Tests of physics beyond the standard model with the *g* factor of few-electron ions

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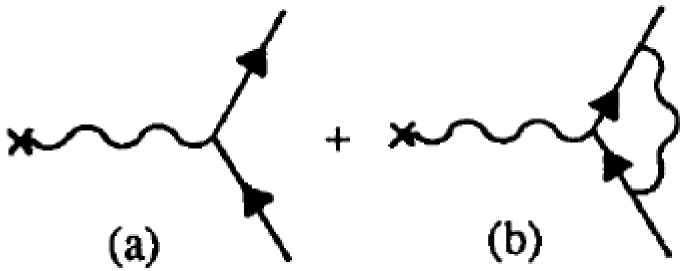
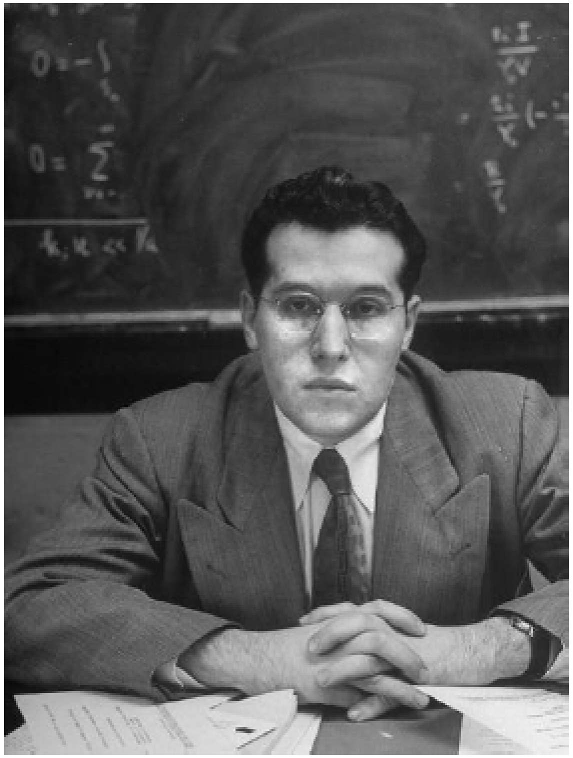
International Conference on Precision Physics and Fundamental Physical Constants

Precision theory of the *g* factor: the free electron

Energy of interaction with an external **magnetic field**: at the one-loop level, it is corrected by the vertex diagram – or Schwinger term:





$$\Delta E = -\langle \vec{\mu} \rangle \cdot \vec{B} \,,$$

with the magnetic moment *μ*, the Bohr magneton $\mu_B = \frac{e\hbar}{2mc}$

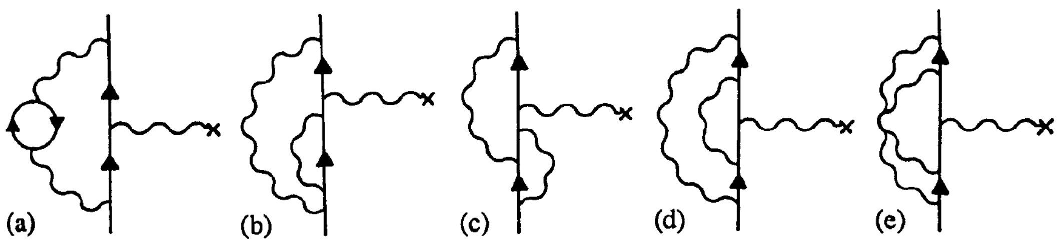
$$\langle \vec{\mu} \rangle = 2 \left(1 + \frac{\alpha}{2\pi}\right) \mu_B \langle \vec{S} \rangle = g \mu_B \langle \vec{S} \rangle \,.$$

Thus the *g* factor of the free electron up to one-loop order

$$g_{\text{free}} = 2 + \frac{\alpha}{\pi} \approx 2(1 + 0.00116141)$$

Here, *α* is the fine-structure constant

Two-loop diagrams:



A. Peterman, Helv. Phys. Act **30**, 407 (1957);
C. M. Sommerfield, Ann. Phys. **5**, 26 (1958)

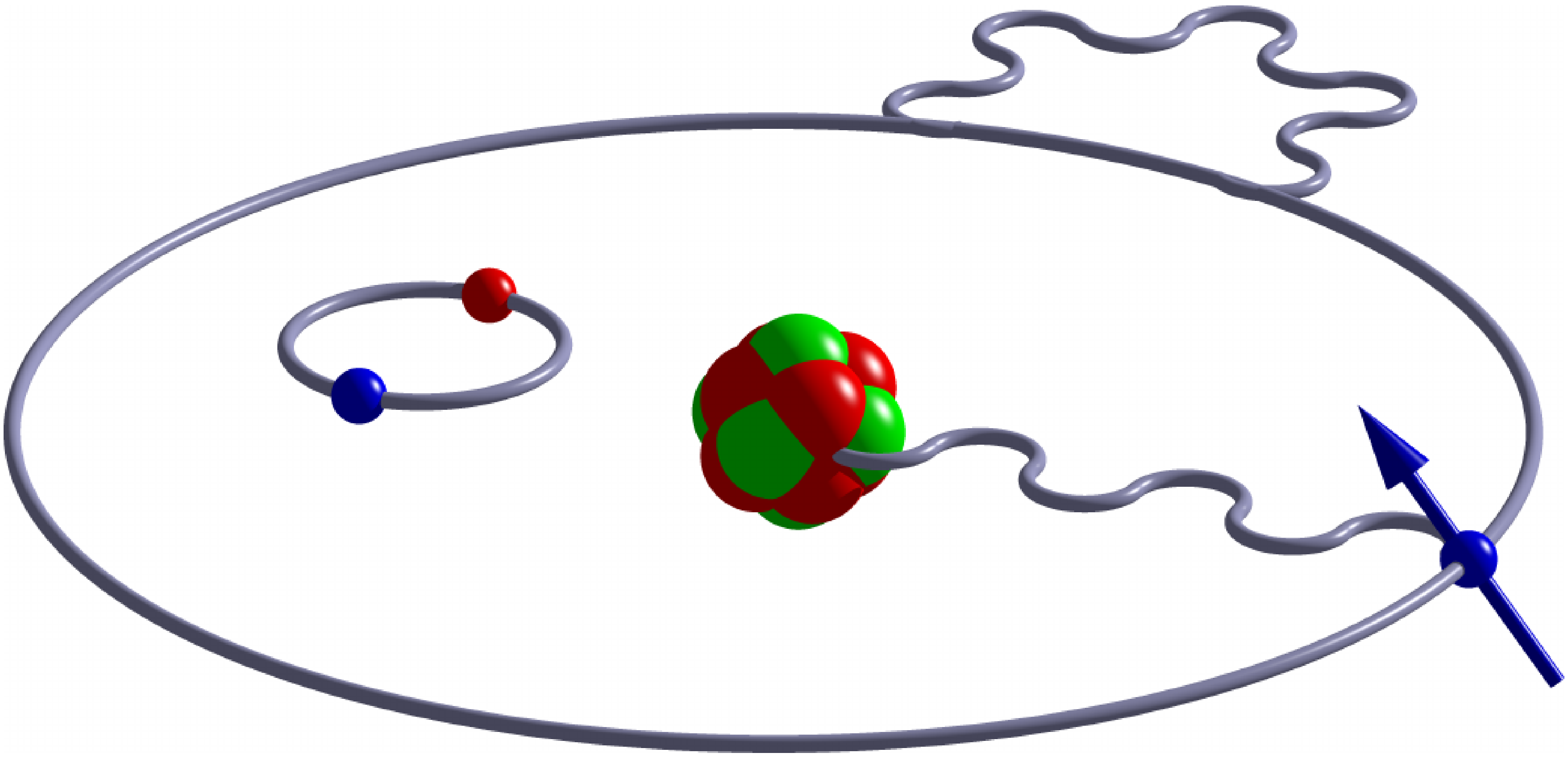
Three^{+} -loop diagrams:

S. Laporta, E. Remiddi, Phys. Lett. B **379**, 283 (1996)
T. Aoyama, M. Hayakawa, T. Kinoshita, M. Nio, Phys. Rev. Lett. **109**, 111807 (2012)
S. Volkov, talk yesterday, +Phys. Part. Nuclei **53**, 805 (2022)

One oft the most accurate values for the fine-structure constant: g_{exp} and corresponding multi-loop free-electron QED calculations

D. Hanneke, S. Fogwell, and G. Gabrielse, Phys. Rev. Lett. **100**, 120801 (2008);
X. Fan, T. G. Myers, B. A. D. Sukra, and G. Gabrielse, arXiv:2209.13084

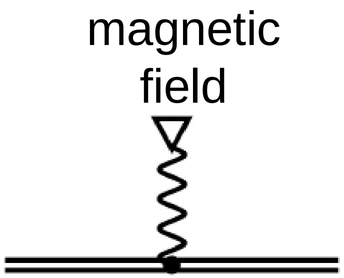
The bound-electron *g* factor



“High-energy atomic physics”:

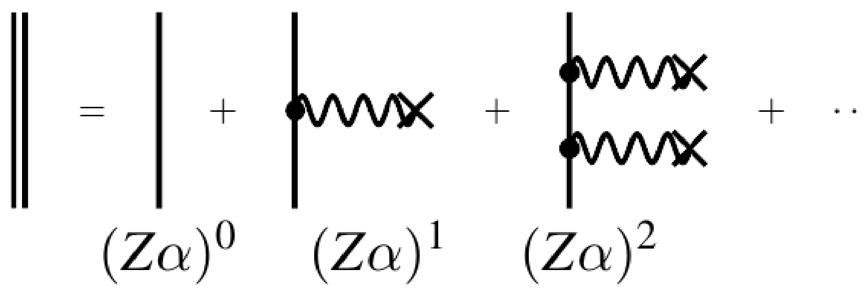
Binding energy according to Bohr: $E_{1s} = \frac{1}{2} m_{\mathrm{e}} c^2 (Z\alpha)^2 \approx m_{\mathrm{e}} c^2$ for high *Z*;
Coupling constant between the electron and the nucleus: $\alpha \rightarrow Z\alpha$

For a Coulomb potential (pointlike nucleus), the Dirac *g*-factor for the *1s* state (G. Breit, 1928):



$$g_{\rm D} = \frac{2}{3}\left(1 + 2\sqrt{1-(Z\alpha)^2}\right) = 2 - \frac{2}{3}(Z\alpha)^2 - \frac{1}{6}(Z\alpha)^4 + \ldots$$

Double line: Coulomb-Dirac (wave function or) propagator:



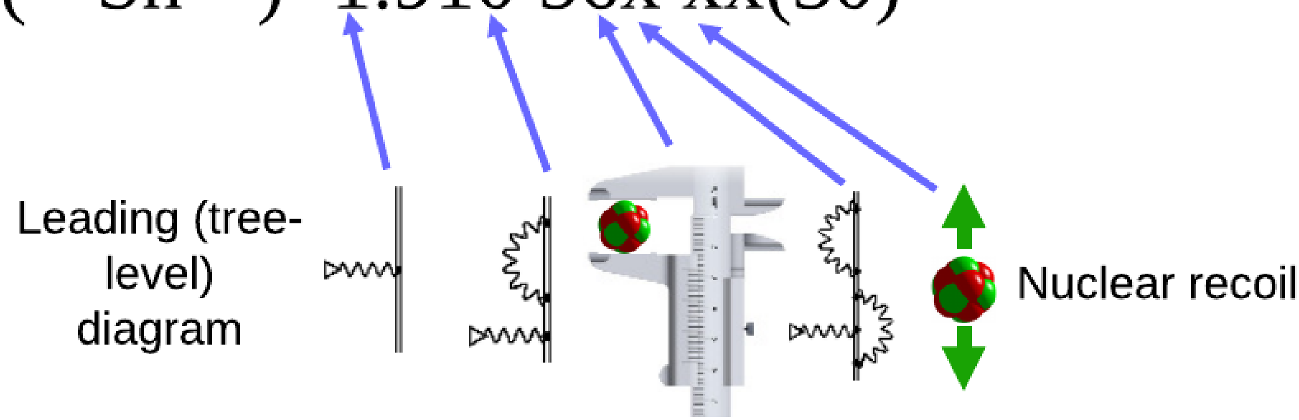
with an arbitrary number of interactions with the (strong) Coulomb potential of the nucleus (Furry picture)

A number of QED and nuclear corrections contribute to the theoretical g factor:

$$g_{\rm th} = g_{\rm D} + \delta g_{\rm 1L} + \delta g_{\rm 2L} + \delta g_{\rm FS} + \delta g_{\rm rec} + \delta g_{\rm NP} \,,$$

1L: one-loop QED: self-energy (SE) and vacuum polarization (VP),
2L: two-loop QED: SE-SE, VP-VP, SE-VP,
FS: nuclear finite-size,
rec: nuclear recoil,
NP: nuclear polarizability, etc.

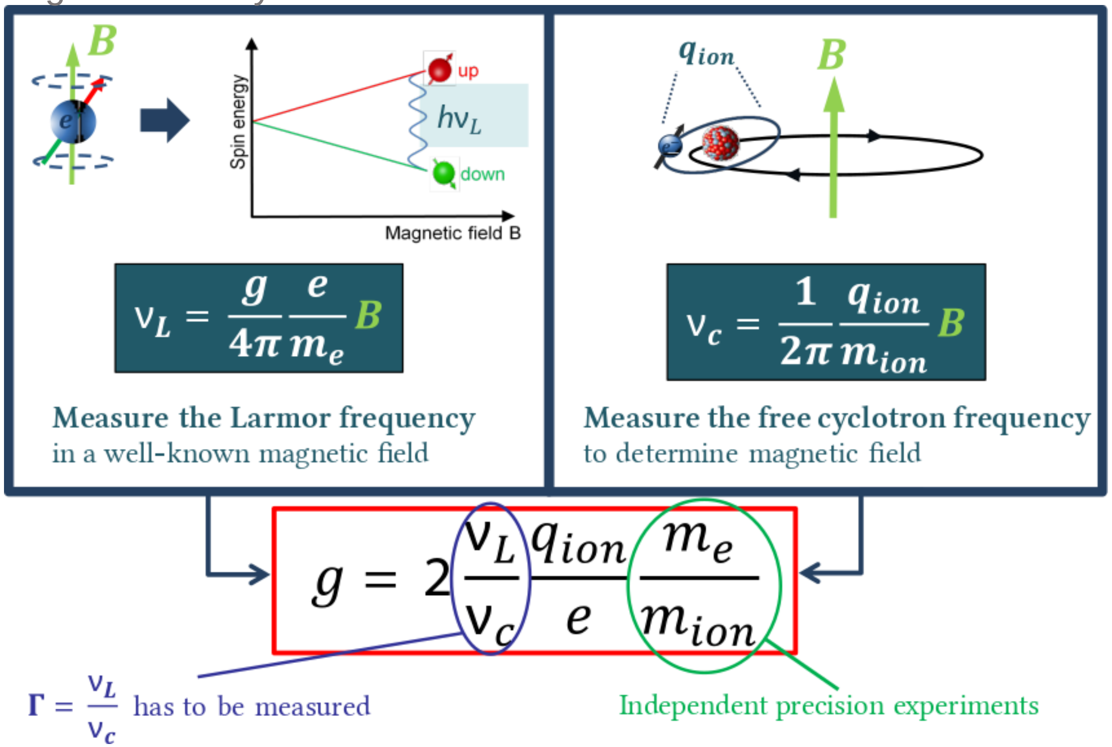
1-e^- ion: $g(^{118}\mathrm{Sn}^{49+})$=1.910 56x xx(30)



(See the previous talk of Vladimir Yerokhin & of Bastian Sikora yesterday)

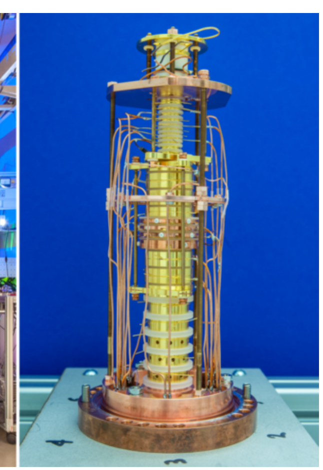
Penning trap measurements of the *g* factor

Figure courtesy of Tim Sailer



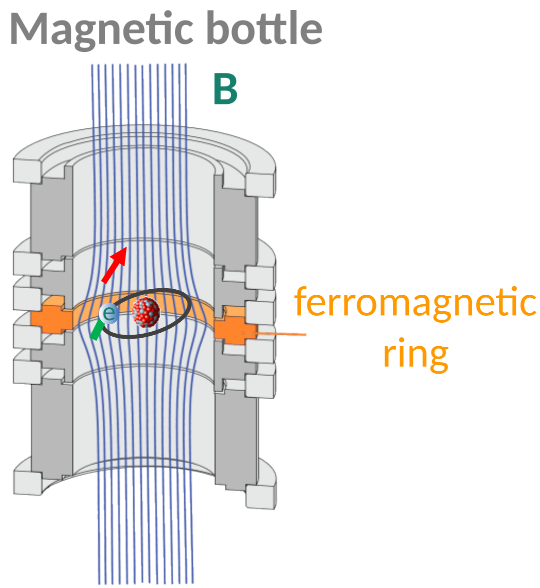
- Measure Larmor frequency ν_L and cyclotron frequency ν_c to "cancel" *B*-field
- Input: ion mass and electron mass to determine *g*-factor

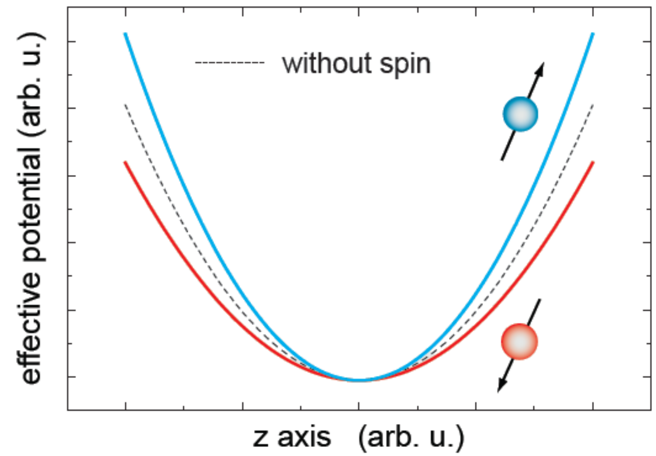
- Experiments by the **ALPHATRAP** group of Sven Sturm, Klaus Blaum (see talk of Charlotte König before)
- So far achieved: detailed tests of **QED** in strong Coulomb fields; theory by Czarnecki, Karshenboim, Milstein, Pachucki, Shabaev, Yerokhin *et al.*
- World-record determination of the **electron mass**



Spin state detection: continuous Stern-Gerlach effect

Analysis Trap

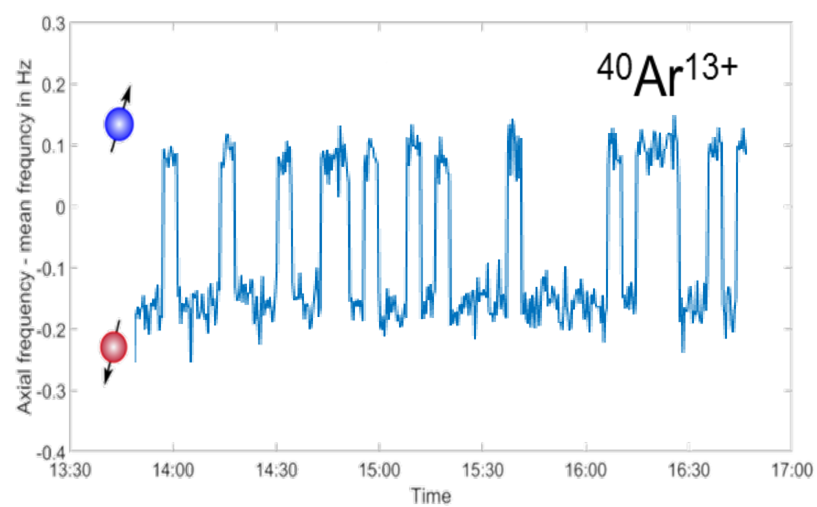




Continuous Stern-Gerlach effect:

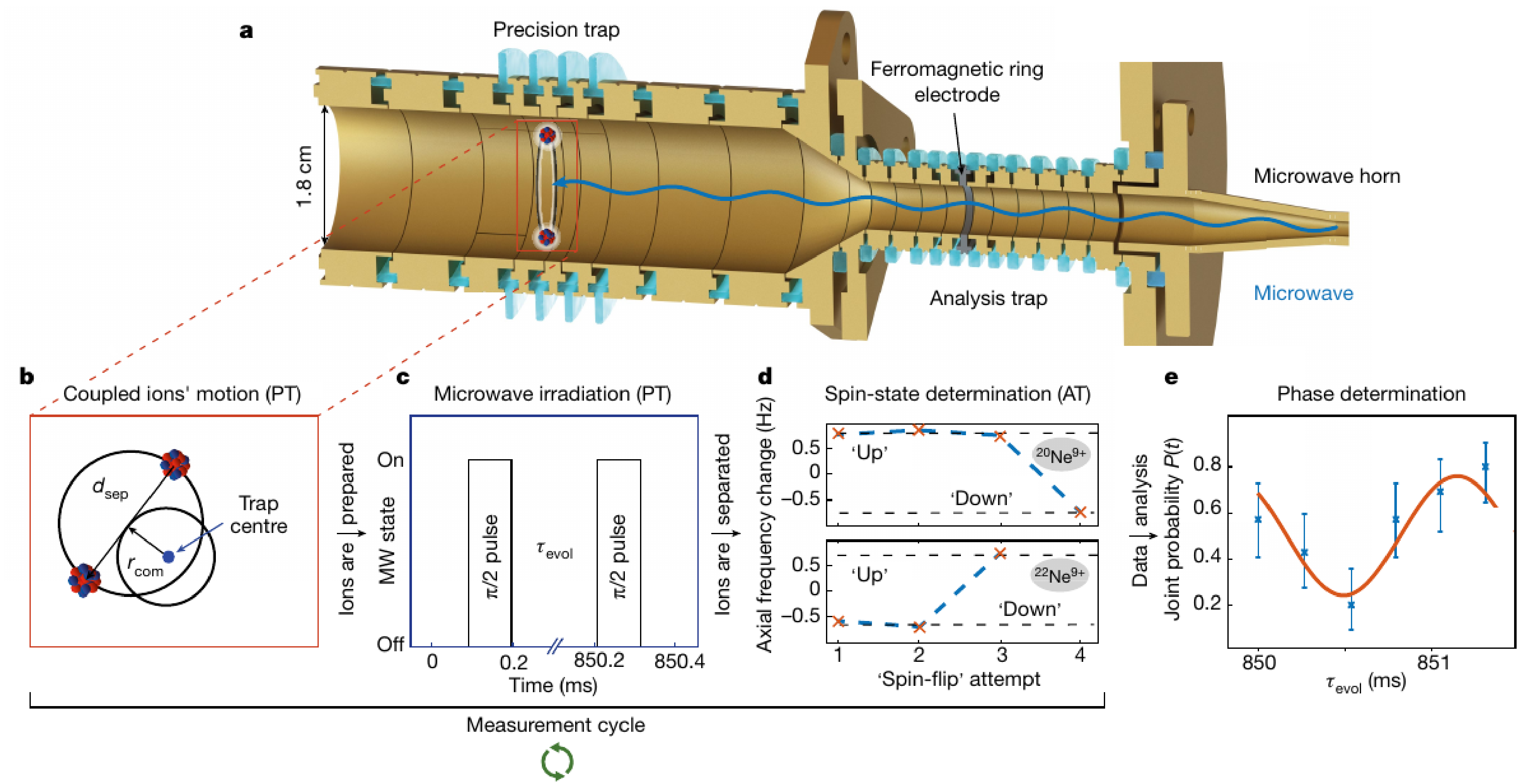
axial frequency offset between “up” and “down” spin orientation

$$\Delta\nu_z \approx \frac{B_2 g \mu_B}{4\pi^2 m_{ion} \nu_z}$$



Slide courtesy of Tim Sailer

Precision measurement of the Larmor frequency difference:



The joint spin-flip probability P(t) displays the beating of the Larmor frequencies, $\Delta\nu_L$

Theoretical isotope shift of $^{20}Ne^{9+}$ – $^{22}Ne^{9+}$ g factor:

	$^{20}Ne^{9+}$	$^{22}Ne^{9+}$	Isotope shift
Dirac	1.996 445 170 898(2)	1.996 445 170 898(2)	0
QED (α)	0.002 325 473 302	0.002 325 473 302	0
QED (α^2)	-0.000 003 547 970(117)	-0.000 003 547 970(117)	0
QED (α^n, n>=3)	0.000 000 029 524	0.000 000 029 524	0
Nuclear size	4.762(7)e-9	4.596(12)e-9	**1.66(11)e-10**
Recoil, non-QED	1.46093e-7	1.32810e-7	13.283e-9
Recoil, QED	4.78e-10	4.34e-10	**4.4e-11**
Recoil, (α(m/M))	-1.13e-10	-1.03e-10	-1e-11
Recoil, $(m/M)^2$	-4.4e-11	-3.6e-11	-8e-12
Total	1.998 767 276 921(117)	1.998 767 263 446(117)	13.474(11) e-9

Experiment: $\Delta\nu_L$=758.752(30)$_{stat}$(56)$_{sys}$ Hz → Δg=13.47524(53)(99)$_{sys}$ e-9

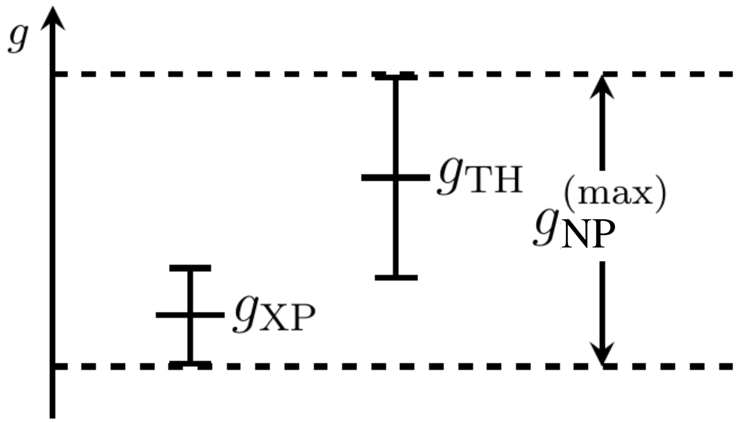
T. Sailer, V. Debierre, Z. Harman, F. Heiße, C. König, J. Morgner, B. Tu, **A. V. Volotka**, C. H. Keitel, K. Blaum, S. Sturm, Nature **606**, 479 (2022)

- Perfect agreement of theory and experiment: the QED theory of recoil is confirmed

Theory has a larger uncertainty than experiment;
Largest error bar stems from the **uncertainty of the nuclear radii,** and also their difference δR_{rms}=0.0530(34) fm
[radii from optical spectroscopy and muonic atom x-ray spectroscopy, I. Angeli, K. Marinova, At. Data Nucl. Data Tables **99**, 69 (2013)]

- Turn this around: **extract** radius difference: δR_{rms}=0.0533(4) fm: order-of-magnitude improvement!

- ...or if you are not interested in nuclear radii, but rather something new:



The (dis)agreement of theory and experiment gives room to **new physics** (NP)

Test of new physics through the bound-electron *g* factor

- Various beyond-standard model scenarios: talk of Zoltán Trócsányi
- A proposed fifth fundamental force

  Massive spinless boson φ (mass range unknown)

  Couples electrons to nucleons according to Yukawa potential
- Relevance to high-energy physics

  Electroweak hierarchy problem: Electroweak force >> Gravitational force

  Possible solution: mixing of a **“relaxion”** with the Higgs boson:

  as the early Universe expanded, the expansion has weakened this coupling, ‘relaxing’ the Higgs mass down to its currently observed value

  Relaxion: **light dark matter candidate**

  Known to be addressable by low-energy experiments since their proposal:

  P. W. Graham, D. E. Kaplan, S. Rajendran, Phys. Rev. Lett. **115**, 221801 (2011)

- Yukawa potential seen by electrons

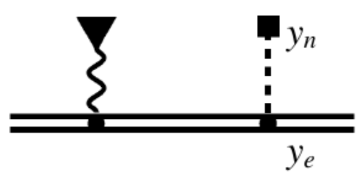
$$V_\phi(\mathbf{r}) = -\hbar c\, \alpha_{\mathrm{NP}} (A - Z) \frac{\mathrm{e}^{-\frac{m_\phi c}{\hbar}|\mathbf{r}|}}{|\mathbf{r}|}$$

  $\alpha_{NP} = y_e y_n / 4\pi$: coupling constant

  m_φ: mass of the boson

$$\Delta g_{1s} = -\frac{4}{3}\alpha_{\mathrm{NP}} \frac{(Z\alpha)}{\gamma} A \left(1 + \frac{m_\phi}{2Z\alpha m_e}\right)^{-2\gamma} \times \left[3 - 2\frac{(Z\alpha)^2}{1+\gamma} - \frac{2\gamma}{1 + \frac{m_\phi}{2Z\alpha m_e}}\right]$$

- Correction to *g* factor:



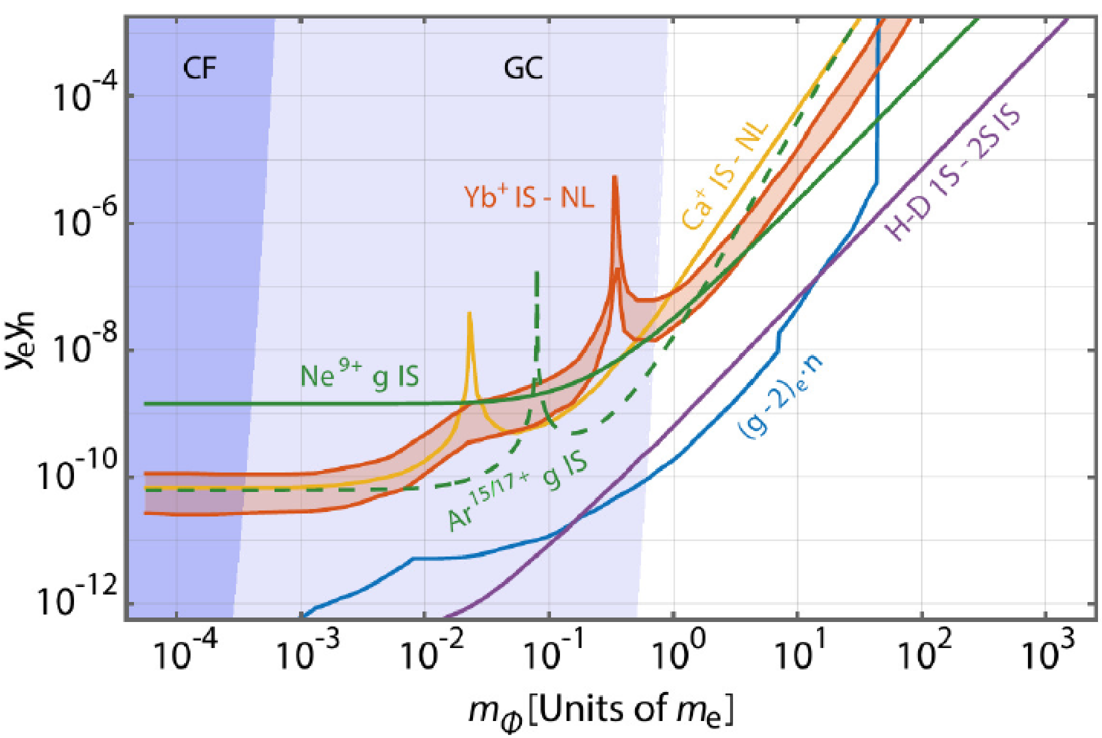
V. Debierre, Z. Harman, C. H. Keitel, Phys. Lett. B **807**, 135527 (2020)

Very recent **ALPHATRAP** experiment on the isotope shift (IS) of H-like Ne,

$$\Delta g = g({}^{20}_{10}\mathrm{Ne}^{9+}) - g({}^{22}_{10}\mathrm{Ne}^{9+}):$$

- **Differences** of *g* factors of two similar ions can be measured **very accurately**
- Different isotopes: different number of **neutrons**, sensitivity to such new physics
- Same number of protons: **QED** ("old physics") largely **cancels**

Bounds on the coupling strength extracted from the experimental Δ*g*:



Ne^{9+} *g* IS: our bound

IS-NL: isotope shift nonlinearity (King plot) experiments

Ca+ : J. Berengut et al., Phys. Rev. Lett. **120**, 091801 (2018);

Yb+ : I. Counts et al., Phys. Rev. Lett. **125**, 123002 (2020)

H-D: hydrogen-deuterium isotope shift, laser spectroscopy

C. Delaunay et al., Phys. Rev. D **96**, 115002 (2017)

T. Sailer, V. Debierre, Z. Harman, F. Heiße, C. König, J. Morgner, B. Tu, A. V. Volotka, C. H. Keitel, K. Blaum, S. Sturm, Nature **606**, 479 (2022)

$Ar^{15/17+}$: projected **improved** bound from g factor isotope shift of H-like and **Li-like** Ar

- **Problem:** nuclear parameters (e.g. $\langle r^2 \rangle$) are not known accurately
- **Solution:** weighted difference of H- and Li-like ions (same Z):

$$\delta_\Xi g = g(2s) - \Xi g(1s) \, ,$$

with the weight Ξ theoretically chosen to suppress nuclear size effects

- Simplest approximation: $\Xi = \frac{1}{8} = 0.125$ – because: $|\psi_{ns}(r=0)|^2 \propto \frac{1}{n^3}$
- Accurate formula (incl. relativity, QED and $e^- - e^-$ interaction:

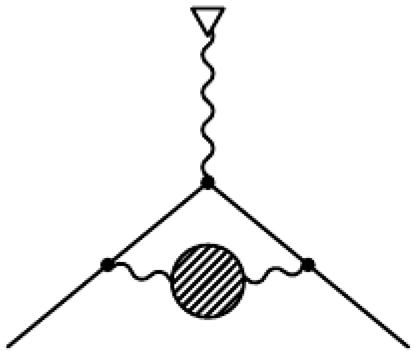
$$\Xi = 2^{-2\gamma-1} \left[1 + \frac{3}{16}(Z\alpha)^2 \right] \left(1 - \frac{2851}{1000}\frac{1}{Z} + \frac{107}{100}\frac{1}{Z^2} \right) \, ,$$

where $\gamma = \sqrt{1-(Z\alpha)^2}$

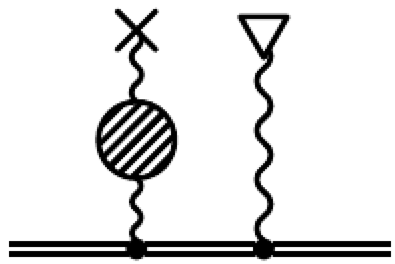
V. A. Yerokhin, E. Berseneva, Z. Harman *et al.*, Phys. Rev. Lett. **116**, 100801 (2016); Phys. Rev. A **94**, 022502 (2016)
V. M. Shabaev, D. A. Glazov, M. B. Shabaeva, V. A. Yerokhin, G. Plunien, and G. Soff Phys. Rev. A **65**, 062104 (2002)
S. G. Karshenboim, Z. Phys. D **39**, 109 (1997); S. G. Karshenboim, V. G. Ivanov, Phys. Lett B **524**, 259 (2002)

Standard model = QED?

Hadronic vacuum polarization



Free electron (muon):
2-loop effect



Bound electron:
1-loop effect

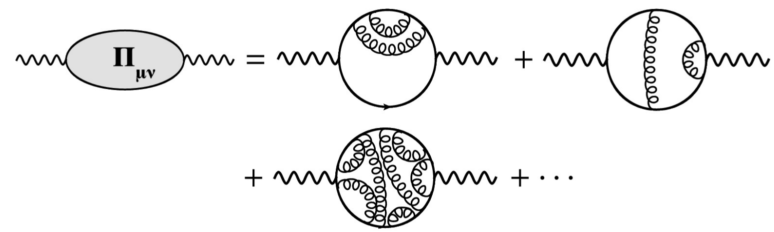


Figure from Goecke et al.,
Phys. Lett. B **704**, 211 (2011)

$$\Delta g^{\mathrm{had.}}_{\mathrm{point}}(1s) = -16 B_1 C_1 m_{\mathrm{e}}^2 (Z\alpha)^4 + \frac{512 B_1 C_1^{3/2} m_{\mathrm{e}}^3 (Z\alpha)^5}{9} - \frac{16 B_1 C_1 m_{\mathrm{e}}^2 (Z\alpha)^6}{3} \Big[2 + 30 C_1 m_{\mathrm{e}}^2 - 3 \ln\Big(2 m_{\mathrm{e}} Z\alpha \sqrt{C_1} \Big) \Big] + \mathcal{O}\left((Z\alpha)^7 \right)$$

→ **Negligible effect on the Ne g factor isotope shift** at the current level of uncertainties

B_1=0.0023092, C_1=3.9925370 GeV^{-2}
from $e^- e^+$ → hadrons experiments
(see talk by Zoltán Fodor yesterday)

E. Dizer, Z. Harman, arXiv:2303.07973 (2023);
S. Breidenbach, E. Dizer, H. Cakir, and Z. Harman, Phys. Rev. A **106**, 042805 (2022)
Related: S. G. Karshenboim, V. A. Shelyuto, Eur. Phys. J. D **75**, 49 (2021);
J. L. Friar, J. Martorell, and D. W. L. Sprung, Phys. Rev. A **59**, 4061 (1999)

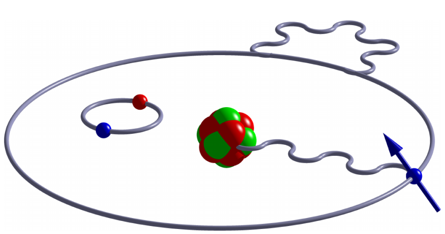
Summary

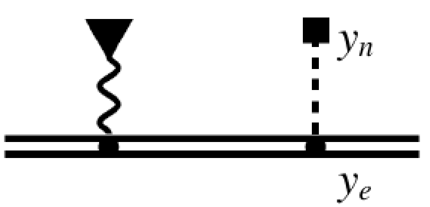
(Isotopic) difference of g factors can be measured more accurately than the absolute *g* factor of a single isotope or ion: 1e-13 vs. 1e-11

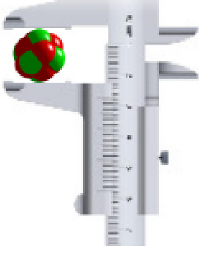
Current application: **test** of the (QED) theory of **nuclear recoil**, OR measuring the difference of **nuclear radii**, OR **constraining new physics**

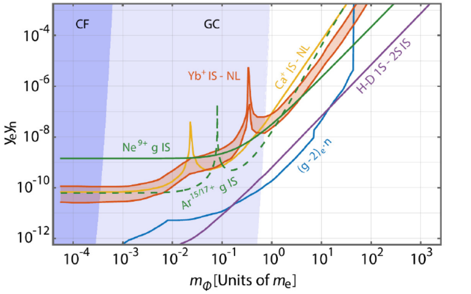
You have to know the „**old physics“** well: few-electron ions

Future applications: even **more competitive bounds; other new boson-mediated interactions**









Bedankt voor uw andacht!
感谢聆听，欢迎提问！
Dziękuję za uwagę!
Grazie per l’attenzione!
İlginiz için teşekkürler!
ध्यान देने के लिए आपका धन्यवाद।
Köszönöm a figyelmet!
Merci à tous pour votre attention !
Muchas gracias por su atención!
Mulţumesc pentru atenţie!
Obrigado pela atenção!
Спасибо за внимание!
Thank you for your attention!
Vielen Dank für Ihre Aufmerksamkeit!