fits for  $\{H_\pm, H_0\}$  defined in both **CLN** and **BGL** parameterisations



# CLN parameterisation

- Helicity amplitudes defined in terms of **form factors** and **form factor ratios** to represent full form factor expression  $\mathcal{F}^2(w)$
- Standard CLN has four **free parameters**

$[\eta_{EW} h_{A_1}(1)] |V_{cb}|$ : Overall normalisation  
 $\rho^2$ : Slope term  
 $R_1(1)$ : Ratio 1 term at  $w=1$   
 $R_2(1)$ : Ratio 2 term at  $w=1$

$$h_{A_1}(w) = h_{A_1}(1) [1 - 8\rho^2 z(w) + (53\rho^2 - 15)z(w)^2 - (231\rho^2 - 91)z(w)^3],$$

$$R_1(w) = \frac{h_V(w)}{h_{A_1}(w)} = R_1(1) - 0.12(w-1) + 0.05(w-1)^2$$

$$R_2(w) = \frac{h_{A_3}(w) + r h_{A_2}(w)}{h_{A_1}(w)} = R_2(1) + 0.11(w-1) - 0.06(w-1)^2$$

$$\frac{d\Gamma}{dw} \propto (\text{phase space}) |V_{cb}|^2 \mathcal{F}^2(w)$$

$$\mathcal{F}^2(w) = h_{A_1}^2(w) \left(1 + 4 \frac{w}{w+1} \frac{1 - 2wr + r^2}{(1-r^2)}\right)^{-1} \times \left[ 2 \frac{1 - 2wr + r^2}{(1-r)^2} \left(1 + R_1^2(w) \frac{w-1}{w+1}\right) + (1 + (1 - R_2(w)) \frac{w-1}{1-r})^2 \right],$$

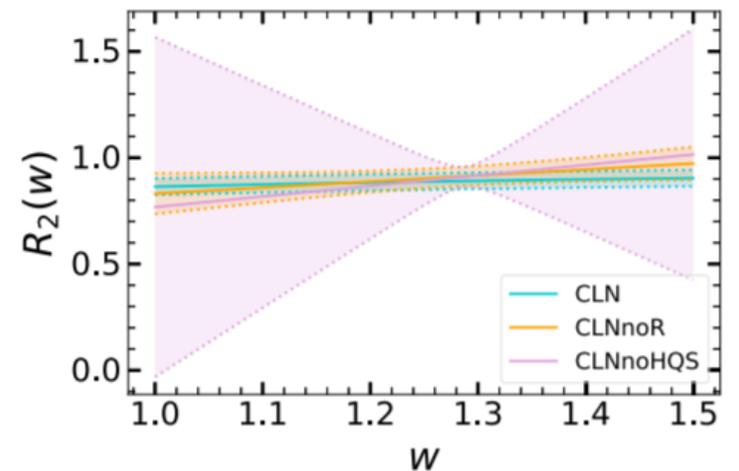
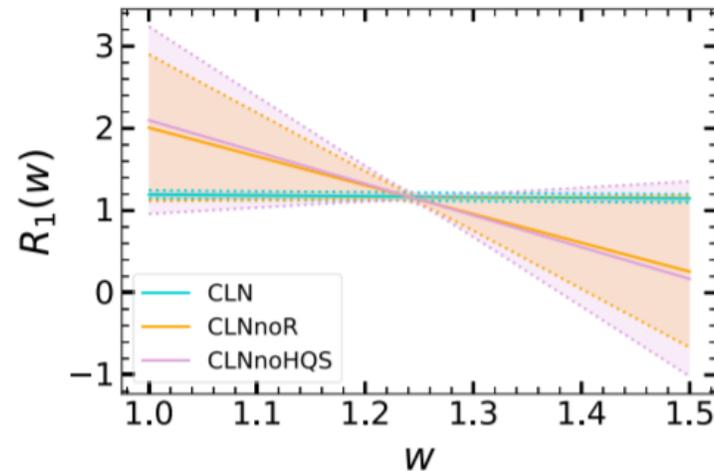
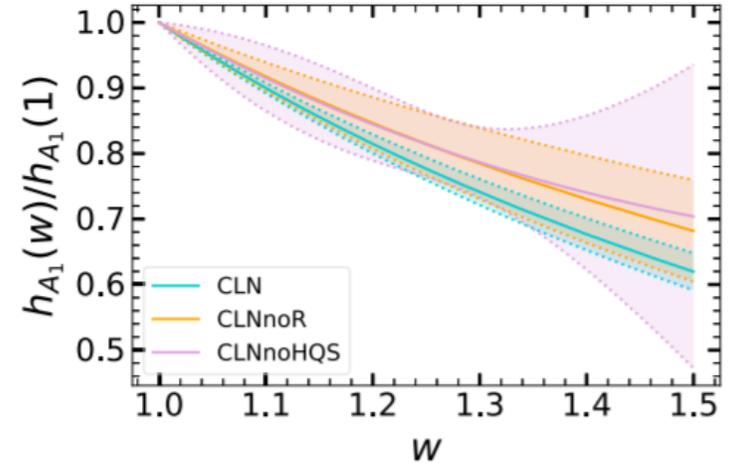
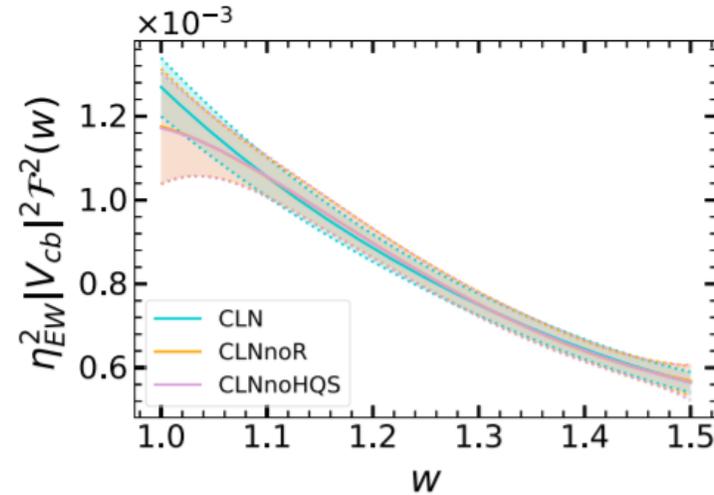
- QCD sum rule constraints can be removed to add  $R'_1(1)$  and  $R'_2(1)$  as new free parameters in ratios (**CLNnoR**)
- Heavy quark symmetry can then also be removed to add curvature term  $c_{D^*}$  in  $h_{A_1}(w)$  (**CLNnoHQS**)

# Fitting in CLN

- Fitting to the 2019 untagged Belle data set shows convergence with the different configurations.
- Large uncertainties are seen due to increased number of free parameters in the  $\chi^2$  fit but results consistent.

Configuration	$\mathcal{F}(1)\eta_{EW} V_{cb} $ (Val. $\pm$ stat. $\pm$ sys.)
CLN	$35.63 \pm 0.18 \pm 0.79$
CLNnoR	$34.29 \pm 0.66 \pm 1.33$
CLNnoHQS	$34.23 \pm 0.63 \pm 1.32$

\* Branching fractions and resonances have been updated to match Fermilab-MILC study, see backup slides



Ferlewicz et. al (2021): [Revisiting fits... doi.org/10.1103/PhysRevD.103.073005](https://doi.org/10.1103/PhysRevD.103.073005)

# BGL parameterisation

- The helicity amplitudes in BGL are defined by power series

$$H_0(w) = \mathcal{F}_1(w) / \sqrt{q^2},$$

$$H_{\pm}(w) = f(w) \mp m_{B^0} m_{D^{*\pm}} \sqrt{w^2 - 1} g(w)$$

$$f(z) = \frac{1}{P_{1+}(z)\phi_f(z)} \sum_{n=0}^{\infty} a_n^f z^n,$$

$$g(z) = \frac{1}{P_{1-}(z)\phi_g(z)} \sum_{n=0}^{\infty} a_n^g z^n$$

$$\mathcal{F}_1(z) = \frac{1}{P_{1+}(z)\phi_{\mathcal{F}_1}(z)} \sum_{n=0}^{\infty} a_n^{\mathcal{F}_1} z^n,$$

BGL( $n_f, n_g, n_{\mathcal{F}_1}$ )

- Each coefficient  $a_n^i$  is a free parameter

- Setting  $\tilde{a}_n^i = \eta_{EW} |V_{cb}| a_n^i$  reduces normalisation D.O.F.

- Another free parameter removed through  $\mathcal{F}_1(0) = (m_B - m_{D^*})f(0)$

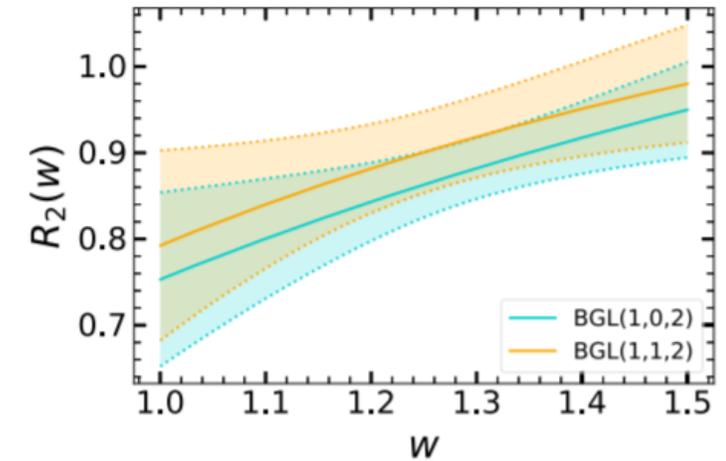
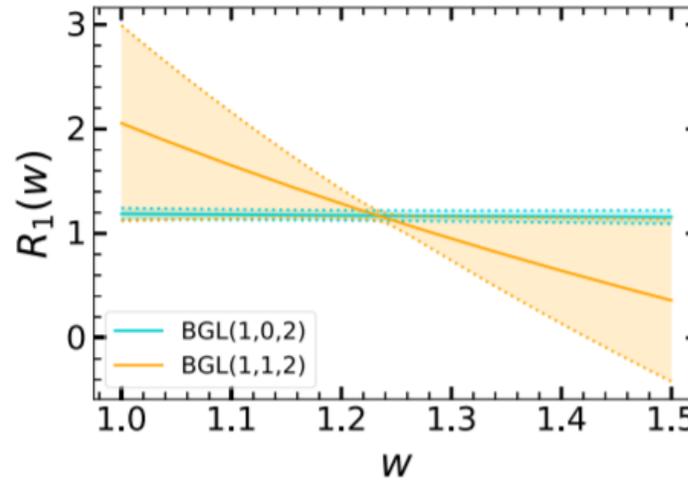
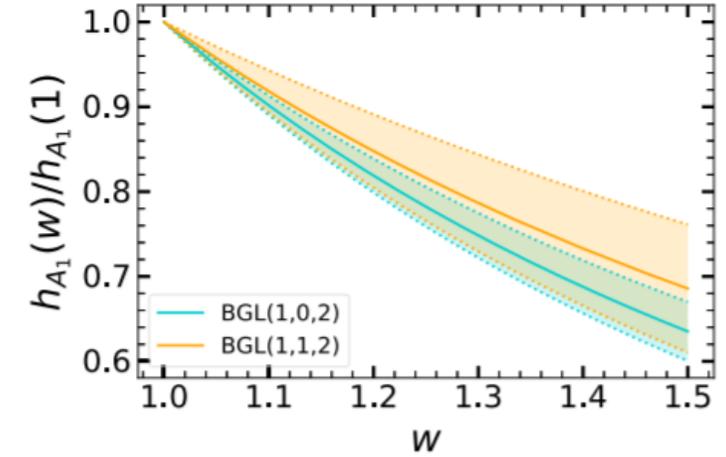
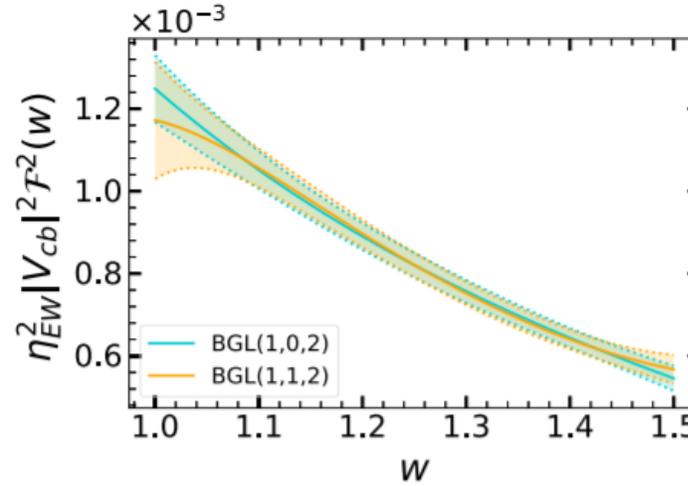
- $\mathcal{F}(1)\eta_{EW}|V_{cb}| = \frac{1}{2m_{B^0}m_{D^{*\pm}}} \frac{|\tilde{a}_0^f|}{P_f(0)\phi_f(0)}$ , where  $\mathcal{F}(1) = h_{A_1}(1)$

# Fitting in BGL

- Belle 2019 study used BGL(1,0,2).
- Higher orders were found to have high correlations, risks of reaching end points and non-convergence
- Truncation error was added and is the leading uncertainty

$$B^0 \rightarrow D^{*-} \ell^+ \nu_\ell \quad |V_{cb}| = (38.4 \pm 0.7 \pm 0.5 \pm 1.0) \times 10^{-3}$$

Stat. error
Sys. error
Error from truncation (for PDG)



Configuration	$\mathcal{F}(1)\eta_{EW} V_{cb} $ (Val. $\pm$ stat. $\pm$ sys.)
BGL(1,0,2)	$35.35 \pm 0.28 \pm 0.95$
BGL(1,1,2)	$33.92 \pm 0.58 \pm 1.25$

# Variation from covariance

- Standard  $\chi^2$  minimisation algorithm for fitting expected results from theory,  $N_{\text{exp}}$ , to observed data,  $N_{\text{obs}}$ , given covariance in data,  $C$ :

$$\chi^2 = \sum_{i,j} (N_i^{\text{obs}} - N_i^{\text{exp}}) C_{ij}^{-1} (N_j^{\text{obs}} - N_j^{\text{exp}})$$

- High systematic correlations related to the overall normalisation may cause a bias in the fit results
- The measured systematic covariance matrix cannot be added to the statistical covariance matrix in a naïve way

Source	$\eta_{\text{EW}} \mathcal{F}(1)  V_{cb} $ [%]	$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)$ [%]
Slow pion efficiency	0.79	1.57
Lepton ID combined	0.67	1.33
$\mathcal{B}(B \rightarrow D^{**} \ell \nu)$	0.05	0.10
$B \rightarrow D^{**} \ell \nu$ form factors	0.08	0.16
$f_{+-}/f_{00}$	0.56	1.05
Fake $e/\mu$	0.07	0.11
K/ $\pi$ ID	0.39	0.77
Fast track efficiency	0.53	1.05
$N(\Upsilon(4S))$	0.69	1.37
$B^0$ lifetime	0.13	0.26
$\mathcal{B}(D^{*+} \rightarrow D^0 \pi_s^+)$	0.37	0.74
$\mathcal{B}(D^0 \rightarrow K \pi)$	0.51	1.02
Total systematic error	1.65	3.26

Covariance:	$C = C_{\text{stat.}}$	$C = C_{\text{stat.}} + C_{\text{sys.}}$
CLN		
$\rho^2$	$1.09 \pm 0.04$	$1.16 \pm 0.04$
$R_1(1)$	$1.20 \pm 0.03$	$1.18 \pm 0.04$
$R_2(1)$	$0.86 \pm 0.02$	$0.85 \pm 0.03$
$\mathcal{F}(1) \eta_{\text{EW}}  V_{cb}  \times 10^3$	$35.3 \pm 0.2$	$36.4 \pm 0.7$
$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)$	4.93%	5.09%
$\chi^2/\text{ndf}$	41/36	24/36
BGL(1,0,2)		
$\tilde{a}_0^f \times 10^3$	$0.506 \pm 0.004$	$0.53 \pm 0.01$
$\tilde{a}_1^f \times 10^3$	$0.6 \pm 0.2$	$0.2 \pm 0.3$
$\tilde{a}_1^{F_1} \times 10^3$	$0.30 \pm 0.07$	$0.2 \pm 0.1$
$\tilde{a}_2^{F_1} \times 10^3$	$-3.8 \pm 1.4$	$-2.6 \pm 1.8$
$\tilde{a}_0^g \times 10^3$	$0.92 \pm 0.02$	$0.92 \pm 0.03$
$\mathcal{F}(1) \eta_{\text{EW}}  V_{cb}  \times 10^3$	$34.9 \pm 0.3$	$36.4 \pm 0.8$
$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)$	4.93%	5.09%
$\chi^2/\text{ndf}$	39/35	24/35

PDG 2018: 4.9%

Lower  $\chi^2$  values

- Need a way to avoid bias by separating the covariance matrices while maintaining covariance properties  $\Rightarrow$  Cholesky decomposition

# The Cholesky decomposition toy MC

- Separate statistical and systematic covariance matrices
- Include systematic uncertainty via toy MC method and Cholesky decomposition

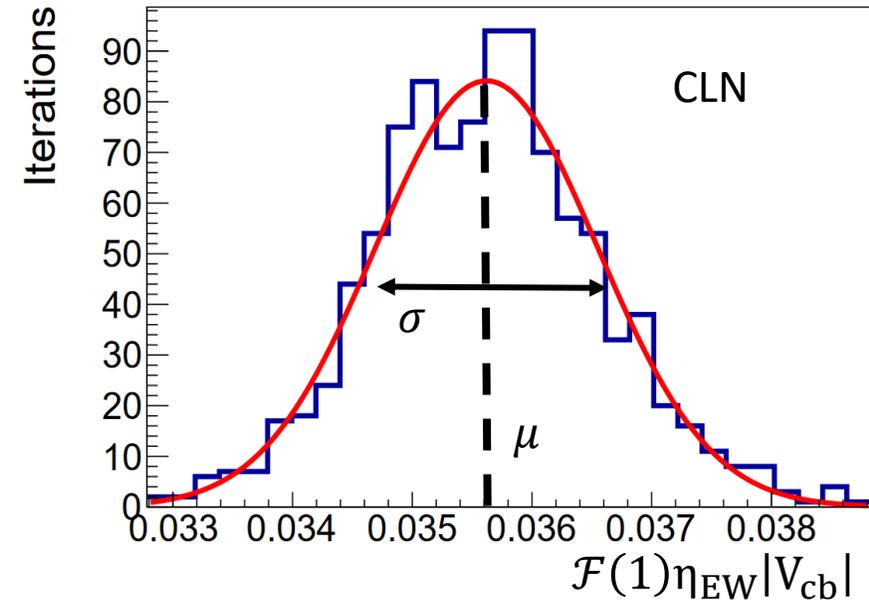
$$\mathcal{C}_{40 \times 40} = LL^T = \left[ \begin{array}{c} \text{Lower triangular matrix} \end{array} \right] \left[ \begin{array}{c} \text{Upper triangular matrix} \end{array} \right]$$

Positive definite covariance matrix

$$u = \text{gaus}(\mu = 0, \sigma = 1)_{1 \times 40}$$

$$N'_{obs} = N_{obs} + Lu$$

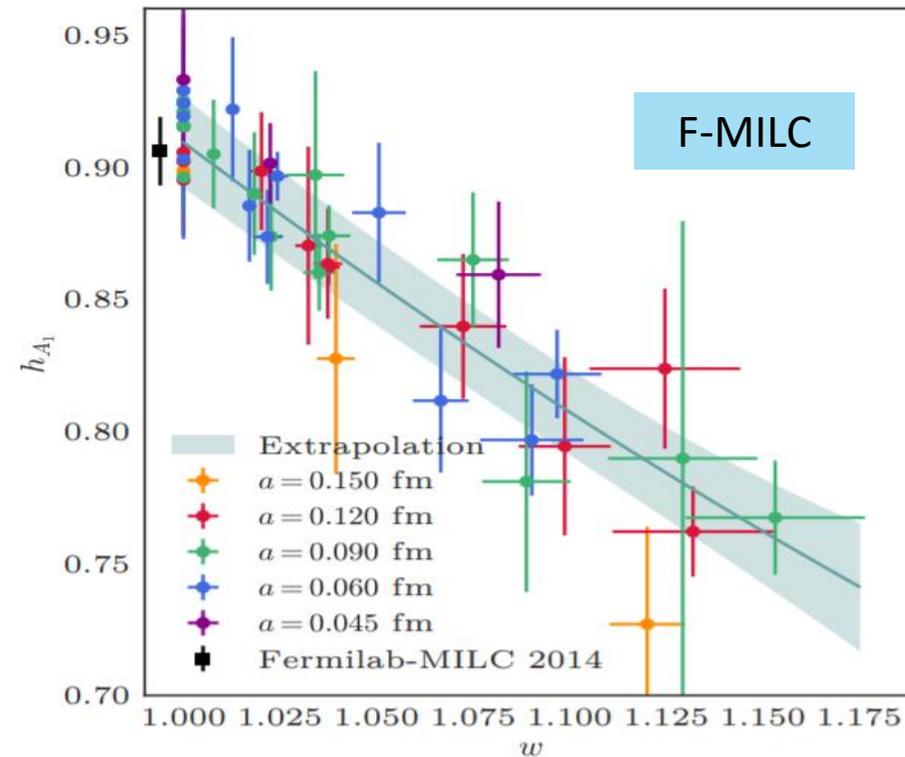
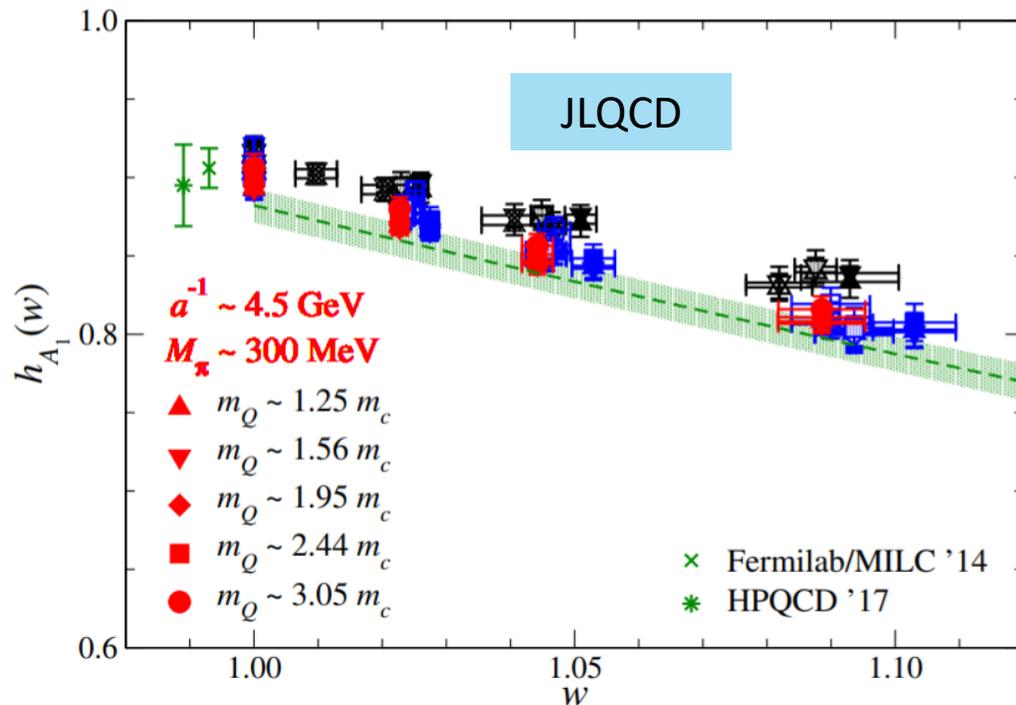
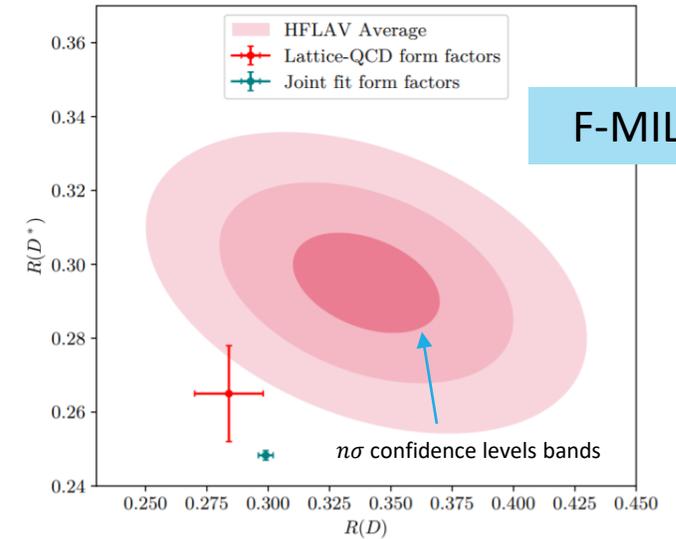
- Generate  $N'_{obs}$  and minimise  $\chi^2$  many times to obtain distribution of free parameter values ( $\mathcal{C} = \mathcal{C}_{stat.}$  in  $\chi^2$  formula)
- Assume a Gaussian to obtain mean value and sys. uncertainty



- Results obtained through this method agree with 2019 Belle study while using the measured systematic covariance matrix

# Lattice QCD synthetic data

- Synthetic data points generated from fit to extrapolated lattice spacing
- F-MILC has used these results with  $B \rightarrow D^{(*)}\tau\nu$  measurements to determine the impact of using these values for  $R(D^{(*)})$  and sees tension
- Comparing to light cone sum rules shows good agreement



# Inclusion of JLQCD input

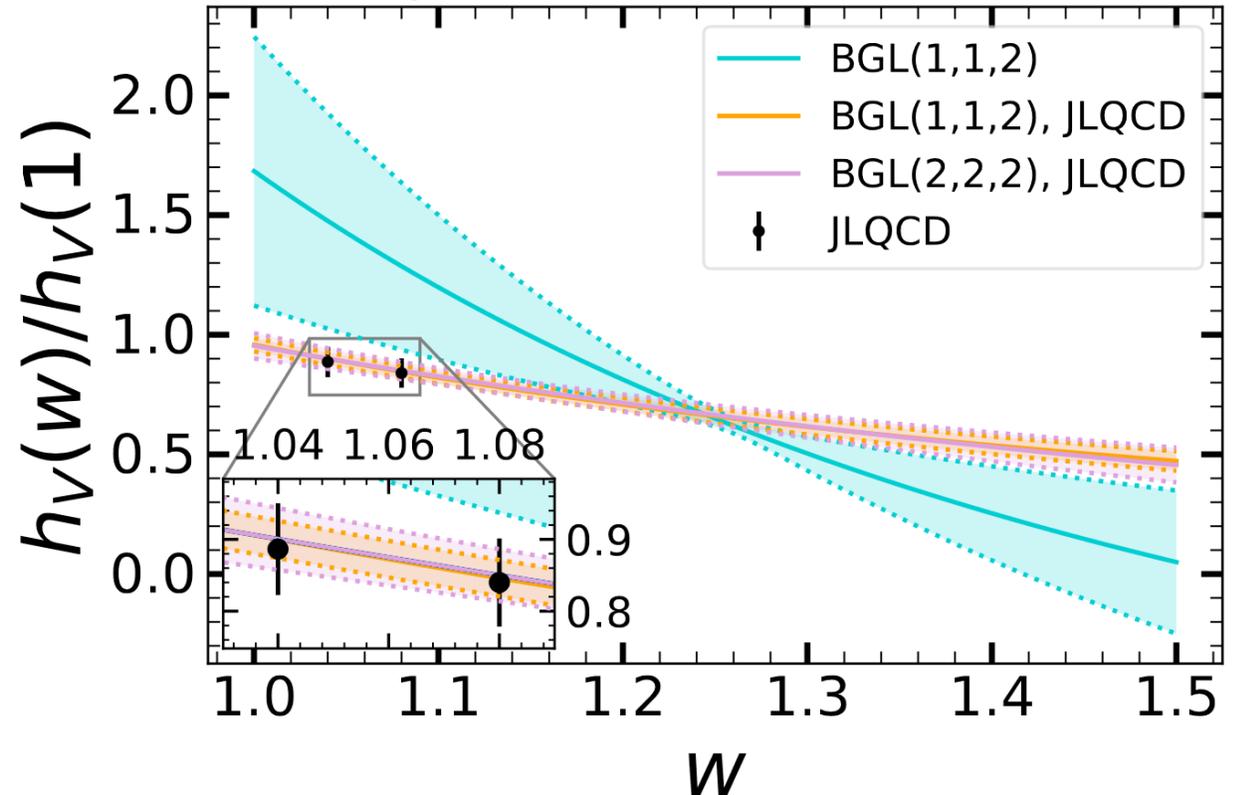
- To make a joint fit, add a term to the  $\chi^2$  for LQCD.

$$\chi^2 = \sum_{ij} (N_i^{obs} - N_i^{fit}) C_{ij}^{-1} (N_j^{obs} - N_j^{fit}) + \sum_{ij} (f_i(x^{LQCD}) - f_i(x^{fit})) C'_{ij}^{-1} (f_j(x^{LQCD}) - f_j(x^{fit}))$$

$f_i$  is a function corresponding of the LQCD inputs,  $x^{LQCD}$  and compared with the free parameters,  $f_i(x^{fit})$

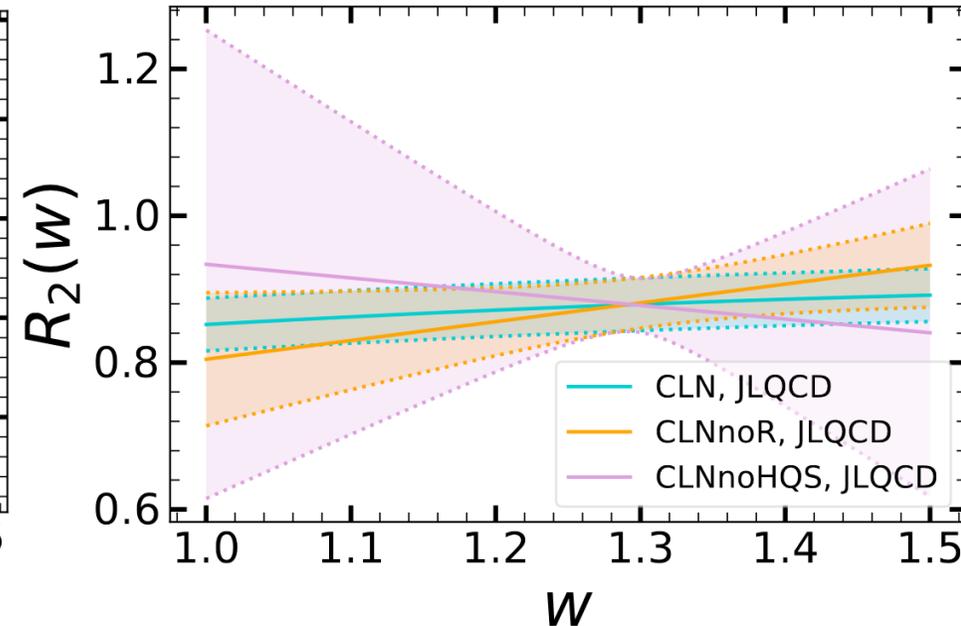
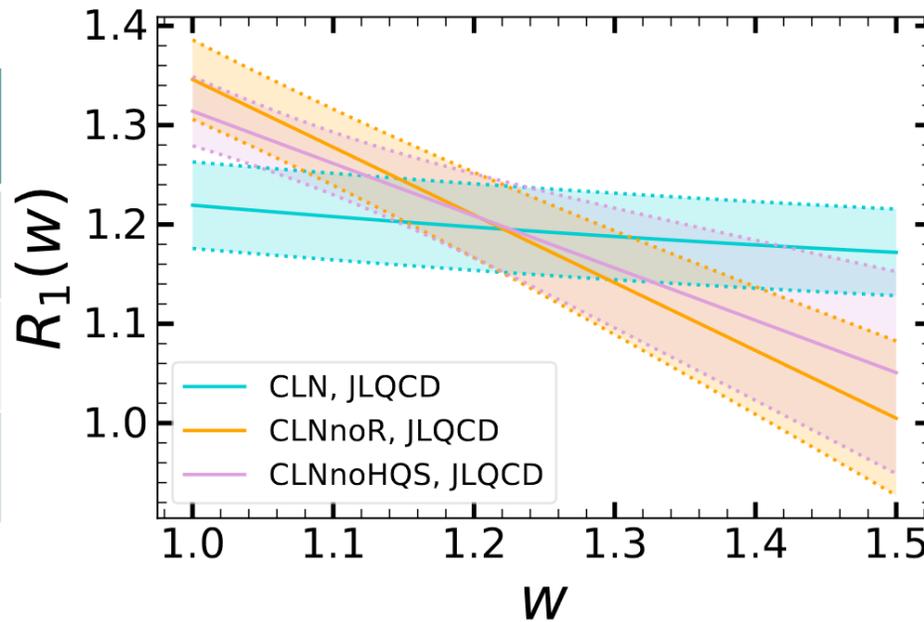
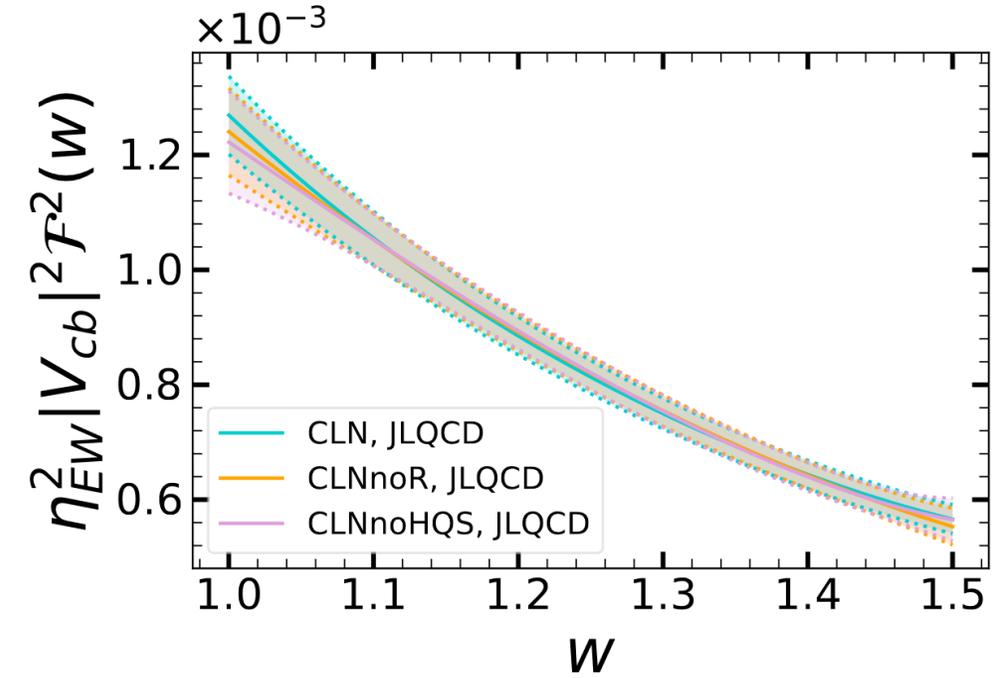
- Two points each used for  $h_{A_1}(w)$  and  $h_V(w)$  at  $w = 1.04, 1.08$ , including correlations
- Convergent, consistent results with reduced uncertainty across higher order (n=2) BGL configurations

Configuration	$\mathcal{F}(1)\eta_{EW} V_{cb} $ (Val. $\pm$ stat. $\pm$ sys.)
BGL(1,0,2)	$35.30 \pm 0.24 \pm 0.88$
BGL(1,1,2)	$35.22 \pm 0.24 \pm 0.86$
BGL(2,2,2)	$35.13 \pm 0.49 \pm 0.98$



# CLN with JLQCD input

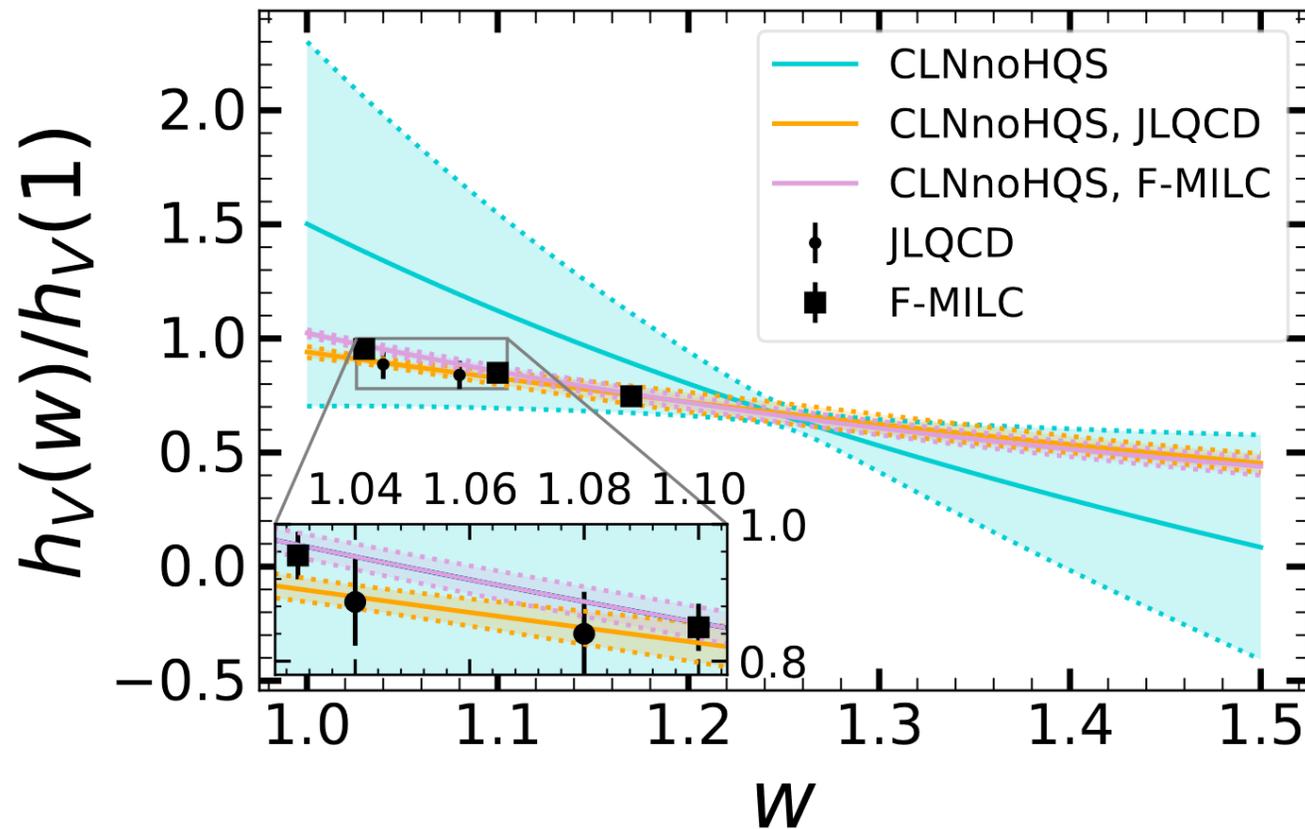
- CLN also shows consistency for different configurations.
- $|V_{cb}|$  remains consistent despite high uncertainty in  $R_2(w)$  when HQS and QCD sum rule theoretical assumptions released



Config.	$\mathcal{F}(1)\eta_{EW} V_{cb} $ (Val. $\pm$ stat. $\pm$ sys.)
CLN	$35.63 \pm 0.17 \pm 0.78$
CLNnoR	$35.22 \pm 0.23 \pm 0.84$
CLNnoHQS	$34.96 \pm 0.31 \pm 0.90$

# Inclusion of Fermilab-MILC input

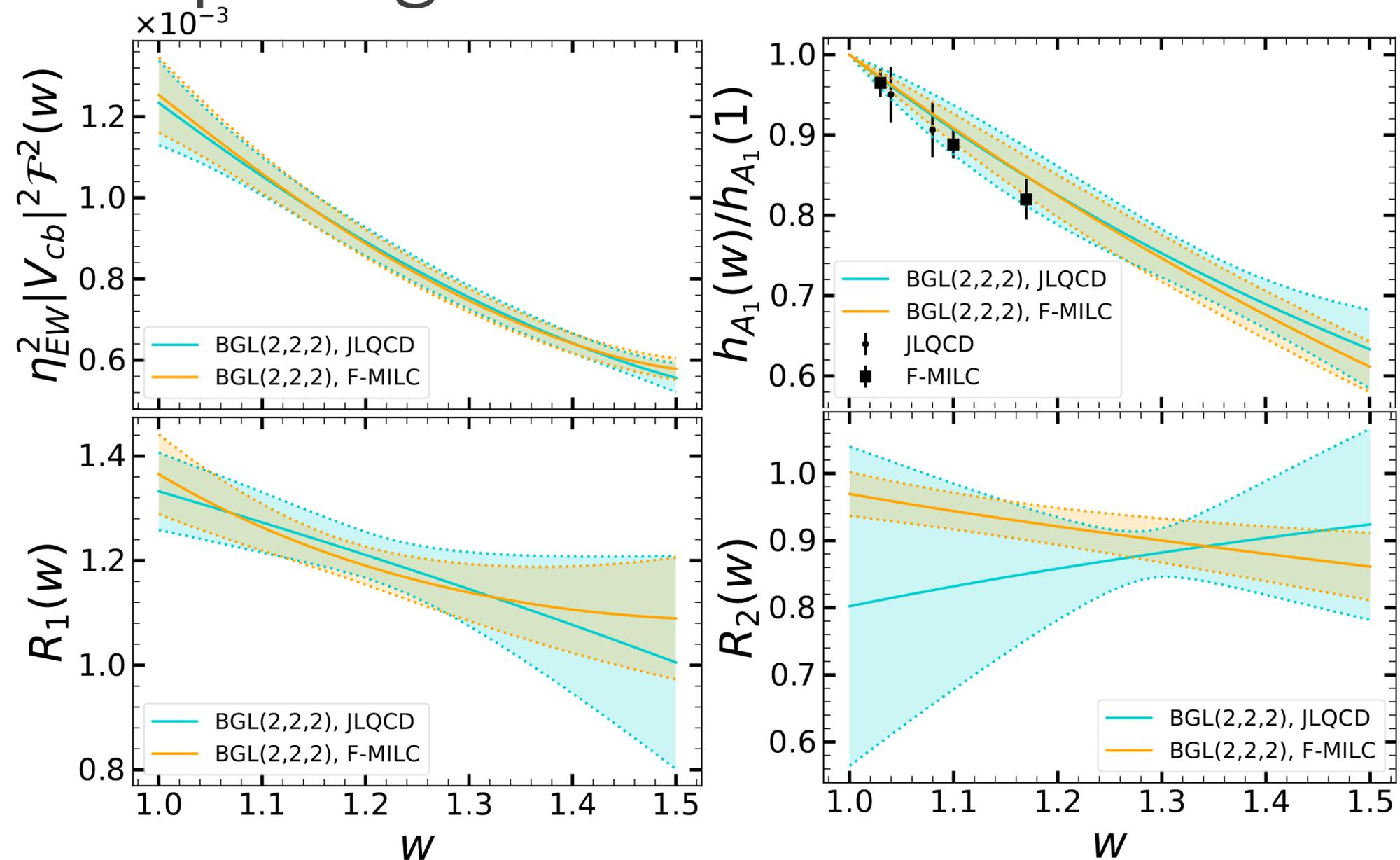
- Nine data points in total for  $g, f, \mathcal{F}_1$  at  $w = 1.03, 1.10, 1.17$
- Results consistent with JLQCD, slightly reduced uncertainty



Configuration	$\mathcal{F}(1)\eta_{EW} V_{cb} $ (Val. $\pm$ stat. $\pm$ sys.)
BGL(1,0,2)	$35.66 \pm 0.20 \pm 0.80$
BGL(1,1,2)	$35.62 \pm 0.20 \pm 0.81$
BGL(2,2,2)	$35.40 \pm 0.48 \pm 0.82$
CLN	$35.74 \pm 0.16 \pm 0.78$
CLNnoR	$35.57 \pm 0.22 \pm 0.84$
CLNnoHQS	$35.57 \pm 0.21 \pm 0.84$
F-MILC combined LQCD+Belle in BGL	$34.93 \pm 0.75$ (val. $\pm$ total uncert.)

- $\chi^2$  for fits increase due to large terms in inverse covariance matrix, leading to small p-values (also reported by authors)

# Comparing form factors

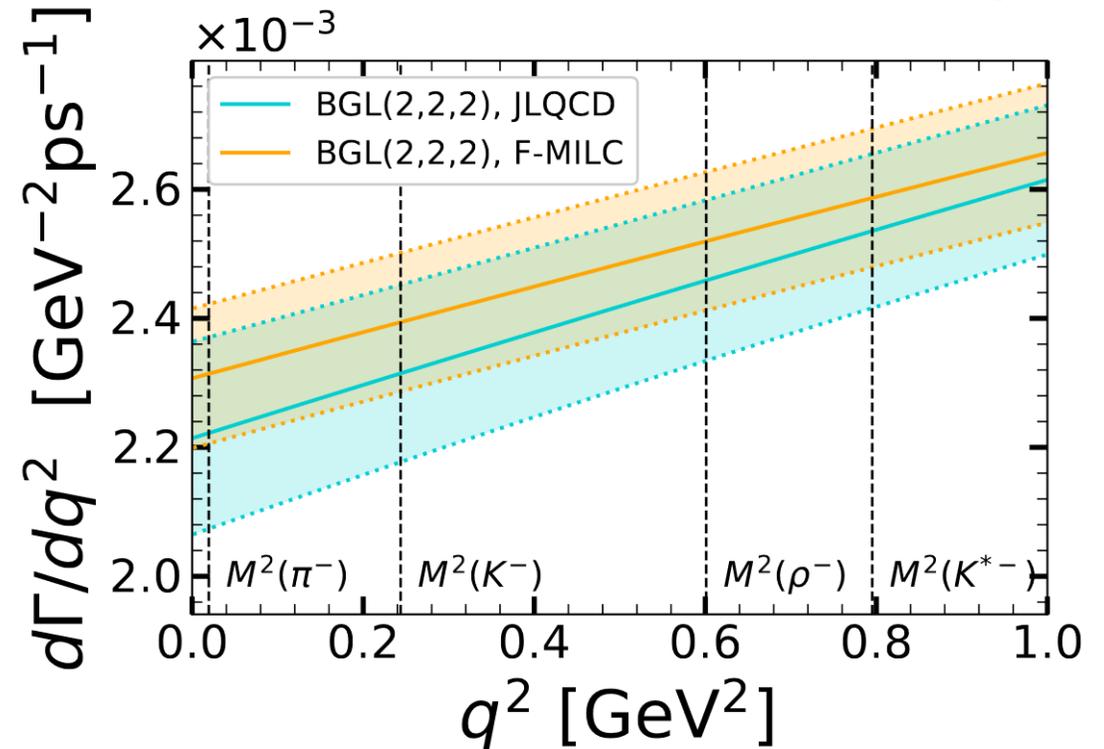
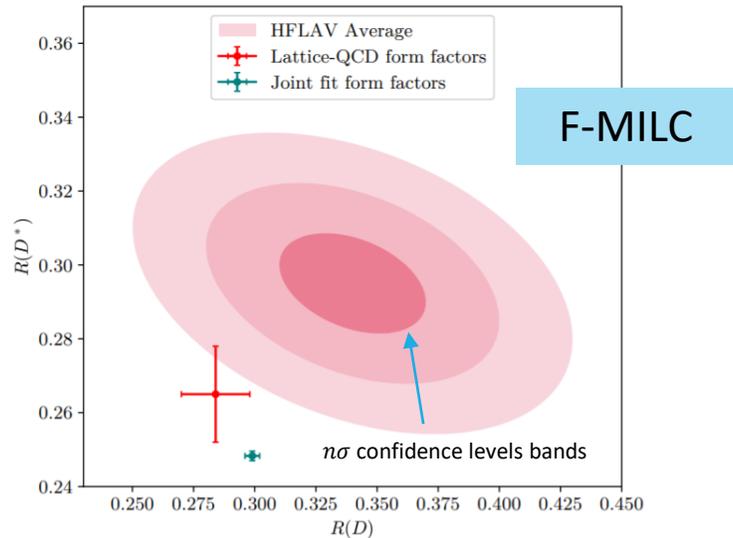


- Studies can be performed to compare results between data, JLQCD and F-MILC inputs at low hadronic recoil

# Implications

- Inclusion of LQCD points will drive down systematic uncertainty in  $|V_{cb}|$  measurements
- Form factors also used with  $\tau$  channel in  $R(D^{(*)})$  measurements will have less model-dependence
- QCD factorisation studies will have more precise input (Separate result from Belle collaboration coming soon)

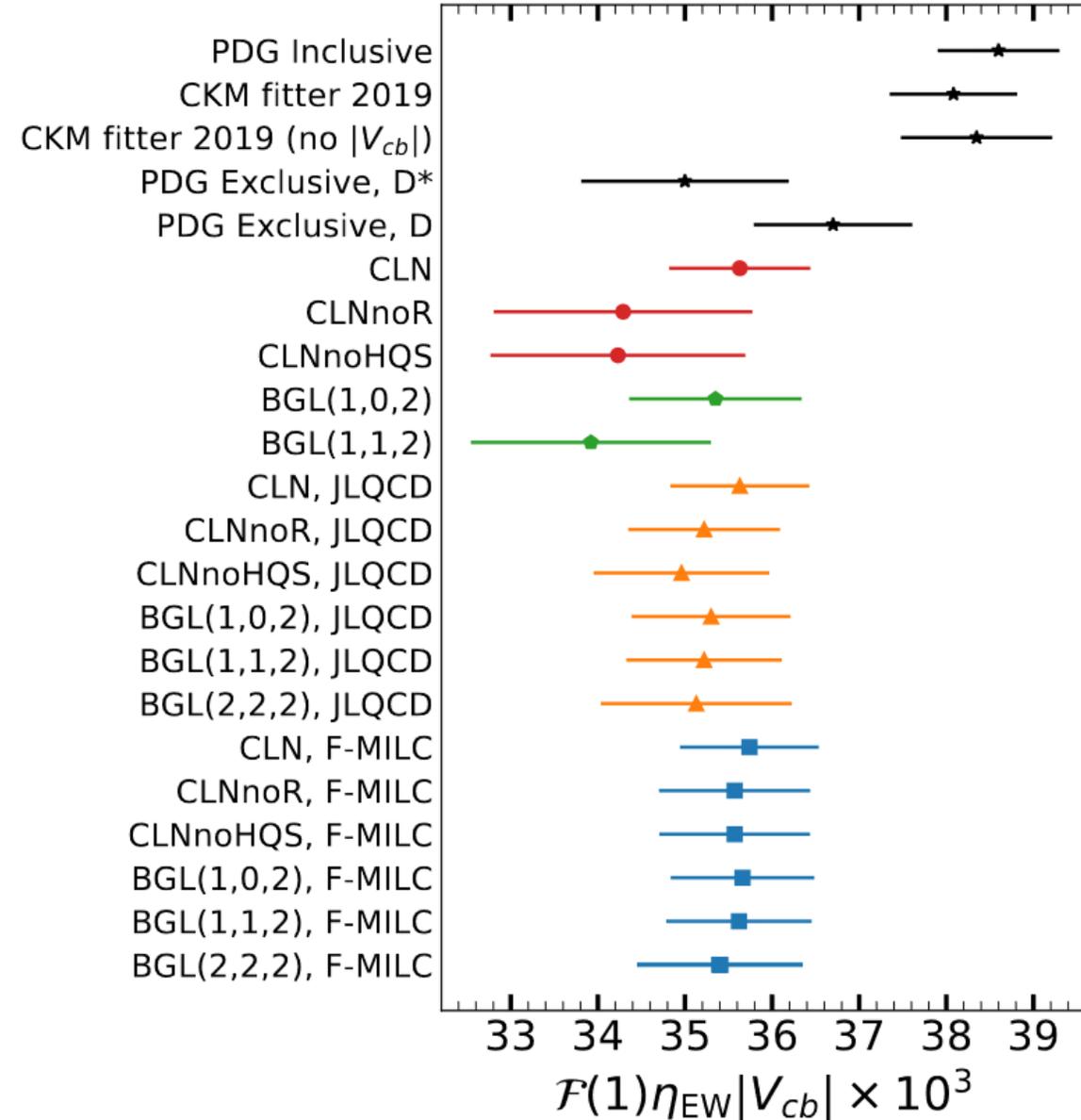
$$|a_1(D^*h)| = \left[ \left( 6\pi^2 \tau_B |V_{uq}| f_h^2 X_h \right)^{-1} \frac{\Gamma(\bar{B}_d \rightarrow D^{*+} h^-)}{d\Gamma(\bar{B}_d \rightarrow D^{*+} \ell^- \bar{\nu})/dq^2|_{q^2=m_h^2}} \right]^{\frac{1}{2}}$$



# Conclusion

- Model-independent results for  $B \rightarrow D^* l \nu$  decays
- Results using new Fermilab-MILC and JLQCD inputs are consistent
- Nominal result using BGL(2,2,2) with F-MILC is
 
$$\mathcal{F}(1)\eta_{EW}|V_{cb}| = 35.40 \pm 0.48 \pm 0.82$$

*(val ± stat. ± sys.)*
- Inclusive-exclusive tension remains with higher confidence.
- Future studies will be able to make use of these LQCD calculations to tighten systematic uncertainties on exclusive  $|V_{cb}|$ ,  $R(D^{(*)})$  and hadronic factorisation



# Back up slides

# Updated input values

Common Input	
$\eta_{EW}$	1.0066
$h_{A_1}(1) = \mathcal{F}(1)$	$0.909 \pm 0.017$
$h_V(1)$	$1.270 \pm 0.046$
$m_{B^0}$	$5.27963 \pm 0.00015 \text{ GeV}/c^2$
$m_{D^{*\pm}}$	$2.01026 \pm 0.00005 \text{ GeV}/c^2$
$\tau_{B^0}$	$(1.520 \pm 0.004) \times 10^{-12} \text{ s}$
$\mathcal{B}(D^{*+} \rightarrow D^0 \pi^+)$	$0.677 \pm 0.005$
$\mathcal{B}(D^0 \rightarrow K^- \pi^+)$	$0.0391 \pm 0.0003$
$G_F$	$1.16637 \times 10^{-5} \text{ GeV}^{-2} (\hbar c)^3$
$2n_{B\bar{B}} f_{00} \text{ (Belle)}$	$(750 \pm 11) \times 10^6$
BGL Input	
$B_c^* 1^+$ masses	6.739 $\text{GeV}/c^2$
	6.750 $\text{GeV}/c^2$
	7.145 $\text{GeV}/c^2$
	7.150 $\text{GeV}/c^2$
$B_c^* 1^-$ masses	6.329 $\text{GeV}/c^2$
	6.920 $\text{GeV}/c^2$
	7.020 $\text{GeV}/c^2$
$n_I$	2.6
$\chi^T(+u)$	$5.131 \times 10^{-4} (\text{GeV}/c^2)^{-2}$
$\chi^T(-u)$	$3.894 \times 10^{-4} (\text{GeV}/c^2)^{-2}$

Parameter	CLN	CLNnoR	CLNnoHQS
$\rho^2$	$1.09 \pm 0.04 \pm 0.05$	$0.89 \pm 0.09 \pm 0.15$	$0.93 \pm 0.38 \pm 0.39$
$R_1(1)$	$1.20 \pm 0.03 \pm 0.02$	$2.01 \pm 0.43 \pm 0.47$	$2.10 \pm 0.60 \pm 0.51$
$R_2(1)$	$0.86 \pm 0.02 \pm 0.01$	$0.83 \pm 0.05 \pm 0.04$	$0.77 \pm 0.42 \pm 0.36$
$R'_1(1)$	-0.12 (fixed)	$-3.50 \pm 1.76 \pm 1.87$	$-3.86 \pm 2.46 \pm 2.10$
$R'_2(1)$	0.11 (fixed)	$0.28 \pm 0.13 \pm 0.15$	$0.50 \pm 1.47 \pm 1.26$
$c_{D^*}$	$\rho^2$ (fixed)	$\rho^2$ (fixed)	$1.00 \pm 1.73 \pm 1.54$
$\mathcal{F}(1)\eta_{\text{EW}} V_{cb}  \times 10^3$	$35.63 \pm 0.18 \pm 0.79$	$34.29 \pm 0.66 \pm 1.33$	$34.23 \pm 0.63 \pm 1.32$
$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)$	5.04%	5.04%	5.04%
$\chi^2/\text{ndf}$	40.8/36	34.8/34	34.7/33
p-value	0.27	0.43	0.39

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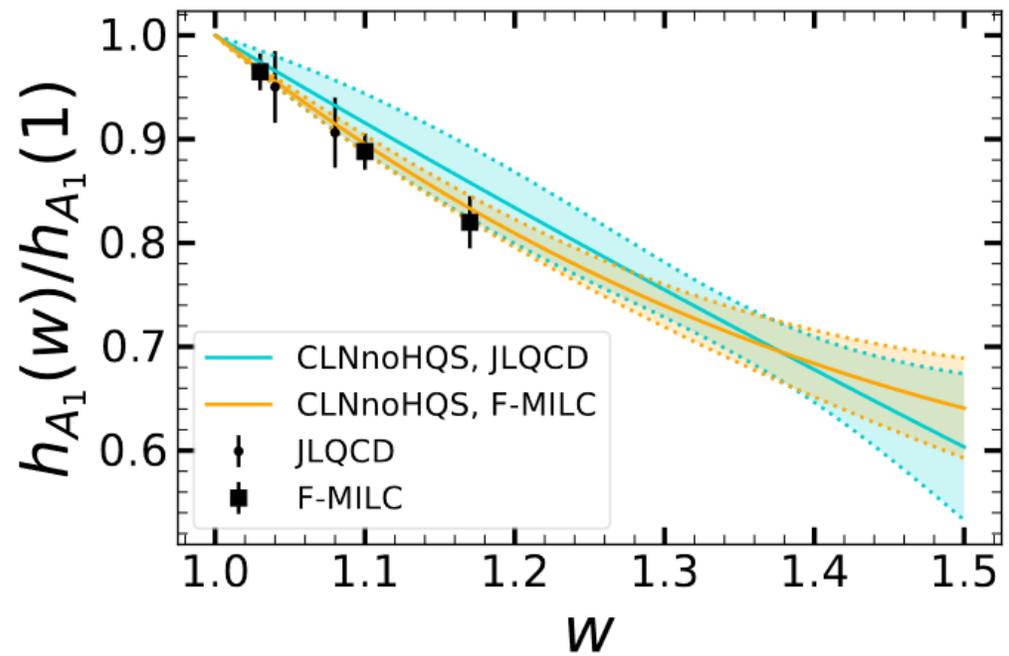
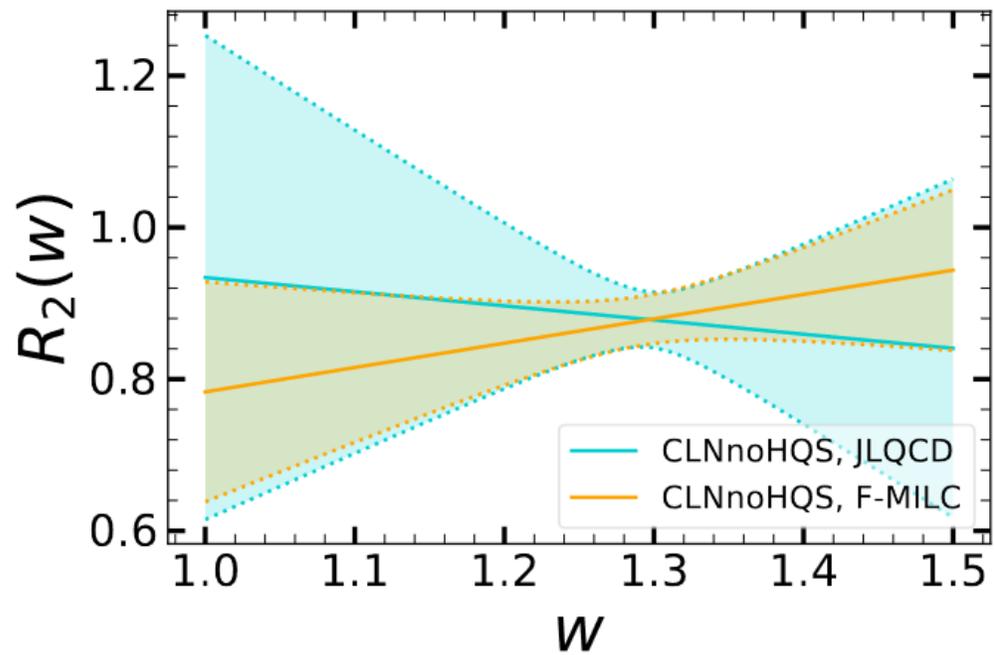
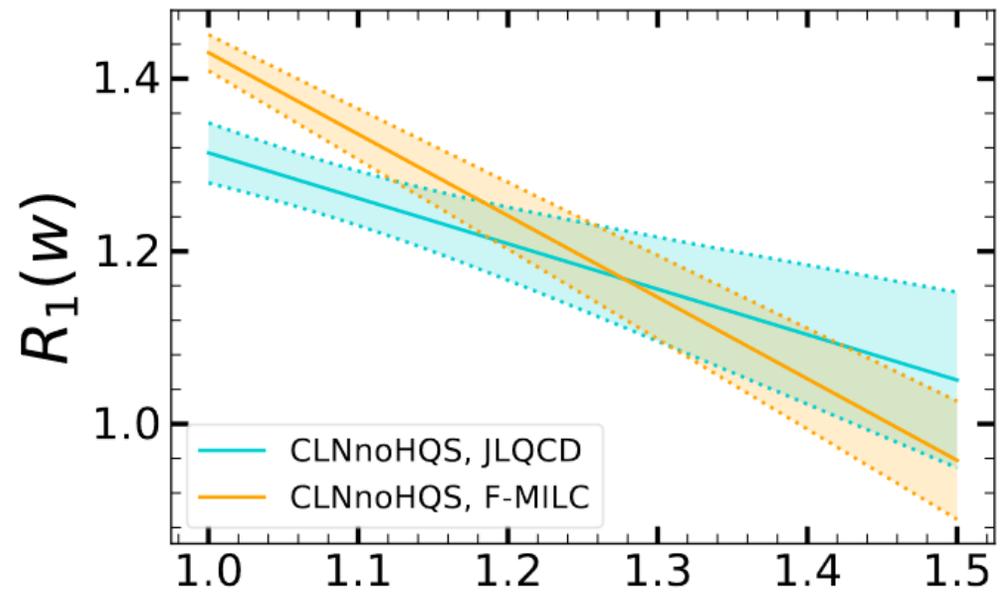
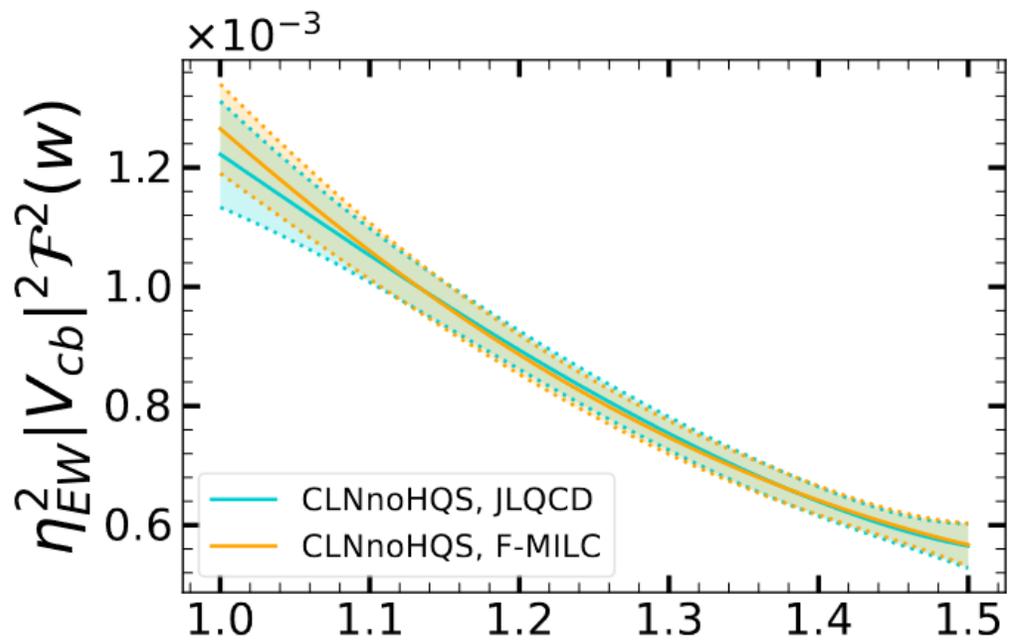
Parameter	CLN, JLQCD	CLNnoR, JLQCD	CLNnoHQS, JLQCD
$\rho^2$	$1.10 \pm 0.03 \pm 0.05$	$1.04 \pm 0.04 \pm 0.07$	$0.86 \pm 0.17 \pm 0.23$
$R_1(1)$	$1.22 \pm 0.03 \pm 0.02$	$1.35 \pm 0.02 \pm 0.02$	$1.31 \pm 0.02 \pm 0.02$
$R_2(1)$	$0.85 \pm 0.02 \pm 0.01$	$0.80 \pm 0.05 \pm 0.04$	$0.93 \pm 0.17 \pm 0.15$
$R'_1(1)$	-0.12 (fixed)	$-0.68 \pm 0.10 \pm 0.06$	$-0.53 \pm 0.11 \pm 0.11$
$R'_2(1)$	0.11 (fixed)	$0.26 \pm 0.13 \pm 0.13$	$-0.19 \pm 0.58 \pm 0.50$
$c_{D^*}$	$\rho^2$ (fixed)	$\rho^2$ (fixed)	$0.20 \pm 0.71 \pm 0.76$
$\mathcal{F}(1)\eta_{\text{EW}} V_{cb}  \times 10^3$	$35.63 \pm 0.17 \pm 0.78$	$35.22 \pm 0.23 \pm 0.84$	$34.96 \pm 0.31 \pm 0.96$
$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)$	5.04%	5.04%	5.04%
$\chi^2/\text{ndf}$	45.4/40	40.2/38	39.4/37
p-value	0.26	0.37	0.36

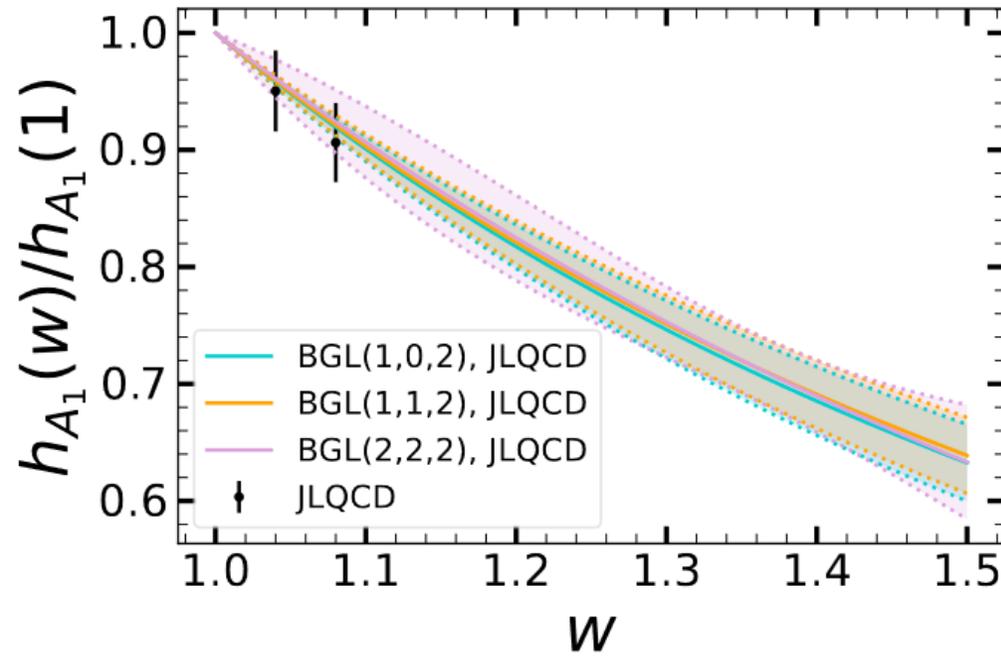
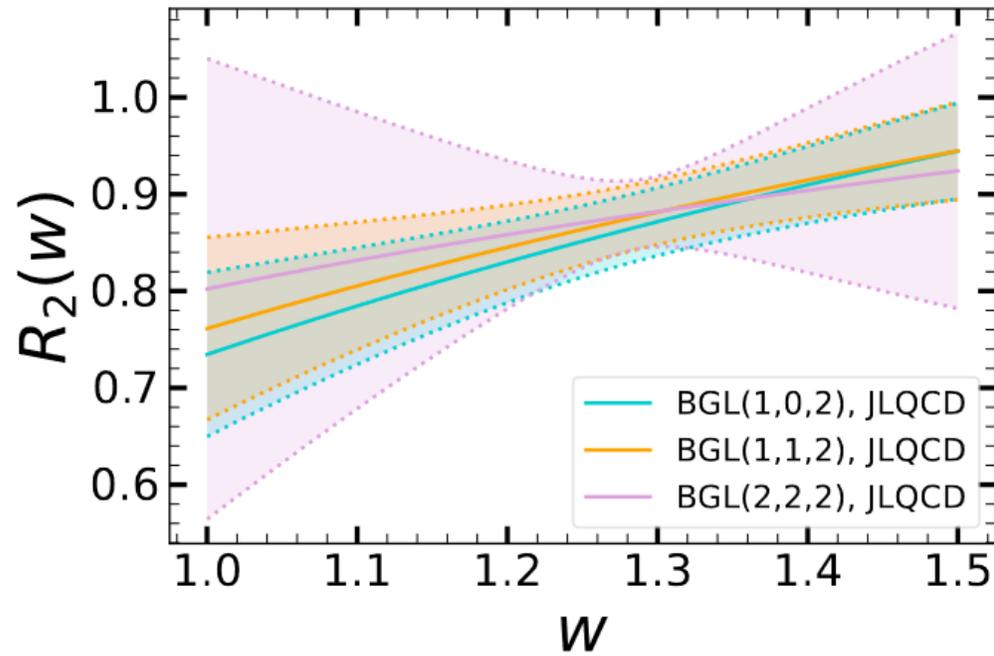
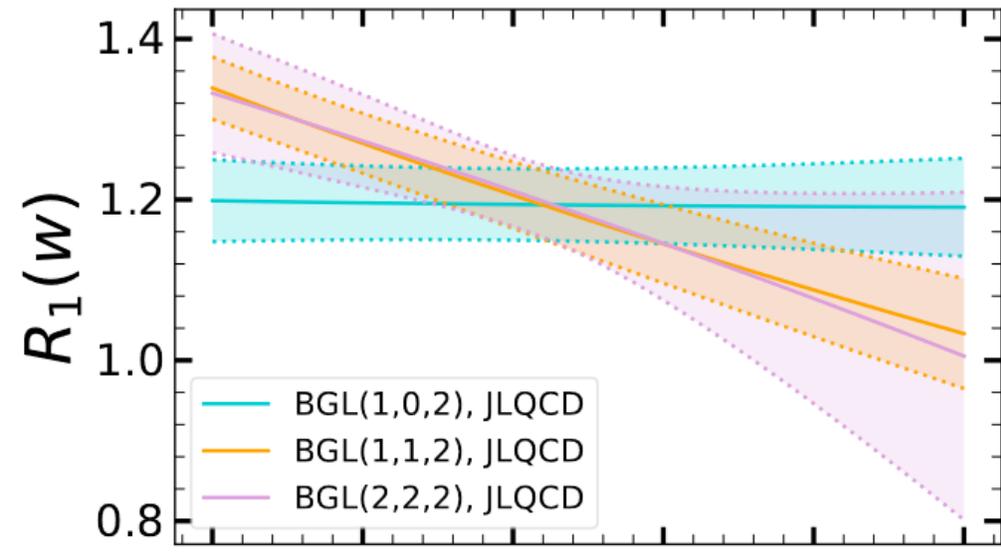
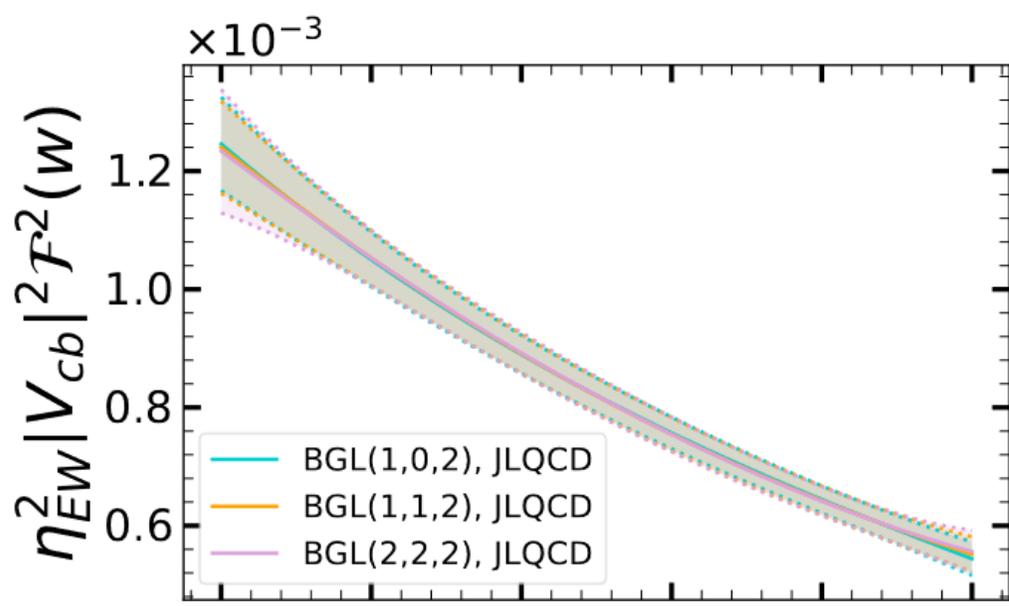
Parameter	CLN, F-MILC	CLNnoR, F-MILC	CLNnoHQS, F-MILC
$\rho^2$	$1.17 \pm 0.03 \pm 0.04$	$1.10 \pm 0.03 \pm 0.06$	$1.15 \pm 0.05 \pm 0.05$
$R_1(1)$	$1.29 \pm 0.02 \pm 0.01$	$1.43 \pm 0.01 \pm 0.01$	$1.43 \pm 0.01 \pm 0.01$
$R_2(1)$	$0.81 \pm 0.02 \pm 0.01$	$0.87 \pm 0.05 \pm 0.04$	$0.78 \pm 0.08 \pm 0.06$
$R'_1(1)$	$-0.12$ (fixed)	$-0.92 \pm 0.07 \pm 0.03$	$-0.94 \pm 0.07 \pm 0.04$
$R'_2(1)$	$0.11$ (fixed)	$0.02 \pm 0.13 \pm 0.12$	$0.32 \pm 0.27 \pm 0.22$
$c_{D^*}$	$\rho^2$ (fixed)	$\rho^2$ (fixed)	$1.21 \pm 0.26 \pm 0.16$
$\mathcal{F}(1)\eta_{EW} V_{cb}  \times 10^3$	$35.74 \pm 0.16 \pm 0.78$	$35.57 \pm 0.22 \pm 0.84$	$35.57 \pm 0.21 \pm 0.84$
$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)$	5.04%	5.04%	5.04%
$\chi^2/\text{ndf}$	379.5/45	361.1/43	362.2/42
p-value	0.0	0.0	0.0

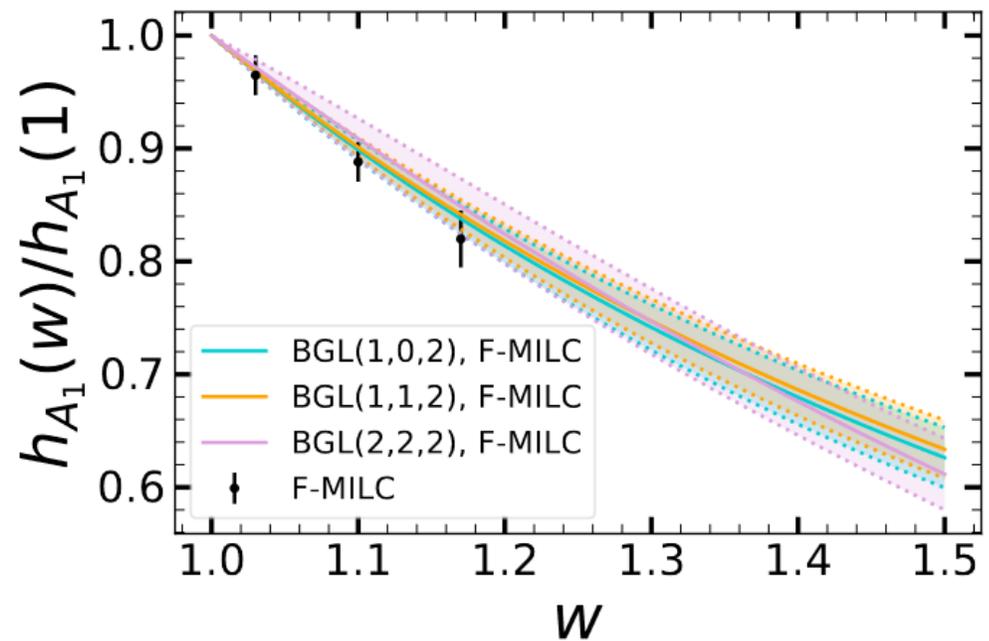
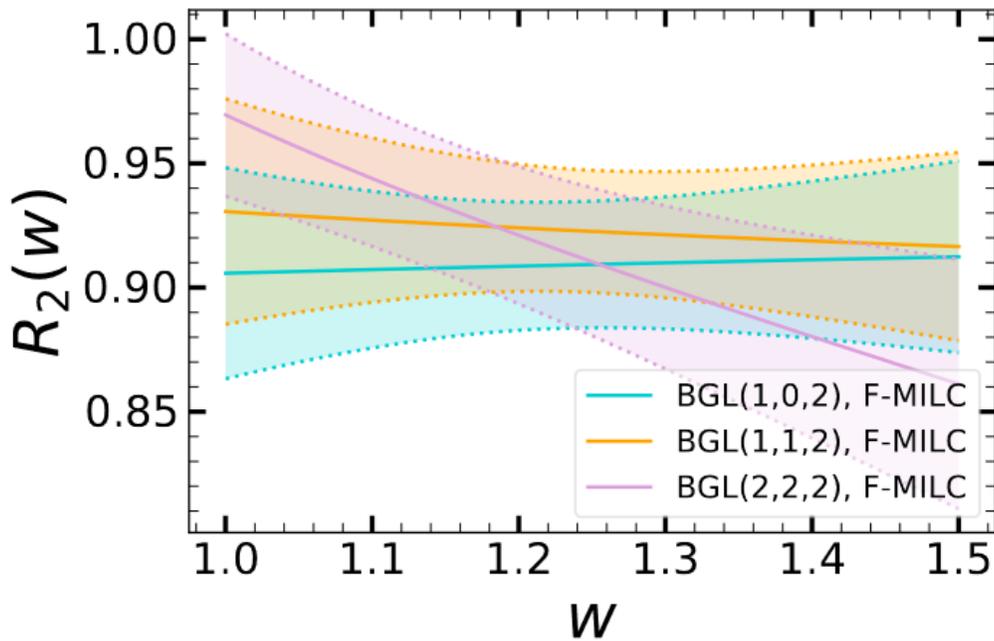
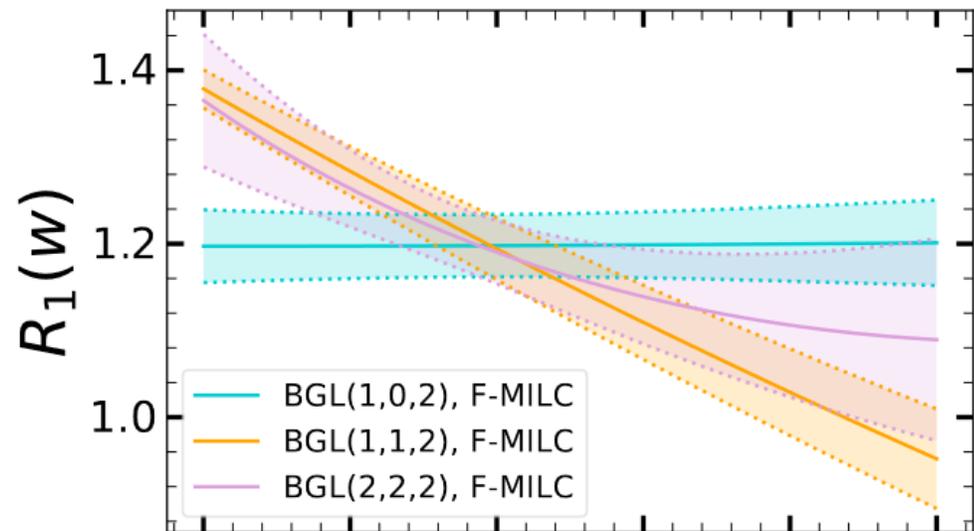
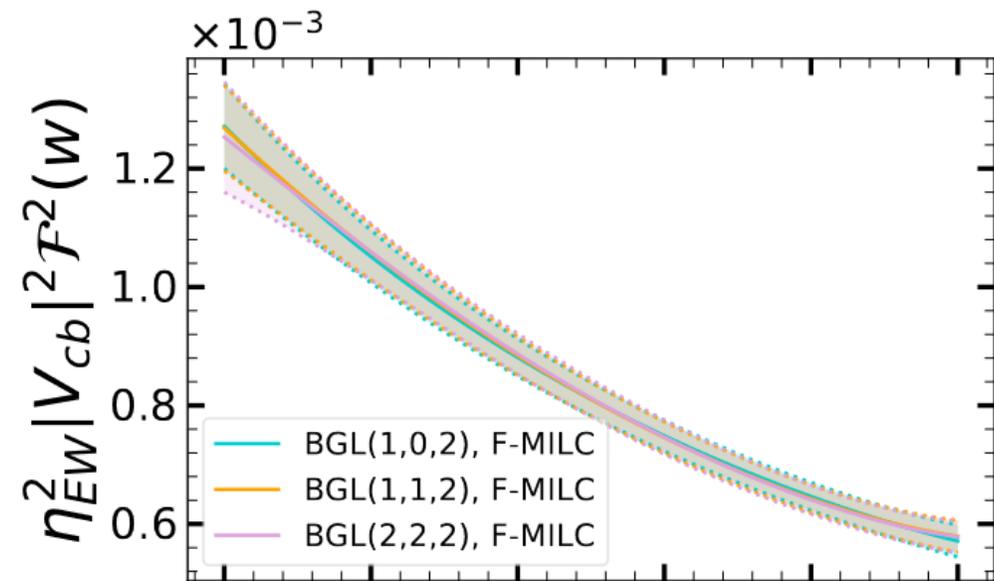
Parameter $\times 10^3$	BGL(1,0,2)	BGL(1,1,2)
$\tilde{a}_0^f$	$0.478 \pm 0.004 \pm 0.013$	$0.458 \pm 0.008 \pm 0.017$
$\tilde{a}_1^f$	$0.53 \pm 0.19 \pm 0.32$	$1.38 \pm 0.33 \pm 0.53$
$\tilde{a}_2^f$	0.0 (fixed)	0.0 (fixed)
$\tilde{a}_0^g$	$1.08 \pm 0.02 \pm 0.02$	$2.07 \pm 0.32 \pm 0.32$
$\tilde{a}_1^g$	0.0 (fixed)	$-35.46 \pm 11.26 \pm 11.20$
$\tilde{a}_2^g$	0.0 (fixed)	0.0 (fixed)
$\tilde{a}_1^{\mathcal{F}_1}$	$0.27 \pm 0.07 \pm 0.09$	$0.36 \pm 0.08 \pm 0.11$
$\tilde{a}_2^{\mathcal{F}_1}$	$-3.61 \pm 1.35 \pm 1.27$	$-3.51 \pm 1.44 \pm 1.28$
$\mathcal{F}(1)\eta_{EW} V_{cb} $	$35.35 \pm 0.28 \pm 0.95$	$33.92 \pm 0.58 \pm 1.25$
$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)$	5.04%	5.04%
$\chi^2/\text{ndf}$	38.7/35	35.4/34
p-value	0.31	0.4

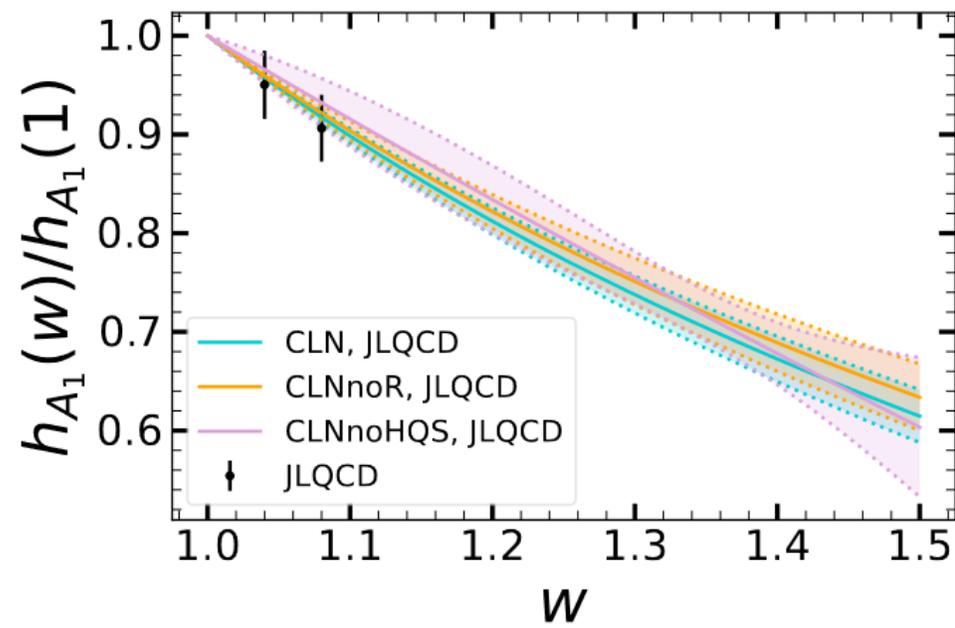
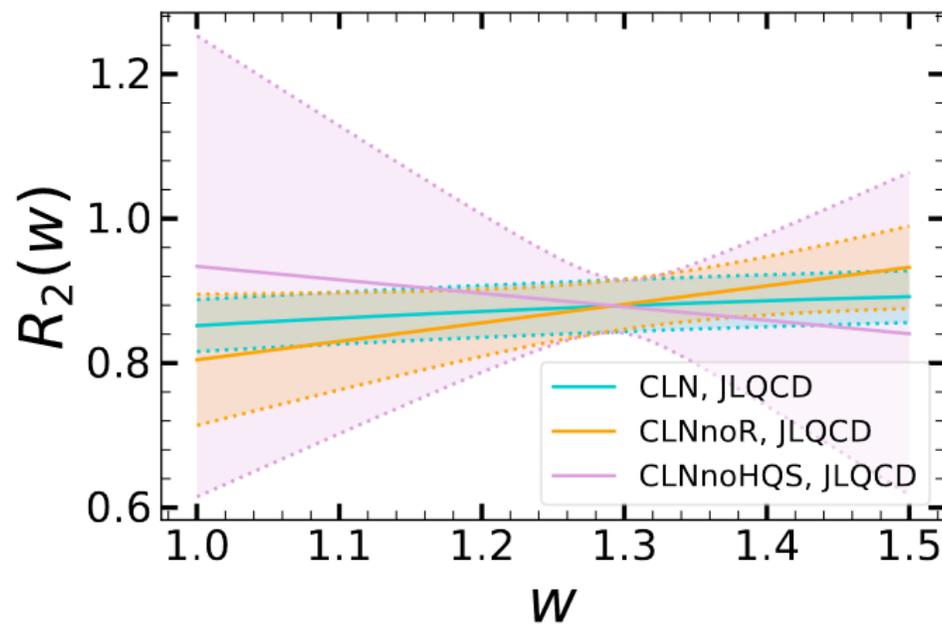
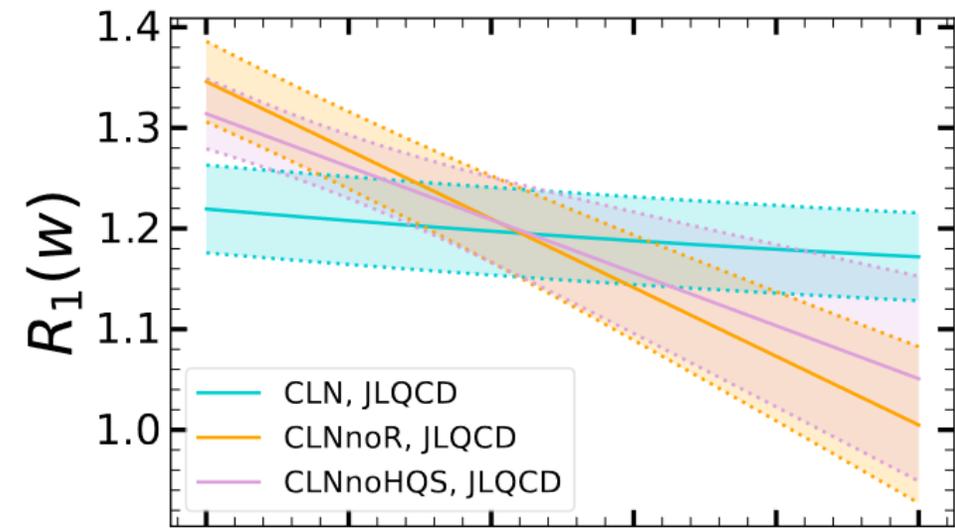
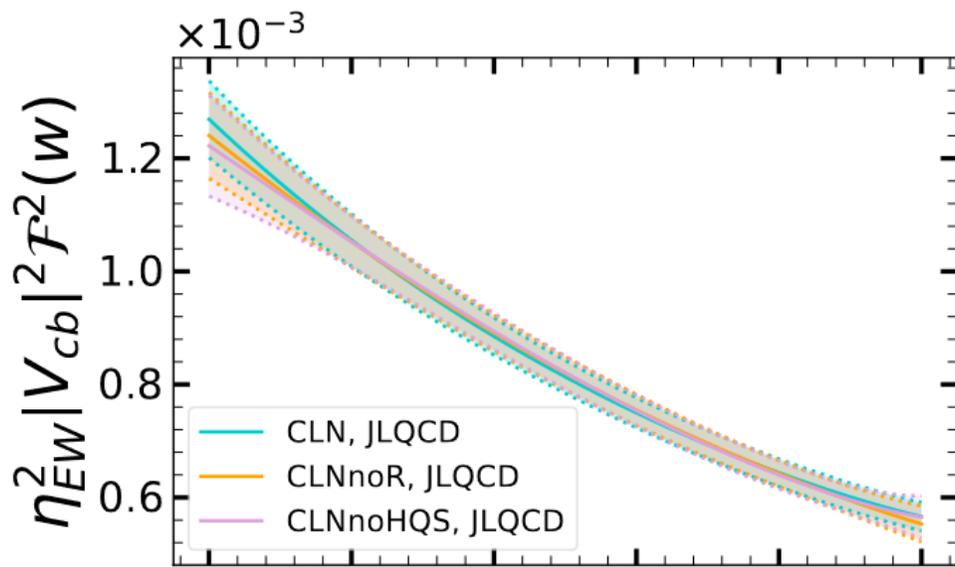
Parameter $\times 10^3$	BGL(1,0,2), JLQCD	BGL(1,1,2), JLQCD	BGL(2,2,2), JLQCD
$\tilde{a}_0^f$	$0.477 \pm 0.003 \pm 0.012$	$0.476 \pm 0.003 \pm 0.012$	$0.475 \pm 0.007 \pm 0.013$
$\tilde{a}_1^f$	$0.50 \pm 0.17 \pm 0.29$	$0.59 \pm 0.16 \pm 0.29$	$0.82 \pm 0.86 \pm 0.96$
$\tilde{a}_2^f$	0.0 (fixed)	0.0 (fixed)	$-5.53 \pm 21.29 \pm 20.01$
$\tilde{a}_0^g$	$1.10 \pm 0.02 \pm 0.02$	$1.23 \pm 0.02 \pm 0.03$	$1.22 \pm 0.04 \pm 0.05$
$\tilde{a}_1^g$	0.0 (fixed)	$-4.73 \pm 0.81 \pm 0.81$	$-3.19 \pm 1.50 \pm 1.98$
$\tilde{a}_2^g$	0.0 (fixed)	0.0 (fixed)	$-36.68 \pm 14.71 \pm 10.69$
$\tilde{a}_1^{\mathcal{F}1}$	$0.28 \pm 0.06 \pm 0.07$	$0.27 \pm 0.06 \pm 0.08$	$0.27 \pm 0.08 \pm 0.09$
$\tilde{a}_2^{\mathcal{F}1}$	$-3.85 \pm 1.08 \pm 1.04$	$-3.35 \pm 1.16 \pm 1.21$	$-3.08 \pm 1.49 \pm 1.18$
$\mathcal{F}(1)\eta_{EW} V_{cb} $	$35.30 \pm 0.24 \pm 0.88$	$35.22 \pm 0.24 \pm 0.86$	$35.13 \pm 0.49 \pm 0.98$
$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)$	5.04%	5.04%	5.04%
$\chi^2/\text{ndf}$	43.9/39	40.4/38	39.5/36
p-value	0.27	0.36	0.32

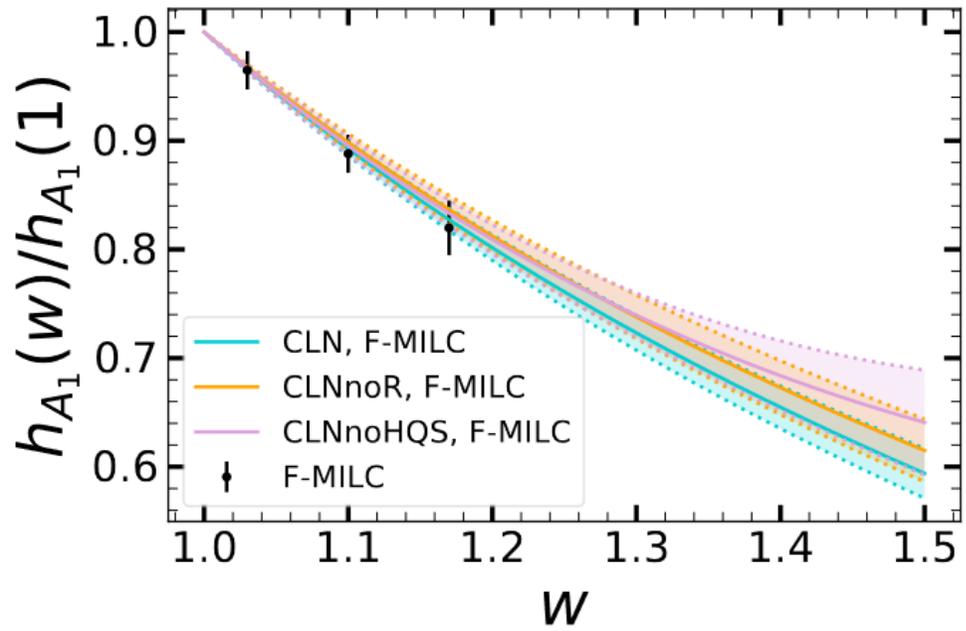
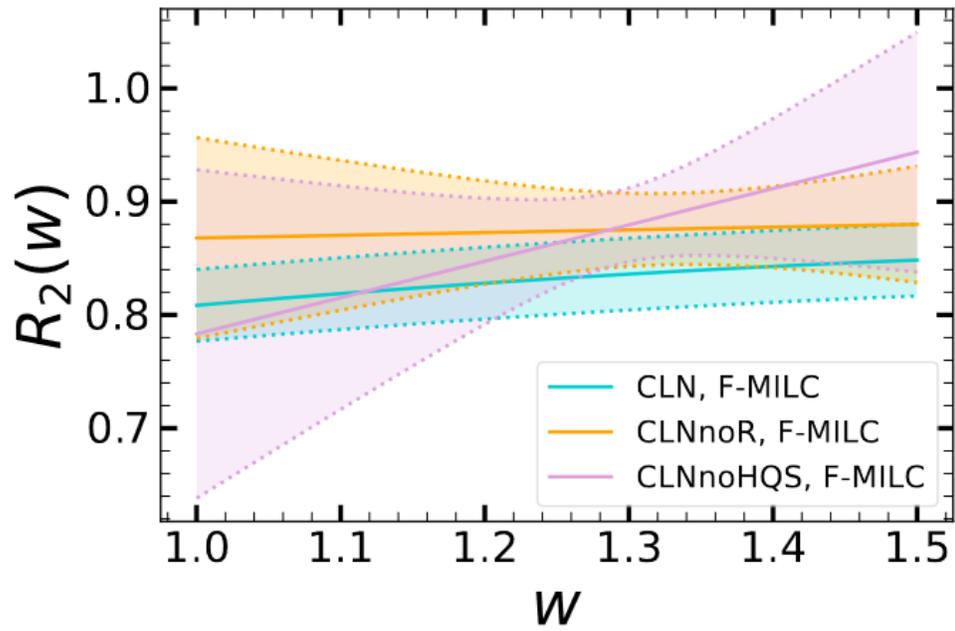
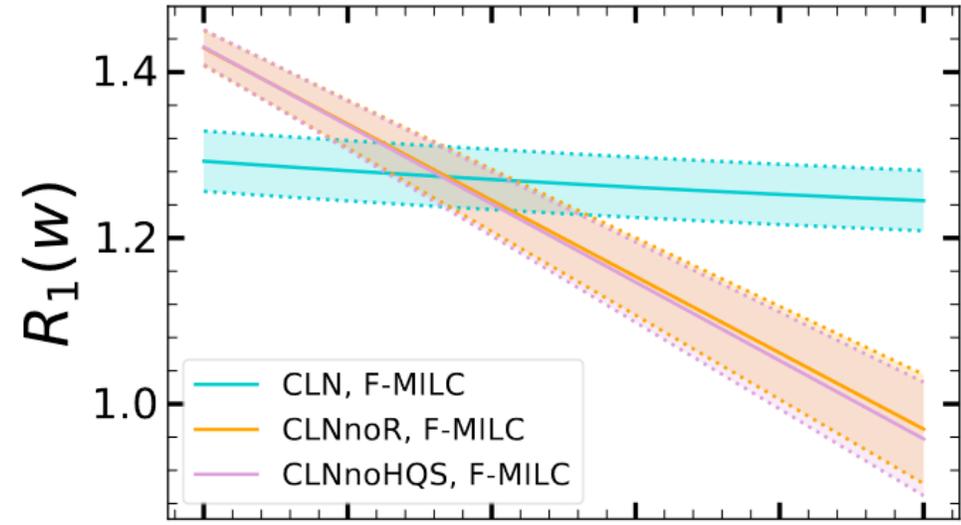
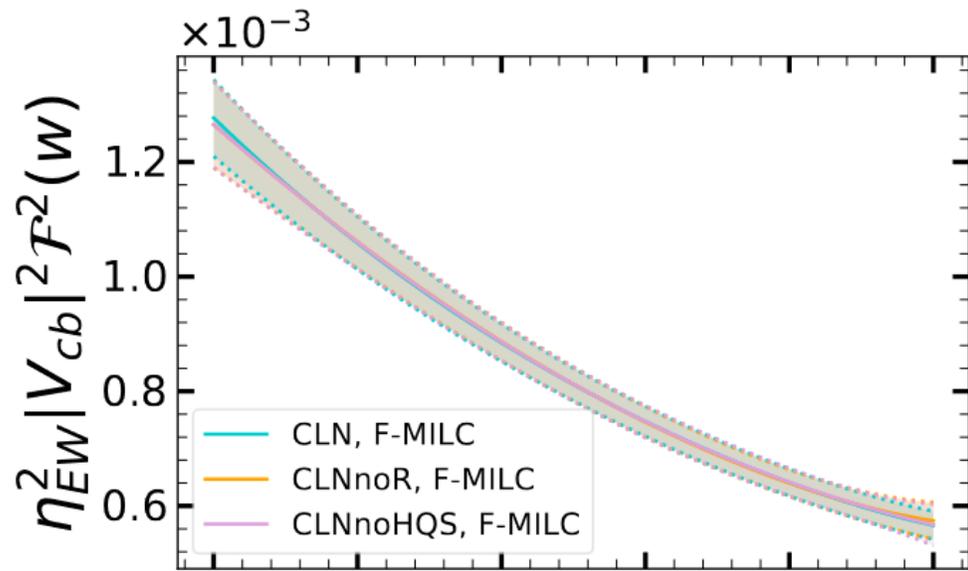
Parameter $\times 10^3$	BGL(1,0,2), F-MILC	BGL(1,1,2), F-MILC	BGL(2,2,2), F-MILC
$\tilde{a}_0^f$	$0.482 \pm 0.003 \pm 0.011$	$0.481 \pm 0.003 \pm 0.011$	$0.478 \pm 0.006 \pm 0.011$
$\tilde{a}_1^f$	$0.42 \pm 0.14 \pm 0.24$	$0.52 \pm 0.14 \pm 0.23$	$1.03 \pm 0.45 \pm 0.49$
$\tilde{a}_2^f$	0.0 (fixed)	0.0 (fixed)	$-14.83 \pm 7.81 \pm 6.80$
$\tilde{a}_0^g$	$1.11 \pm 0.02 \pm 0.02$	$1.28 \pm 0.01 \pm 0.02$	$1.26 \pm 0.03 \pm 0.04$
$\tilde{a}_1^g$	0.0 (fixed)	$-6.95 \pm 0.46 \pm 0.65$	$-7.06 \pm 1.07 \pm 1.10$
$\tilde{a}_2^g$	0.0 (fixed)	0.0 (fixed)	$36.41 \pm 15.34 \pm 9.59$
$\tilde{a}_1^{\mathcal{F}1}$	$0.09 \pm 0.03 \pm 0.05$	$0.08 \pm 0.03 \pm 0.05$	$0.13 \pm 0.08 \pm 0.08$
$\tilde{a}_2^{\mathcal{F}1}$	$-0.01 \pm 0.66 \pm 0.57$	$0.37 \pm 0.68 \pm 0.62$	$-0.28 \pm 1.15 \pm 1.12$
$\mathcal{F}(1)\eta_{EW} V_{cb} $	$35.66 \pm 0.20 \pm 0.80$	$35.62 \pm 0.20 \pm 0.81$	$35.40 \pm 0.48 \pm 0.82$
$\mathcal{B}(B^0 \rightarrow D^{*-} \ell^+ \nu_\ell)$	5.04%	5.04%	5.04%
$\chi^2/\text{ndf}$	89.1/44	62.4/43	61.8/41
p-value	0.0	0.03	0.02











### MILC inputs covariance table

	g(1.03)	g(1.10)	g(1.17)	f(1.03)	f(1.10)	f(1.17)	F1(1.03)	F1(1.10)	F1(1.17)
0	0.000184	0.0001691	0.0001482	0.0001777	0.0001981	0.0002432	0.0006775	0.000947	0.001304
1	0.0001691	0.0001803	0.0001858	0.0001551	0.0002835	0.00056	0.0005879	0.001184	0.002344
2	0.0001482	0.0001858	0.0002769	0.0001338	0.0003933	0.001073	0.0005101	0.001501	0.004229
3	0.0001777	0.0001551	0.0001338	0.01126	0.01057	0.009259	0.03786	0.03787	0.03513
4	0.0001981	0.0002835	0.0003933	0.01057	0.01186	0.01381	0.03528	0.04111	0.04938
5	0.0002432	0.00056	0.001073	0.009259	0.01381	0.02615	0.03071	0.04624	0.09071
6	0.0006775	0.0005879	0.0005101	0.03786	0.03528	0.03071	0.1303	0.136	0.1318
7	0.000947	0.001184	0.001501	0.03787	0.04111	0.04624	0.136	0.176	0.2241
8	0.001304	0.002344	0.004229	0.03513	0.04938	0.09071	0.1318	0.2241	0.449

### MILC inputs inverse covariance table

	0	1	2	3	4	5	6	7	8
0	6.649e+04	-8.153e+04	1.697e+04	3488	-503.8	690.6	-1734	784.8	-167.2
1	-8.153e+04	1.196e+05	-3.491e+04	-5813	1228	-767.9	2641	-1335	307.4
2	1.697e+04	-3.491e+04	1.897e+04	3200	-1026	32.17	-1121	586.3	-153.3
3	3488	-5813	3200	5.431e+04	-2.278e+04	3752	-1.601e+04	6124	-869.9
4	-503.8	1228	-1026	-2.278e+04	1.194e+04	-2569	6467	-2895	539.9
5	690.6	-767.9	32.17	3752	-2569	829.8	-1040	593.8	-168.2
6	-1734	2641	-1121	-1.601e+04	6467	-1040	4827	-1857	264.2
7	784.8	-1335	586.3	6124	-2895	593.8	-1857	844.8	-158.1
8	-167.2	307.4	-153.3	-869.9	539.9	-168.2	264.2	-158.1	46.61

# CLN results with MILC

