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CONNECTING SCIENCES

Slow-roll inflation in Palatini $F(R)$ gravity

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based on JHEP 06 (2022) 106
with
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Euroopa Liit
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Eesti tuleviku heaks

Dark Universe

• Summary •

Gravity preliminaries

The properties of torsion-free spacetime are essentially described by:

- the affine connection: $\Gamma^{\lambda}_{\alpha\beta}$
- the metric tensor: $g_{\mu\nu}$

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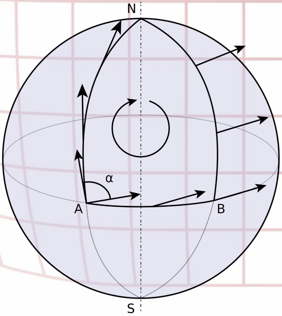
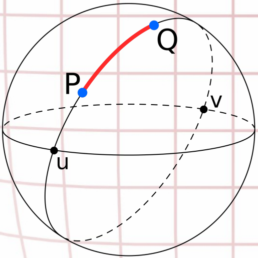
$\Gamma^{\lambda}_{\alpha\beta}$ describes the parallel transport of tensor fields along a given curve. If the spacetime is curved, parallel transport around a closed path, after a full cycle, results in a finite mismatch.



The properties of torsion-free spacetime are essentially described by:

- the affine connection: $\Gamma^{\lambda}_{\alpha\beta}$
- the metric tensor: $g_{\mu\nu}$

$g_{\mu\nu}$ allows us to introduce the notion of distance.



• Gravity preliminaries •

The properties of torsion-free spacetime are essentially described by:

- the affine connection: $\Gamma^{\lambda}_{\alpha\beta}$ → parallel transport
- the metric tensor: $g_{\mu\nu}$ → distance

The connection coefficients and metric tensor are fundamentally independent quantities. They exhibit no *a priori* known relationship. If they are to have any relationship, it must derive from

- additional constraints (metric formalism $\nabla_\alpha g_{\mu\nu} = 0$)
- EoM (Palatini formalism)

• Motivation •

- Inflation with R^2 term in the Palatini formalism, Enckell et al. 1810.05536

$$S_J = \int d^4x\sqrt{-g_J}\left[\frac{1}{2}\left(R_J + \alpha R_J^2\right) - \frac{1}{2}g_J^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V(\phi)\right] \qquad \boxed{M_P = 1}$$

- Main results:
  - ◇ universal asymptotically flat Einstein-frame scalar potential

$$U = \frac{V}{1+8\alpha V} = \frac{U^0}{1+8\alpha U^0} \xrightarrow{\alpha\to\infty} \frac{1}{8\alpha}$$

  $\ldots^0$ means the same quantity but for $\alpha = 0$
  - ◇ simple inflationary preditions:

$$N_e \simeq N_e^0$$
$$A_s = A_s^0$$
$$n_s = n_s^0$$
$$r = \frac{r_0}{1 + 12\pi^2\alpha A_s^{exp} r^0} \xrightarrow{\alpha\to\infty} \frac{1}{12\pi^2\alpha A_s^{exp}}$$

  - ◇ $\alpha \gg 1 \Rightarrow r \to 0$ regardless of the initial V
- Q's: What makes $F(R) = R + \alpha R^2$ special? Is there any $F(R) \Rightarrow r \to 0$?

Palatini $F(R)$

- we start with Palatini $F(R)$ action alone

$$S_J = \int \mathrm{d}^4x \sqrt{-g_J} \frac{1}{2} F(R_J(\Gamma))$$

- we rewrite the $F(R)$ term using the auxiliary field ζ

$$\begin{aligned} S_J &= \int \mathrm{d}^4x \sqrt{-g_J} \left[\frac{1}{2} F'(\zeta) R_J(\Gamma) - V_J(\zeta) \right] \\ V_J(\zeta) &= \frac{-F(\zeta)}{2} + \frac{\zeta F'(\zeta)}{2} \qquad F' = \frac{\partial F}{\partial \zeta} \end{aligned}$$

- we move to the Einstein frame:

$$\left. \begin{array}{l} g^E_{\mu\nu} = F' g^J_{\mu\nu} \\ \Gamma^E = \Gamma^J \end{array} \right\} \Rightarrow \left\{ \begin{array}{l} g^J_{\mu\nu} = g^E_{\mu\nu}/F' \quad \Rightarrow \quad \sqrt{-g^J} = \sqrt{-g^E}/(F')^2 \\ R^J_{\mu\nu} = R^E_{\mu\nu} \quad \Rightarrow \quad R_J = F' R_E \end{array} \right.$$

$$\begin{aligned} S_E &= \int \mathrm{d}^4x \sqrt{-g_E} \left[\frac{R_E}{2} - V_E(\zeta) \right] \\ V_E(\zeta) &= -\frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)} \end{aligned}$$

- N.B. ζ stays auxiliary!
  Not dynamical like in metric gravity (cf. Starobinsky model)

• Palatini $F(R)$ •

- ζ's EoM: $V_E'(\zeta) = 0 \rightarrow$ a bit of albegra:

$$G(\zeta) = \frac{1}{4}\left[2F(\zeta) - \zeta F'(\zeta)\right] = 0$$

$$\zeta = \zeta^* \quad \rightarrow \quad F(\zeta^*) = \frac{1}{2}\zeta^* F'(\zeta^*)$$

with $\boxed{F', F'' \neq 0}$

- inserting $\zeta = \zeta^*$ in V_E we obtain

$$V_E(\zeta^*) = -\frac{\zeta^* F'(\zeta^*)}{2F'(\zeta^*)^2} + \frac{\zeta^*}{4F'(\zeta^*)} = \frac{1}{4}\frac{\zeta^*}{F'(\zeta^*)}$$

i.e. a CC

- therefore pure Palatini $F(R)$ is equivalent to GR + CC
- completely different from metric gravity!

• Palatini $F(R) + \mathcal{L}(\phi)$ •

- We add a scalar field to the previous setup

$$S_J = \int \mathrm{d}^4x \sqrt{-g_J} \left[\frac{1}{2} F(R_J(\Gamma)) + \mathcal{L}(\phi) \right]$$

$$\mathcal{L}(\phi) = -\frac{1}{2} k(\phi) g_J^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$$

- again introducing ζ and moving to the Einstein frame

$$S_E = \int \mathrm{d}^4x \sqrt{-g_E} \left[\frac{R_E}{2} - \frac{1}{2} g_E^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - U(\chi, \zeta) \right]$$

$$\frac{\partial \chi}{\partial \phi} = \sqrt{\frac{k(\phi)}{F'(\zeta)}} \quad \text{(canonically normalized scalar)}$$

$$U(\chi, \zeta) = V_E(\zeta) + \frac{V(\phi(\chi))}{F'(\zeta)^2}$$

$$V_E(\zeta) = -\frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)}$$

- N.B. ζ stays auxiliary! Not dynamical like in metric gravity

EoM for ζ

- The full EoM for ζ is

$$G(\zeta) = \frac{1}{2} k(\phi)\, \partial^\mu \phi \partial_\mu \phi\, F'(\zeta) + V(\phi)$$

with

$$G(\zeta) = \frac{1}{4}\left[2F(\zeta) - \zeta F'(\zeta)\right]$$

- The standard procedure would be now to solve the EoM and determine $\zeta(\phi, \partial^\mu \phi \partial_\mu \phi)$ and insert it back into the action.

- However not always solvable for any $F(R) \Rightarrow$ SR approximation $\Rightarrow$

$$G(\zeta) = V(\phi)$$

$\Rightarrow$ still not always solvable for any $F(R)$

- On the other hand it is still possible to perform inflationary computations. The trick is to use the auxiliary field ζ as a computational variable and $G = V$ as a constraint.

$U(\zeta)$

- first of all $V \to G$ in U:

$$
\begin{aligned}
U(\chi,\zeta) &= \frac{V(\phi(\chi))}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)} \\
&= \frac{G(\zeta)}{F'(\zeta)^2} - \frac{F(\zeta)}{2F'(\zeta)^2} + \frac{\zeta}{2F'(\zeta)} \\
&= \cancel{\frac{F(\zeta)}{2F'(\zeta)^2}} - \frac{\zeta}{4F'(\zeta)} - \cancel{\frac{F(\zeta)}{2F'(\zeta)^2}} + \frac{\zeta}{2F'(\zeta)} \\
&= \boxed{\frac{1}{4}\frac{\zeta}{F'(\zeta)} = U(\zeta)}
\end{aligned}
$$

- valid for any $F(R)$ and $V(\phi)$
- what changes is the actual *solution* for ζ
- also valid for the pure $F(R)$ case ($\mathcal{L}(\phi) = 0$)
- $F(R) = R + \alpha R^2$ is the only $F(R)$ $\Rightarrow$ universally flat ($\neq 0$) U for $\alpha \gg 1$

• Calculus •

- SR computations → we need derivatives of U
- we start with the 1st derivative:

$$\frac{\partial}{\partial\chi}U(\zeta) = \boxed{\frac{\partial\zeta}{\partial\chi}}\frac{\partial}{\partial\zeta}U(\zeta) \quad \leftarrow \text{we need this!}$$

- $G(\zeta) = V(\phi) \Rightarrow \phi = V^{-1}(G)$, the inverse function of $V(\phi)$

$$\frac{\partial\zeta}{\partial\chi} = \frac{\partial\phi}{\partial\chi}\frac{\partial\zeta}{\partial\phi} = \dots$$

$$= \boxed{\sqrt{\frac{F'(\zeta)}{k(V^{-1}(G))}}\frac{1}{\frac{\partial G}{\partial\zeta}\frac{\partial V^{-1}}{\partial G}}} = \boxed{g(\zeta)}$$

- This allows us to easily express higher derivatives:

$$\frac{\partial^2}{\partial\chi^2}U(\zeta) = g(\zeta)\frac{\partial}{\partial\zeta}\left(g(\zeta)\frac{\partial U}{\partial\zeta}\right) = gg'U' + g^2U'' \ , \ \dots$$

where primes denote derivatives w.r.t. ζ.

- we have a method for computing SR parameters

• Inflationary observables •

- SR parameters

$$\epsilon(\zeta) = \frac{1}{2}\left(\frac{\partial U/\partial\chi}{U}\right)^2 = \frac{1}{2}g^2\left(\frac{U'}{U}\right)^2$$

$$\eta(\zeta) = \frac{\partial^2 U/\partial\chi^2}{U} = \frac{gg'U' + g^2 U''}{U}$$

- observables

$$N_e = \int_{\chi_f}^{\chi_N} \frac{U}{\partial U/\partial\chi}\mathrm{d}\chi = \int_{\zeta_f}^{\zeta_N} \frac{U}{g^2\,U'}\mathrm{d}\zeta$$

$$r(\zeta) = 16\epsilon(\zeta) = 8g^2\left(\frac{U'}{U}\right)^2$$

$$n_s(\zeta) = 1 + 2\eta(\zeta) - 6\epsilon(\zeta) = 1 + \frac{2g}{U^2}\left(g'U'U + gU''U - 3gU'^2\right)$$

$$A_s(\zeta) = \frac{U}{24\pi^2\epsilon(\zeta)} = \frac{U^3}{12\pi^2 g^2 U'^2}$$

- N.B. U contains info from $F(R)$, while V-info is in g

• Examples •

- We checked $R + \alpha R^2 \rightarrow$ OK! $\rightarrow$ backup slides

- We study the test scenario $n \neq 2$:

$$F(R) = R + \alpha R^n \,, \quad k(\phi) = 1 \,, \quad V(\phi) = \frac{m^2}{2}\phi^2$$

• Analytic solution •

We can compute the phenomenological parameters:

$$N_e = \left[\frac{\zeta\left(n-(n-1)\right)}{8m^2}\, {}_2F_1\left(1, \frac{1}{n-1}; \frac{n}{n-1}; (n-2)\alpha\zeta^{n-1}\right)\right]_{\zeta=\zeta_f}^{\zeta=\zeta_N}$$

$$r(\zeta_N) = \frac{64m^2}{\zeta_N}\frac{1+\alpha(2-n)\zeta_N^{n-1}}{1+\alpha n\zeta_N^{n-1}}$$

$$n_s(\zeta_N) = 1-\frac{8m^2}{\zeta_N}\frac{2+\alpha(n-2)(n-3)\zeta_N^{n-1}}{1+\alpha(2-n)n\zeta_N^{n-1}}$$

$$A_s(\zeta_N) = \frac{1}{384\pi^2 m^2}\frac{\zeta_N^2}{1+\alpha(2-n)\zeta_N^{n-1}}$$

where we used the hypergeometric function

$${}_2F_1(a,b,c,z) = \sum_{k=0}^{\infty}\frac{(a)_n(b)_n}{(c)_n}\frac{z^n}{n!}$$

with

$$(q)_n = \frac{\Gamma(q+n)}{\Gamma(q)}$$

the (rising) Pochhammer symbol

• $G > 0$ problem •

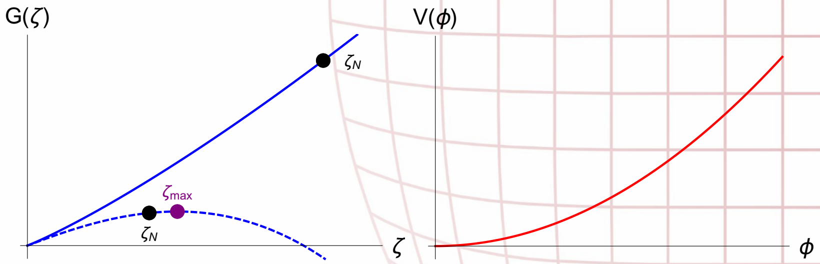


Figure: $G(\zeta)$ (left) and $V(\phi) = \phi^2$ (right) for $F(R) = R + R^n$ with $n = 3/2$ (continuous) and $n = 5/2$ (dashed).

n=3/2 For each ϕ, even though we cannot compute it analytically, we can always find a ζ so that $G(\zeta) = V(\phi) \Rightarrow$ limit: $\boxed{\zeta_N \to \infty}$

n=5/2 For each ϕ, we cannot always find a ζ so that $G(\zeta) = V(\phi)$ $\to$ effective description: inflation before $\zeta_{max} \Rightarrow$ limit: $\boxed{\zeta_N = \zeta_{max}}$

• Example 1. $n < 2$ •

More readable expressions considering the limit $\zeta_N \to \infty$ i.e. $\boxed{|n-2|\alpha \to \infty}$
In such a limit we can approximate the number of e-folds as

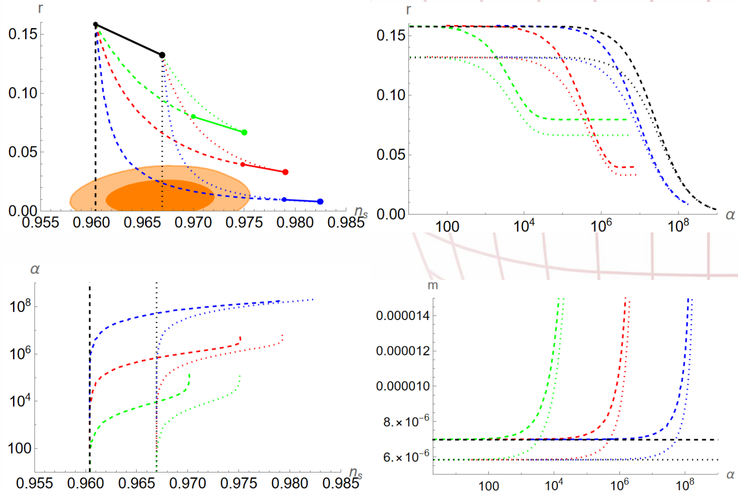
$$N_e \sim \frac{n}{8m^2}\zeta_N$$

obtaining

$$\begin{aligned} r(\zeta_N) &\simeq \frac{8(2-n)}{N_e} \\ n_s(\zeta_N) &\simeq 1 - \frac{3-n}{N_e} \\ A_s(\zeta_N) &\simeq \frac{2^{2-3n}\left(\frac{n}{N_e}\right)^{n-3}}{3\pi^2(2-n)} \frac{\left(m^2\right)^{2-n}}{\alpha} \end{aligned}$$

N.B. Valid only for $\boxed{n \neq 2}$

• $n < 2$ •



- ● $\alpha = 0$
- ■ $n = 2$
- ■ $n = \frac{31}{16}$
- ■ $n = \frac{7}{4}$
- ■ $n = \frac{3}{2}$
- - - $N_e = 50$
- · · $N_e = 60$
- ■ Planck & BICEP

• Example 2. $n > 2$ •

- rough upper limit for $\alpha = \bar{\alpha}$ when $\zeta_N = \zeta_{\max}$.
- The limit is only rough because η has a pole at $\zeta = \zeta_{\max}$ meaning the loss of validity of the slow-roll approximation.
- Therefore the actual upper limit $\bar{\alpha}$ takes place for ζ_N not equal, but slightly smaller than $\zeta_{\max}$.
- we can still provide useful estimates by using $\zeta_N = \zeta_{\max}$.

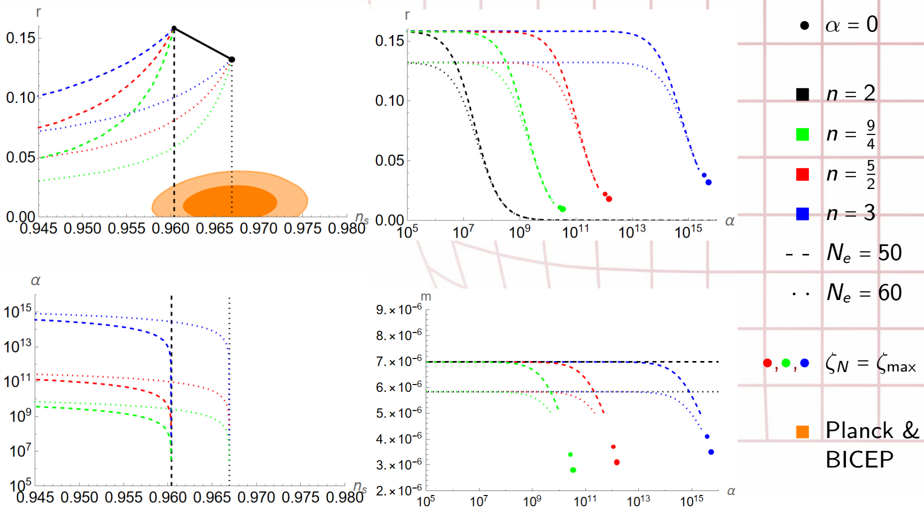
$$m_{\bar{\alpha}}^2 \simeq \frac{n(\bar{\alpha}(n-2)n)^{-\frac{2}{n-1}}}{384\pi^2(n-1)A_s}$$

$$N_e \sim (\bar{\alpha}(n-2)n)^{\frac{1}{n-1}} \frac{48\pi^2(n-1)A_s}{n} \left[n + (1-n)\,{}_2F_1\left(1, \frac{1}{n-1}; \frac{n}{n-1}; \frac{1}{n}\right)\right] \rightarrow$$

$\rightarrow$ we can use it as a definition for $\bar{\alpha}$

$$r_{\bar{\alpha}} \simeq \frac{(n-2)(\bar{\alpha}(n-2)n)^{\frac{1}{1-n}}}{6\pi^2(n-1)A_s}$$

• $n > 2$ •


r
0.15
0.10
0.05
0.00
0.945 0.950 0.955 0.960 0.965 0.970 0.975 0.980
n_s
r
0.15
0.10
0.05
0.00
10^5 10^7 10^9 10^{11} 10^{13} 10^{15}
α
α
10^{15}
10^{13}
10^{11}
10^9
10^7
10^5
0.945 0.950 0.955 0.960 0.965 0.970 0.975 0.980
n_s
m
$9.\times10^{-6}$
$8.\times10^{-6}$
$7.\times10^{-6}$
$6.\times10^{-6}$
$5.\times10^{-6}$
$4.\times10^{-6}$
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$2.\times10^{-6}$
10^5 10^7 10^9 10^{11} 10^{13} 10^{15}
α
$\alpha = 0$
$n = 2$
$n = \frac{9}{4}$
$n = \frac{5}{2}$
$n = 3$
$N_e = 50$
$N_e = 60$
$\zeta_N = \zeta_{max}$
Planck & BICEP

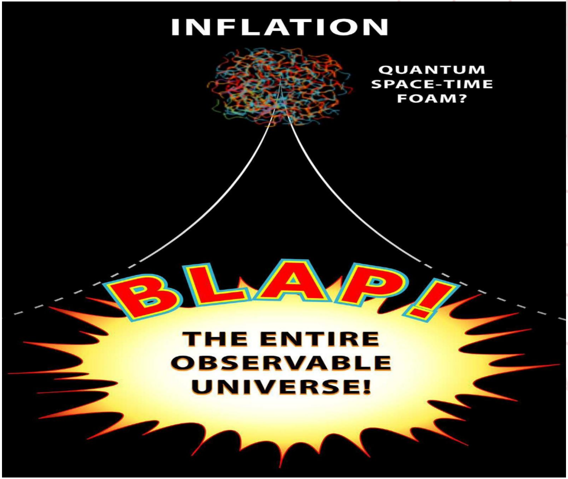
Conclusions

- We studied single field inflation embedded in Palatini $F(R)$ gravity
- We explained why Palatini $R + R^2$ is so unique
- We found a method to perform inflationary computations even though the EoM of ζ is not solvable.
- We tested the method on a couple of examples
- Unexpected outcome/future outlook:
  - $n = 2$ quite unstable configuration
  - $R + R^n$, $n > 2$ is problematic $\Rightarrow$ hint for a Palatini UV theory of gravity?

Grazie! - Thank you! - Aitäh!

BACKUP SLIDES

• Inflation •



- Solution for:
  - ◇ horizon problem
  - ◇ flatness problem
- Realizable with a constant Λ:

$$\sqrt{-g}\mathcal{L} = \sqrt{-g}\left[\frac{M_P^2}{2}R - \Lambda^4\right]$$

$$\Downarrow$$

$$\text{EoM} \rightarrow a(t) \sim e^{\frac{\Lambda^2}{M_P}t}$$

- PROBLEM: it never stops!
- SOLVED: ϕ with a quasi-flat V
- assumptions: GUT, SUGRA, ...
  $\rightarrow V(\phi) \rightarrow$ predictions
- our starting point:
  Palatini $F(R)$ gravity

• Inflationary computations •

Assuming SR, the inflationary dynamics is described by the SR parameters and the N_e. Assuming that we solved EoM $\zeta(\phi)$ and field redef. $\phi(\chi)$, the SR parameters are

$$\epsilon \equiv \frac{1}{2}\left(\frac{1}{U}\frac{\mathrm{d}U}{\mathrm{d}\chi}\right)^2 , \quad \eta \equiv \frac{1}{U}\frac{\mathrm{d}^2 U}{\mathrm{d}\chi^2} ,$$

and the number of e-folds as

$$N = \int_{\chi_f}^{\chi_N} \mathrm{d}\chi \, U \left(\frac{\mathrm{d}U}{\mathrm{d}\chi}\right)^{-1} ,$$

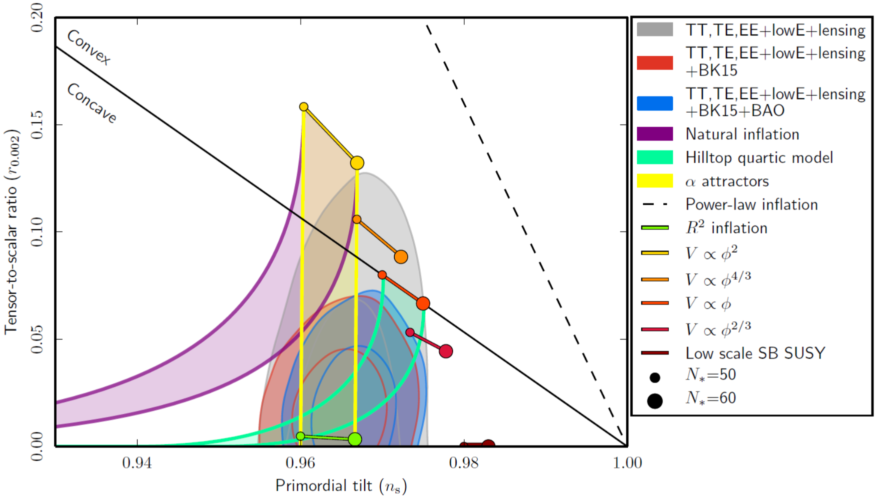
where the field value at the end of inflation, χ_f, is defined via $\epsilon(\chi_f) = 1$. The field value χ_i at the time a given scale left the horizon is given by the corresponding N. To reproduce the correct A_s, the potential has to satisfy

$$\ln\left(10^{10} A_s\right) = 3.044 \pm 0.014 \qquad \text{where} \qquad A_s = \frac{1}{24\pi^2}\frac{U(\chi_N)}{\epsilon(\chi_N)}$$

and the other two main observables, i.e. the spectral index and the tensor-to-scalar ratio are expressed as

$$n_s \simeq 1 + 2\eta - 6\epsilon$$
$$r \simeq 16\epsilon$$

• Planck 2018 data •



- FLAT potentials are strongly FAVORED!!!

• Metric vs Palatini formulation •

Einstein-Hilbert action: $\sqrt{-g^E}\mathcal{L}^E = \sqrt{-g^E}\left[\frac{1}{2}R_E\right]$

Metric

No torsion $\Rightarrow \Gamma^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\beta\alpha}$

$\boxed{\nabla_\alpha g_{\mu\nu} = 0} \Rightarrow \Gamma = \text{Levi-Civita } \Gamma = \bar{\Gamma}$

$$\bar{\Gamma}^{\lambda}_{\alpha\beta} = \frac{1}{2}g^{\lambda\rho}\left(\partial_\alpha g_{\beta\rho} + \partial_\beta g_{\rho\alpha} - \partial_\rho g_{\alpha\beta}\right)$$

Palatini

No torsion $\Rightarrow \Gamma^{\lambda}_{\alpha\beta} = \Gamma^{\lambda}_{\beta\alpha}$

$\boxed{\Gamma\text{'s EoM}} \rightarrow \nabla_\alpha\left[\sqrt{-g}\,g_{\mu\nu}\right] = 0$

$$\Gamma^{\lambda}_{\alpha\beta} = \bar{\Gamma}^{\lambda}_{\alpha\beta}$$

- In non-minimally coupled theories, metric and Palatini formalism give different physical theories. (Koivisto & Kurki-Suonio: arXiv:0509422)

• Metric vs Palatini formulation •

Jordan frame:
$$\sqrt{-g^J}\mathcal{L}^J = \sqrt{-g^J}\left[\frac{1}{2}F(R_J) - \frac{1}{2}(\partial\phi)^2 - V(\phi)\right]$$
$$= \sqrt{-g^J}\left[\frac{1}{2}f(\zeta)R_J - \frac{1}{2}(\partial\phi)^2 - V(\phi,\zeta)\right]$$

Metric

$$\nabla_\alpha g^J_{\mu\nu} = 0 \Rightarrow \Gamma = \text{Levi-Civita } \Gamma = \bar{\Gamma}$$

$$\bar{\Gamma}^\lambda_{\alpha\beta} = \frac{1}{2}g^{\lambda\rho}\left(\partial_\alpha g_{\beta\rho} + \partial_\beta g_{\rho\alpha} - \partial_\rho g_{\alpha\beta}\right)$$

Palatini

$$\Gamma\text{'s EoM: } \nabla_\alpha\left[f(\zeta)\sqrt{-g^J}g^J_{\mu\nu}\right] = 0$$

$$\Gamma^\lambda_{\alpha\beta} = \bar{\Gamma}^\lambda_{\alpha\beta} + \delta^\lambda_\alpha\partial_\beta\omega + \delta^\lambda_\beta\partial_\alpha\omega - g_{\alpha\beta}\partial^\lambda\omega$$

$$\omega(\zeta) = \ln\Omega(\zeta), \qquad g^E_{\mu\nu} = \Omega(\zeta)^2\, g^J_{\mu\nu}, \qquad \Omega(\zeta)^2 = f(\zeta) = F'(\zeta)$$

Einstein frame:
$$\sqrt{-g^E}\mathcal{L}^E = \sqrt{-g^E}\left[\frac{1}{2}R_E - K(\phi,\zeta) - U(\phi,\zeta)\right]$$

Metric ($\Gamma_E = \bar{\Gamma}_E$)

$K(\phi,\zeta) \to 2$ dyn. fields: ϕ & ζ

Palatini ($\Gamma_E = \bar{\Gamma}_E$)

$K(\phi,\zeta) = K(\phi) \to \phi$ dyn. & ζ aux.

$G > 0$ problem

$$G(\zeta) = \frac{1}{4}\left[2F(\zeta) - \zeta F'(\zeta)\right] = V(\phi)$$

- For $\zeta \to +\infty$, if $F(\zeta) \approx \zeta^n \Rightarrow G(\zeta) \approx (2-n)\zeta^n$
  $\Rightarrow$ with $n > 2 \Rightarrow G \to -\infty$
- Problem: $V(\infty) \to \infty \Rightarrow$ no real values for ζ

- Solutions:

$n < 2 \ \Rightarrow G(+\infty) \to +\infty \Rightarrow$ OK!

$n > 2 \ \ G(0^+) \sim \zeta \Rightarrow$ G is a first a crescent function which reaches a local max and then decreases towards $-\infty$
$\Rightarrow$ we need to ensure that inflation happens within $\zeta = 0$ and ζ =local max. (in order to avoid also the bijectivity problem)
$\Rightarrow$ the model is only an effective description

• Reproduce R^2 results •

- $F(R) = R + \alpha R^2$, $k(\phi) = 1$
- Let's check: $G(\zeta) = V(\phi)$

$$G(\zeta) = \frac{1}{4}\left[2F(\zeta) - \zeta F'(\zeta)\right] = \frac{1}{4}\left[2\zeta + 2\alpha\zeta^2 - \zeta(1+2\alpha\zeta)\right]$$
$$= \boxed{\frac{1}{4}\zeta = V(\phi)} = U^0 \quad \leftarrow \text{no } \alpha!!!$$

- Einstein frame scalar potential:

$$U = \frac{1}{4}\frac{\zeta}{F'(\zeta)} = \frac{\zeta}{4+8\alpha\zeta} = \frac{U^0}{1+8\alpha U^0} \quad \to \text{OK!}$$

- g function

$$g = \sqrt{F'(\zeta)}\frac{1}{\frac{\partial G}{\partial \zeta}\frac{\partial V^{-1}}{\partial G}} = \sqrt{1+2\alpha\zeta}\frac{1}{\frac{\partial V^{-1}}{\partial \zeta}}$$

- inflationary observables

$$r(\zeta) = 8g^2\left(\frac{U'}{U}\right)^2 = \left(\frac{1}{\frac{\partial V^{-1}}{\partial \zeta}}\right)^2 \frac{8}{\zeta^2}\left(\frac{1}{1+2\alpha\zeta}\right) = \frac{r^0}{1+8\alpha U^0} \to \text{OK!}$$

analogously we get $N_e = N_e^0$, $A_s = A_s^0$ and $n_s = n_s^0$

• Beyond SR •

After some manipulations, the full Einstein frame EoMs read:

$$\ddot{\phi} + 3H\dot{\phi} + \frac{V'(\phi)}{F'(\zeta)k(\phi)} = \frac{\dot{\phi}\dot{\zeta}F''(\zeta)}{F'(\zeta)} - \frac{1}{2}\frac{k'(\phi)}{k(\phi)}\dot{\phi}^2$$

$$3H^2 = \frac{1}{2}\frac{\dot{\phi}^2}{F'(\zeta)}k(\phi) + U(\phi,\zeta)$$

$$-\frac{1}{2}\dot{\phi}^2 F'(\zeta)k(\phi) + 2V(\phi) - 2G(\zeta) = 0$$

These can be used to also derive

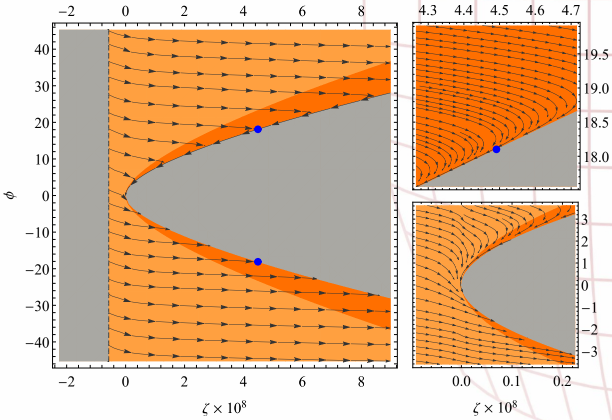
$$\dot{H} = -\frac{1}{2}\frac{\dot{\phi}^2}{F'(\zeta)}k(\phi)$$

$$\epsilon_H \equiv -\frac{\dot{H}}{H^2} = \frac{12V(\phi) - 6F(\zeta) + 3\zeta F'(\zeta)}{6V(\phi) - 3F(\zeta) + 2\zeta F'(\zeta)}$$

$$\dot{\zeta} = \frac{3H\dot{\phi}^2 F'(\zeta)k(\phi) + 3V'(\phi)\dot{\phi}}{2G'(\zeta) + \frac{3}{2}\dot{\phi}^2 F''(\zeta)k(\phi)}$$

N.B. Even though ζ is only auxiliary, it has an implicit time dependence via ϕ

• Beyond SR. $n < 2$ •



- $n = 3/2$, $\alpha = 8710$, $m = 1.15 \cdot 10^{-5}$

  $\sim$ for any $1 < n < 2$
- $H > 0$, $\dot{\phi} < 0$ branch

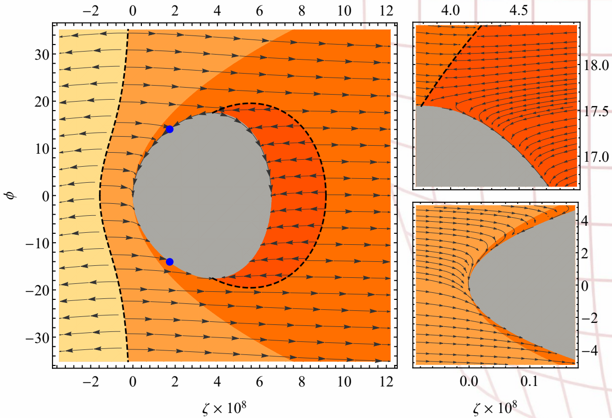
■ $F' < 0$ or no ζ's EoM

■ dark $\rightarrow \epsilon_H < 1$

● $N_e = 50$, $n_s = 0.967$, $r = 0.096$

- $\dot{\phi} > 0 \rightarrow$ mirror with respect to x-axis
  $H < 0 \rightarrow$ switch the direction of the flow. not reached smoothly when $1\ \phi$
- $\phi > 0$: trajectory sharp turns into SR slow-roll
- $\phi < 0$: trajectory enters SR in $\dot{\phi} > 0$ branch

• Beyond SR. $n > 2$ •



- $n = 3$, $\alpha = 2.32 \cdot 10^{14}$, $m = 6.40 \cdot 10^{-6}$

  $\sim$ for any $n > 2$

- $H > 0$, $\dot{\phi} < 0$ branch

■ no ζ's EoM

■ dark $\to \epsilon_H < 1$

● $N_e = 50$, $n_s = 0.952$, $r = 0.116$

- - $\dot{\zeta} \to \infty$: trajectories diverging from there. cannot be crossed
  left: $G' < 0 \to$ in SR only if ϕ not too big
  right: $F'' < 0 \to$ *crazy* isolated inflationary region on the left

- SR is not an attractor of EoM
- illness of the $n > 2$ case