Recoil and Radiative-Recoil Corrections in Muonium

Gregory S. Adkins[a], Jonathan Gomprecht[a], Yanxi Li[a], and Evan Shinn[a]
[a] *Franklin & Marshall College, Lancaster, Pennsylvania*

Muonium, the $e^-\mu^+$ bound system, is presently the subject of intense experimental activity. The MUSEUM collaboration at J-PARC is mounting an experiment to measure the muonium ground state hyperfine splitting with an uncertainty goal of $5\,\mathrm{Hz}$ $(1.2\,\mathrm{ppb})$ [1]. The MuMASS collaboration at PSI is working on new measurements of the 1S-2S interval and the $n = 2$ Lamb shift. The 1S-2S experiment has a final uncertainty goal of $10\,\mathrm{kHz}$ $(4\,\mathrm{ppt})$ [2, 3]. For the Lamb shift, a new measurement has already reduced the uncertainty by an order of magnitude compared to previous measurements [4] with prospects for significant additional improvement. The leading uncertainties in the QED calculations for these intervals are due to uncalculated recoil and radiative-recoil corrections [3, 5, 6]. It is important to reduce these theoretical uncertainties in order to make the best use of improved experimental results.

In this talk I will report on new results for the recoil and radiative-recoil corrections to muonium energy levels at orders $(Z\alpha)^6$ and $\alpha(Z\alpha)^5$, respectively [7]. These results are exact in the particle masses, eliminating the need for an expansion in the small mass ratio m_e/m_μ. The calculations of the required "hard" integrals (the ones involving relativistic momenta) were done using the integration-by-parts identities in terms of a small set of master integrals, which were evaluated using the method of differential equations [8]. Calculations involving the "soft" (non-relativistic) scale were performed using NRQED. Progress on using the same methods for the calculation of recoil and radiative-recoil contributions at orders $(Z\alpha)^7$, $\alpha(Z\alpha)^6$, and $\alpha^2(Z\alpha)^5$ will be discussed.

---

[1] S. Kanda *et al.*, Phys. Lett. B **815** (2021) 136154.
[2] B. Ohayon, Z. Burkley, and P. Crivelli, SciPost Phys. Proc. **5** (2021) 029.
[3] I. Cortinovis *et al.*, arXiv:2301.12883 [physics.atom-ph] (2023).
[4] B. Ohayon *et al.*, Phys. Rev. Lett. **128** (2022) 011802.
[5] E. Tiesinga, P. J. Mohr, D. B. Newell, and B. N. Taylor, Rev. Mod. Phys. **93** (2021) 025010.
[6] G. Janka, B. Ohayon, and P. Crivelli, EPJ Web of Conferences **262** (2022) 01001.
[7] G. S. Adkins, J. Gomprecht, Y. Li, and E. Shinn, Phys. Rev. Lett. **130** (2023) 023004.
[8] J. Blümlein and C. Schneider, arXiv:2203.13015 [hep-ph] (2022)