Path integral formalism for radiative corrections in bound-state QED

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The theory of radiative corrections in bound-state quantum electrodynamics (QED) is developed using Feynman's path integral formalism[1,2]. As a first step, we derive the free Dirac propagator in spherical coordinates. Next, we derive the Dirac-Coulomb Green's function (DCGF) in the Furry picture by reducing it in a basis such that the effective action becomes similar to that of the nonrelativistic hydrogen atom. As such, the DCGF is obtained in closed form along with the energy spectrum of the bound states. In the final step, first-order Lamb shift, characterized by vacuum-polarization and self-energy shift, is calculated. The lowest-order vacuum polarization correction to the energy levels of bound electrons is calculated using perturbative path integral formalism. Starting from an interparticle classical action, we arrive directly at the propagators of QED. The energy level shifts are then calculated from the perturbative shift of poles of the Green's functions obtained. Finally, we derive the self-energy corrected propagator using Schwinger-Dyson equations through the path integral formalism. From the Feynman amplitude of this propagator, the energy shift is determined by complex contour integrals. The existing divergences are treated by separating the energy shift into zero-, one-, and many-potential terms following existing works [3,4]. The many potential term being UV-finite is calculated using partial-wave expansion, the zero- and one-potential terms are regularized using the regularized expressions for self-energy and vertex function in the momentum space and are evaluated numerically using b-spline codes[5].

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