Nuclear Radii Constrain the Isospin-Breaking Corrections to V_{ud}

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Based on:
Chien-Yeah Seng, MG
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2304.03800
2212.02681
2211.10214

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Universality, Completeness & CKM unitarity

Fermi constant from muon lifetime: $G_F = G_\mu = 1.1663788(7) \times 10^{-5} GeV^{-2}$

$$\mathscr{L}_{e\mu} = -2\sqrt{2}G_\mu \bar{e}\gamma_\alpha \nu_{eL} \cdot \bar{\nu_{\mu L}}\gamma^\alpha \mu + \mathrm{h.c.}$$

SM: same W-coupling to LH leptons and quarks, but strength shared between 3 generations

$$\mathscr{L}_{eq} = -\sqrt{2}G_\mu \bar{e}\gamma_\mu \nu_{eL} \cdot \bar{U}_i \gamma^\mu (1-\gamma_5) V_{ij} D_j + \mathrm{h.c.}$$

$$U_i = (u, c, t)^T$$
$$D_j = (d, s, b)^T$$

Universality + Completeness of SM (only 3 gen's) —> unitary CKM matrix $V^\dagger V = 1$

Top-row unitarity condition: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$

At low energy accessible via β-decas of hadrons, e.g. $n \to pe\bar{\nu}$

$$\mathscr{L}_{e\nu pn} = -\sqrt{2}G_\mu V_{ud} \bar{e}\gamma_\mu \nu_L \cdot \bar{p}\gamma^\mu (g_V^{pn} - g_A^{pn}\gamma_5) n + \mathrm{h.c.}$$

Conserved vector current: $g_V^{pn} = 1 + O((m_d - m_u)^2)$ but $g_A^{ud} = 1 \to g_A^{pn} \approx 1.276$

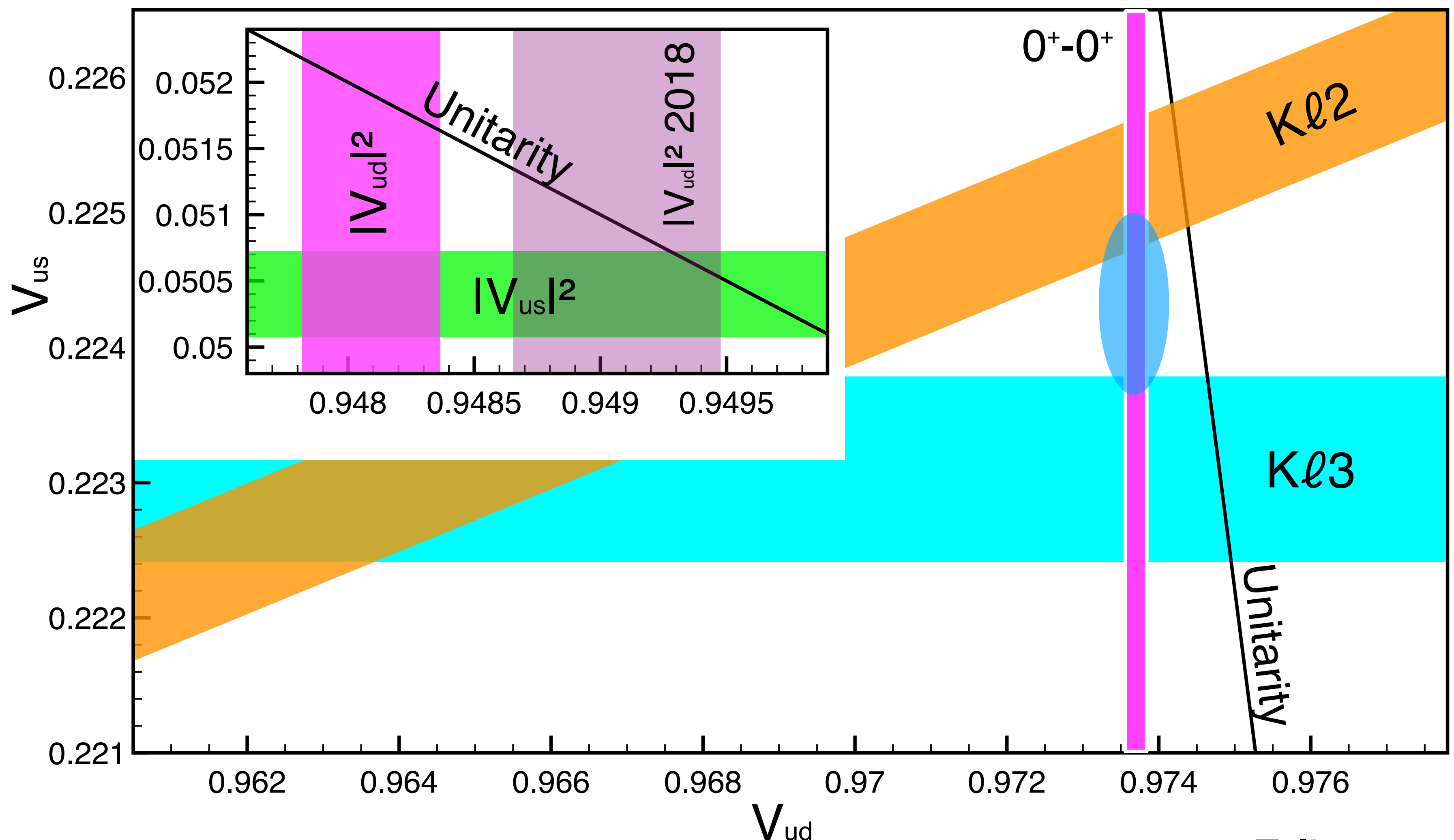
Precise measurements of g_V —> precision tests of EW sector of SM (currently 0.02%)

Get rid of g_A —> superallowed nuclear decays between states $J^P = 0^+$

Top-row CKM unitarity deficit

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 0.9985(6)_{V_{ud}}(4)_{V_{us}}$$

$\sim 0.95 \qquad \sim 0.05 \qquad \sim 10^{-5}$



Talks:
Neutron decay by Hartmut Abele
V_{ub}, V_{cb} **by Christoph Schwanda**

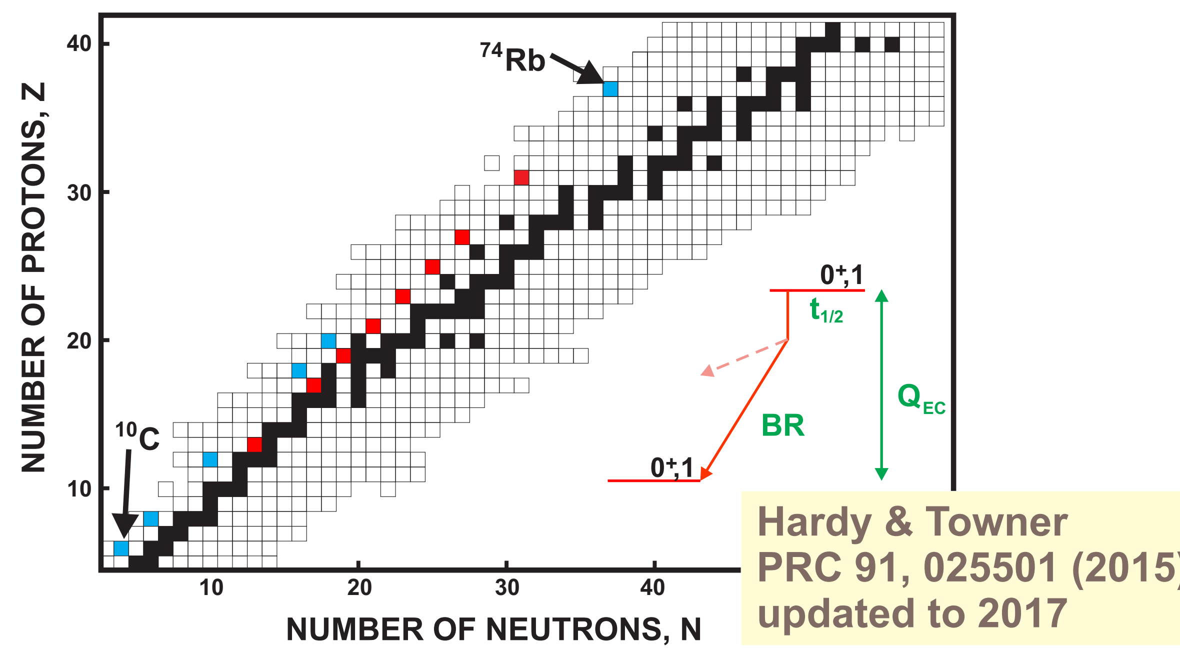
Inconsistencies between measurements of V_{ud} and V_{us} and SM predictions
Most precise V_{ud} from superallowed nuclear decays

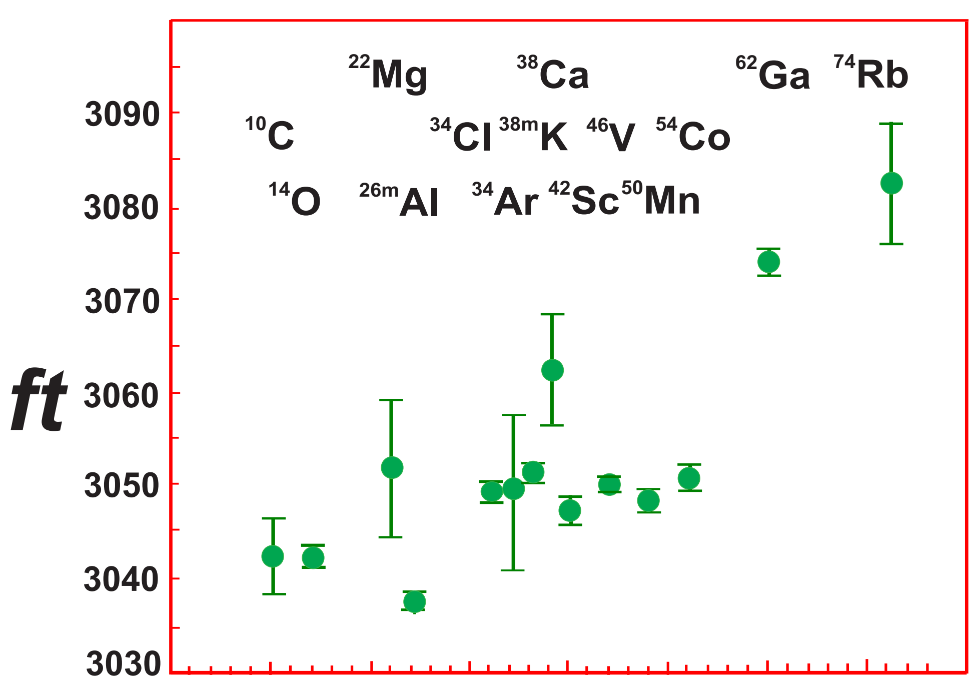
Precise V_{ud} from superallowed decays

Superallowed 0^+-0^+ nuclear decays:
- only conserved vector current (unlike the neutron decay and other mirror decays)
- many decays (unlike pion decay)
- all decay rates should be the same modulo phase space

Experiment: **f** - phase space (Q value) and **t** - partial half-life ($t_{1/2}$, branching ratio)

- **8 cases with *ft*-values measured to <0.05% precision; 6 more cases with 0.05-0.3% precision.**
- **~220 individual measurements with compatible precision**





ft values: same within ~2% but not exactly!
Reason: SU(2) slightly broken

a. RC (e.m. interaction does not conserve isospin)
b. Nuclear WF are not SU(2) symmetric
   (proton and neutron distribution not the same)

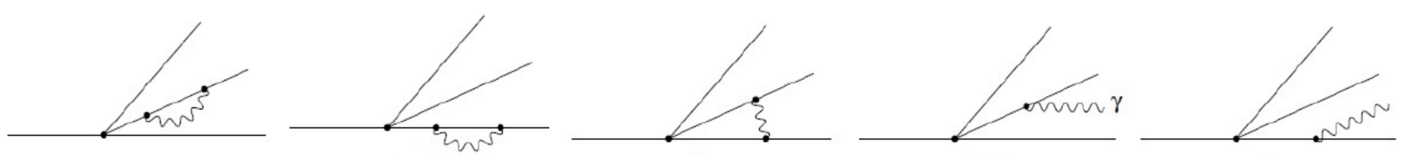
Precise V_{ud} from superallowed decays

Modified ft-values to include these effects

$$\mathcal{F}t = ft(1+\delta'_R)[1-(\delta_C - \delta_{NS})]$$

$$|V_{ud}|^2 = \frac{2984.432(3)}{\mathcal{F}t(1+\Delta_R^V)}$$

δ'_R - "outer" correction (depends on e-energy) - QED

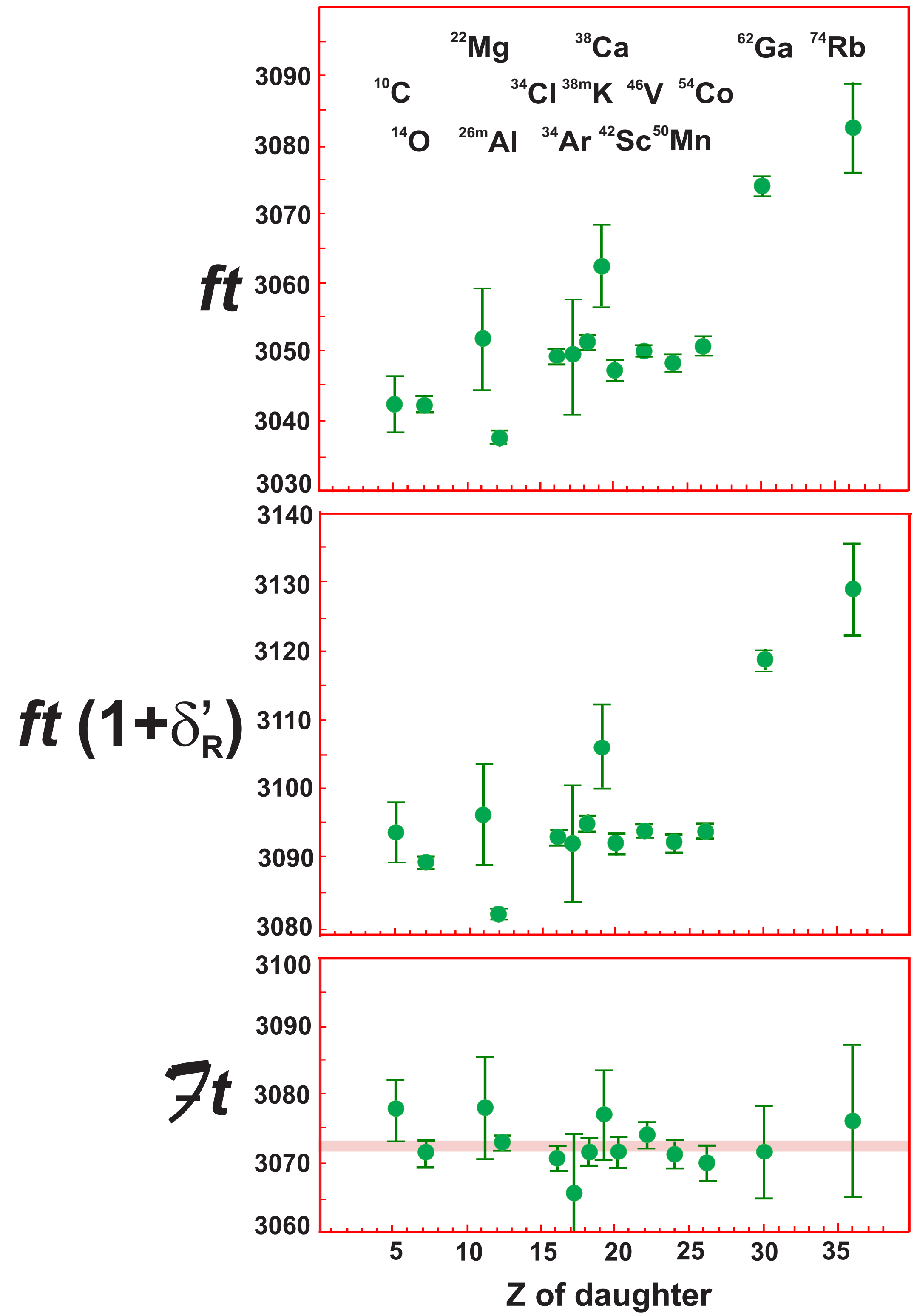


δ_C - SU(2) breaking in the nuclear matrix elements

δ_{NS} - RC depending on the nuclear structure

Average of 14 decays - 0.02%

$$\overline{\mathcal{F}t} = 3072.1 \pm 0.7$$



Hardy, Towner 1973 - 2020

Isospin symmetry breaking in superallowed β-decay

Tree-level Fermi matrix element

$$M_F = \langle f | \tau^+ | i \rangle$$

τ^+ — Isospin operator

$|i\rangle, |f\rangle$ — members of T=1 isotriplet

If isospin symmetry were exact, $M_F \to M_0 = \sqrt{2}$

Isospin symmetry is broken in nuclear states (e.g. Coulomb, nucleon mass difference, …)

In presence of isospin symmetry breaking (ISB):

$$|M_F|^2 = |M_0|^2 (1 - \delta_C)$$

ISB correction is crucial for V_{ud} extraction

HT: shell model with *phenomenological* Woods-Saxon potential locally adjusted to:

- Masses of the isotriplet T=1, 0+ (IMME)
- Neutron and proton separation energies
- Known proton radii of stable isotopes

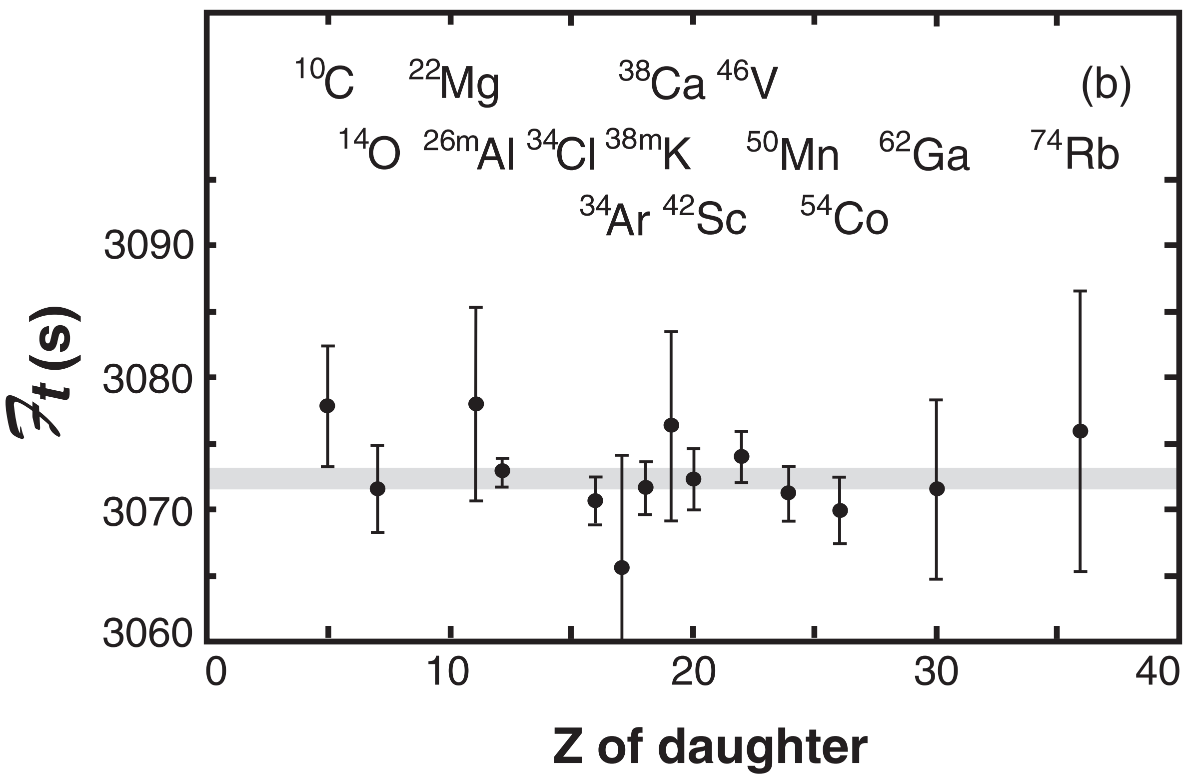
TABLE X. Corrections δ'_R, δ_{NS}, and δ_C that are applied to experimental ft values to obtain $\mathcal{F}t$ values.

Parent nucleus	δ'_R (%)	δ_{NS} (%)	δ_{C1} (%)	δ_{C2} (%)	δ_C (%)
$T_z = -1$					
^{10}C	1.679	−0.345(35)	0.010(10)	0.165(15)	0.175(18)
^{14}O	1.543	−0.245(50)	0.055(20)	0.275(15)	0.330(25)
^{18}Ne	1.506	−0.290(35)	0.155(30)	0.405(25)	0.560(39)
^{22}Mg	1.466	−0.225(20)	0.010(10)	0.370(20)	0.380(22)
^{26}Si	1.439	−0.215(20)	0.030(10)	0.405(25)	0.435(27)
^{30}S	1.423	−0.185(15)	0.155(20)	0.700(20)	0.855(28)
^{34}Ar	1.412	−0.180(15)	0.030(10)	0.665(55)	0.695(56)
^{38}Ca	1.414	−0.175(15)	0.020(10)	0.745(70)	0.765(71)
^{42}Ti	1.427	−0.235(20)	0.105(20)	0.835(75)	0.940(78)
$T_z = 0$					
^{26m}Al	1.478	0.005(20)	0.030(10)	0.280(15)	0.310(18)
^{34}Cl	1.443	−0.085(15)	0.100(10)	0.550(45)	0.650(46)
^{38m}K	1.440	−0.100(15)	0.105(20)	0.565(50)	0.670(54)
^{42}Sc	1.453	0.035(20)	0.020(10)	0.645(55)	0.665(56)
^{46}V	1.445	−0.035(10)	0.075(30)	0.545(55)	0.620(63)
^{50}Mn	1.444	−0.040(10)	0.035(20)	0.610(50)	0.645(54)
^{54}Co	1.443	−0.035(10)	0.050(30)	0.720(60)	0.770(67)
^{62}Ga	1.459	−0.045(20)	0.275(55)	1.20(20)	1.48(21)
^{66}As	1.468	−0.060(20)	0.195(45)	1.35(40)	1.55(40)
^{70}Br	1.486	−0.085(25)	0.445(40)	1.25(25)	1.70(25)
^{74}Rb	1.499	−0.075(30)	0.115(60)	1.50(26)	1.62(27)

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

$$\delta_C \sim 0.17\% - 1.6\%!$$

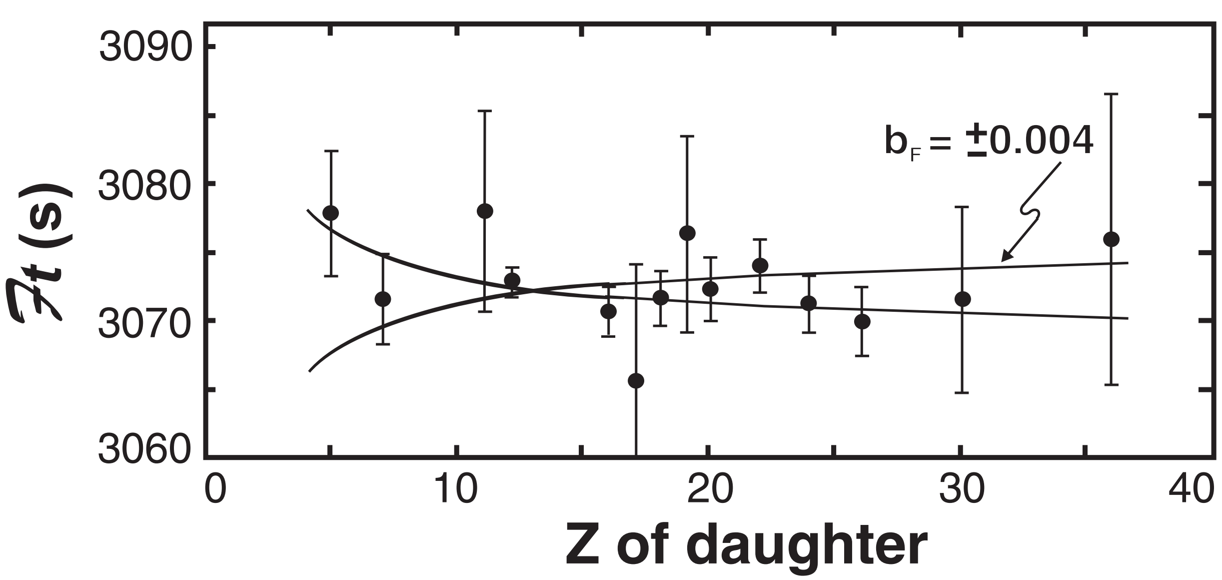
ISB vs. scalar interactions?



Once all corrections are included:
CVC —> Ft constant

δ_C particularly important for alignment!

Fit to 14 transitions:
Ft constant within 0.02% if using SM-WS



If BSM scalar currents present: “Fierz interference” b_F

$$\mathcal{F}t^{SM} \rightarrow \mathcal{F}t^{SM}\left(1 + b_F \frac{m_e}{\langle E_e \rangle}\right)$$

$Q_{EC} \uparrow$ with Z —> effect of $b_F \downarrow$ with Z
Introduces nonlinearity in the Ft plot

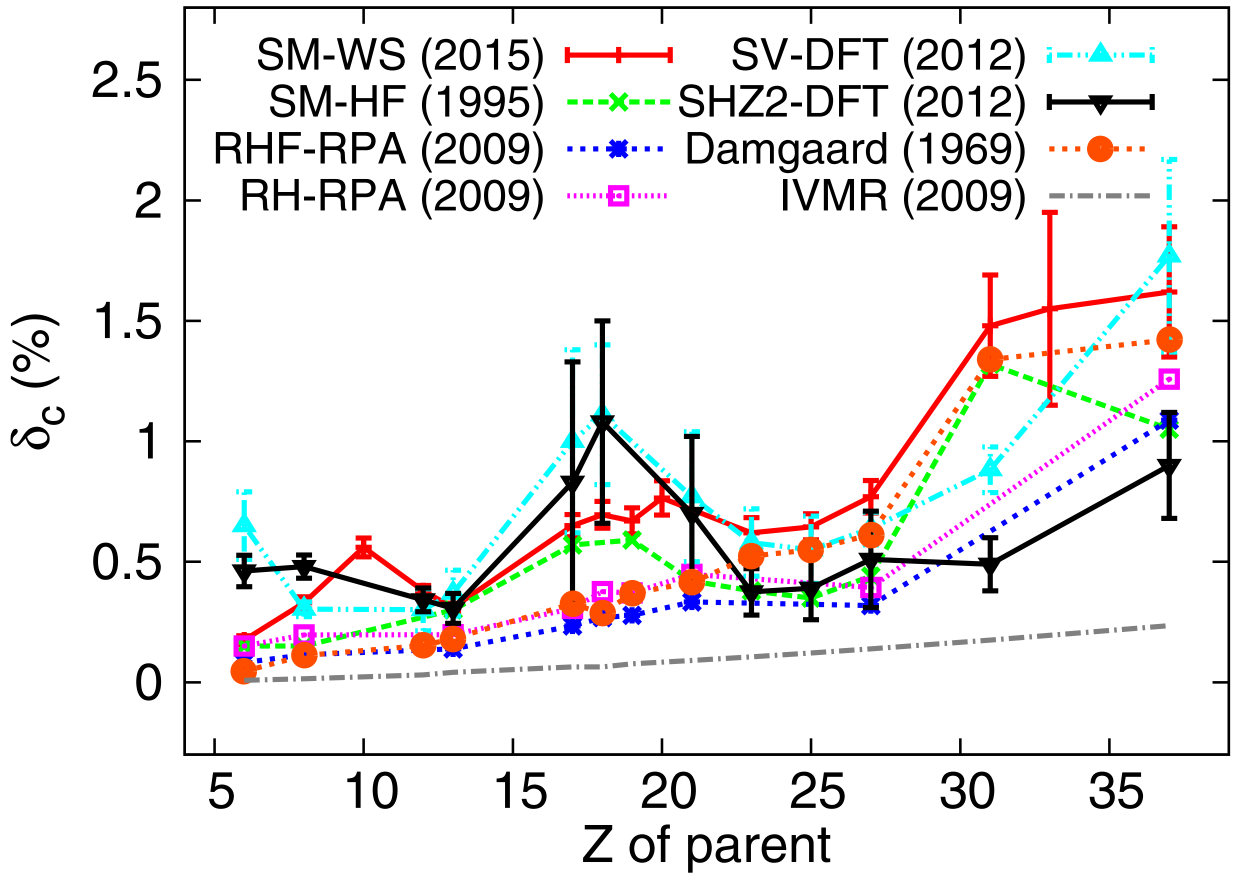
$b_F = -0.0028(26)$ ~ consistent with 0

Nuclear model comparison for δ_C

J. Hardy, I. Towner, Phys.Rev. C 91 (2014), 025501

	SM-WS	SM-HF	RPA PKO1	RPA DD-ME2	RPA PC-F1	IVMR[a]	DFT
$T_z = -1$							
^{10}C	0.175	0.225	0.082	0.150	0.109	0.147	0.650
^{14}O	0.330	0.310	0.114	0.197	0.150		0.303
^{22}Mg	0.380	0.260					0.301
^{34}Ar	0.695	0.540	0.268	0.376	0.379		
^{38}Ca	0.765	0.620	0.313	0.441	0.347		
$T_z = 0$							
^{26m}Al	0.310	0.440	0.139	0.198	0.159		0.370
^{34}Cl	0.650	0.695	0.234	0.307	0.316		
^{38m}K	0.670	0.745	0.278	0.371	0.294	0.434	
^{42}Sc	0.665	0.640	0.333	0.448	0.345		0.770
^{46}V	0.620	0.600					0.580
^{50}Mn	0.645	0.610					0.550
^{54}Co	0.770	0.685	0.319	0.393	0.339		0.638
^{62}Ga	1.475	1.205					0.882
^{74}Rb	1.615	1.405	1.088	1.258	0.668		1.770
χ^2/ν	1.4	6.4	4.9	3.7	6.1		4.3[b]

L. Xayavong, N.A. Smirnova, Phys.Rev. C 97 (2018), 024324



HT: χ^2 as criterion to prefer SM-WS; V_{ud} and limits on BSM strongly depend on nuclear model

Nuclear community embarked on ab-initio δ_C calculations (NCSM, GFMC, CC, IMSRG)
Especially interesting for light nuclei accessible to different techniques!

Phenomenological constraints on δ_C?

Idea: δ_C dominated by Coulomb repulsion between protons (hence C)

Coulomb interaction generates both δ_C and ISB combinations of nuclear radii

Auerbach 0811.4742; 2101.06199; Seng, MG 2208.03037; 2304.03800; 2212.02681

Nuclear Hamiltonian: $H = H_0 + V_{\text{ISB}} \approx H_0 + V_C$

Coulomb potential for uniformly charged sphere $$V_C \approx -\frac{Ze^2}{4\pi R_C^3}\sum_{i=1}^{A}\left(\frac{1}{2}r_i^2 - \frac{3}{2}R_C^2\right)\left(\frac{1}{2} - \hat{T}_z(i)\right)$$

ISB due to IV monopole, $V_{\text{ISB}} \approx \frac{Ze^2}{8\pi R^3}\sum_i r_i^2 \hat{T}_z(i) = \frac{Ze^2}{8\pi R^3}\hat{M}_0^{(1)}$

Same operator generates nuclear radii $$R_{p/n,\phi} = \sqrt{\frac{1}{X}\langle\phi|\sum_{i=1}^{A} r_i^2\left(\frac{1}{2} \mp \hat{T}_z(i)\right)|\phi\rangle}$$

Phenomenological constraints on δ_C?

$0^+, T=1, T_z=-1$ → $0^+, T=1, T_z=0$ → $0^+, T=1, T_z=1$

E.g. ${}^{42}_{22}\mathrm{Ti} \rightarrow {}^{42}_{21}\mathrm{Sc} \rightarrow {}^{42}_{20}\mathrm{Ca}$

ISB-sensitive combinations of radii: Wigner-Eckart theorem

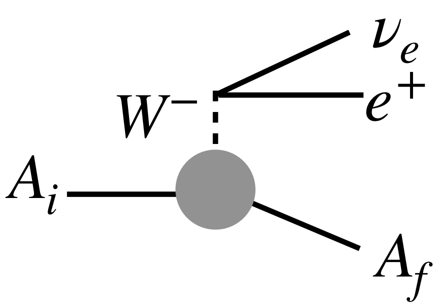
$$\Delta M_A^{(1)} \equiv \langle f | M_{+1}^{(1)} | i \rangle + \langle f | M_0^{(1)} | f \rangle$$

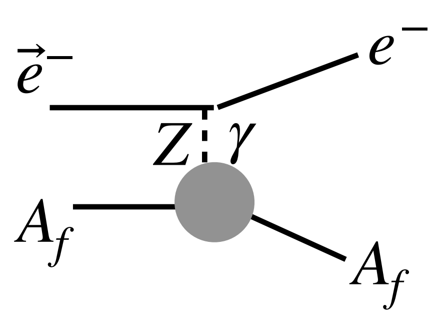
$$\Delta M_B^{(1)} \equiv \frac{1}{2}\left(Z_1 R_{p,1}^2 + Z_{-1} R_{p,-1}^2\right) - Z_0 R_{p,0}^2$$

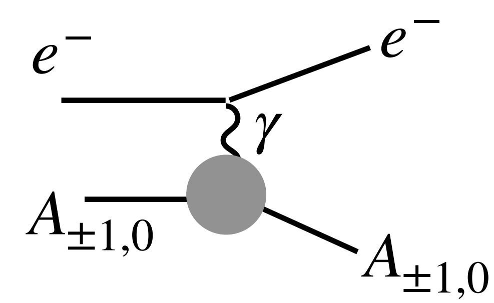
Transition radius From β spectrum

Neutron skin From PVES

Charge radii from atomic spectra and electron scattering







$$F_{CW}(Q^2) = 1 - R_{CW}^2 Q^2/6 + \ldots$$

$$A^{PV} = -\frac{G_F Q^2}{4\sqrt{2}\pi\alpha}\frac{Q_W}{Z}\frac{F_{NW}(Q^2)}{F_{Ch}(Q^2)}$$

$$F_{Ch}(Q^2) = 1 - R_{Ch}^2 Q^2/6 + \ldots$$

Since N ≠ Z for $T_z = \pm 1$ factors $Z_{\pm 1,0}$ remove the symmetry energy to isolate ISB

(Usually PVES —> neutron skins —> symmetry energy —> nuclear EOS —> nuclear astrophysics)

Electroweak radii constrain ISB in superallowed β-decay

Employ the correct isospin formalism by **Schwenk, Miller 0805.0603; 0910.2790**

δ_C and radii expressed via the same set of matrix elements

$$\delta_{\rm C} = \frac{1}{3}\sum_a \frac{|\langle a;0||V||g;1\rangle|^2}{(E_{a,0}-E_{g,1})^2} + \frac{1}{2}\sum_{a\neq g} \frac{|\langle a;1||V||g;1\rangle|^2}{(E_{a,1}-E_{g,1})^2} - \frac{5}{6}\sum_a \frac{|\langle a;2||V||g;1\rangle|^2}{(E_{a,2}-E_{g,1})^2} + \mathcal{O}(V^3)$$

$$\Delta M_A^{(1)} = \frac{1}{3}\Gamma_0 + \frac{1}{2}\Gamma_1 + \frac{7}{6}\Gamma_2$$

$$\Delta M_B^{(1)} = \frac{2}{3}\Gamma_0 - \Gamma_1 + \frac{1}{3}\Gamma_2$$

$$\Gamma_T = -\sum_a \frac{|\langle a;T||V||g;1\rangle|^2}{E_{a,T}-E_{g,1}}$$

Different scaling with ISB: $\delta_C \sim \mathrm{ISB}^2,\ \Delta M_A^{(1)} \sim \mathrm{ISB}^1,\ \Delta M_B^{(1)} \sim \mathrm{ISB}^3$

Compare to IMME (masses across an isomultiplet)

$$E(a,T,T_z) = \mathtt{a}(a,T) + \mathtt{b}(a,T)T_z + \mathtt{c}(a,T)T_z^2$$

$$\mathtt{b} \sim \langle a;T,T_z|V^{(1)}|a;T,T_z\rangle\ ,\ \ \mathtt{c} \sim \langle a;T,T_z|V^{(2)}|a;T,T_z\rangle$$

Unlike δ_C, $\Delta M_{A,B}^{(1)}$ — IMME only depends on diagonal m.e. — indirect constraint

Electroweak radii constrain ISB in superallowed β-decay

For numerical analysis: lowest isovector monopole resonance dominates
One ISB matrix element, one energy splitting

Model for δ_C → prediction for $\Delta M^{(1)}_{A,B}$

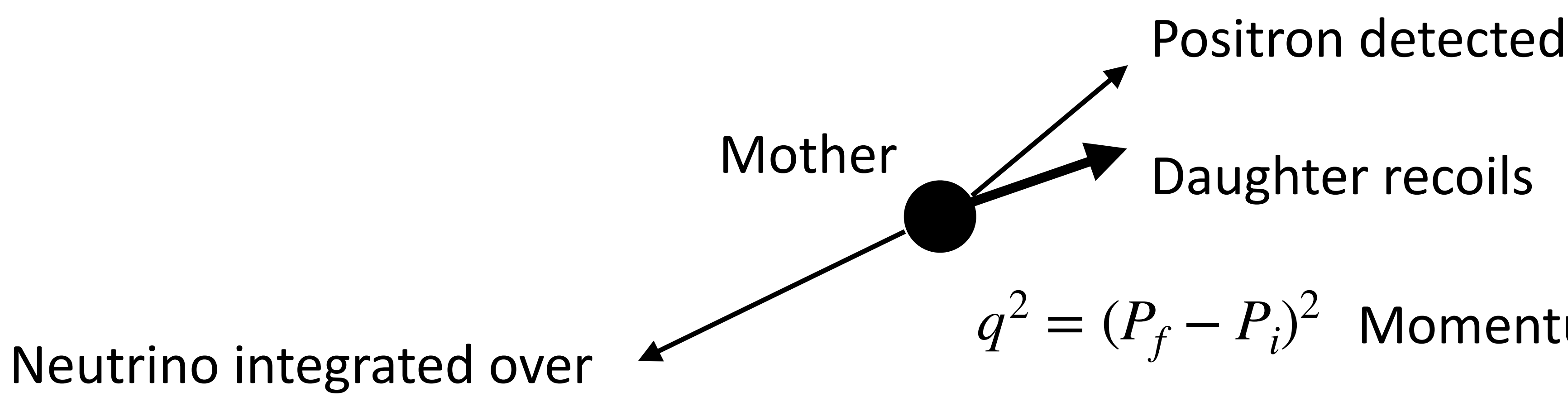
Transitions	δ_C (%)					$\Delta M^{(1)}_A$ (fm^2)					$\Delta M^{(1)}_B$ (fm^2)				
	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro	WS	DFT	HF	RPA	Micro
^{26m}Al $\to$ ^{26}Mg	0.310	0.329	0.30	0.139	0.08	-2.2	-2.3	-2.1	-1.0	-0.6	-0.12	-0.12	-0.11	-0.05	-0.03
^{34}Cl $\to$ ^{34}S	0.613	0.75	0.57	0.234	0.13	-5.0	-6.1	-4.6	-1.9	-1.0	-0.17	-0.21	-0.16	-0.06	-0.04
^{38m}K $\to$ ^{38}Ar	0.628	1.7	0.59	0.278	0.15	-5.4	-14.6	-5.1	-2.4	-1.3	-0.15	-0.42	-0.15	-0.07	-0.04
^{42}Sc $\to$ ^{42}Ca	0.690	0.77	0.42	0.333	0.18	-6.2	-6.9	-3.8	-3.0	-1.6	-0.15	-0.17	-0.09	-0.07	-0.04
^{46}V $\to$ ^{46}Ti	0.620	0.563	0.38	/	0.21	-5.8	-5.3	-3.6	/	-2.0	-0.12	-0.11	-0.08	/	-0.04
^{50}Mn $\to$ ^{50}Cr	0.660	0.476	0.35	/	0.24	-6.4	-4.6	-3.4	/	-2.4	-0.12	-0.09	-0.06	/	-0.04
^{54}Co $\to$ ^{54}Fe	0.770	0.586	0.44	0.319	0.28	-7.8	-5.9	-4.4	-3.2	-2.8	-0.13	-0.10	-0.07	-0.05	-0.05

Can discriminate models if independent information on nuclear radii is available
ΔM_A from measured radii —> test models for δ_C

Charge radii across superallowed isotriplets?
Some are known (but difficult — unstable isotopes, some g.s. are not 0^+)
Typically, precision is not enough to make a quantitative statement — need to improve!

Charge radii + isospin symmetry —> nuclear recoil

We said that ft-values are experimental — but not quite!
A few theory ingredients are absorbed: Coulomb distortions + Nuclear form factor



$q^2 = (P_f - P_i)^2$ Momentum transfer

Integrating over neutrino momenta = integrating over q^2

$$ft \equiv ft(q^2 = 0) \int_{\min}^{\max} \frac{F_{CW}(q^2)\, dq^2}{q^2_{\max} - q^2_{\min}}$$

Usual approach: assume $F_{CW} \approx F_{Ch}^{\text{daughter}}$ —> $R_{CW} = R_{Ch,1}$

But R_{CW} can be expressed via charge radii assuming approximate isospin symmetry

$$R^2_{\text{CW}} = R^2_{\text{Ch},1} + Z_0 (R^2_{\text{Ch},0} - R^2_{\text{Ch},1}) = R^2_{\text{Ch},1} + \frac{Z_{-1}}{2} (R^2_{\text{Ch},-1} - R^2_{\text{Ch},1})$$

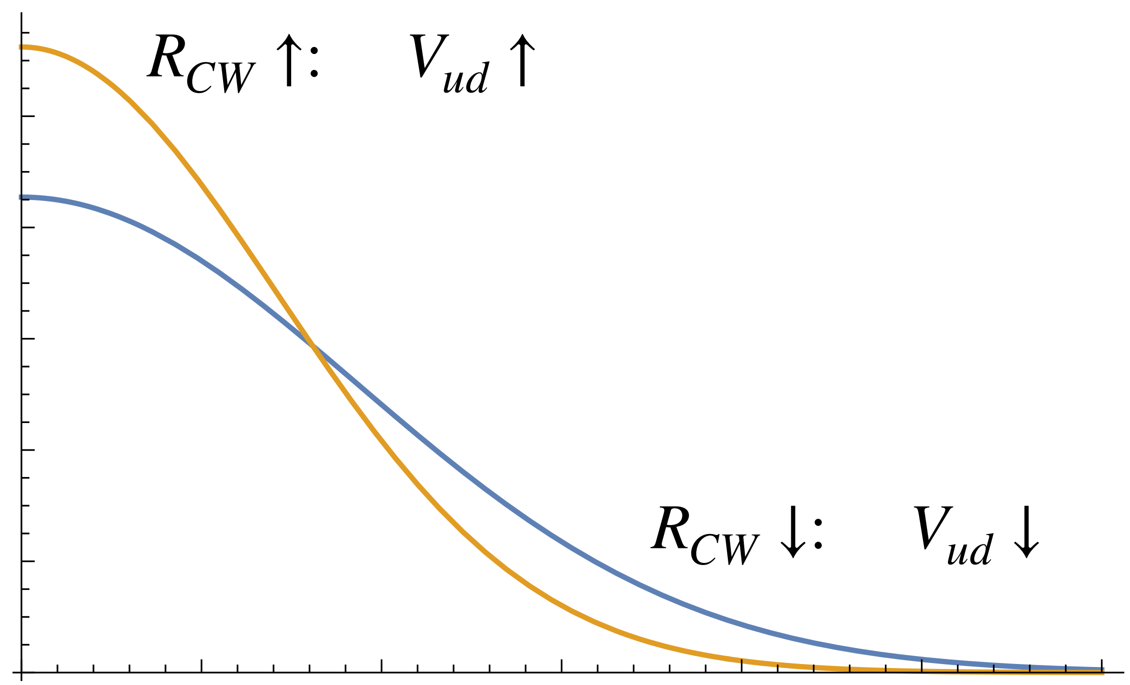
Seng 2212.02681

Charge radii + isospin symmetry —> nuclear recoil

A	$R_{\mathrm{Ch},-1}$ (fm)	$R_{\mathrm{Ch},0}$ (fm)	$R_{\mathrm{Ch},1}$ (fm)	$R^2_{\mathrm{Ch},1}$ (fm^2)	R^2_{CW} (fm^2)
10	$^{10}_{6}$C	$^{10}_{5}$B(ex)	$^{10}_{4}$Be: 2.3550(170)[a]	5.546(80)	N/A
14	$^{14}_{8}$O	$^{14}_{7}$N(ex)	$^{14}_{6}$C: 2.50 25(87)[a]	6.263(44)	N/A
18	$^{18}_{10}$Ne: 2.9714(76)[a]	$^{18}_{9}$F(ex)	$^{18}_{8}$O: 2.77 26(56)[a]	7.687(31)	13.40(53)
22	$^{22}_{12}$Mg: 3.0691(89)[b]	$^{22}_{11}$Na(ex)	$^{22}_{10}$Ne: 2.9525(40)[a]	8.717(24)	12.93(71)
26	$^{26}_{14}$Si	$^{26m}_{13}$Al	$^{26}_{12}$Mg: 3.0337(18)[a]	9.203(11)	N/A
30	$^{30}_{16}$S	$^{30}_{15}$P(ex)	$^{30}_{14}$Si: 3.1336(40)[a]	9.819(25)	N/A
34	$^{34}_{18}$Ar: 3.3654(40)[a]	$^{34}_{17}$Cl	$^{34}_{16}$S: 3.2847(21)[a]	10.789(14)	15.62(54)
38	$^{38}_{20}$Ca: 3.467(1)[c]	$^{38m}_{19}$K: 3.437(4)[d]	$^{38}_{18}$Ar: 3.4028(19)[a]	11.579(13)	15.99(28)
42	$^{42}_{22}$Ti	$^{42}_{21}$Sc: 3.5702(238)[a]	$^{42}_{20}$Ca: 3.5081(21)[a]	12.307(15)	21.5(3.6)
46	$^{46}_{24}$Cr	$^{46}_{23}$V	$^{46}_{22}$Ti: 3.6070(22)[a]	13.010(16)	N/A
50	$^{50}_{26}$Fe	$^{50}_{25}$Mn: 3.7120(196)[a]	$^{50}_{24}$Cr: 3.6588(65)[a]	13.387(48)	23.2(3.8)
54	$^{54}_{28}$Ni: 3.738(4)[e]	$^{54}_{27}$Co	$^{54}_{26}$Fe: 3.6933(19)[a]	13.640(14)	18.29(92)
62	$^{62}_{32}$Ge	$^{62}_{31}$Ga	$^{62}_{30}$Zn: 3.9031(69)[b]	15.234(54)	N/A
66	$^{66}_{34}$Se	$^{66}_{33}$As	$^{66}_{32}$Ge	N/A	N/A
70	$^{70}_{36}$Kr	$^{70}_{35}$Br	$^{70}_{34}$Se	N/A	N/A
74	$^{74}_{38}$Sr	$^{74}_{37}$Rb: 4.1935(172)[b]	$^{74}_{36}$Kr: 4.1870(41)[a]	17.531(34)	19.5(5.5)

Charge radii + isospin symmetry —> nuclear recoil

Total decay rate $\sim ft\,|V_{ud}|^2 \sim |V_{ud}|^2 \int_0^{Q^2_{EC}} dQ^2 F_{CW}(Q^2)$



Only total rate measured — if radius underestimated, V_{ud} will come out smaller

Systematic shift by up to 0.1% to some ft values —> may resolve CKM deficit!

Estimate from isospin symmetry — but isospin symmetry broken, how credible?

Theory strategy: compute all radii AND δ_C — check pattern, compare to available data, motivate exp.

Discussion & Caveats

Precision tests of SM with CKM unitarity at few 10^{-4} —> 2-3σ deficit —> V_{ud} guilty!

V_{ud} from 0^+ nuclear decays: nuclear corrections to warrant 0.02% precision, $ft \to \mathscr{F}t$

Crucial for constant Ft: isospin-breaking correction δ_C —> affects bounds on BSM too!

δ_C not directly measurable —> theory input necessary - ab initio era commenced, but…

Proposal: look for phenomenological tests for nuclear theory calculations

Precise nuclear charge radii —> input to δ_C and ft-values via CC radius R_{CW}

Highest precision from μ atom spectra (reference radius) + isotope shifts (King's plot)

Caveat: for needed precision —> nuclear polarization

Talks: Anna Vyatkina, Igor Valuev

Historically: Tables of nuclear radii based on old Rinker-Späth nuclear polarization

e.g. Angeli-Marinova, Fricke-Heilig

Similar physics: Δ_{NP} in atoms <—> δ_{NS} in β-decays

Recent insights from dispersion theory prospective for δ_{NS}

Seng, MG 2211.10214

Follows progress in Δ_{NP} in light μ atoms

Neutron skins of stable daughters from PVES — feasibility studies at MESA @ Mainz