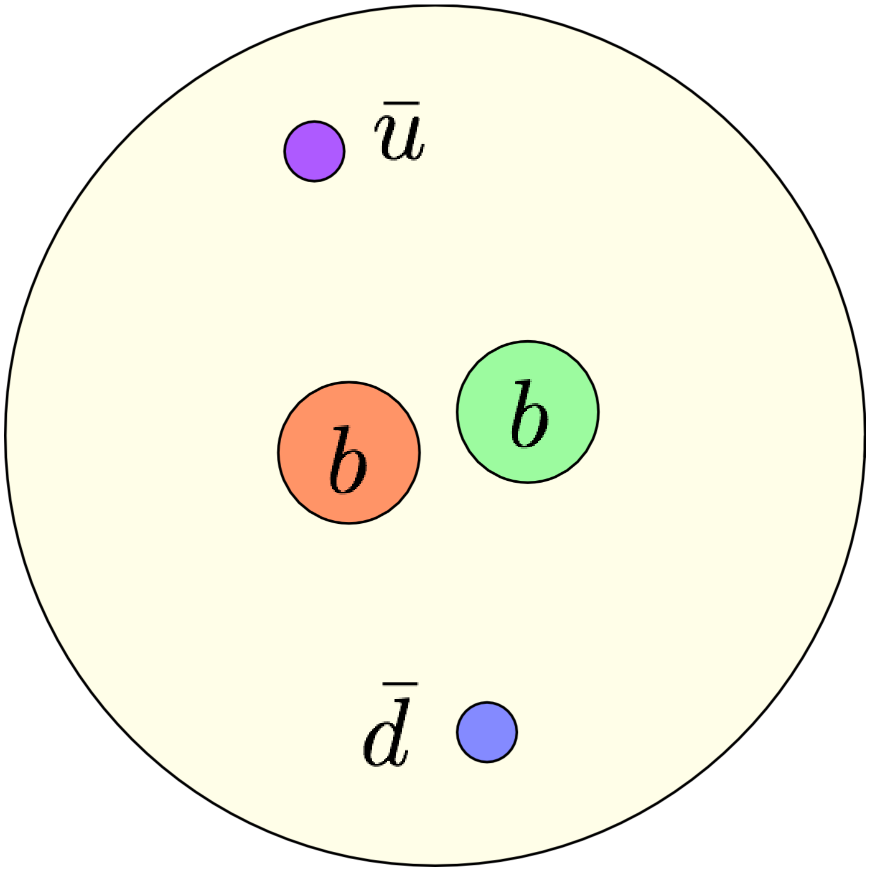
Stable tetraquarks; bound-electron g



PSAS 2018, Vienna

Andrzej Czarnecki University of Alberta

May 17, 2018

Outline:

- two quarks and two anti-quarks: like Ps-molecule
- effective anti-quark --> resulting new type of hadron
- PLB 778 (2018) 233 with Bo Leng and M. B. Voloshin

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Stability of tetrons

Andrzej Czarnecki[a], Bo Leng[a], M.B. Voloshin[b,c,d,*]

- g-factor of a bound electron: new level of precision

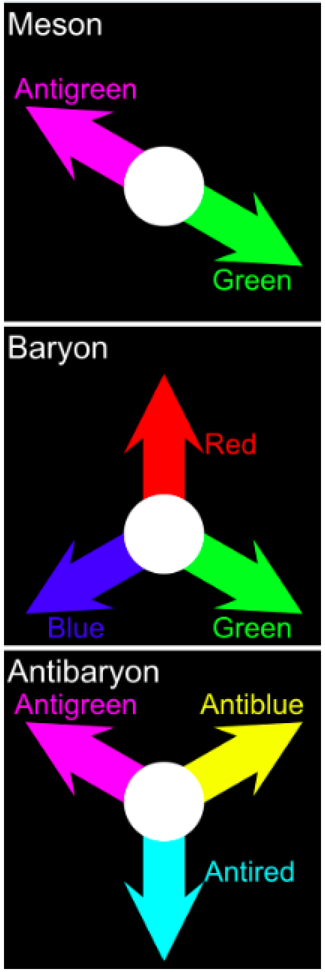
PHYSICAL REVIEW LETTERS **120,** 043203 (2018)

Two-Loop Binding Corrections to the Electron Gyromagnetic Factor

Andrzej Czarnecki,[1] Matthew Dowling,[1] Jan Piclum,[1,2] and Robert Szafron[1,3]

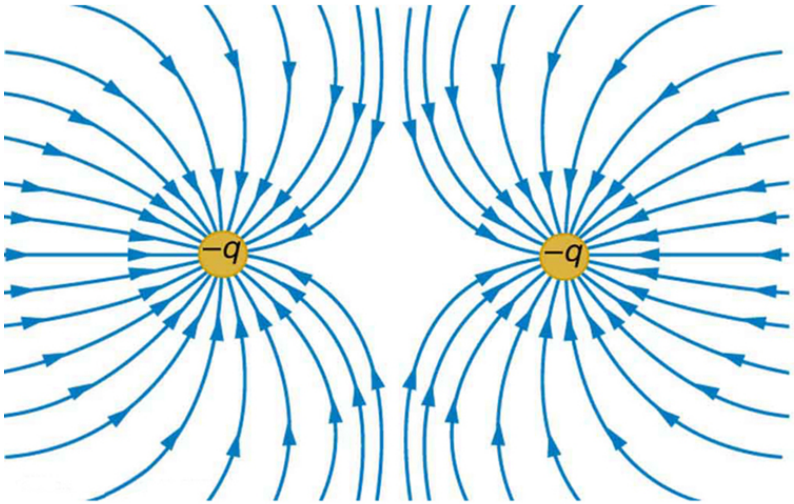
Color interaction between two heavy quarks

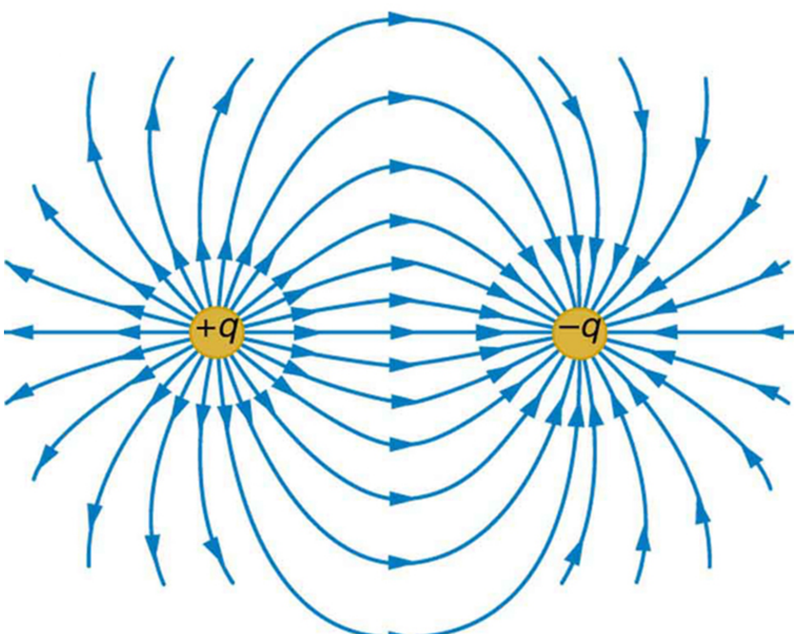
Hadrons have no net color:



From https://en.wikipedia.org/wiki/Hadron

Field point of view: electrostatics





Electric fields:

$$\boldsymbol{E} = \frac{q\boldsymbol{r}}{4\pi\epsilon_0 r^3}$$ q at the origin

$$\boldsymbol{E}' = \frac{q'\left(\boldsymbol{r}-\boldsymbol{r}'\right)}{4\pi\epsilon_0\left|\boldsymbol{r}-\boldsymbol{r}'\right|^3}$$ q' at r'

Energy density:

$$\frac{\epsilon_0}{2}\left(\boldsymbol{E}+\boldsymbol{E}'\right)^2$$

Interaction energy:

$$U_{\text{int}} = \frac{\epsilon_0}{2}\int 2\boldsymbol{E}'\cdot\boldsymbol{E}\mathrm{d}^3r$$

$$= \frac{\epsilon_0 q' q}{(4\pi\epsilon_0)^2}\int\frac{(\boldsymbol{r}-\boldsymbol{r}')\cdot\boldsymbol{r}}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|^3 r^3}\mathrm{d}^3r$$

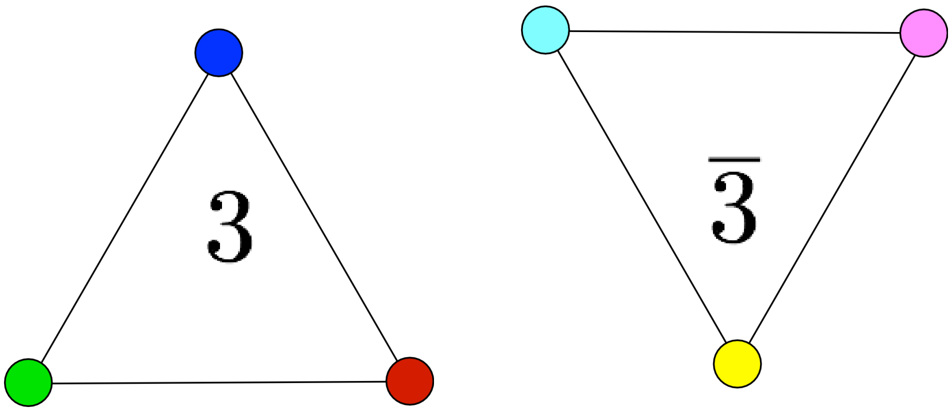
$$= \frac{q'q}{16\pi^2\epsilon_0}\int\frac{r-r'\cos\theta}{\left|\boldsymbol{r}-\boldsymbol{r}'\right|^3 r^2}\mathrm{d}^3r \quad \boxed{= \frac{q'q}{4\pi\epsilon_0 r'}}$$

Similar cancellation happens in QCD

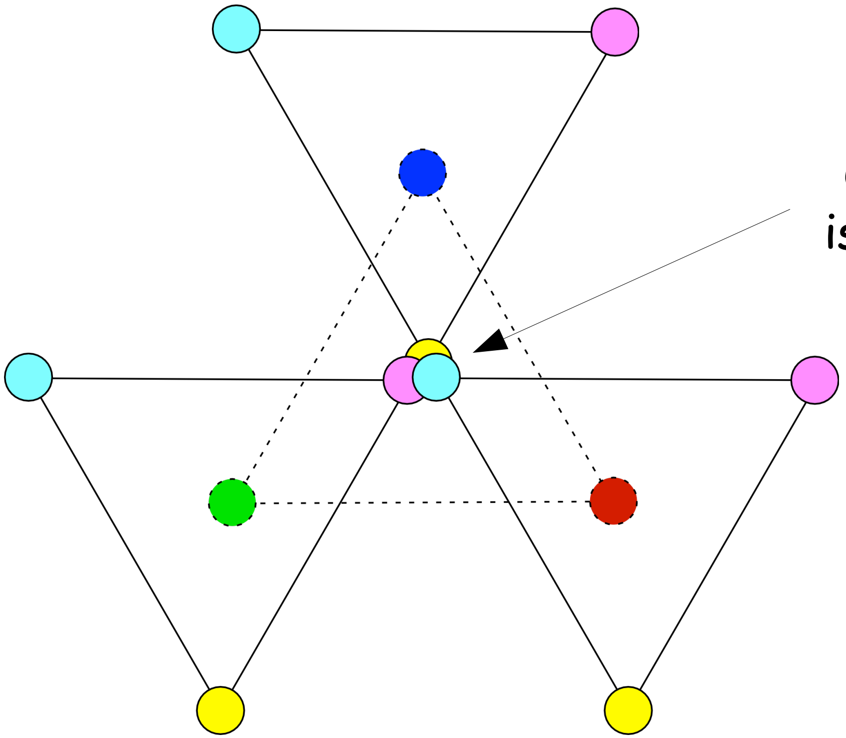
One-gluon-exchange approximation: the same spatial dependence.

However, the color charge opens more options.

Example: quark-antiquark states



Quark-antiquark: color singlet or octet

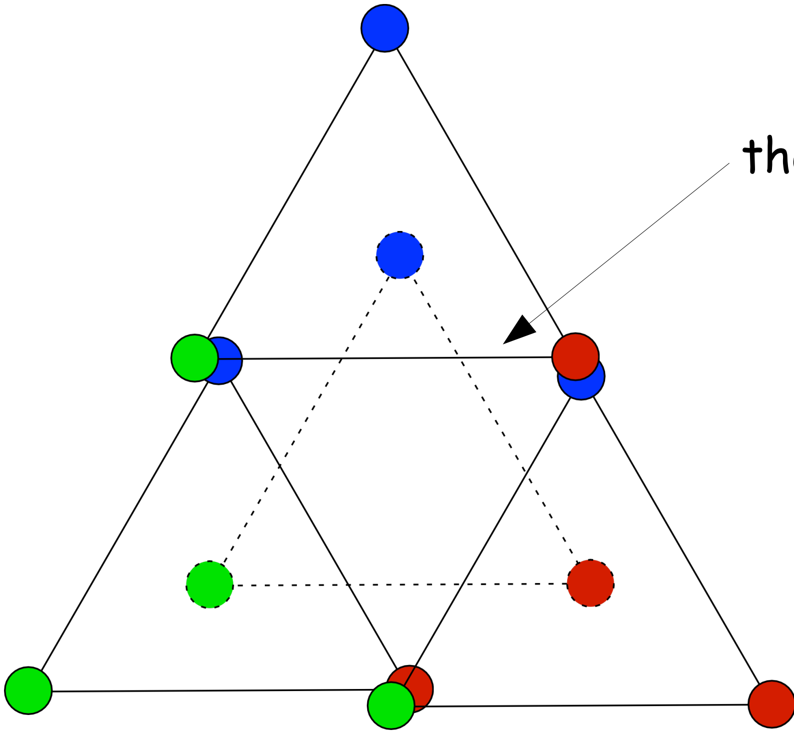


One of these three states, singlet, is the least colorful: most attractive

(the most complete cancellation of fields)

$$3 \otimes \overline{3} = 1 \oplus 8$$

Quark-quark: anti-triplet or sextet



Emerging anti-triplet:
the least colorful combination;
attractive.

However, the cancellation is not as complete as in quark-antiquark pairs, so the attraction is not as strong.
An interesting competition between heavy and lighter degrees of freedom!

$$3 \otimes 3 = \overline{3} \oplus 6$$

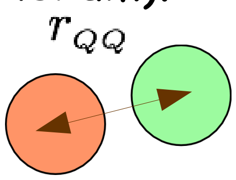
Attractive potential between heavy quarks

Singlet: quark-antiquark $-\frac{N_c^2-1}{2N_c}\frac{\alpha_s}{r} \xrightarrow{N_c=3} -\frac{4}{3}\frac{\alpha_s}{r}$ $\sim N_c$ for $N_c \to \infty$

Anti-triplet: quark-quark $-\frac{N_c+1}{2N_c}\frac{\alpha_s}{r} \xrightarrow{N_c=3} -\frac{2}{3}\frac{\alpha_s}{r}$

If the QQ are sufficiently heavy, they form a compact bound state that, from far away, looks like an antiquark: can bind two antiquarks like in an antibaryon (resembling helium).

Bohr radius of the QQ state: $r_{QQ} \sim \frac{1}{\alpha_s M}$



Interaction of the QQ with antiquarks

First consider a solvable model with heavy antiquarks:
single-gluon exchanges --> Coulomb-like potential.

$$V_{qQ} = -\frac{N_c \alpha_s}{2} \left(\frac{1}{r_q} + \frac{1}{r_{q'}} \right)$$

again $\sim N_c$
(total color cancellation)

Characteristic distance of the antiquarks from the QQ-nucleus:

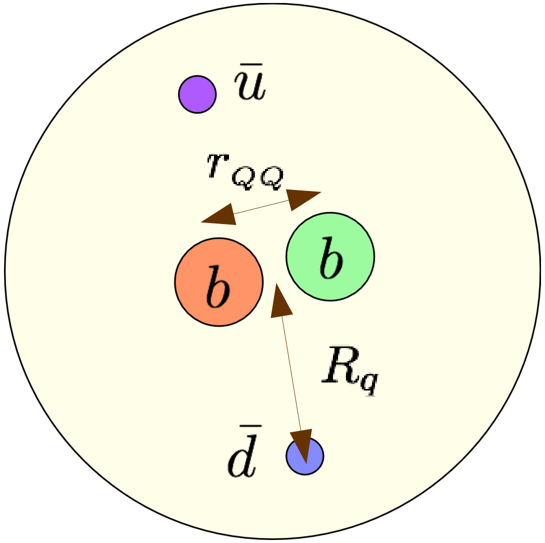
$$R_q \sim \frac{1}{N_c \alpha_s m}$$

No auto-dissociation provided that

$$R_q \gg r_{QQ}$$

Auto-dissociation when

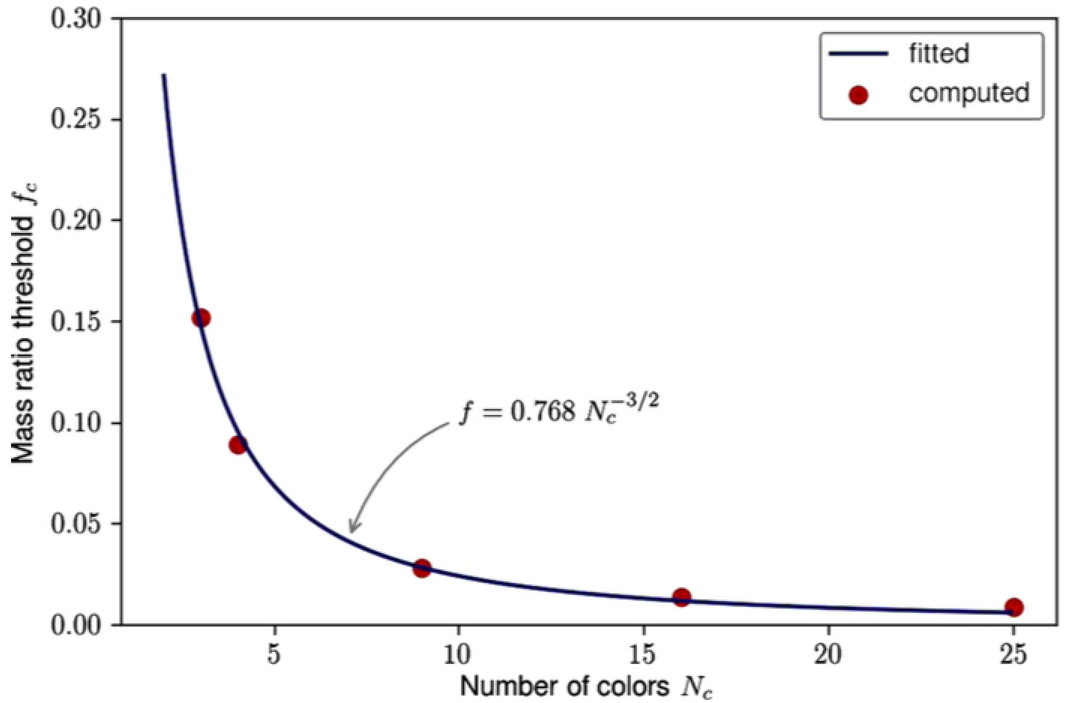
$$\frac{N_c^3 m^2}{M^2} \sim 1$$



Results of variational calculation

Critical mass ratio f as a function of the number of colors: confirms breakup when

$$\frac{N_c^3 m^2}{M^2} \sim 1$$



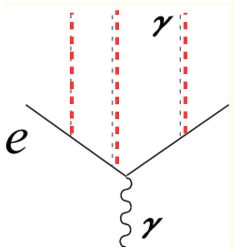
Based on Mariusz Puchalski's code.

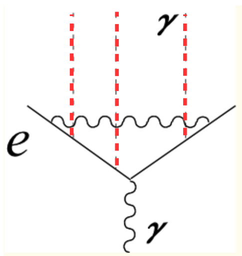
Conclusion: tetrons only possible when Q=b and q=u,d, maybe s.

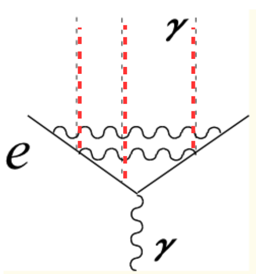
Different approach: Karliner+Rosner PRL **119**, 202001 (2017) 1707.07666
Eichten+Quigg PRL **119**, 202002 (2017) 1707.09575

Bound-electron *g*-factor

Bound-electron g-2: binding and loops







$$g = 2 - \frac{2\,(Z\alpha)^2}{3} - \frac{(Z\alpha)^4}{6} + \ldots$$

Breit 1928

$$+\frac{\alpha}{\pi}\left[1 + \frac{(Z\alpha)^2}{6} + (Z\alpha)^4\,(a_{41}\ln Z\alpha + a_{40}) + \ldots\right]$$

Pachucki, Jentschura, Yerokhin (2004)

$$+\left(\frac{\alpha}{\pi}\right)^2\left[-0.65..\left(1 + \frac{(Z\alpha)^2}{6}\right) + (Z\alpha)^4\,(b_{41}\ln Z\alpha + b_{40}) + \ldots\right]$$

two-loop corrections

Pachucki, AC Jentschura, Yerokhin (2005)

Next stage:

$$\frac{\alpha}{\pi}(Z\alpha)^5 \quad \text{and} \quad \left(\frac{\alpha}{\pi}\right)^2 (Z\alpha)^5$$

One- and two-loop binding corrections at (Zα)^5

One-loop: Pachucki+Puchalski, PRA96 (2017) 032503
conceptual breakthrough

$$\Delta g \sim \alpha \, (Z\alpha)^5$$

$$\Delta g \sim \alpha^2 \, (Z\alpha)^5$$

Sources of $\alpha^2 (Z\alpha)^5$ effects:
- additional short-distance potential generated by Lamb-shift diagrams
- energy-dependence of the Lamb shift (and B-field change of energy)
- modification of the electron response to the B-field
- modification of the B-field by the Coulomb field

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$$a_e = -1.912245764926445574152647167439830054060873390658725345\ldots\left(\frac{\alpha}{\pi}\right)^4$$

Crucial tool: Laporta algorithm

Integration by parts

↓

Identities among loop integrals
with various powers of propagators

↓

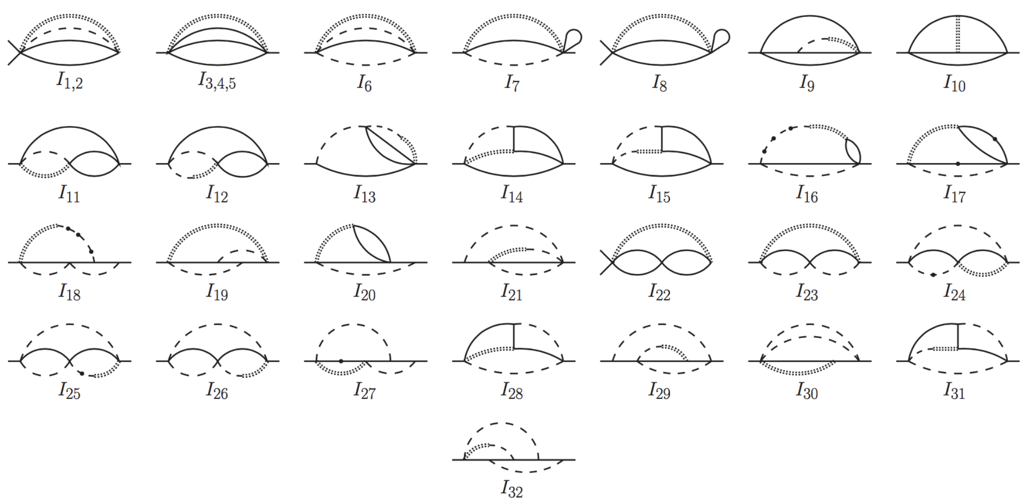
System of difference equations

↓

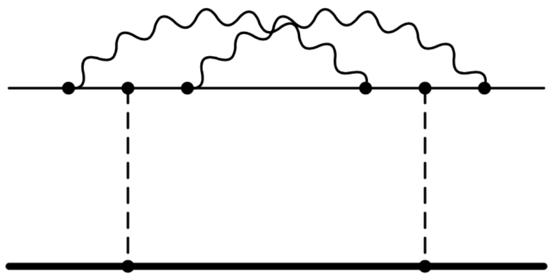
Solution in terms of a basis of master integrals

Milestone papers:

Tkachev PLB 100 (1981) 65
Chetyrkin and Tkachev NPB 192 (1981) 159
Laporta IJMP 15 (2000) 5087
Smirnov JHEP 0810 (2008) 107



Short-distance potential at $O(\alpha^2 (Z\alpha)^2)$

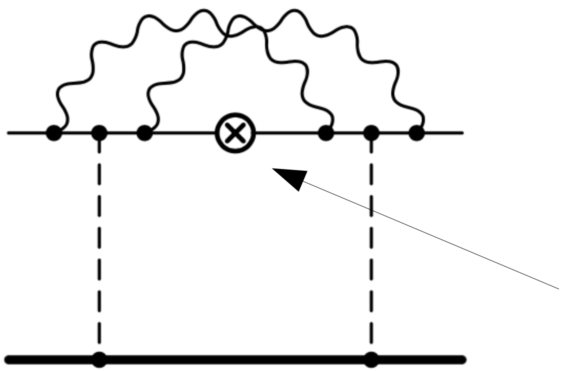


+ 18 other diagrams with two-loop self-energy

First computed for Lamb shift in hydrogen:
Pachucki, PRL 72, 3154 (1994);
Eides and Shelyuto, PRA 52, 954 (1995).

Improved precision by reduction to master integrals:
Dowling, Mondejar, Piclum, AC, PRA 81, 022509 (2010).

Modified electron response to the B-field

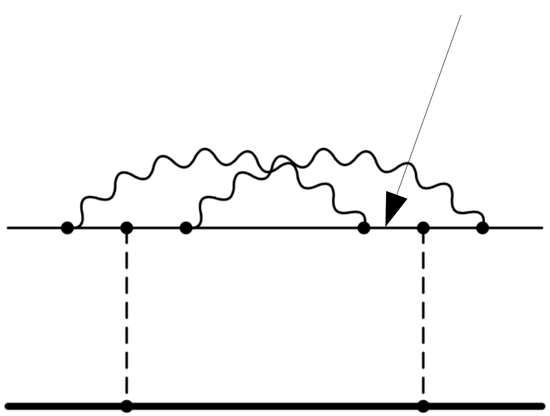


External magnetic field

A group of about a hundred 3-loop diagrams: automatically generated from Lamb. Reduced to the same master integrals.

Gauge dependent: need g_3 to cancel the ξ-dependence.

Propagators depend on the energy of the electron



The energy is shifted in the magnetic field,

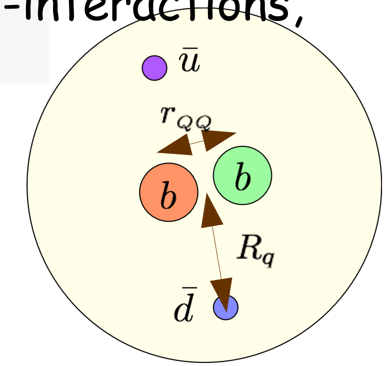
$$\Delta E = -\boldsymbol{\mu} \cdot \boldsymbol{B} = \frac{e}{2m} g \boldsymbol{s} \cdot \boldsymbol{B}$$

$$g_3 = g \frac{\mathrm{d\,Lamb}}{\mathrm{d}E} = \left(2 + \frac{\alpha}{\pi}\right) \frac{\mathrm{d\,Lamb}}{\mathrm{d}E}$$

Summary

Tetrons: a new type of hadrons; stable with respect to strong-interactions, very narrow states (decay only weakly).

Studied with variational methods developed in atomic theory.



Bound-electron g-factor: effects $\alpha^2 (Z\alpha)^5$ are now under control.

$$g^{(2)} = \left(\frac{\alpha}{\pi}\right)^2 \left[b_{00}\left(1 + \frac{\alpha_Z^2}{6}\right) + \alpha_Z^4\left(b_{40} + b_{41}L\right) + \alpha_Z^5 b_{50} + \dots\right]$$

$$b_{50} = b_{50}^{\mathrm{VP}} + \Delta b_{50}$$

$$\Delta b_{50} = 4.7304(9)$$

Extra slides

Result: short-distance potential

$$\Delta V = c\,\alpha^2\,(Z\alpha)^2\,\delta^3(\boldsymbol{r})$$

Lamb shift

$$\Delta E = -\frac{7.72381(4)}{\pi}\alpha^2\,(Z\alpha)^5\,m$$

Correction to g

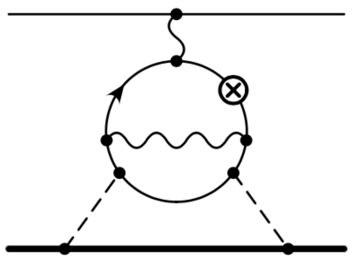
$$\Delta g = \frac{4\Delta E}{m}$$

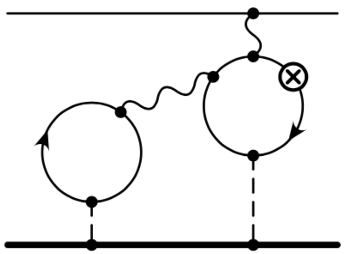
Karshenboim, PLA 266 (2000) 380

Coulomb field modifies the magnetic field

Karshenboim, Milstein, PLB 549 (2002) 321

"Magnetic loop": a vacuum-polarization effect,





$$g^{\mathrm{MLPH}} = \left(-\frac{7543}{16200} - \frac{303587}{10125\pi} + \frac{92368}{2025\pi} \ln 2 \right) \alpha^2 \alpha_Z^5 \qquad g^{\mathrm{MLVP}} = \left(\frac{628}{8505\pi} - \frac{1}{54} \right) \alpha^2 \alpha_Z^5$$

Note: technically simpler --> analytical result.
This part is numerically small.

TABLE I. Bound electron g factor for helium, carbon, and silicon ions. The error related to missing higher order contributions is estimated by $(\alpha/\pi)^2\alpha_Z^6\ln^3\alpha_Z^{-2}$.

Contribution	$^4He^+$	$^{12}C^{5+}$	$^{28}Si^{13+}$	Source
Dirac/Breit value	1.999 857 988 825 37(6)	1.998 721 354 392 1(6)	1.993 023 571 557(3)	Ref. [20]
+ other known corrections	2.002 177 406 711 41(55)	2.001 041 590 168 6(12)	1.995 348 957 825(39)	Refs. [16,22][a]
g^{SE}	0.000 000 000 000 02	0.000 000 000 005 0	0.000 000 000 348	This work
g^{LBL}	−0.000 000 000 000 01	−0.000 000 000 001 5	−0.000 000 000 102	This work
g^{ML}	0.000 000 000 000 00	0.000 000 000 000 6	0.000 000 000 038	This work
$(\alpha/\pi)^2\alpha_Z^6\ln^3\alpha_Z^{-2}$	0.000 000 000 000 00(3)	0.000 000 000 000 0(93)	0.000 000 000 000(583)	
Total	2.002 177 406 711 42(55)	2.001 041 590 172 7(94)	1.995 348 958 109(584)	