Status of the CP violation study in $H \to \tau^+ \tau^- \gamma$

Part 1: Introduction

Goal: Probe the CP violating $H\tau\tau$ Lagrangian

CP-even CP-odd

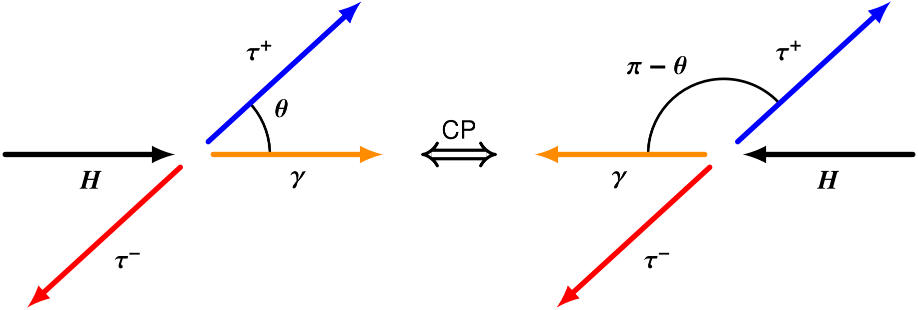
$$\mathscr{L}_{H\tau\tau} = -\frac{m_\tau}{v}\,\bar{\tau}\left(a_\tau + i\gamma^5\, b_\tau\right)\tau\, H$$

Standard Model $\Longrightarrow$ $a_\tau = 1$ $b_\tau = 0$

New Physics $\Longrightarrow$ $a_\tau \neq 1$ $b_\tau \neq 0$

Idea: Use angular distribution of $H \to \tau^+ \tau^- \gamma$

Gottfried-Jackson (GJ) frame
Center-of-momentum frame of $\tau^+ \tau^-$



Hypothesis: CP violation $\Leftrightarrow$ **asymmetry** under $\theta \leftrightarrow \pi - \theta$ exchange

Confirmation: Analytical calculation confirms the hypothesis.

Question: How well can it be studied experimentally?

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Gottfried-Jackson (GJ) frame
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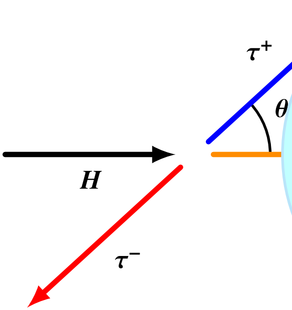

- This idea is *not* new.
- We propose *a new way to probe this idea*.

(1) 1302.2159
(2) 1303.2230
(3) 1308.0422
(4) 1405.6723
(5) 1408.0342
(6) 2001.06516
(7) 2109.04426
. . . . . .

Hypothesis: CP violation ⇔ **asymmetry** under $\theta \leftrightarrow \pi - \theta$ exchange

Confirmation: Analytical calculation confirms the hypothesis.

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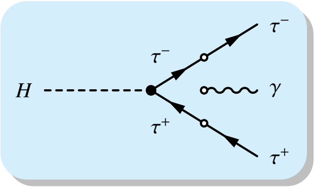
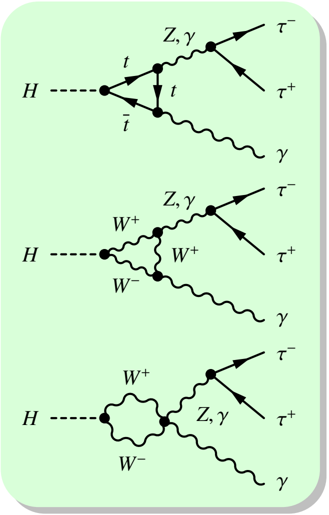
Part 2: Origin of CP asymmetry

Feynman Diagrams and Amplitude for $H \to \tau^+ \tau^- \gamma$



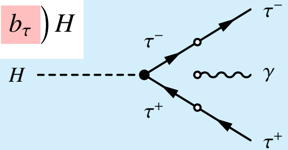
$$\mathscr{M}_{\tau\tau\gamma} = \mathscr{M}^{(\mathrm{Yuk})}_{\tau\tau\gamma} + \mathscr{M}^{(Z\gamma)}_{\tau\tau\gamma} + \mathscr{M}^{(\gamma\gamma)}_{\tau\tau\gamma}$$



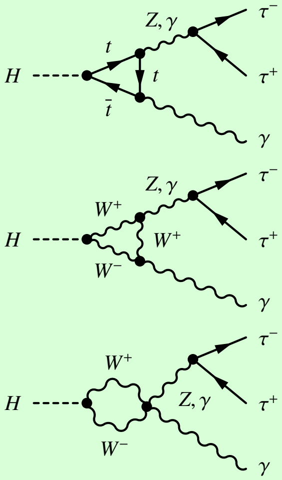
1-loop SM box diagrams
neglected: smaller contribution.

Feynman Diagrams and Amplitude for $H \to \tau^+ \tau^- \gamma$

$$\mathscr{L}_{H\tau\tau} = -\frac{m_\tau}{v}\,\bar{\tau}\left(a_\tau + i\,\gamma^5\, b_\tau\right) H$$



$$\mathscr{M}_{\tau\tau\gamma} = \mathscr{M}^{(\mathrm{Yuk})}_{\tau\tau\gamma} + \mathscr{M}^{(Z\gamma)}_{\tau\tau\gamma} + \mathscr{M}^{(\gamma\gamma)}_{\tau\tau\gamma}$$



$$\mathscr{L}_{H\mathcal{V}\gamma} = \frac{H}{4\,v}\Big(2\,A_2^{Z\gamma}\,F^{\mu\nu}Z_{\mu\nu} + 2\,A_3^{Z\gamma}\,F^{\mu\nu}\widetilde{Z}_{\mu\nu} + A_2^{\gamma\gamma}\,F^{\mu\nu}F_{\mu\nu} + A_3^{\gamma\gamma}\,F^{\mu\nu}\widetilde{F}_{\mu\nu}\Big),$$

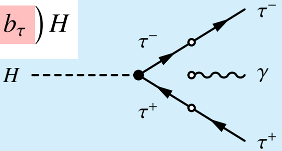
where $\mathcal{V}_{\mu\nu} = \partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu, \quad \widetilde{\mathcal{V}}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathcal{V}^{\rho\sigma},$

for $\mathcal{V} = Z, \gamma$.

1-loop SM box diagrams neglected: smaller contribution.

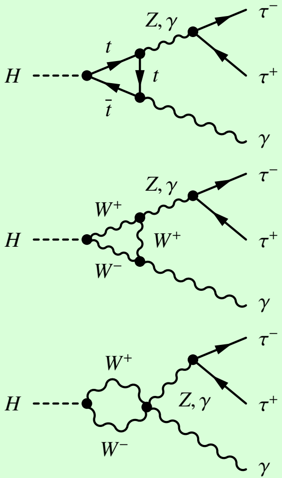
Feynman Diagrams and Amplitude for $H \to \tau^+ \tau^- \gamma$

$$\mathscr{L}_{H\tau\tau} = -\frac{m_\tau}{v}\,\bar{\tau}\left(a_\tau + i\gamma^5\, b_\tau \right) H$$



$$\mathscr{M}_{\tau\tau\gamma} = \mathscr{M}^{(\mathrm{Yuk})}_{\tau\tau\gamma} + \mathscr{M}^{(Z\gamma)}_{\tau\tau\gamma} + \mathscr{M}^{(\gamma\gamma)}_{\tau\tau\gamma}$$

SM loop effects only



$$\mathscr{L}_{HV\gamma} = \frac{H}{4\,v}\Big(2\, A_2^{Z\gamma} F^{\mu\nu} Z_{\mu\nu} + 2\, A_3^{Z\gamma} F^{\mu\nu} \widetilde{Z}_{\mu\nu} + A_2^{\gamma\gamma} F^{\mu\nu} F_{\mu\nu} + A_3^{\gamma\gamma} F^{\mu\nu} \widetilde{F}_{\mu\nu}\Big),$$

where $\mathcal{V}_{\mu\nu} = \partial_\mu \mathcal{V}_\nu - \partial_\nu \mathcal{V}_\mu$, $\quad \widetilde{\mathcal{V}}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\mathcal{V}^{\rho\sigma}$,

for $\mathcal{V} = Z, \gamma$. *We shall consider* $A_3^{\gamma\gamma} = 0 = A_3^{Z\gamma}$.

1-loop SM box diagrams neglected: smaller contribution.

Kinematics: Only 2 independent variables

❖ Only 3 Lorentz invariant mass-squares possible,

$$m_{+-}^2 \equiv (p_H - p_0)^2 = (p_+ + p_-)^2 \,, \qquad \Longrightarrow 4\,m_\tau^2 \leqslant m_{+-}^2 \leqslant m_H^2$$
$$m_{+0}^2 \equiv (p_H - p_-)^2 = (p_+ + p_0)^2 \,, \qquad \Longrightarrow m_\tau^2 \leqslant m_{+0}^2 \leqslant (m_H - m_\tau)^2$$
$$m_{-0}^2 \equiv (p_H - p_+)^2 = (p_- + p_0)^2 \,. \qquad \Longrightarrow m_\tau^2 \leqslant m_{-0}^2 \leqslant (m_H - m_\tau)^2$$

Note: $m_{+-}^2 + m_{+0}^2 + m_{-0}^2 = m_H^2 + 2\,m_\tau^2.$ $\Longrightarrow$ Only 2 *independent* mass-squares.

❖ In the GJ frame,

$$\left.\begin{aligned} m_{+0}^2 &= M^2 - M'^2 \cos\theta, \\ m_{-0}^2 &= M^2 + M'^2 \cos\theta, \end{aligned}\right\} \Longrightarrow \begin{cases} & \theta \leftrightarrow \pi - \theta \\ \equiv & \cos\theta \leftrightarrow -\cos\theta \\ \equiv & m_{+0}^2 \leftrightarrow m_{-0}^2 \end{cases}$$

where $M^2 = \frac{1}{2}\left(m_H^2 + 2\,m_\tau^2 - m_{+-}^2\right), \quad M'^2 = \frac{1}{2}\left(m_H^2 - m_{+-}^2\right)\sqrt{1 - 4\,m_\tau^2/m_{+-}^2}.$

Choice of independent variables in $H \to \tau^+ \tau^- \gamma$

	$\left(m_{+0}^2, m_{-0}^2\right)$	$\left(m_{+-}^2, \cos\theta\right)$	$\left(E_+, E_-\right)$
Differential Decay rate	$\dfrac{d^2\Gamma_{\tau\tau\gamma}}{dm_{+0}^2\, dm_{-0}^2}$	$\dfrac{d^2\Gamma_{\tau\tau\gamma}}{dm_{+-}^2\, d\cos\theta}$	$\dfrac{d^2\Gamma_{\tau\tau\gamma}}{dE_+\, dE_-}$
Frame of reference	Any frame	GJ frame	H rest frame
Need to boost?	No	Yes	Yes

- $E_\pm$ = energy of $\tau^\pm$ in H rest frame. $m_{\pm 0}^2 = m_H^2 - 2\, m_H\, E_\pm$ & $m_{+0}^2 \leftrightarrow m_{-0}^2 \equiv E_+ \leftrightarrow E_-$
- Differential decay rate is frame dependent:

$$\left(\frac{d^2\Gamma_{\tau\tau\gamma}}{dm_{+0}^2\, dm_{-0}^2}\right)_{\text{H rest}} = \frac{\left|\mathcal{M}_{\tau\tau\gamma}\right|^2}{256\,\pi^3\, m_H^3},$$

$$\left(\frac{d^2\Gamma_{\tau\tau\gamma}}{dm_{+-}^2\, d\cos\theta}\right)_{\text{H rest}} = \frac{m_H^2 - m_{+-}^2}{512\,\pi^3\, m_H^3} \sqrt{1 - \frac{4\, m_\tau^2}{m_{+-}^2}}\, \left|\mathcal{M}_{\tau\tau\gamma}\right|^2,$$

$$\left(\frac{d^2\Gamma_{\tau\tau\gamma}}{dm_{+-}^2\, d\cos\theta}\right)_{\text{GJ}} = \frac{m_{+-}\left(m_H^2 - m_{+-}^2\right)}{256\,\pi^3\, m_H^2\left(m_H^2 + m_{+-}^2\right)} \sqrt{1 - \frac{4\, m_\tau^2}{m_{+-}^2}}\, \left|\mathcal{M}_{\tau\tau\gamma}\right|^2.$$

Amplitude squared

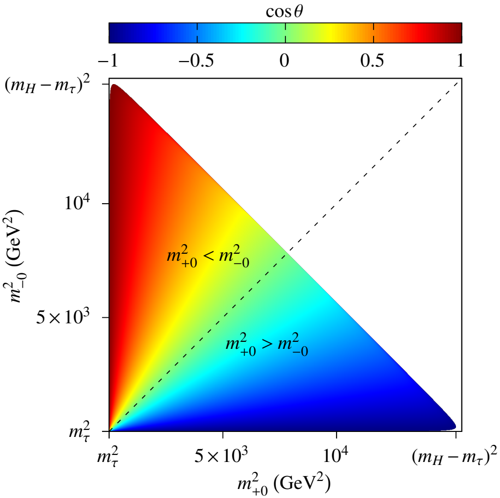
$$\left|\mathscr{M}_{\tau\tau\gamma}\right|^2 = \left|\mathscr{M}_{\tau\tau\gamma}^{(\mathrm{Yuk})}\right|^2 + \left|\mathscr{M}_{\tau\tau\gamma}^{(Z\gamma)}\right|^2 + \left|\mathscr{M}_{\tau\tau\gamma}^{(\gamma\gamma)}\right|^2 + 2\,\mathrm{Re}\left(\mathscr{M}_{\tau\tau\gamma}^{(\gamma\gamma)}\,\mathscr{M}_{\tau\tau\gamma}^{(Z\gamma)*}\right)$$
$$+ \; 2\,\mathrm{Re}\left(\mathscr{M}_{\tau\tau\gamma}^{(\mathrm{Yuk})}\,\mathscr{M}_{\tau\tau\gamma}^{(Z\gamma)*}\right) + 2\,\mathrm{Re}\left(\mathscr{M}_{\tau\tau\gamma}^{(\mathrm{Yuk})}\,\mathscr{M}_{\tau\tau\gamma}^{(\gamma\gamma)*}\right),$$

↓

Source of forward-backward asymmetry
Has term linear in b_τ

Part 3: Defining various Dalitz Plot Asymmetries

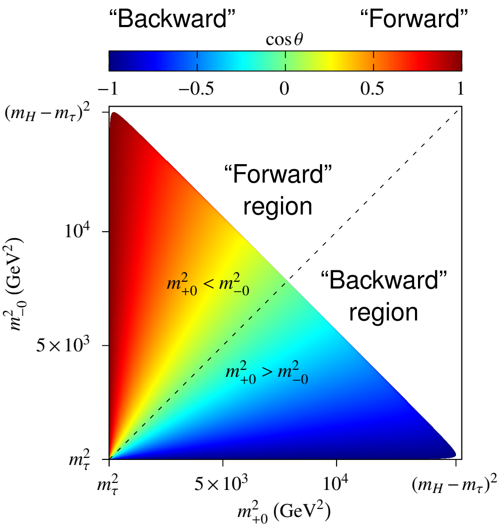
Dalitz Plot: Notations, Regions & Expectations



- Let $\mathcal{D}\left(m^2_{+0}, m^2_{-0}\right) \equiv \dfrac{\mathrm{d}^2\Gamma_{\tau\tau\gamma}}{\mathrm{d}m^2_{+0}\,\mathrm{d}m^2_{-0}}$ denote **distribution of events in the** m^2_{+0} vs. m^2_{-0} **Dalitz plot.**

- Area of the Dalitz plot $\propto$ Available phase space

Dalitz Plot: Notations, Regions & Expectations



Notation:

Region	“Forward”	“Backward”
$\cos\theta$	$[0, 1]$	$[-1, 0]$
$m_{\pm0}^2$	$m_{+0}^2 < m_{-0}^2$	$m_{+0}^2 > m_{-0}^2$
Distribution	$\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right)$	$\mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right)$
No. of events	N_F	N_B

Expectation: CP violation $(b_\tau \neq 0) \implies$

- $\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right) \neq \mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right)$
- $N_F \neq N_B$

Dalitz Plot Asymmetries: Quantify CP violation

- **Non-integrated or distribution asymmetry:** Compare the distribution of events *across* the Dalitz plot in the "forward" and "backward" regions.

$$\mathcal{A}\left(m_{+0}^2, m_{-0}^2\right) = \frac{\left|\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right) - \mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right)\right|}{\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right) + \mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right)}.$$

- **Integrated asymmetry:** Count and compare the number of events *contained inside* the Dalitz plot in the "forward" and "backward" regions.

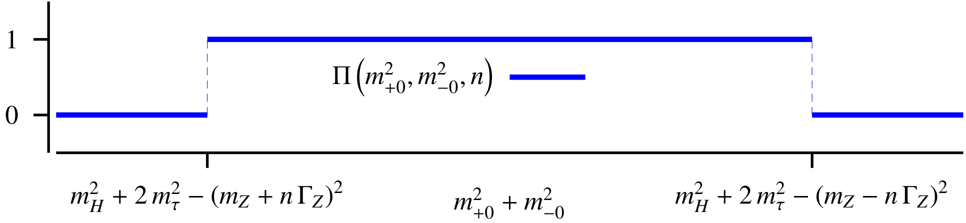
$$a_{\mathrm{FB}} = \frac{\left|\iint_{m_\tau^2}^{(m_H - m_\tau)^2} \left[\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right) - \mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right)\right] \mathrm{d}m_{+0}^2 \, \mathrm{d}m_{-0}^2\right|}{\iint_{m_\tau^2}^{(m_H - m_\tau)^2} \mathcal{D}\left(m_{+0}^2, m_{-0}^2\right) \mathrm{d}m_{+0}^2 \, \mathrm{d}m_{-0}^2} = \frac{|N_F - N_B|}{N_F + N_B} = a_{\mathrm{DP}}.$$

Dalitz Plot Asymmetries: Quantify CP violation

❖ **Regional integrated asymmetries:** Count and compare the number of events in certain '*islands*' of the Dalitz plot, e.g. $a'_{\rm FB}$ which specifically probes region around Z-pole,

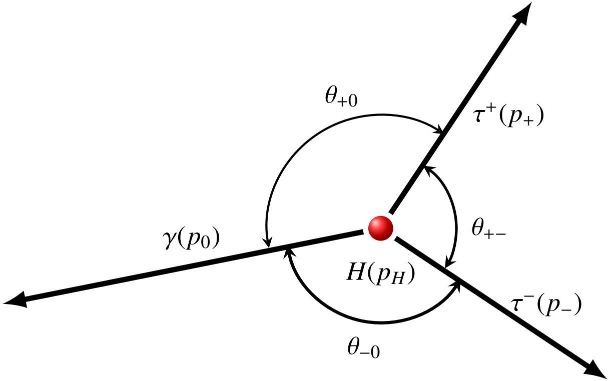
$$a'_{\rm FB}(n) = \frac{\left| \displaystyle\iint_{m_\tau^2}^{(m_H - m_\tau)^2} \left[\mathcal{D}\left(m_{+0}^2 < m_{-0}^2\right) - \mathcal{D}\left(m_{+0}^2 > m_{-0}^2\right)\right] \Pi\left(m_{+0}^2, m_{-0}^2, n\right) \mathrm{d}m_{+0}^2\, \mathrm{d}m_{-0}^2 \right|}{\displaystyle\iint_{m_\tau^2}^{(m_H - m_\tau)^2} \mathcal{D}\left(m_{+0}^2, m_{-0}^2\right) \Pi\left(m_{+0}^2, m_{-0}^2, n\right) \mathrm{d}m_{+0}^2\, \mathrm{d}m_{-0}^2}.$$

where the rectangular function $\Pi\left(m_{+0}^2, m_{-0}^2, n\right)$ is graphically shown below.



Some additional considerations in H rest frame

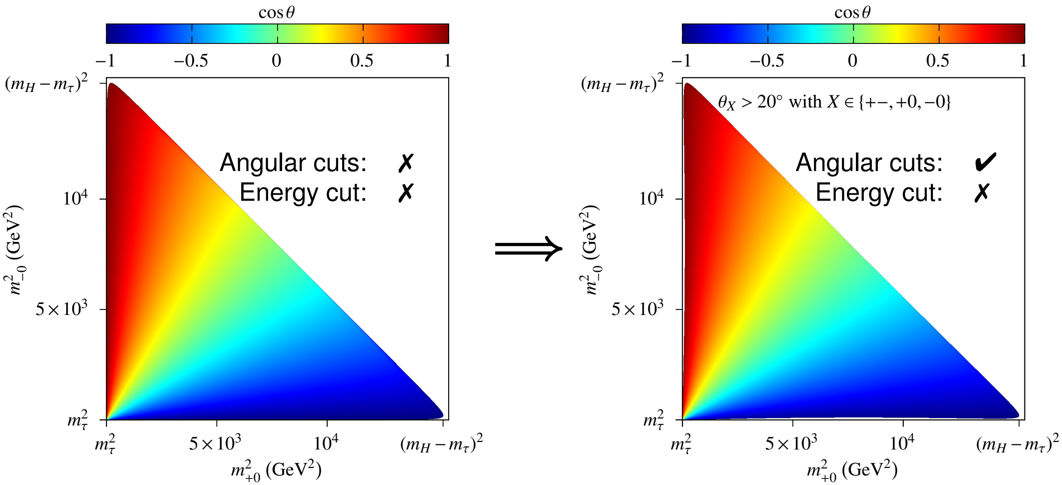
❖ Photon energy, $E_\gamma > E_\gamma^{\text{cut}} = 20$ GeV.



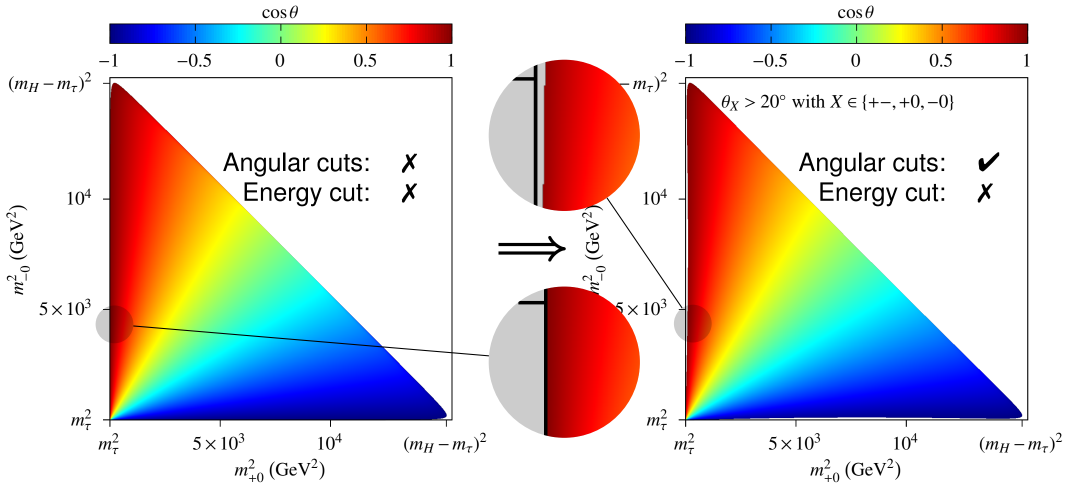
❖ Angles between outgoing particles, θ_X with $X \in \{+-, +0, -0\}$, be such that

$$\theta_X > 5^\circ, \text{ or } 10^\circ, \text{ or } 15^\circ \text{ etc.}$$

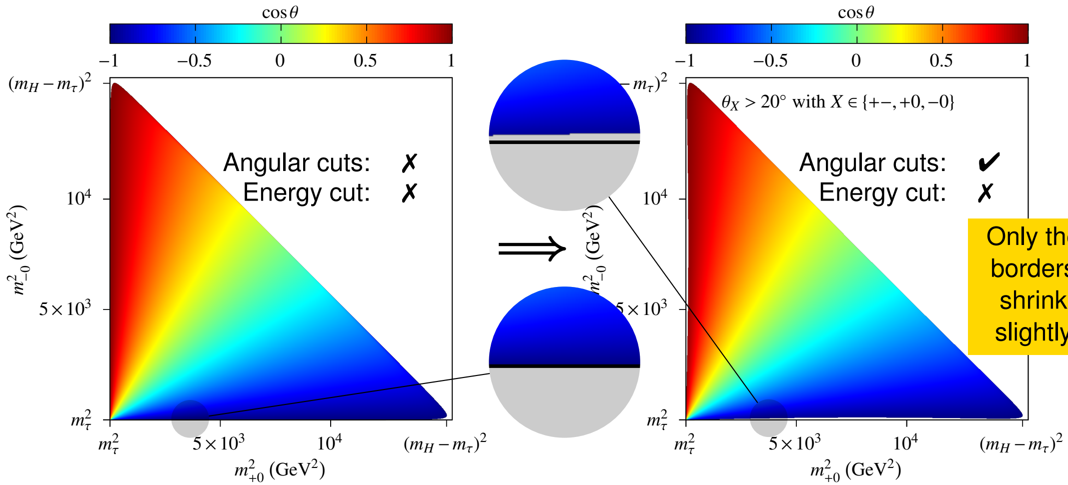
Impact of considerations in H rest frame on phase space or the Dalitz plot region



Impact of considerations in H rest frame on phase space or the Dalitz plot region

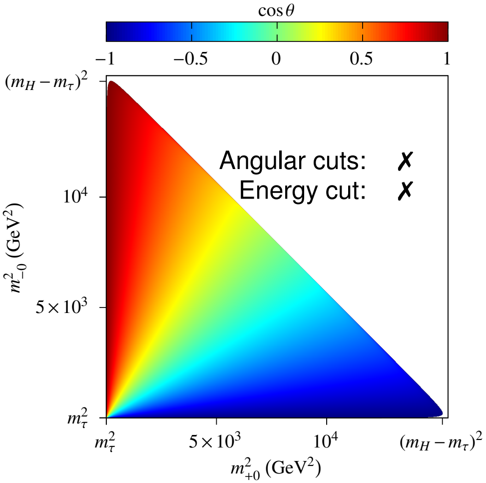


Impact of considerations in H rest frame on phase space or the Dalitz plot region

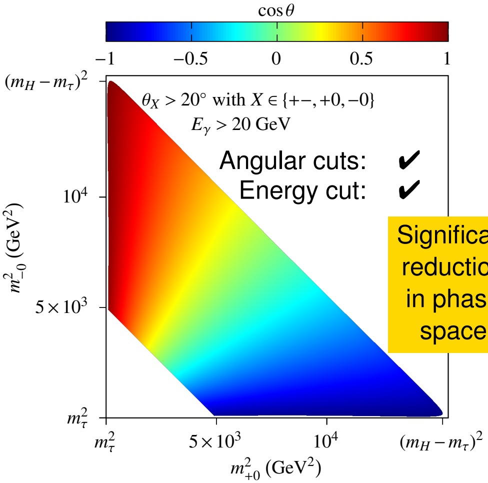


Only the borders shrink slightly

Impact of considerations in H rest frame on phase space or the Dalitz plot region



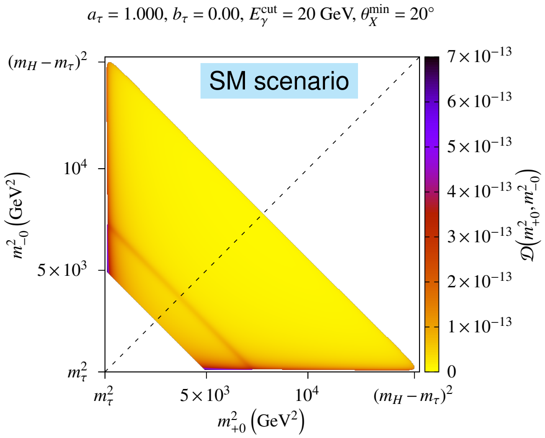
$\Longrightarrow$

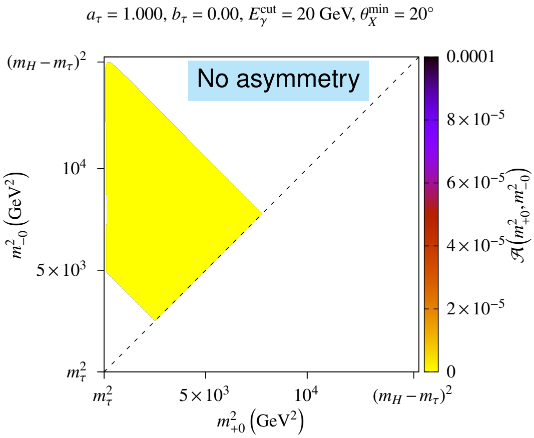


Significant reduction in phase space

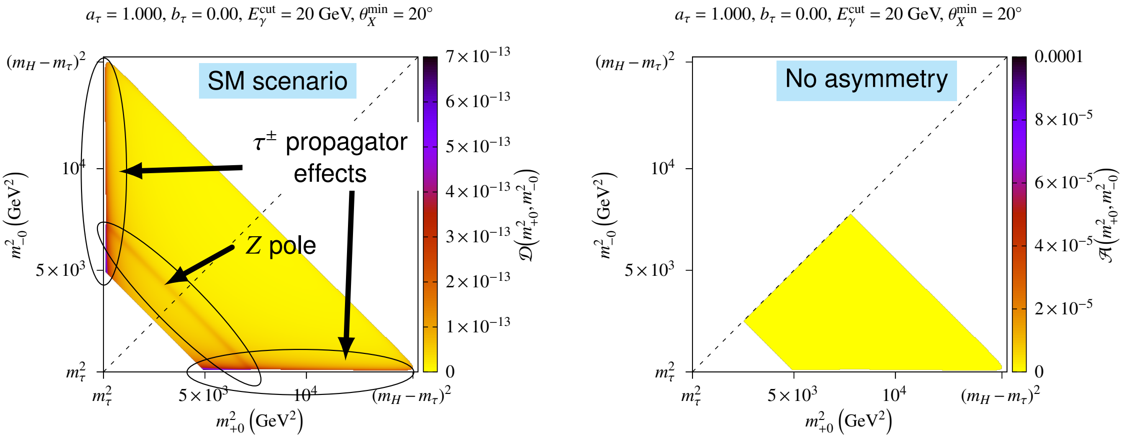
Part 4: A look at some Dalitz Plot Asymmetries

Analytical Dalitz plot distributions and asymmetry





Analytical Dalitz plot distributions and asymmetry



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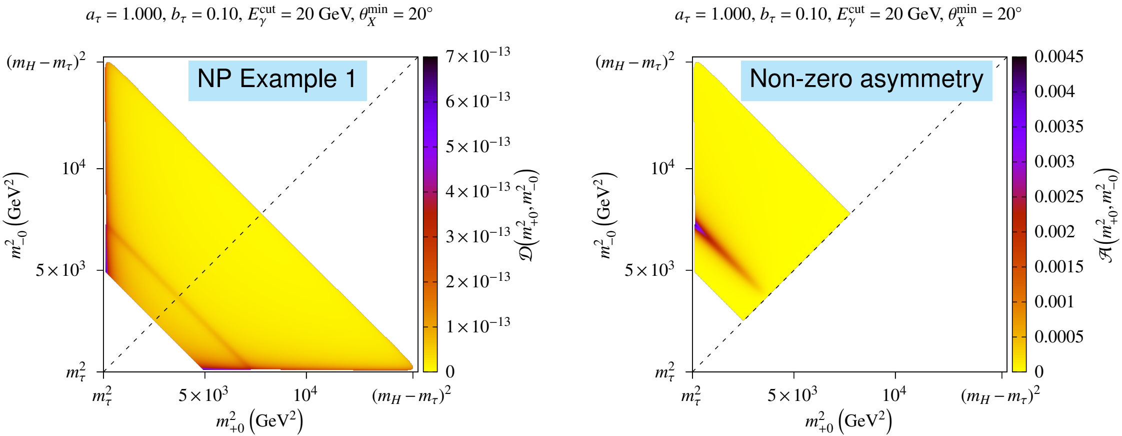
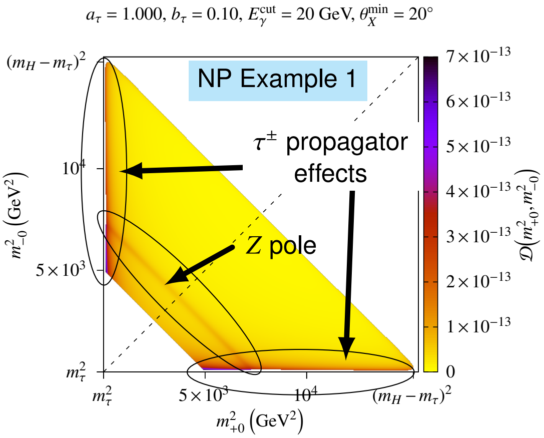
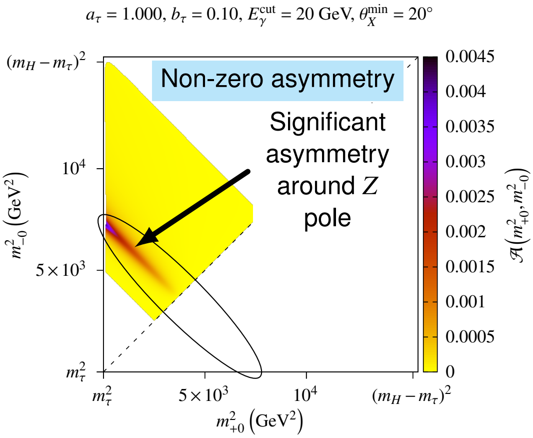
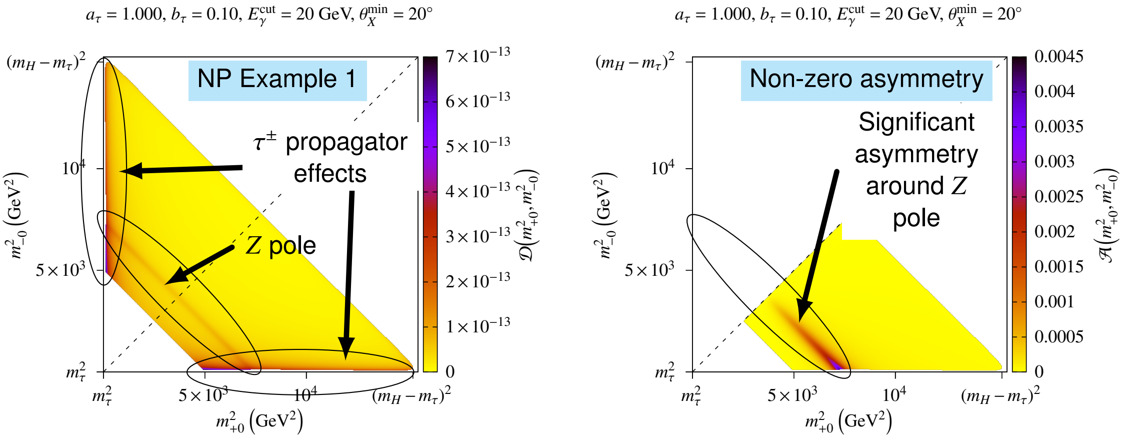


Analytical Dalitz plot distributions and asymmetry





Analytical Dalitz plot distributions and asymmetry



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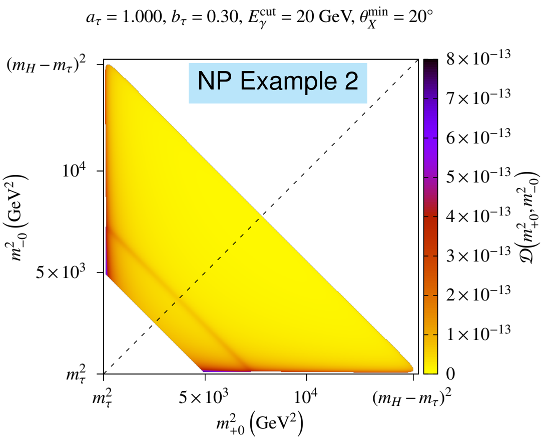
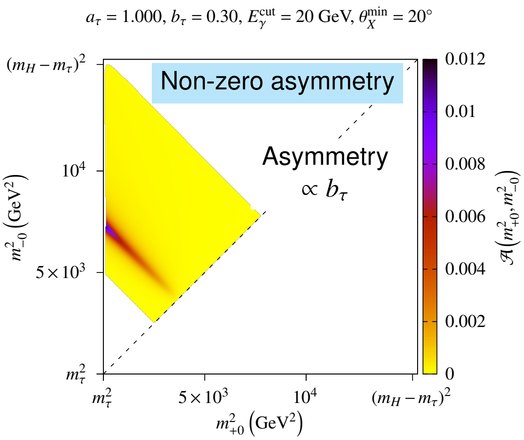
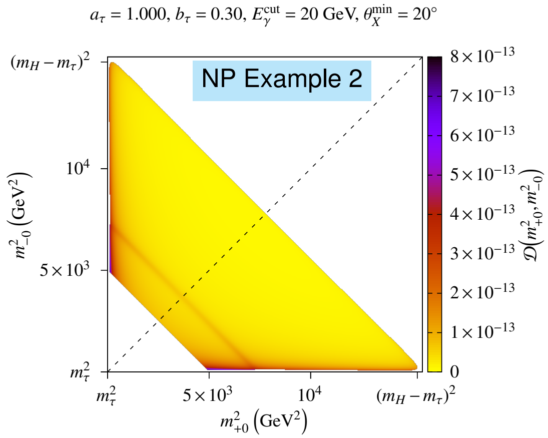
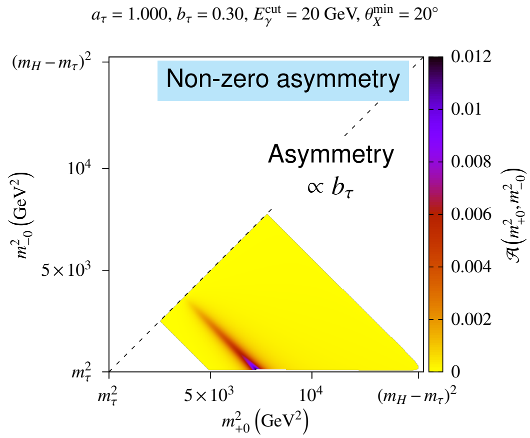




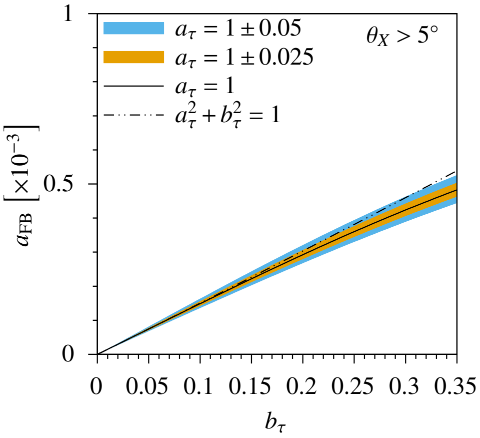
Analytical Dalitz plot distributions and asymmetry

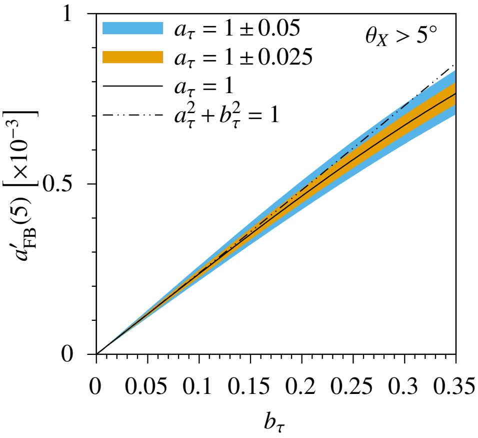




Integrated Asymmetry and Regional Asymmetry

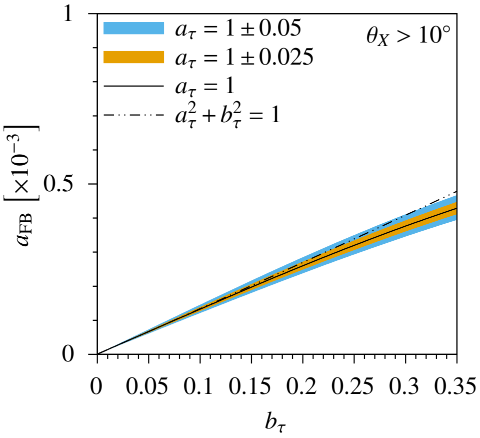
Focusing around Z pole yields larger asymmetry



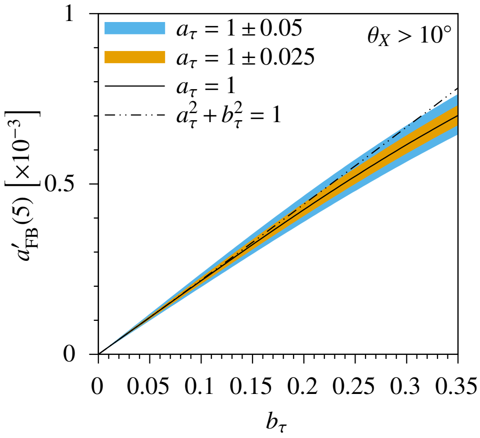


Integrated Asymmetry and Regional Asymmetry

Larger angular cuts reduce observable asymmetry

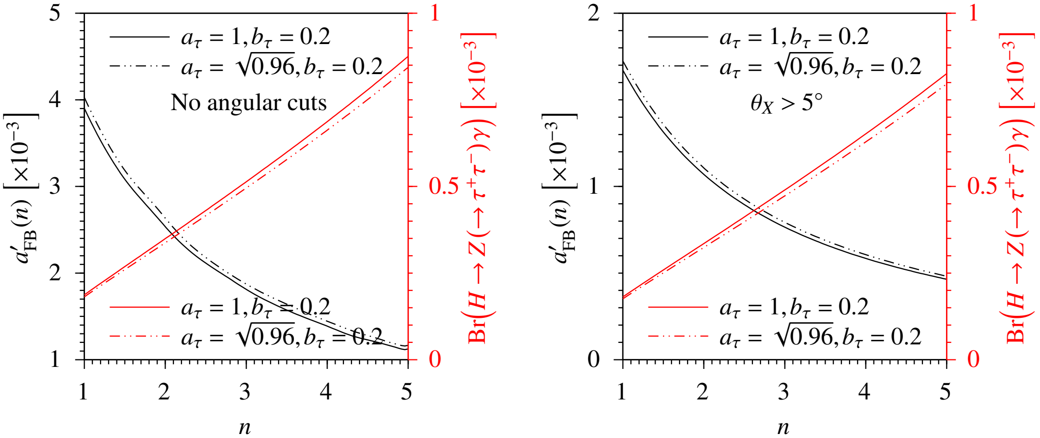


Focusing around Z pole yields larger asymmetry



Asymmetry around Z pole & corresponding branching ratio

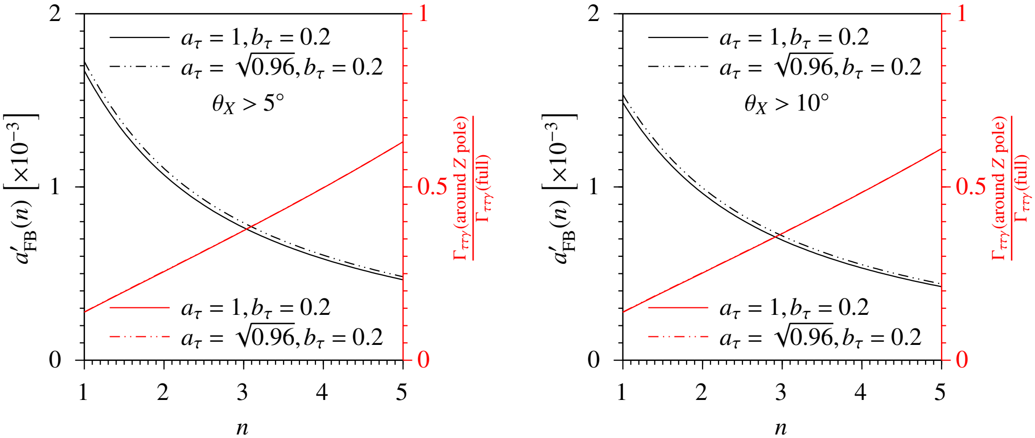
Angular cut $\implies$ phase space reduction → Decreased branching ratio



Angular cut $\implies$ Regions with highest asymmetry get sliced → Smaller asymmetry

Asymmetry around Z pole vis-á-vis reduction in decay rate

Considering only those events surrounding Z pole $\implies$ drastic reduction in phase space



There is also some compensation due to presence of Z pole.

Part 5: Conclusion & Outlook

Feasibility study for experimental prospect. . .

- **In summary**
  - (1) CP violation $(b_\tau \neq 0) \implies$ Forward-Backward asymmetry in Gottfried-Jackson frame
  - (2) Forward-Backward asymmetry ≡ Asymmetry in m^2_{+0} vs. m^2_{-0} Dalitz plot under $m^2_{+0} \leftrightarrow m^2_{-0}$:

    $\mathcal{A}\left(m^2_{+0}, m^2_{-0}\right) \neq 0,$ — full distribution asymmetry

    $a_{\rm FB} \neq 0,$ — full integrated asymmetry

    $a'_{\rm FB}(n) \neq 0.$ — asymmetry around Z pole

    [All asymmetries $\sim \mathcal{O}\left(10^{-3}\right)$]

  - (3) m^2_{+0} vs. m^2_{-0} Dalitz plot: can be obtained in *any frame of reference*
  - (4) Asymmetry is prominent surrounding the Z pole

- **Feasibility: Can these asymmetries be probed in ongoing or future experiments?**
- **Prospect: What range of b_τ would get constrained from such experimental studies?**

. . . **Thank You** . . .