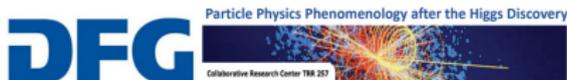


The semileptonic $b \rightarrow d\ell^+\ell^-$ transitions

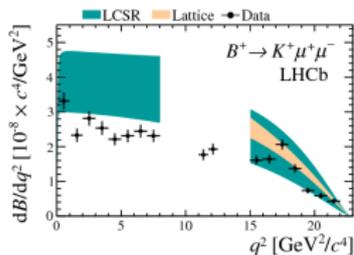
Aleksey Rusov

University of Siegen, Germany

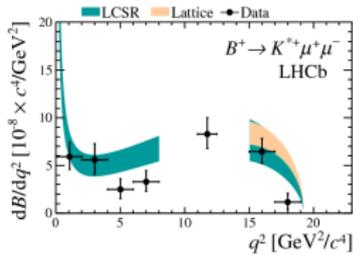


Workshop "Anomalies and Precision in the Belle II Era",
Vienna, 6-8 September 2021

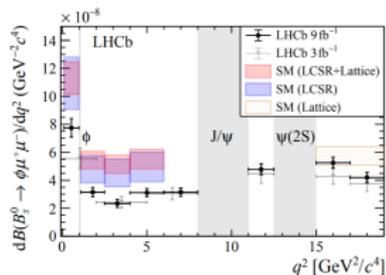
Anomalies in $b \rightarrow sl^+l^-$



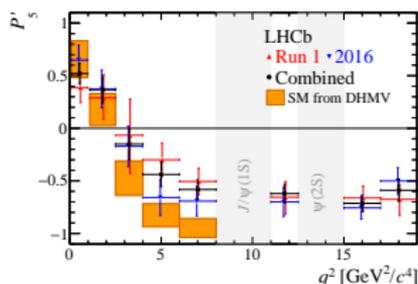
[JHEP 1406 (2014) 133]



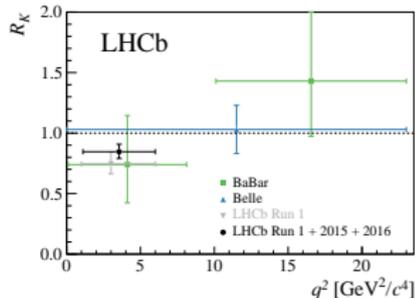
[JHEP 1406 (2014) 133]



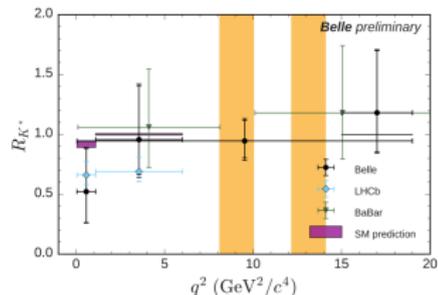
[ArXiv:2105.14007]



[ArXiv: 2003.04831]

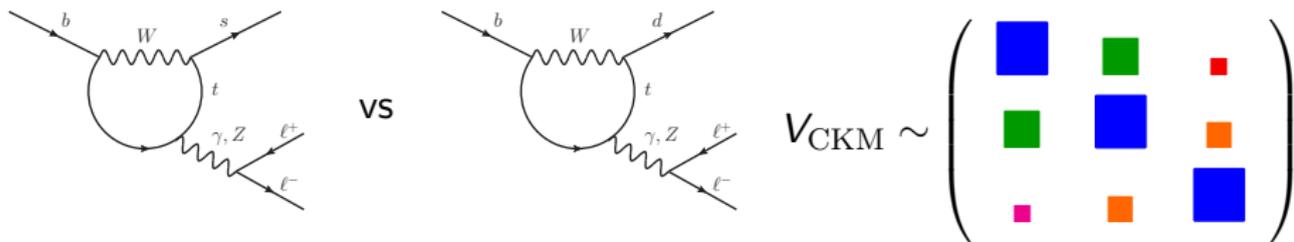


[Phys.Rev.Lett. 122 (2019) 19]
[ArXiv:2103.11769]



[Markus Prim, Moriond EW 2019]

$b \rightarrow d$ vs $b \rightarrow s$



- Additionally CKM suppressed

$$\left| \frac{V_{tb} V_{td}^*}{V_{tb} V_{ts}^*} \right| \approx 0.22 \quad \Rightarrow \quad \left| \frac{V_{tb} V_{td}^*}{V_{tb} V_{ts}^*} \right|^2 \approx 0.05$$

- $b \rightarrow d$ transitions induce **non-vanishing** direct **CP-asymmetry**

▷ In $b \rightarrow s$:

$$|V_{tb} V_{ts}^*| \sim |V_{cb} V_{cs}^*| \sim \lambda^2 \gg |V_{ub} V_{us}^*| \sim \lambda^4$$

▷ In $b \rightarrow d$:

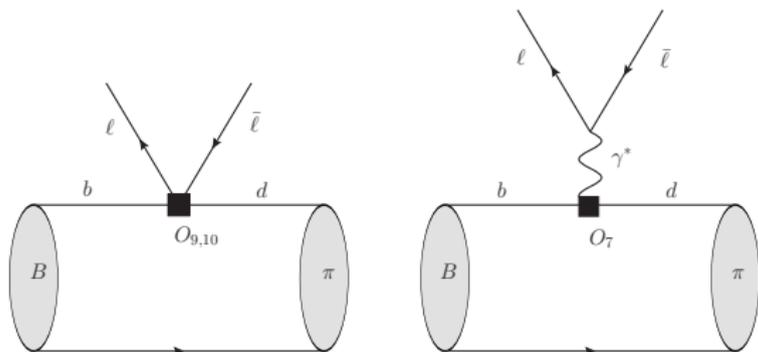
$$|V_{tb} V_{td}^*| \sim |V_{cb} V_{cd}^*| \sim |V_{ub} V_{ud}^*| \sim \lambda^3$$

- Also sensitive to contribution from **New Physics (NP)**

The $B \rightarrow \pi \ell^+ \ell^-$ decays

- Have been measured by the **LHCb** collaboration

[JHEP 12 (2012) 125; JHEP 10 (2015) 034]



- $$\mathcal{H}_{\text{eff}}^{b \rightarrow d} = \frac{4G_F}{\sqrt{2}} \left(\lambda_u \sum_{i=1}^2 C_i \mathcal{O}_i^u + \lambda_c \sum_{i=1}^2 C_i \mathcal{O}_i^c - \lambda_t \sum_{i=3}^{10} C_i \mathcal{O}_i \right) + \text{h.c.}$$

$$\lambda_p = V_{pb} V_{pd}^* \quad p = u, c, t$$

Operators basis

$$\mathcal{O}_9 = \frac{\alpha_{\text{em}}}{4\pi} (\bar{d}_L \gamma^\mu b_L) (\bar{\ell} \gamma_\mu \ell), \quad \mathcal{O}_{10} = \frac{\alpha_{\text{em}}}{4\pi} (\bar{d}_L \gamma^\mu b_L) (\bar{\ell} \gamma_\mu \gamma_5 \ell),$$

$$\mathcal{O}_{7\gamma} = -\frac{e m_b}{16\pi^2} (\bar{d}_L \sigma^{\mu\nu} b_R) F_{\mu\nu},$$

$$\mathcal{O}_1^u = (\bar{d}_L \gamma_\mu u_L) (\bar{u}_L \gamma^\mu b_L), \quad \mathcal{O}_2^u = (\bar{d}_L^i \gamma_\mu u_L^j) (\bar{u}_L^j \gamma^\mu b_L^i),$$

$$\mathcal{O}_1^c = (\bar{d}_L \gamma_\mu c_L) (\bar{c}_L \gamma^\mu b_L), \quad \mathcal{O}_2^c = (\bar{d}_L^i \gamma_\mu c_L^j) (\bar{c}_L^j \gamma^\mu b_L^i),$$

$$\mathcal{O}_3 = (\bar{d}_L \gamma_\mu b_L) \sum_q (\bar{q}_L \gamma^\mu q_L), \quad \mathcal{O}_4 = (\bar{d}_L^i \gamma_\mu b_L^j) \sum_q (\bar{q}_L^j \gamma^\mu q_L^i),$$

$$\mathcal{O}_5 = (\bar{d}_L \gamma_\mu b_L) \sum_q (\bar{q}_R \gamma^\mu q_R), \quad \mathcal{O}_6 = (\bar{d}_L^i \gamma_\mu b_L^j) \sum_q (\bar{q}_R^j \gamma^\mu q_R^i),$$

$$\mathcal{O}_{8g} = -\frac{g_s m_b}{16\pi^2} (\bar{d}_L^i \sigma_{\mu\nu} (T^a)^{ij} b_R^j) G^{a\mu\nu}$$

Hadronic input

Form Factors

$$\langle \pi(p) | \bar{q} \gamma^\mu b | B(p+q) \rangle = f_{B\pi}^+(q^2) (2p^\mu + q^\mu) + \left(f_{B\pi}^+(q^2) - f_{B\pi}^0(q^2) \right) \frac{m_B^2 - m_\pi^2}{q^2} q^\mu$$

$$\langle \pi(p) | \bar{q} \sigma^{\mu\nu} q_\nu b | B(p+q) \rangle = \frac{i f_{B\pi}^T(q^2)}{m_B + m_\pi} \left[2q^2 p^\mu + \left(q^2 - (m_B^2 - m_\pi^2) \right) q^\mu \right]$$

Nonlocal effects via correlation functions

$$\begin{aligned} \mathcal{H}_{B\pi, \mu}^{(p)} = & i \int d^4x e^{iqx} \langle \pi(p) | T \left\{ j_\mu^{\text{em}}(x), \left[C_1 \mathcal{O}_1^p(0) + C_2 \mathcal{O}_2^p(0) \right. \right. \\ & \left. \left. + \sum_{k=3-6,8g} C_k \mathcal{O}_k(0) \right] \right\} | B(p+q) \rangle = [(p \cdot q) q_\mu - q^2 p_\mu] \mathcal{H}_{B\pi}^{(p)}(q^2) \end{aligned}$$

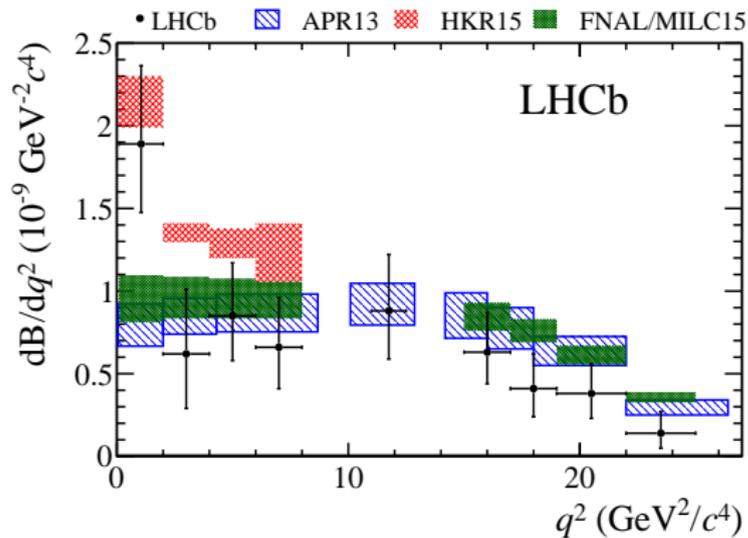
The amplitude

$$\begin{aligned}
 A(B \rightarrow \pi \ell^+ \ell^-) = & \frac{G_F}{\sqrt{2}} \frac{\alpha_{\text{em}}}{\pi} \lambda_t f_{B\pi}^+(q^2) \left[(\bar{\ell} \gamma^\mu \ell) p_\mu \left(C_9 + \frac{2m_b}{m_B + m_P} C_7^{\text{eff}} \frac{f_{B\pi}^T(q^2)}{f_{B\pi}^+(q^2)} \right) \right. \\
 & \left. + (\bar{\ell} \gamma^\mu \gamma_5 \ell) p_\mu C_{10} - (\bar{\ell} \gamma^\mu \ell) p_\mu \underbrace{\frac{16\pi^2}{f_{B\pi}^+(q^2)} \left(\frac{\lambda_u}{\lambda_t} \mathcal{H}_{B\pi}^{(u)}(q^2) + \frac{\lambda_c}{\lambda_t} \mathcal{H}_{B\pi}^{(c)}(q^2) \right)}_{\Delta C_9(q^2)} \right]
 \end{aligned}$$

- Wilson coefficients C_i at **NLO**
- Form factors $f_{B\pi}^{+,T}(q^2)$ from **LCSR** [A. Khodjamirian, A.V. Rusov (2017)]
- Nonlocal hadronic amplitudes $\mathcal{H}_{BP}^{(u,c)}(q^2)$ via **QCDF, LCSR** and **hadronic dispersion relations** [Ch. Hambrock, A. Khodjamirian, A.V. Rusov (2015)]

Theory vs LHCb measurement

- Differential branching fraction in $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ [LHCb, JHEP 10 (2015) 034]



■ - including non-local effects
and resonances contributions
[Hambrock, Khodjamirian, AR (2015)]

Probing NP in $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays

- Consider NP effective operators

$$\mathcal{O}_{9,L} = (\bar{d}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \mu)$$

$$\mathcal{O}_{10,L} = (\bar{d}\gamma_\mu P_L b)(\bar{\mu}\gamma^\mu \gamma_5 \mu)$$

$$\mathcal{O}_{9,R} = (\bar{d}\gamma_\mu P_R b)(\bar{\mu}\gamma^\mu \mu)$$

$$\mathcal{O}_{10,R} = (\bar{d}\gamma_\mu P_R b)(\bar{\mu}\gamma^\mu \gamma_5 \mu)$$

- The estimates [A.V. Rusov (2019)]

Scenario	1σ -interval
C_9^{NP} only	$[-5.2, -1.9]$
C_{10}^{NP} only	$[+1.4, +6.8]$
$C_9^{\text{NP}} = -C_{10}^{\text{NP}}$	$[-1.8, -0.7] \cup [-7.0, -5.8]$

$$C_9^{\text{NP}} = C_{9,L} + C_{9,R}$$

$$C_{10}^{\text{NP}} = C_{10,L} + C_{10,R}$$

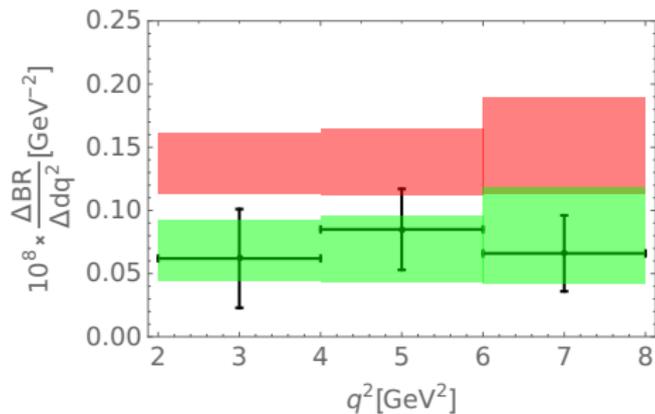
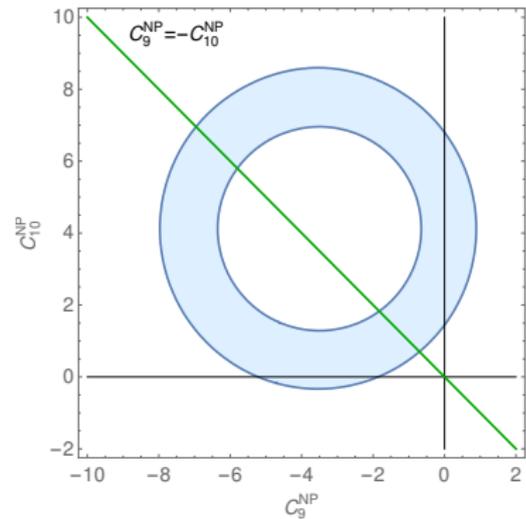
- For example, from global fit of the $b \rightarrow s\ell^+\ell^-$ observables

$$C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} = -0.46 \pm 0.10 \quad [\text{M. Alguero et al. (2019)}]$$

$$C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} = -0.41 \pm 0.10 \quad [\text{A. Arbey et al. (2019)}]$$

$$C_{9\mu}^{\text{NP}} = -C_{10\mu}^{\text{NP}} = -0.53 \pm 0.08 \quad [\text{J. Aebischer et al. (2019)}]$$

Probing NP in $B^\pm \rightarrow \pi^\pm \mu^+ \mu^-$ decays



The LFU in $b \rightarrow d\ell^+\ell^-$

- The LFU ratio: $R_\pi[q_{\min}^2, q_{\max}^2] = \frac{\text{Br}(B \rightarrow \pi\mu^+\mu^-)[q_{\min}^2, q_{\max}^2]}{\text{Br}(B \rightarrow \pi e^+e^-)[q_{\min}^2, q_{\max}^2]}$
- In the SM: $R_\pi^{\text{SM}} \cong 1$
- [Bordone et al., arXiv:2101.11626]:
 - ▷ considering NP effects

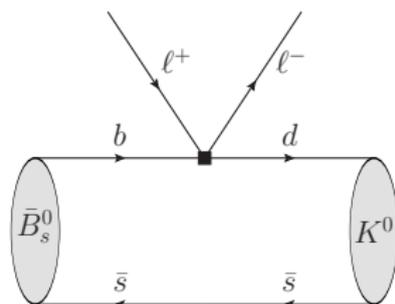
$$R_\pi = R_\pi^{\text{SM}} + b_v \Delta C_9 + (b_v - b_L) \Delta C_{10}$$

- ▷ using $\Delta C_9 = -\Delta C_{10} = -0.43 \pm 0.11$ from $b \rightarrow s$ fit
[J. Fuentes-Martin et al, arXiv:1909.02519]

$$R_\pi[1.1, 8.0] \Big|_{U(2)^5}^{\text{NP}} = 0.79 \pm 0.05_{\text{NP}} \pm 0.02_{\text{LD}}$$

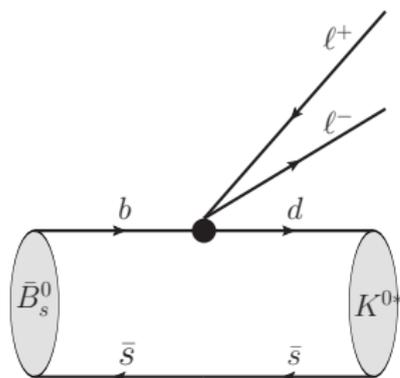
- Measurement of $B^\pm \rightarrow \pi^\pm e^+e^-$?

The $\bar{B}_s^0 \rightarrow K^0 \ell^+ \ell^-$ decays



- Similar to $B \rightarrow \pi \ell^+ \ell^-$ up to $SU(3)_F$ -breaking
- [A. Khodjamirian, A.V. Rusov (2017)]:
 - ▷ New results for the $B_s \rightarrow K$ form factors and corresponding nonlocal hadronic amplitudes
 - ▷ Predictions for the differential branching fraction and CP -asymmetry at large hadronic recoil
- No measurement so far

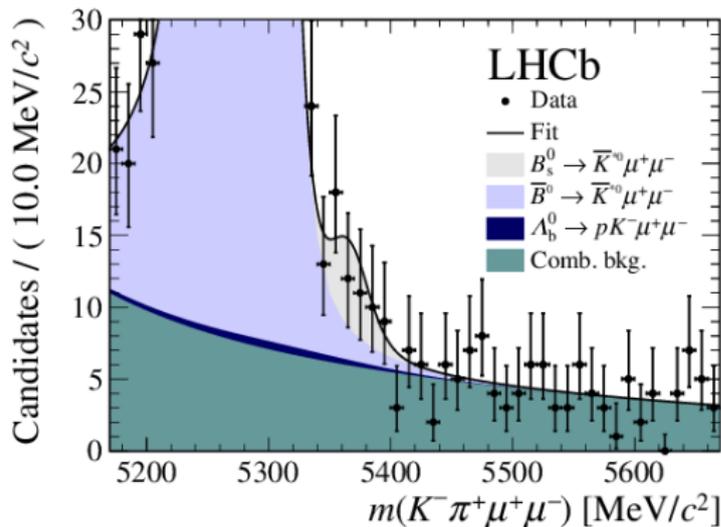
The $B_s^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$ decay



- Vector final state K^{0*} -meson ($J^P = 1^-$)
→ 3 helicity amplitudes
- 7 form factors
6 nonlocal hadronic amplitudes
→ more involved theoretical analysis
- $\text{BR}(B_s^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-) \sim 10^{-8}$

- Richer set of interesting observables
 - ▷ Differential decay rate
 - ▷ **Nonzero** direct CP -asymmetry
 - ▷ Angular distributions (incl. P'_5)

$B_s^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$ decay



First evidence of the $B_s^0 \rightarrow \bar{K}^{*0} \mu^+ \mu^-$ decay (with a significance of 3.4σ)
[LHCb, JHEP 07 (2018) 020]

Conclusion

- $b \rightarrow d\ell^+\ell^-$ processes are potential sources of **New Physics**
- CKM suppressed vs $b \rightarrow s \Rightarrow$ more statistics needed
- More interesting observables (incl. non-vanishing **CP-asymmetry**)
- Not discussed:
 - ▷ $B^0 \rightarrow \pi^+\pi^-\mu^+\mu^-$
 - ▷ $\Lambda_b^0 \rightarrow p\pi^-\mu^+\mu^-$
- Looking forward to more data on
 - ▷ $B^\pm \rightarrow \pi^\pm\mu^+\mu^-$ (incl. diff. **CP-asymmetry**)
 - ▷ $B^\pm \rightarrow \pi^\pm e^+e^-$
 - ▷ $\bar{B}_s^0 \rightarrow K^{0(*)}\mu^+\mu^-$
 - ▷ $B^0 \rightarrow \mu^+\mu^-$

Backup

Form factors from LCSR

- Form factors parametrise hadronic matrix element

$$\langle \pi(p) | \bar{d} \gamma^\mu b | B(p+q) \rangle \sim f_{B\pi}^+(q^2)$$

- Form factors are calculated from QCD Light Cone Sum Rule (LCSR)
- Starting object – correlation function

$$F_{B\pi}^\mu(p, q) = i \int d^4x e^{iqx} \langle \pi^+(p) | T \{ \underbrace{\bar{u}(x) \gamma^\mu b(x)}_{\text{Quark current}}, \underbrace{m_b \bar{b}(0) i \gamma_5 d(0)}_{\text{B-meson interpolating current}} \} | 0 \rangle$$

$$= F_{B\pi}(q^2, (p+q)^2) p^\mu + \tilde{F}_{B\pi}(q^2, (p+q)^2) q^\mu$$

- Region of light-cone dominance ($x^2 \sim 0$): $q^2 \ll m_b^2$, $(p+q)^2 \ll m_b^2$

$$\langle \pi^+(p) | \bar{u}(x) \gamma^\mu \gamma_5 d(0) | 0 \rangle |_{x^2 \rightarrow 0} \sim \int du e^{iupx} \phi_\pi(u) + \dots$$

$\phi_\pi(u)$ - pion light cone distribution amplitude (LCDA)

Form factors from LCSR

- The current accuracy of OPE:

$$\text{OPE} = \left(T_0^{(2)} + (\alpha_s/\pi) T_1^{(2)} + (\alpha_s/\pi)^2 T_2^{(2)} \right) \otimes \varphi_\pi^{(2)} \\ + \left(T_0^{(3)} + (\alpha_s/\pi) T_1^{(3)} \right) \otimes \varphi_\pi^{(3)} + T_0^{(4)} \otimes \varphi_\pi^{(4)} + \langle \bar{q}q \rangle \left(T_0^{(5)} \otimes \varphi_\pi^{(2)} + T_0^{(6)} \otimes \varphi_\pi^{(3)} \right)$$

$T_n^{(k)}$ - hard-scattering kernels, $\varphi_\pi^{(k)}$ - pion LCDAs ($k = 2, 3, \dots$ - twist)

- LO twist 2, 3, 4

[V.Belyaev, A.Khodjamirian, R.Rückl (1993); V.Braun, V.Belyaev, A.Khodjamirian, R.Rückl (1996)]

- NLO $O(\alpha_s)$ twist 2

[A.Khodjamirian, R.Rückl, S.Weinzierl, O. Yakovlev (1997); E.Bagan, P.Ball, V.Braun (1997)]

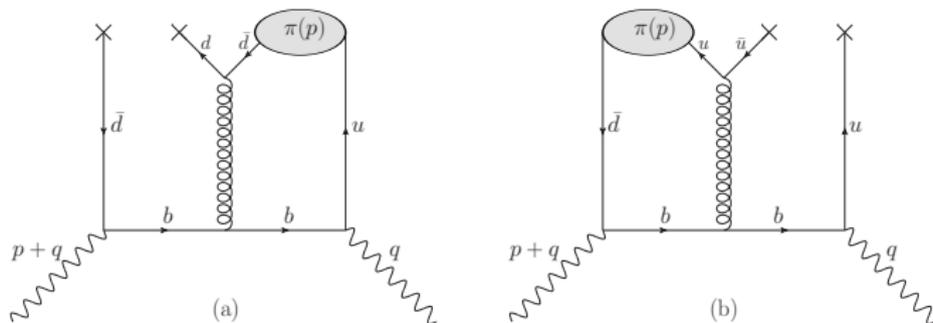
- NLO $O(\alpha_s)$ twist 3

[P. Ball, R. Zwicky (2001); G.Duplancic, A.Khodjamirian, B.Melic, Th.Mannel, N.Offen (2007)]

- Part of NNLO $O(\alpha_s^2\beta_0)$ twist 2 [A. Bharucha (2012)]

- LO twist 5 and twist 6 in factorization approximation [A.V. Rusov (2017)]

Factorizable twist-5 and twist-6 contributions



- In the framework of the factorization approximation

$$[F_{B\pi}(q^2)]_{\text{tw}5,6} \sim \langle \bar{q}q \rangle \left(T_0^{(5)} \otimes \varphi_P^{(2)} + T_0^{(6)} \otimes \varphi_P^{(3)} \right)$$

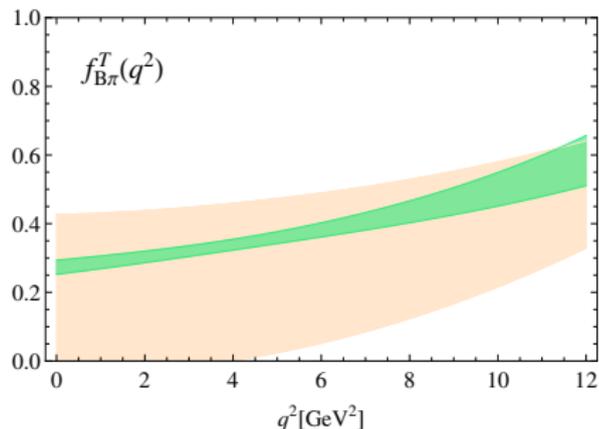
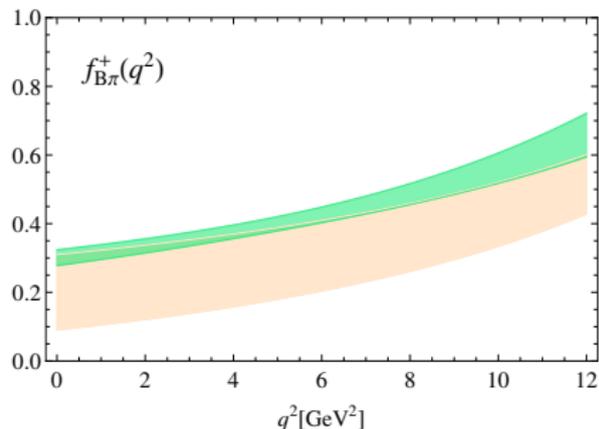
- Calculation reveals [A.V. Rusov (2017)]

$$[f_{B\pi}^+(0)]_{\text{tw}5,6} / f_{B\pi}^+(0) < 0.1\%$$

- \Rightarrow Truncation up to twist-4 contributions is reliable

Results for $B \rightarrow \pi$ form factors

[A. Khodjamirian, A.V. Rusov (2017)]

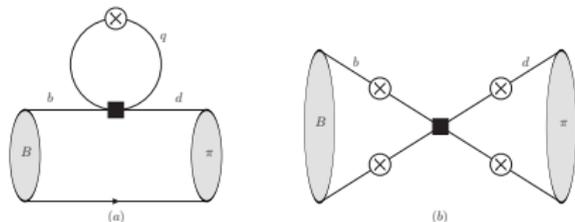


LCSR vs Lattice QCD (extrapolations from high q^2)

[FermiLab Lattice and MILC (2015)]

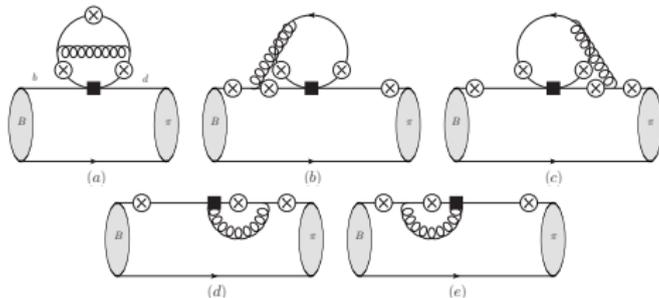
Calculation of $\mathcal{H}_{B\pi}^{(u,c)}(q^2)$ at $q^2 < 0$

- LO, factorizable loop and weak annihilation [M. Beneke, Th. Feldmann, D. Seidel (2001)]



- NLO, factorizable

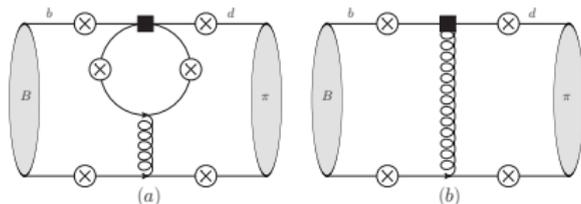
[H.H.Asatryan, H.M. Asatrian, C. Greub, M. Walker (2002)]
 [H.M.Asatrian, K. Bieri, C. Greub, M. Walker (2004)]



Calculation of $\mathcal{H}_{B\pi}^{(u,c)}(q^2)$ at $q^2 < 0$

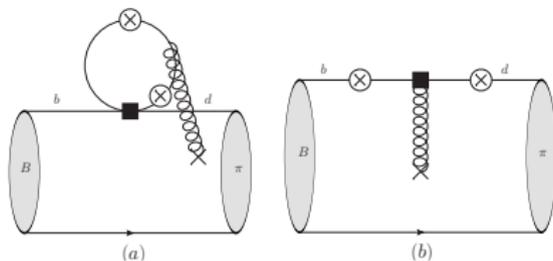
- NLO, nonfactorizable (hard gluons)

[M. Beneke, Th. Feldmann, D. Seidel (2001)]



- Soft gluons

[A. Khodjamirian, Th. Mannel, A.A. Pivovarov, Y.-M. Wang (2010)]
[A. Khodjamirian, Th. Mannel, Y.-M. Wang (2013)]



▷ Need to be updated using new determination of B-meson LCDAs

[see talk by Danny van Dyk]

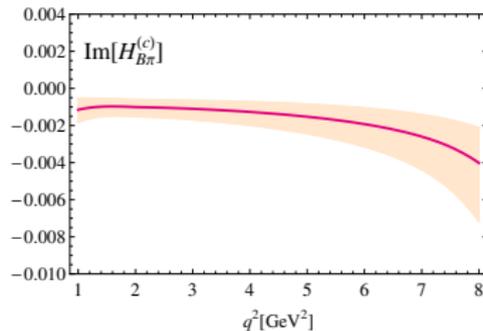
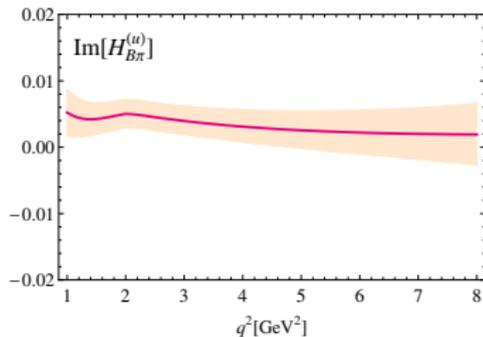
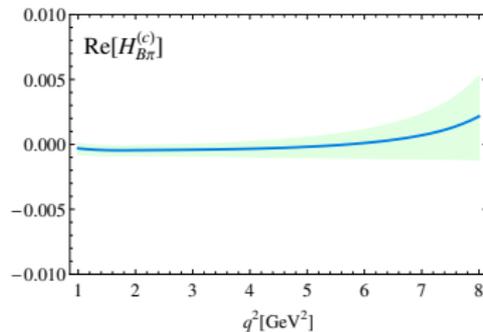
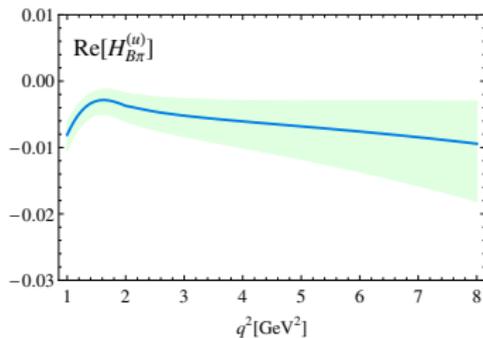
Dispersion relations for $\mathcal{H}_{BP}^{(u,c)}(q^2)$

- The dispersion relations:

$$\mathcal{H}_{BP}^{(u,c)}(q^2) = (q^2 - q_0^2) \left[\sum_{V=\rho,\omega,J/\psi,\psi(2S)} \frac{k_V f_V A_{BVP}^{u,c}}{(m_V^2 - q_0^2)(m_V^2 - q^2 - im_V \Gamma_V^{\text{tot}})} + \int_{s_0^{u,c}}^{\infty} ds \frac{\rho_{BP}^{(u,c)}(s)}{(s - q_0^2)(s - q^2 - i\epsilon)} \right] + \mathcal{H}_{BP}^{(u,c)}(q_0^2)$$

- $A_{BVP}^{u,c} = |A_{BVP}^{u,c}| e^{i\delta_{BVP}^{u,c}}$
- $|A_{BVP}^{u,c}|$ are extracted from nonleptonic $B \rightarrow VP$ decays
- $\delta_{BVP}^{u,c}$ are extracted from the fit of the DR to $\mathcal{H}_{BP}^{(u,c)}(q^2)$ at $q^2 < 0$
- For $\rho_{BP}^{(u,c)}(s)$ one applies quark-hadron duality

Results for $\mathcal{H}_{B\pi}^{(u)}$, $\mathcal{H}_{B\pi}^{(c)}$



Direct CP asymmetry of $B \rightarrow \pi \ell^+ \ell^-$ decays

- Definition of **direct CP asymmetry**

$$\mathcal{A}_{CP}[q_1^2, q_2^2] = \frac{\text{BR}(B^- \rightarrow \pi^- \ell^+ \ell^-; q_1^2 \leq q^2 \leq q_2^2) - \text{BR}(B^+ \rightarrow \pi^+ \ell^+ \ell^-; q_1^2 \leq q^2 \leq q_2^2)}{\text{BR}(B^- \rightarrow \pi^- \ell^+ \ell^-; q_1^2 \leq q^2 \leq q_2^2) + \text{BR}(B^+ \rightarrow \pi^+ \ell^+ \ell^-; q_1^2 \leq q^2 \leq q_2^2)}$$

- LHCb measurement of **total CP -asymmetry** [LHCb, JHEP 10 (2015) 034]

$$\mathcal{A}_{CP}[q_{\min}^2, q_{\max}^2] = -0.11 \pm 0.12 \pm 0.01$$

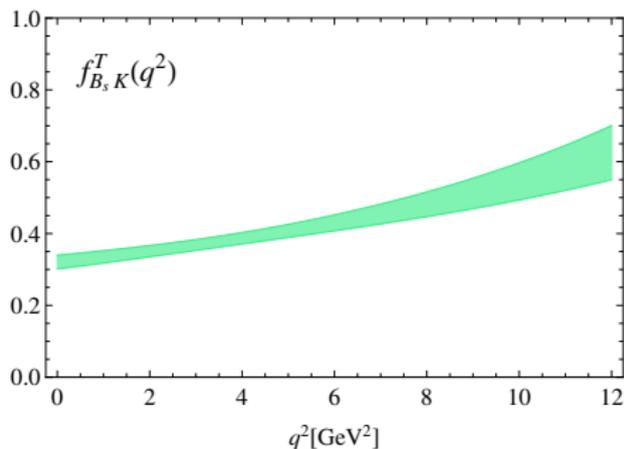
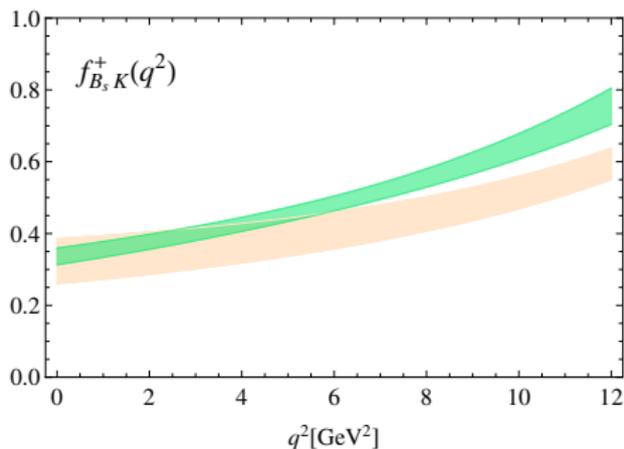
- SM prediction (only at **low q^2 range**) [A. Khodjamirian, A.V. Rusov (2017)]

$$\mathcal{A}_{CP}[1 \text{ GeV}^2, 6 \text{ GeV}^2] = -0.15 \pm 0.11$$

- The measurement of the **binned CP -asymmetry** is anticipated

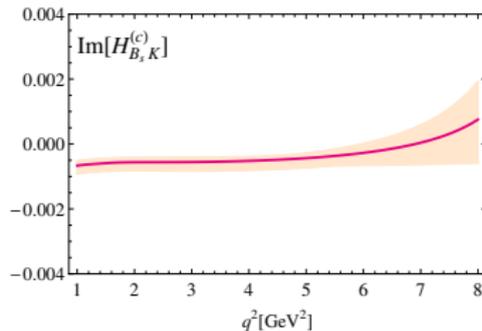
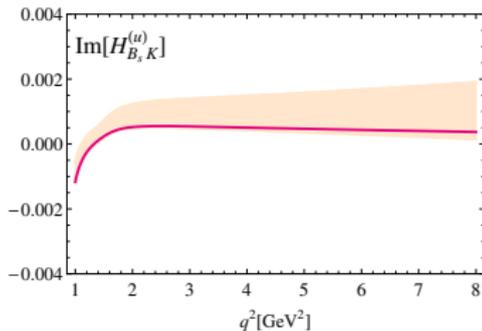
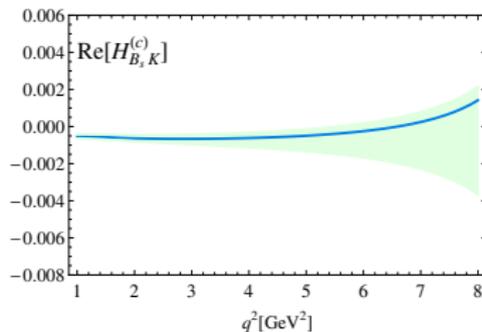
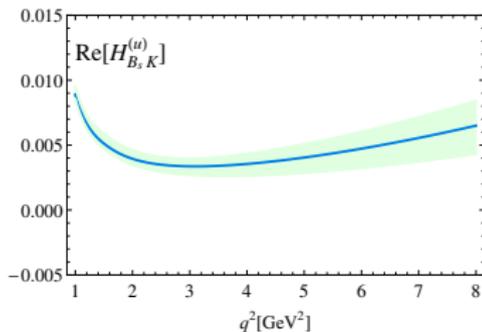
Results for the $B_s \rightarrow K$ form factors

A. Khodjamirian, A.V. Rusov, 1703.04765 [hep-ph]



Lattice QCD results: C.M. Bouchard et al., hep-lat: 1406.2279

Results for $\mathcal{H}_{B_s K}^{(u)}$, $\mathcal{H}_{B_s K}^{(c)}$



Impact on the $B^0 \rightarrow \mu^+ \mu^-$ decays

- The current experimental bounds

$$\text{BR}(B^0 \rightarrow \mu^+ \mu^-) < 2.1 \times 10^{-10} \quad 95\% \text{ CL} \quad [\text{ATLAS (2019)}]$$

$$\text{BR}(B^0 \rightarrow \mu^+ \mu^-) < 3.6 \times 10^{-10} \quad 95\% \text{ CL} \quad [\text{CMS (2019)}]$$

$$\text{BR}(B^0 \rightarrow \mu^+ \mu^-) < 3.4 \times 10^{-10} \quad 95\% \text{ CL} \quad [\text{LHCb (2017)}]$$

- The Standard Model prediction [M. Beneke, C. Bobeth, R. Szafron (2019)]

$$\text{BR}^{\text{SM}}(B^0 \rightarrow \mu^+ \mu^-) = (1.027 \pm 0.051) \times 10^{-10}$$

- The NP estimates

$$\text{BR}^{\text{NP}}(B^0 \rightarrow \mu^+ \mu^-) \simeq (0.6 \pm 0.2) \times 10^{-10} \quad \text{if } C_{10}^{\text{NP}} = C_{10,L}$$

$$\text{BR}^{\text{NP}}(B^0 \rightarrow \mu^+ \mu^-) \simeq (1.8 \pm 0.4) \times 10^{-10} \quad \text{if } C_{10}^{\text{NP}} = C_{10,R}$$

- Not yet possible to resolve the left- and right-handed quark operators

Impact on $B^0 - \bar{B}^0$ -mixing

- Consider as an example the NP model with Z' boson

$$\mathcal{L}_{Z'}^{\text{eff}} = -\frac{1}{2M_{Z'}^2} \left[\left(g_{13}^Q \right)^2 (\bar{d}_L \gamma^\mu b_L)(\bar{d}_L \gamma_\mu b_L) + 2 g_{13}^Q g_{22}^L (\bar{d}_L \gamma^\mu b_L)(\bar{\mu}_L \gamma_\mu \mu_L) \right] + \dots$$

