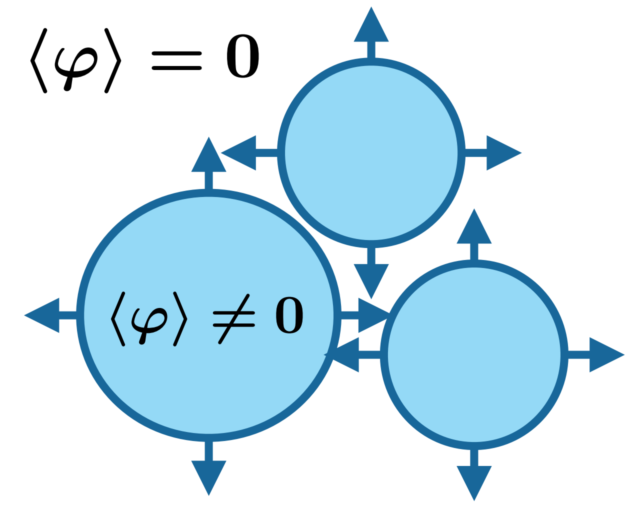
2ヒッグスダブレット模型における対称性の破れ

青木真由美 (金沢大学)

Electroweak Baryogenesis



- **Electroweak Baryogenesis**

  → strong 1st order EW phase transition

- **A first order phase transition is characterized by the nucleation of bubbles of the broken phase.**

The sphaleron process should be decoupled in the broken phase.

$$\frac{\Gamma_{\rm sph}^{\rm broken}}{T_c^3} < H(T_c) \quad \rightarrow \quad \frac{\varphi_c}{T_c} > 1$$

criterion for the “strong” 1st order PT

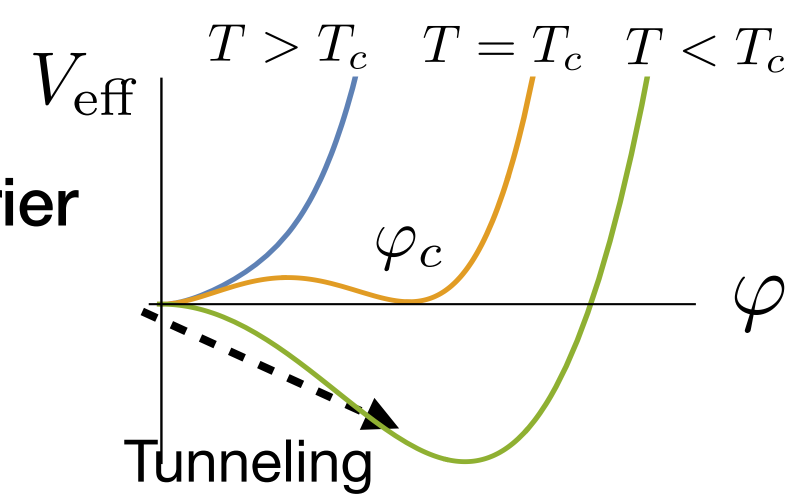
- SM : 1st order PT gives an upper limit on $m_h \lesssim 70\ \mathrm{GeV}$

  $m_h \simeq 125\ \mathrm{GeV}$ → crossover

Electroweak phase transition

❖ **Strong 1st order PT :**

There is a sufficiently high and wide potential barrier separating the two degenerate vacua at T=Tc.



❖ **High temperature expansion :**

$$V_{\rm eff}(\varphi, T) = D(T^2 - T_0^2)\varphi^2 - ET|\varphi|^3 + \frac{\lambda_T}{4}\varphi^4$$

- Two degenerate minima : $\varphi = 0 \text{ and } \varphi_c.$ $\quad \varphi_c = \frac{2ET_c}{\lambda_{T_c}}$

- The condition for the strong 1st order PT : $\quad \frac{\varphi_c}{T_c} = \frac{2E}{\lambda_{T_c}} > 1$

The magnitude of *E* is crucial for the strong 1st order PT.

Electroweak phase transition

❖ **The cubic term arises from the bosonic thermal corrections.**

One-loop thermal potential :

$$V_T = \frac{T^4}{2\pi^2}\left[\sum_f n_f J_F\left(\frac{m_f^2}{T^2}\right) + \sum_B n_B J_B\left(\frac{m_B^2}{T^2}\right)\right]$$

m : field dependent mass

✱ Boson-loop:

$$J_B(y) \approx -\frac{\pi^4}{45} + \frac{\pi^2}{12}y - \frac{\pi}{6}y^{\frac{3}{2}} - \frac{1}{32}\,y^2\log\left(\frac{|y|}{a_b}\right) + \mathcal{O}(y^3)\,,$$

✱ Fermion-loop:

$$J_F(y) \approx -\frac{7\pi^4}{360} + \frac{\pi^2}{24}y + \frac{1}{32}\,y^2\log\left(\frac{|y|}{a_f}\right) + \mathcal{O}(y^3)$$

$$a_b = 16\pi^2\exp(3/2 - 2\gamma_E), \quad a_f = \pi^2\exp(3/2 - 2\gamma_E)$$

Electroweak phase transition

❖ **The cubic term arises from the bosonic thermal corrections.**

SM : $$E = \frac{1}{12\pi v^3}\left(6m_W^3 + 3m_Z^3\right)$$

BSM : $$E = \frac{1}{12\pi v^3}\left(6m_W^3 + 3m_Z^3 + \cdots\right)$$

↑ extra bosonic degree of freedom

❖ **The "non-decoupling effects" is important.**

field dependent mass : $m_\Phi^2(\varphi) = M^2 + \lambda\varphi^2$

- $M^2 > \lambda\varphi^2$ $\quad V_{eff} \ni -|M|^3 T\left(1 + \frac{\lambda\varphi^2}{M^2}\right)^{3/2}$

- $M^2 < \lambda\varphi^2$ $\quad V_{eff} \ni -\lambda^{3/2} T\varphi^3\left(1 + \frac{M^2}{\lambda\varphi^2}\right)^{3/2}$ → **large *E***

Non-decoupling effect is required.

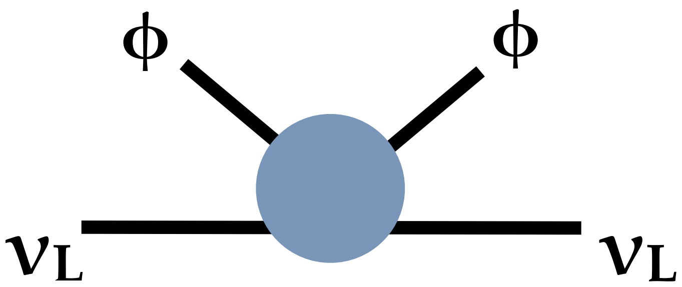
3 loop radiative seesaw model

MA, Kanemura, Seto, PRL(2009)

MA, K. Enomoto , S. Kanemura, Phys.Rev.D 107 (2023) 11

Radiative seesaw model

Neutrino masses are generated via the radiative effect.



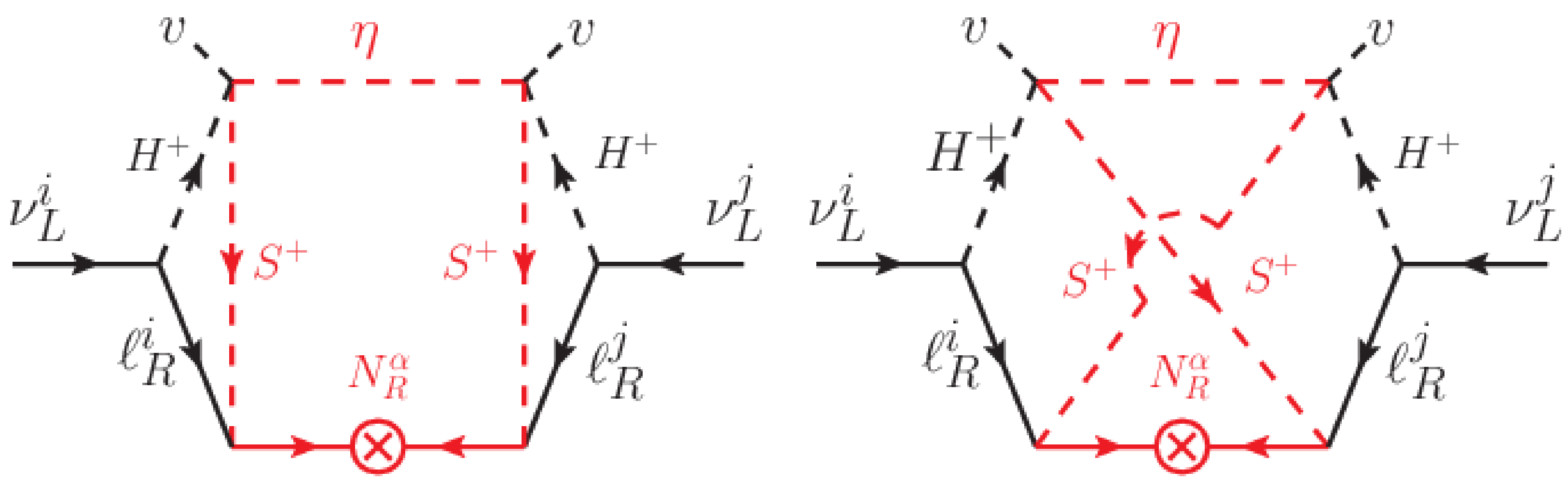
N-loop: $$m_\nu^{ij} = \left(\frac{1}{16\pi^2}\right)^N \frac{f_{ij}}{\Lambda} \langle \phi^0 \rangle^2$$

⇒ Due to the loop suppression factor, Λ can be lower.

Neutrino masses would be explained by the TeV-scale physics.

3-loop radiative seesaw model

MA, Kanemura, Seto, PRL(2009)



	$SU(2)_L$	$U(1)$	Z_2
Φ_1	2	1/2	+
Φ_2	2	1/2	+
S^-	1	-1	$-$
η	1	0	$-$
N_R^α	1	0	$-$

α=1, 2

$$\Phi_1 = \begin{pmatrix} \omega_1^+ \\ \frac{1}{\sqrt{2}}(v_1 + h_1 + iz_1) \end{pmatrix}, \quad \Phi_2 = \begin{pmatrix} \omega_2^+ \\ \frac{1}{\sqrt{2}}(v_2 + h_2 + iz_2) \end{pmatrix}$$

$$v_1^2 + v_2^2 = v^2 \simeq (246\ \text{GeV})^2$$

3-loop radiative seesaw model

* (softly broken) Z_2 symmetry :

$$\Phi_1 \to \Phi_1, \quad \Phi_2 \to -\Phi_2$$

	$SU(2)_L$	$U(1)$	$\mathbf{Z_2}$ (exact)	$\mathbf{Z_2}$ (softly broken)
Φ_1	2	1/2	+	+
Φ_2	2	1/2	+	−
S^-	1	−1	−	−
η	1	0	−	+

❖ **The tree-level potential :**

$$V = -\mu_1^2|\Phi_1|^2 - \mu_2^2|\Phi_2|^2 - (\mu_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.})$$
$$+ \frac{\lambda_1}{2}|\Phi_1|^4 + \frac{\lambda_2}{2}|\Phi_2|^4 + \lambda_3|\Phi_1|^2|\Phi_2|^2 + \lambda_4|\Phi_1^\dagger\Phi_2|^2 + \left\{\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)^2 + \text{h.c.}\right\}$$

Z_2 even

$$+ \mu_s^2|S|^2 + \lambda_s|S|^4 + \frac{1}{2}\mu_\eta^2\eta^2 + \lambda_\eta\eta^4 + \xi|S|^2\frac{\eta^2}{2}$$

Z_2 odd

$$+ \sum_{a=1}^{2}\left(\rho_a|\Phi_a|^2|S|^2 + \sigma_a|\Phi_a|^2\frac{\eta^2}{2}\right) + \sum_{a,b=1}^{2}\{\kappa\ \epsilon_{ab}(\Phi_a^c)^\dagger\Phi_b S^-\eta + \text{h.c.}\}$$

mixing

* **Phases of λ5 and κ can be eliminated.**
* **We neglect the phase.**

Two Higgs Doublet Model

❖ Parameters in the Higgs potential

$$\mu_1^2,\ \mu_2^2,\ \mu_{12}^2,\ \lambda_{1-5}$$

$$\rightarrow\ m_h,\ m_H,\ m_A,\ m_{H^\pm},\ v,\ \tan\beta,\ \cos(\beta-\alpha),\ \mu_{12}^2$$

125 GeV　　246 GeV

❖ Mass eigenstate

h	CP-even neutral (SM-like Higgs boson)
H	CP-even neutral
A	CP-odd neutral
$H^\pm$	Charged

* Mass matrices can be diagonalized by the mixing angles α and β.

$$\begin{pmatrix} \omega_1^\pm \\ \omega_2^\pm \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} \omega^\pm \\ H^\pm \end{pmatrix}, \qquad \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} \cos\beta & -\sin\beta \\ \sin\beta & \cos\beta \end{pmatrix} \begin{pmatrix} z \\ A \end{pmatrix},$$

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix} \qquad \tan\beta = \frac{v_2}{v_1}$$

Two Higgs Doublet Model

❖ **Scalar masses:**

$$m_h^2 = M^2 \cos^2(\beta - \alpha) + (\lambda_1 \sin^2 \alpha \cos^2 \beta + \lambda_2 \cos^2 \alpha \sin^2 \beta - \frac{\lambda}{2} \sin 2\alpha \sin 2\beta) v^2$$

$$m_H^2 = M^2 \sin^2(\beta - \alpha) + (\lambda_1 \cos^2 \alpha \cos^2 \beta + \lambda_2 \sin^2 \alpha \sin^2 \beta - \frac{\lambda}{2} \sin 2\alpha \sin 2\beta) v^2$$

$$m_A^2 = M^2 - \lambda_5 v^2$$

$$m_{H^\pm}^2 = M^2 - (\lambda_4 + \lambda_5) \frac{v^2}{2}$$

$$\lambda = \lambda_3 + \lambda_4 + \lambda_5$$

$$M^2 \equiv \frac{\mu_{12}^2}{\sin \beta \cos \beta}$$

❖ **Alignment limit :** $\sin(\beta - \alpha) = 1$ ➡ $m_{\Phi_i}^2 = M^2 + \lambda_i v^2$

Two Higgs Doublet Model

❖ **Yukawa interactions**

$$-\mathcal{L}_Y = (y_u^a)_{ij}\overline{Q_L'^i}\tilde{\Phi}_a u_R'^j + (y_d^a)_{ij}\overline{Q_L'^i}\Phi_a d_R'^j + (y_\ell^a)_{ij}\overline{L_L'^i}\Phi_a \ell_R'^j + \mathrm{h.c.},$$

$$\parallel$$

$$\overline{L_L'^i}\left\{(y_\ell^1)_{ij}\Phi_1 + (y_\ell^2)_{ij}\Phi_2\right\}\ell_R'^j$$

In general, the Yukawa couplings are not simultaneously diagonalized by the same biunitary transformation.

→ **Flavor Changing Neutral Current**

❖ **Avoid FCNC,**

✓ - **introduce a Z2 symmetry**

- $y_f^1,\ y_f^2$ **can be simultaneously diagonalized. (Yukawa alignment)**

Two Higgs Doublet Model

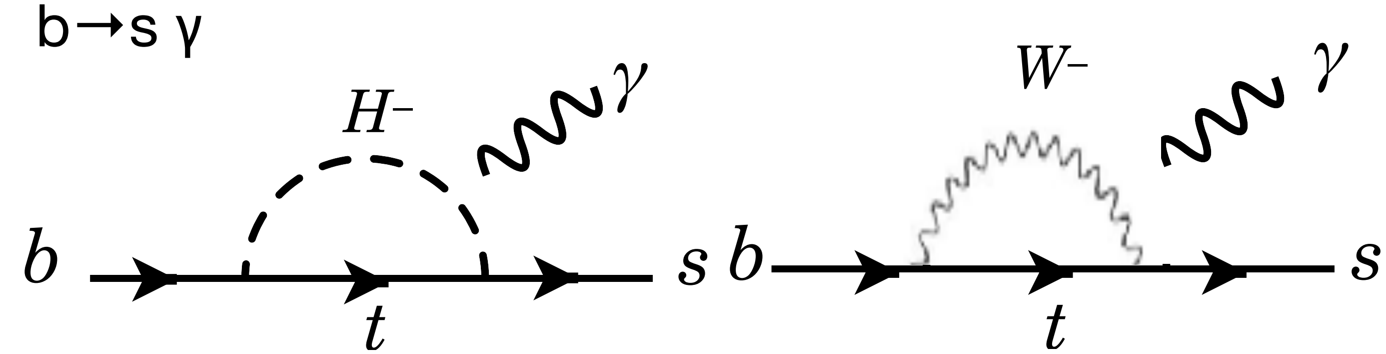
❖ **Four types of Yukawa interactions**

	Φ_1	Φ_2	u_R	d_R	ℓ_R	Q_L, L_L
Type-I	$+$	$-$	$-$	$-$	$-$	$+$
Type-II	$+$	$-$	$-$	$+$	$+$	$+$
Type-X	$+$	$-$	$-$	$-$	$+$	$+$
Type-Y	$+$	$-$	$-$	$+$	$-$	$+$

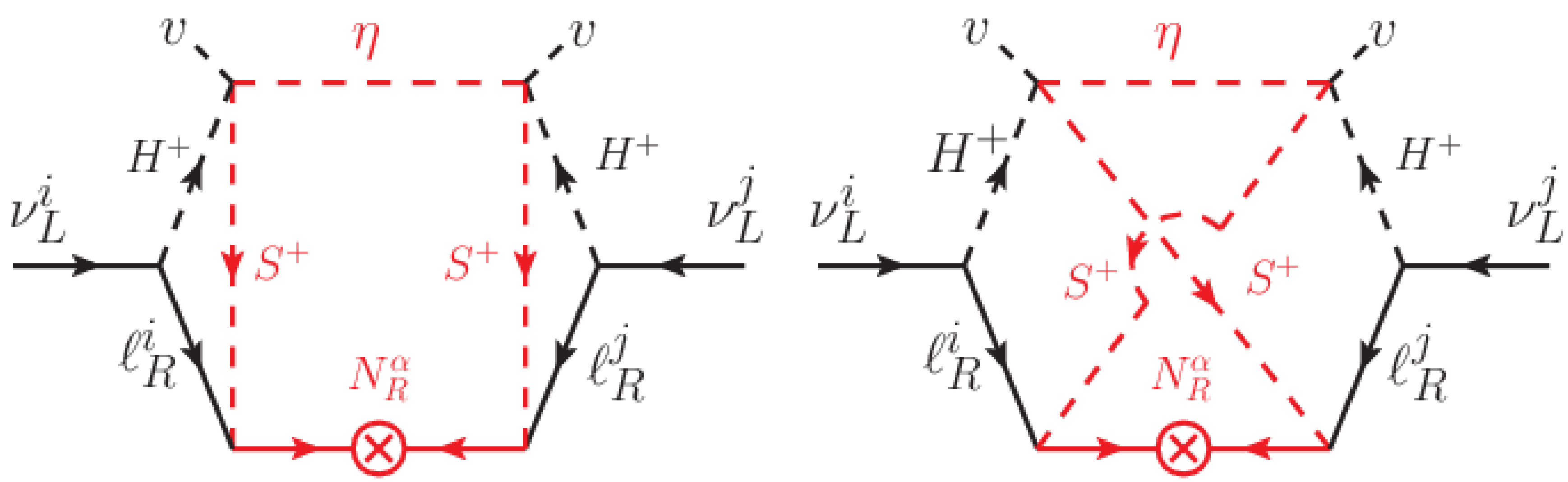
✱ **Type II, Type Y :**

$$m_{H^\pm} \gtrsim 800 \text{ GeV}$$

b→s γ



3-loop radiative seesaw model



$$M_{ij} = \sum_{\alpha=1}^{2} 4\kappa^2 \tan^2\beta (y_{e_i}^{\rm SM} h_i^\alpha)(y_{e_j}^{\rm SM} h_j^\alpha) \left\{ F_{||}(m_{H^\pm}, m_{S^\pm}, m_{N_R^\alpha}, m_\eta) + F_\times (m_{H^\pm}, m_{S^\pm}, m_{N_R^\alpha}, m_\eta) \right\}$$

Possible values turn out to be

$$m_{H^\pm} \simeq 100 \text{ GeV}, \ \kappa \tan\beta = \mathcal{O}(10)$$

Type-X Yukawa interaction

* **Neutrino mass matrix is related to the data by**

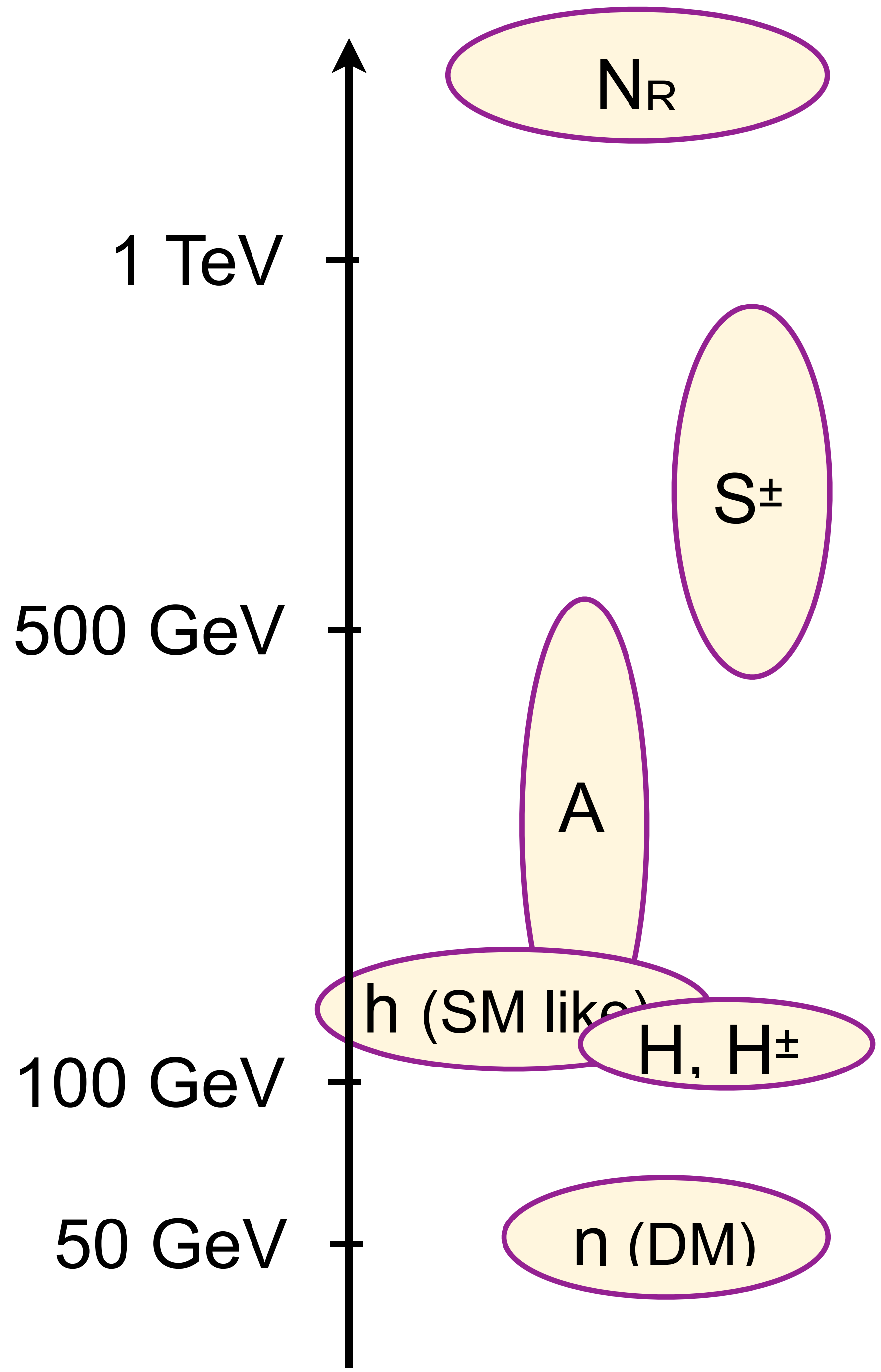
$$M_{ij} = U_{ia} diag(m_1, m_2, m_3) U_{tj}^T$$

U : PMNS matrix

3-loop radiative seesaw model

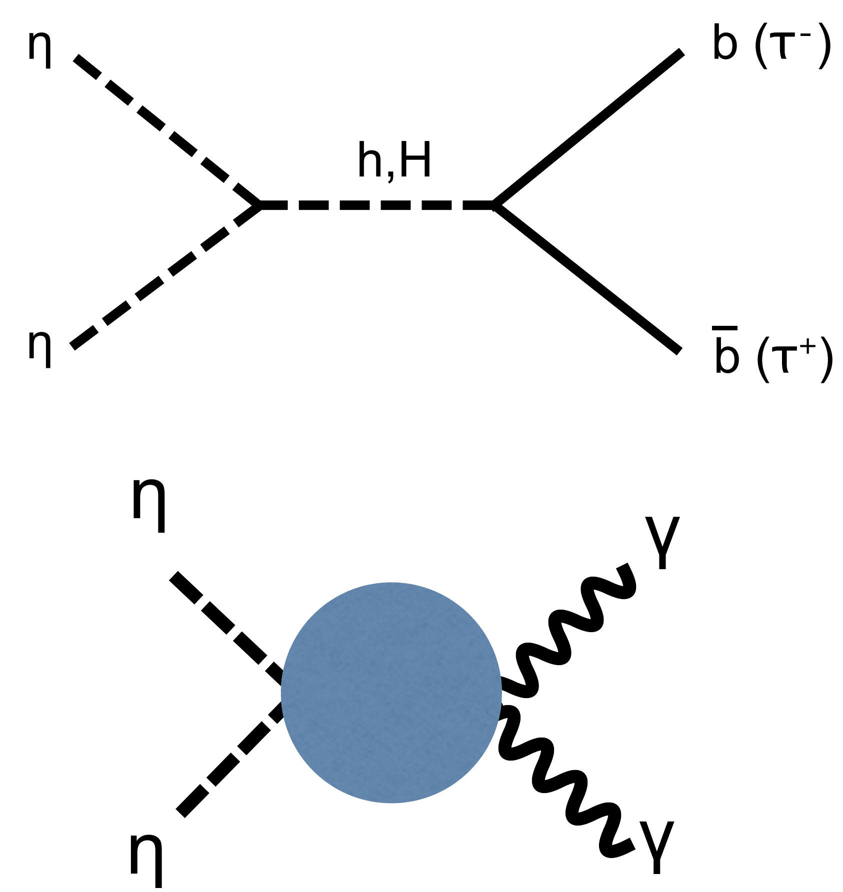
AKS09

All masses are between O(100)GeV-O(1)TeV.



❖ **Relic abundance**

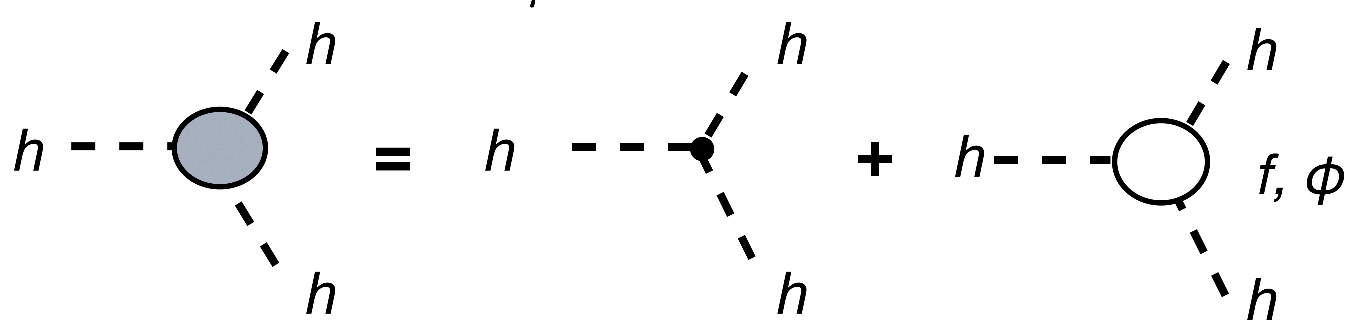
$$\eta\eta \to f^i \bar{f}^i, W^+W^-, ZZ, \gamma\gamma$$



Electroweak Phase Transition

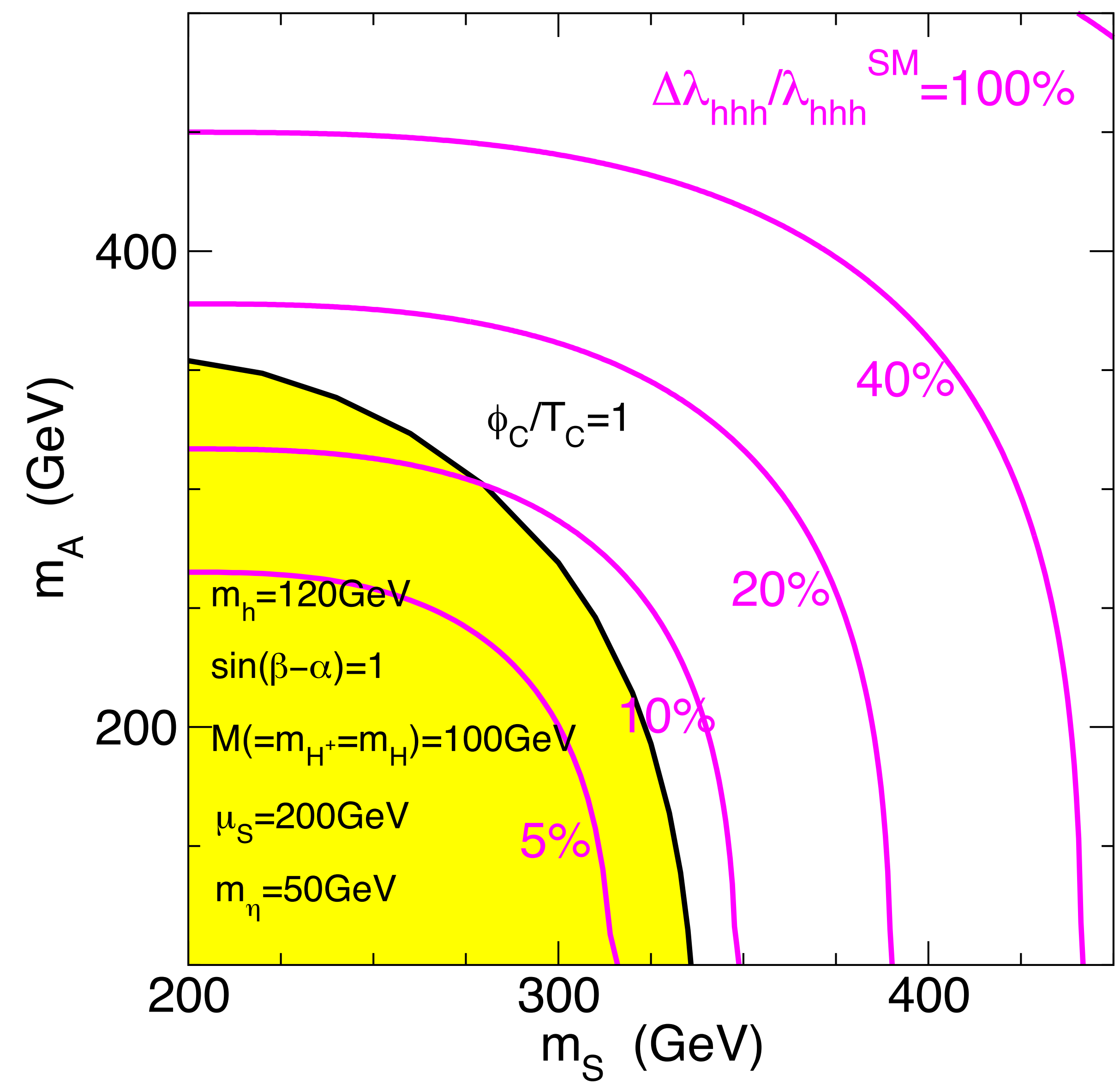
❖ **Higgs self-coupling**

$$\frac{\partial^3}{\partial\varphi^3} V_{eff}[\varphi]\Big|_{\varphi=v} = \lambda_{hhh}$$



$$\delta\lambda_{hhh} \equiv \frac{\lambda_{hhh}^{\rm 2HDM} - \lambda_{hhh}^{\rm SM}}{\lambda_{hhh}^{\rm SM}}$$

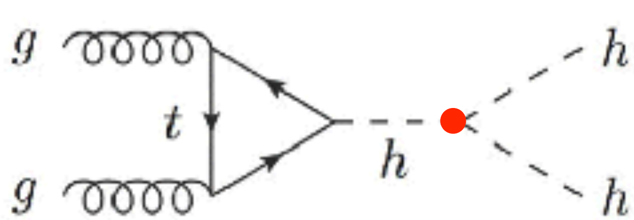
Deviations in the hhh coupling from the SM values



- HL-LHC

√s = 14 TeV , 3000 fb-1

$$\delta\lambda_{hhh} \sim 50\,\%$$



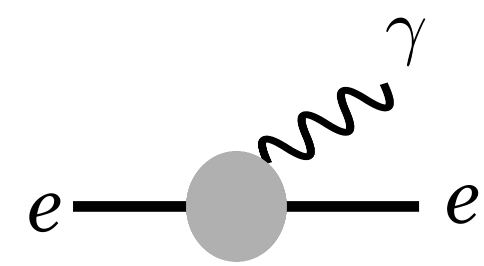
3-loop radiative seesaw model (2023)

MA, K. Enomoto , S. Kanemura, Phys.Rev.D 107 (2023)

EDM constraints

❖ **The EDM measurements put severe constraints on the CPV source.**

$$\mathcal{L}_{\mathrm{EDM}} = -\frac{d_f}{2}\bar{f}\sigma^{\mu\nu}(i\gamma_5)fF_{\mu\nu}$$

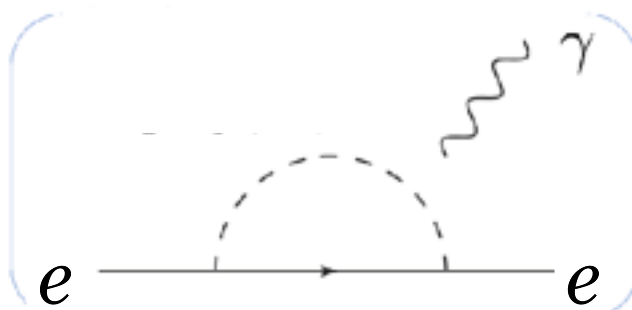


electron EDM : $|d_e| \lesssim 1.1 \times 10^{-29}$ e cm **ACME (2018)**

$|d_e| \lesssim 4.1 \times 10^{-30}$ e cm **JILA (2023)**

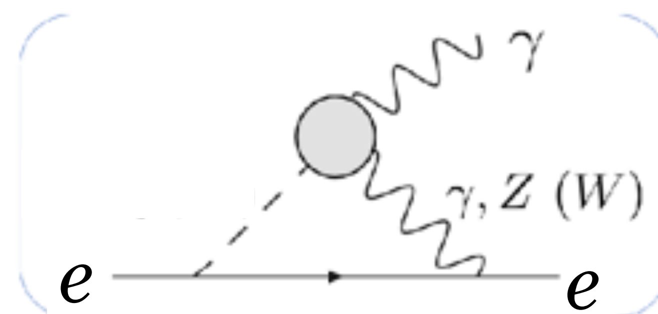
❖ **2HDM**

- **Barr-Zee diagrams give a larger contribution.**



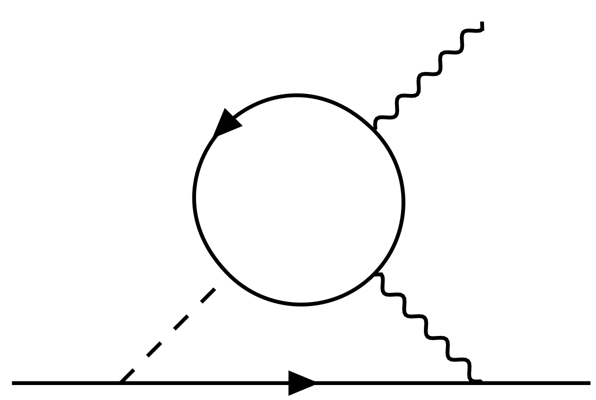
$\propto \frac{1}{(4\pi)^2} y_e^3$

<

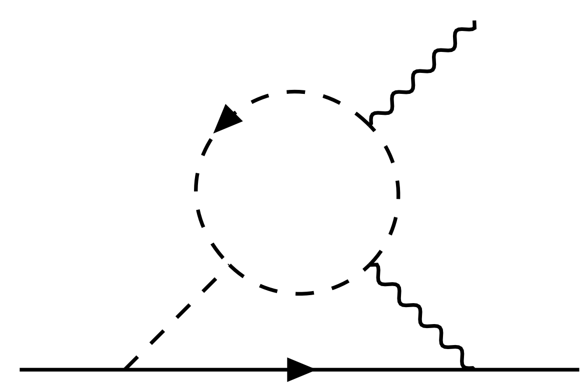


$\propto \frac{1}{(4\pi)^4} y_e$

d_e =



+



< d_e (exp)

Kanemura, Kubota, Yagyu (2020)

3-loop radiative seesaw model (2023)

	$SU(2)_L$	$U(1)$	$\mathbf{Z_2}$ (exact)	$\mathbf{Z_2}$ (softly broken)
Φ_1	2	1/2	$+$	$+$
Φ_2	2	1/2	$+$	$-$
S^-	1	-1	$-$	$-$
η	1	0	$-$	$+$
N_R^α	1	0	$-$	$+$

α=1, 2, 3

3-loop radiative seesaw model (2023)

❖ **Higgs potential**

$$\Phi_1=\begin{pmatrix} G^+ \\ (v+H_1+iG^0)/\sqrt{2}\end{pmatrix},\quad \Phi_2=\begin{pmatrix} H^+ \\ (H_2+iH_3)/\sqrt{2}\end{pmatrix},$$

$$V=\sum_{a=1}^{2}\Big(\mu_a^2|\Phi_a|^2+\frac{\lambda_a}{2}|\Phi_a|^4\Big)+\big(\mu_{12}^2\Phi_1^\dagger\Phi_2+\text{h.c.}\big)+\lambda_3|\Phi_1|^2|\Phi_2|^2+\lambda_4|\Phi_1^\dagger\Phi_2|^2$$
$$+\Big\{\Big(\frac{\lambda_5}{2}(\Phi_1^\dagger\Phi_2)+\lambda_6|\Phi_1|^2+\lambda_7|\Phi_2|^2\Big)(\Phi_1^\dagger\Phi_2)+\text{h.c.}\Big\}+\mu_S^2|S^+|^2+\frac{\mu_\eta^2}{2}\eta^2+\sum_{a=1}^{2}\Big(\rho_a|S^+|^2+\frac{\sigma_a}{2}\eta^2\Big)|\Phi_a|^2$$
$$+\Big\{\Big(\rho_{12}|S^+|^2+\frac{\sigma_{12}}{2}\eta^2\Big)(\Phi_1^\dagger\Phi_2)+2\kappa(\tilde{\Phi}_1^\dagger\Phi_2)S^-\eta+\text{h.c.}\Big\}+\frac{\lambda_S}{4}|S^+|^4+\frac{\lambda_\eta}{4!}\eta^4+\frac{\xi}{2}|S^+|^2\eta^2,$$

* **Complex parameters:**

$$\mu_{12}^2,\ \lambda_5,\ \lambda_6,\ \lambda_7,\ \rho_{12},\ \sigma_{12},\ \kappa.$$

* **The stationary condition of the vacuum**

$$2\mu_1^2=-\lambda_1v^2,\quad 2\mu_{12}^2=-\lambda_6v^2.$$

* **Four independent CP-violating phases:**

$$\lambda_6,\ \lambda_7,\ \rho_{12},\ \sigma_{12}$$

$$\boldsymbol{\theta_6,\ \theta_7,\ \theta_\varrho,\ \theta_\sigma,}$$

3-loop radiative seesaw model (2023)

❖ **Higgs alignment** S. Kanemura, M. Kubota, K. Yagyu (2020)

$$\begin{array}{cc} & \begin{array}{ccc} H_1 & H_2 & H_3 \end{array} \\ M_h^2 = & \begin{pmatrix} \lambda_1 v^2 & \mathrm{Re}[\lambda_6]v^2 & -\mathrm{Im}[\lambda_6]v^2 \\ \mathrm{Re}[\lambda_6]v^2 & M_+^2 & \cancel{-\mathrm{Im}[\lambda_5]v^2/2} \\ -\mathrm{Im}[\lambda_6]v^2 & \cancel{-\mathrm{Im}[\lambda_5]v^2/2} & M_-^2 \end{pmatrix}, \end{array} \begin{array}{c} \\ H_1 \\ H_2 \\ H_3 \end{array}$$

✱ For simplicity, $\lambda_6 = 0$ → Higgs alignment

H_1 is the SM-like Higgs boson at the tree level.

3-loop radiative seesaw model (2023)

- **Yukawa interactions**
  - quarks and charged leptons

$$-\mathcal{L}_Y = (y_u^a)_{ij}\overline{Q'^i_L}\tilde{\Phi}_a u'^j_R + (y_d^a)_{ij}\overline{Q'^i_L}\Phi_a d'^j_R + (y_\ell^a)_{ij}\overline{L'^i_L}\Phi_a \ell'^j_R + \text{h.c.},$$

- introduce a Z2 symmetry

✓ - Yukawa alignment — Pich, Tuzon PRD(2009)

$$(y_f^1)_{ij} \to \left(\frac{m_{f^i}}{v}\right)\delta_{ij}, \quad (y_f^2)_{ij} \to \zeta_{f^i}\left(\frac{m_{f^i}}{v}\right)\delta_{ij}$$

$$\Rightarrow \quad -\mathcal{L}_Y = \frac{m_{f^i}}{v}\overline{f^i_L}f^i_R H_1 + \zeta_{f^i}\underbrace{\left(\frac{m_{f^i}}{v}\right)}_{\text{SM Yukawa}}\delta_{ij}\overline{f^i_L}f^i_R(H_2 + iH_3) + h.c. + \cdots$$

* We assume the flavor universality for quarks.

$$\zeta_u = \zeta_c = \zeta_t \qquad \zeta_d = \zeta_s = \zeta_b \qquad \zeta_e, \quad \zeta_\mu, \quad \zeta_\tau$$

- right-handed neutrino

$$-\mathcal{L}_{\ell SN} = h_i^\alpha \overline{(N_R^\alpha)^c}\ell_R^i S^+ + \text{h.c.} \qquad \alpha = 1, 2, 3$$

EDM constraints

- **Complex parameters:**

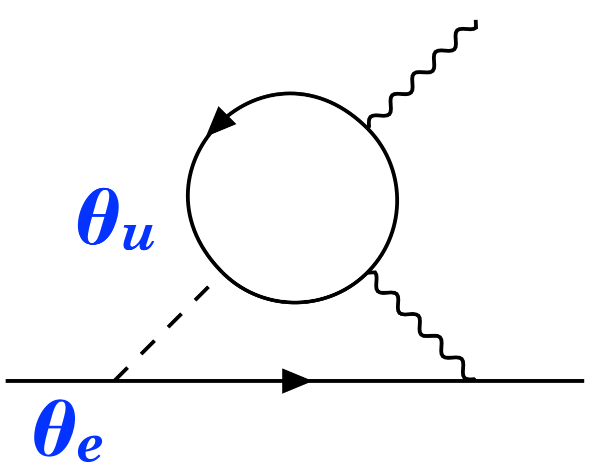
$$\lambda_7,\ \rho_{12},\ \sigma_{12} \qquad \zeta_u,\ \zeta_d,\ \zeta_e,\ \zeta_\mu,\ \zeta_\tau,\ h_i^\alpha$$

$$\boldsymbol{\theta_7,\ \theta_\varrho,\ \theta_\sigma,} \qquad \boldsymbol{\theta_u,\ \theta_d,\ \theta_e,\ \theta_\mu,\ \theta_\tau,}$$

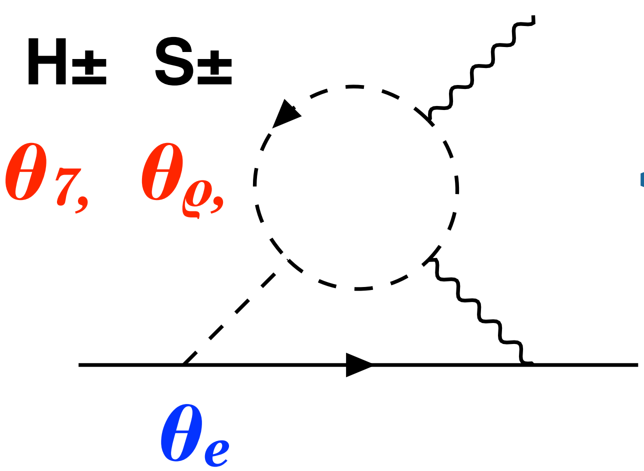
- **Electric Dipole Moment**

  * eEDM can be small by **destructive interference.**

$d_e =$



+



$< d_e$ (exp)

A benchmark scenario

- **Parameters in the Higgs potential**

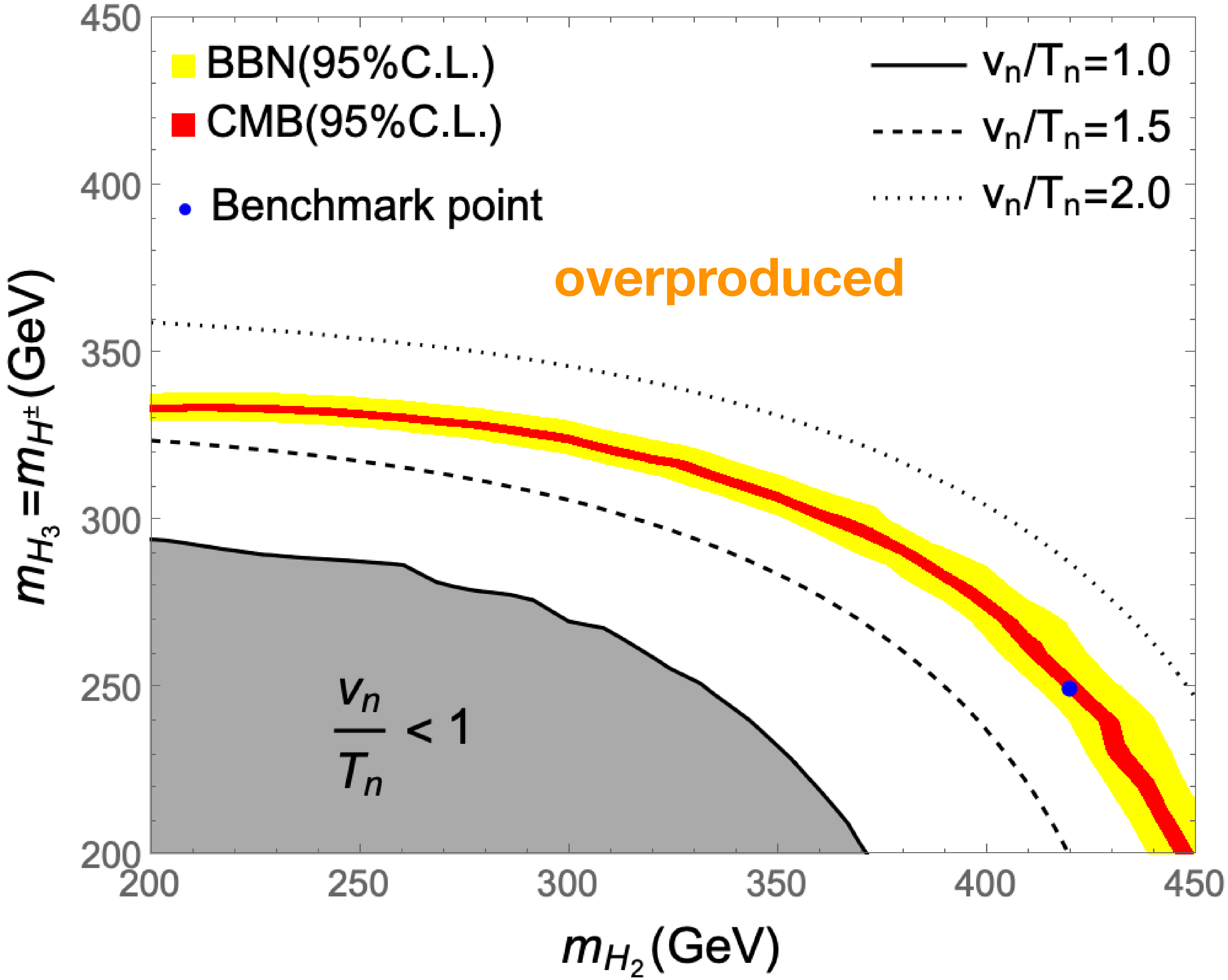
$$\mu_2^2 = (50\ \text{GeV})^2, \quad \mu_{12}^2 = 0, \quad \mu_S^2 = (320\ \text{GeV})^2, \quad \lambda_2 = 0.1, \quad \lambda_6 = 0, \quad |\lambda_7| = 0.828, \quad \theta_7 = -2.34,$$
$$|\rho_{12}| = 0.1, \quad \theta_\rho = -2.94, \quad \rho_2 = 0.1, \quad \sigma_1 = |\sigma_{12}| = 1.1 \times 10^{-3}, \quad \theta_\sigma = 0, \quad \sigma_2 = 0.1, \quad \kappa = 2,$$
$$\lambda_S = 1, \quad \lambda_\eta = 1, \quad \xi = 1.$$

- **Parameters in the Yukawa interaction**

$$\frac{m_u}{v}|\zeta_u| = 2.18 \times 10^{-6}, \quad \frac{m_c}{v}|\zeta_u| = 1.28 \times 10^{-3}, \quad \frac{m_t}{v}|\zeta_u| = 0.174, \quad \theta_u = 0.246,$$
$$\frac{m_d}{v}|\zeta_d| = 4.71 \times 10^{-6}, \quad \frac{m_s}{v}|\zeta_d| = 9.42 \times 10^{-5}, \quad \frac{m_b}{v}|\zeta_d| = 4.21 \times 10^{-3}, \quad \theta_d = 0.246,$$
$$\frac{m_e}{v}|\zeta_e| = 2.5 \times 10^{-4}, \quad \frac{m_\mu}{v}|\zeta_\mu| = 2.5 \times 10^{-4}, \quad \frac{m_\tau}{v}|\zeta_\tau| = 2.5 \times 10^{-3}, \quad \theta_e = \theta_\mu = \theta_\tau = -2.94,$$

$$\begin{pmatrix} h_1^1 & h_2^1 & h_3^1 \\ h_1^2 & h_2^2 & h_3^2 \\ h_1^3 & h_2^3 & h_3^3 \end{pmatrix} = \begin{pmatrix} 1.00\, e^{-0.314i} & 0.196\, e^{0.302i} & 1.04\, e^{-2.39i} \\ 1.08\, e^{-1.88i} & 0.205\, e^{-1.80i} & 1.05\, e^{2.33i} \\ 0.449\, e^{2.74i} & 1.31\, e^{-0.0331i} & 0.100\, e^{0.628i} \end{pmatrix}.$$

Baryon asymmetry



$v_w = 0.1$

BP :
Tn ~ 100 GeV
vn/Tn =1.7

hhh coupling
$\delta\lambda_{hhh}$ ~ 38%

❖ **Ideas to make the EWBG successful :**

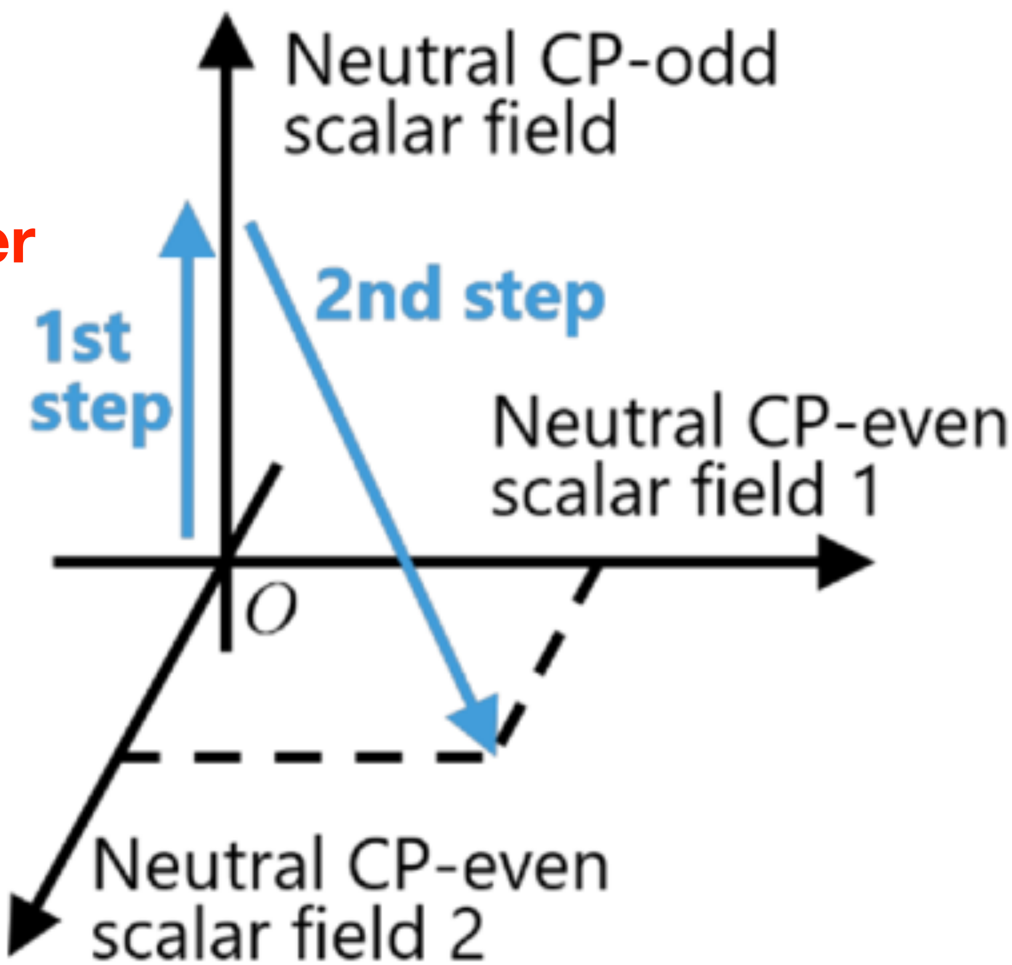
- Destructive interference
- CPV vacuum at intermediate stage

:

Multistep Phase Transition

CP-violating 2HDM

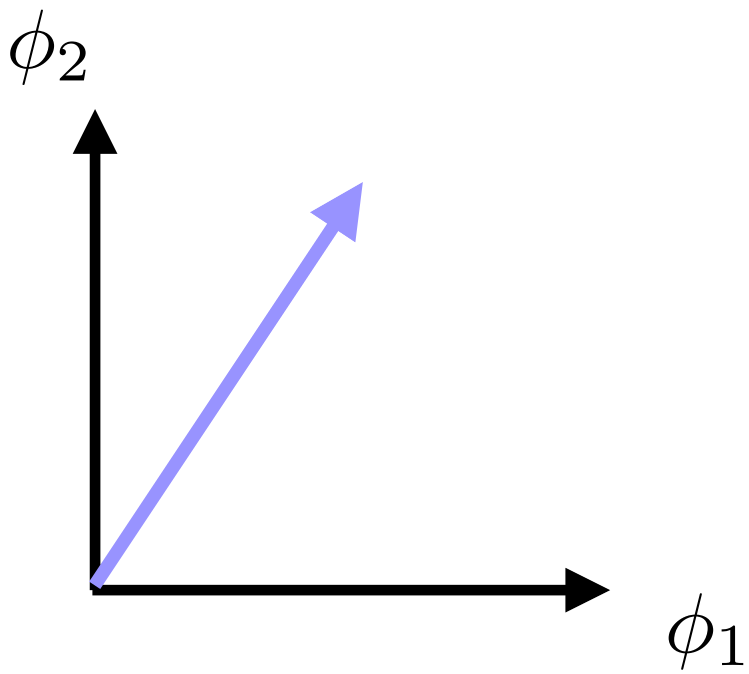
strongly 1st order
Baryogenesis



Multistep Phase Transition

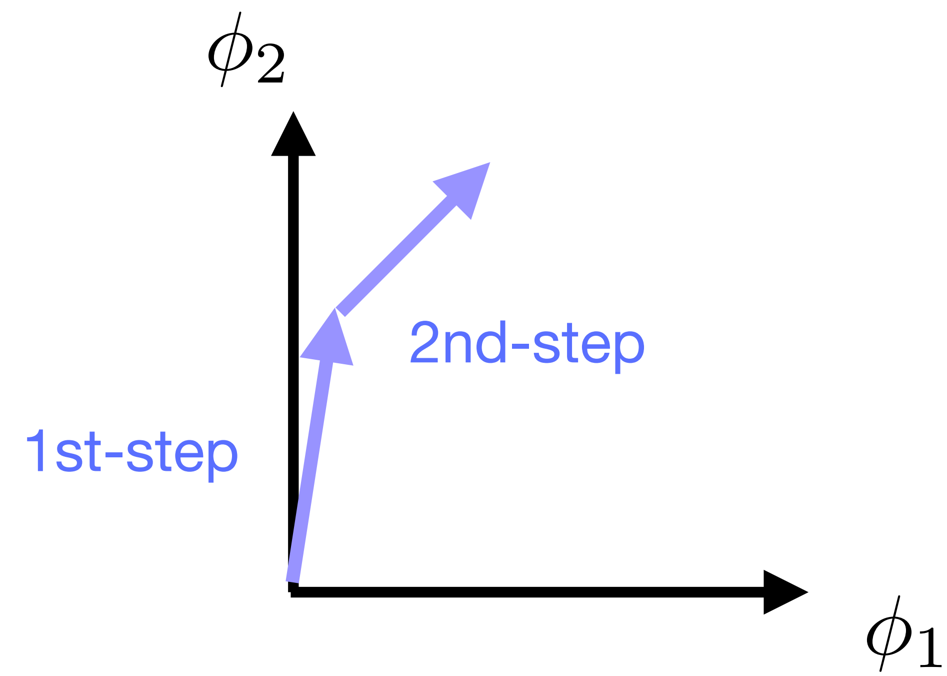
CP-conserving 2HDM

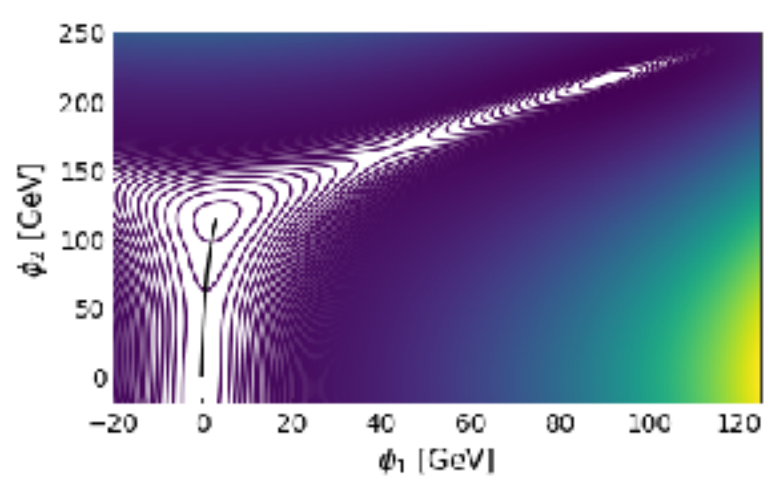
1 step

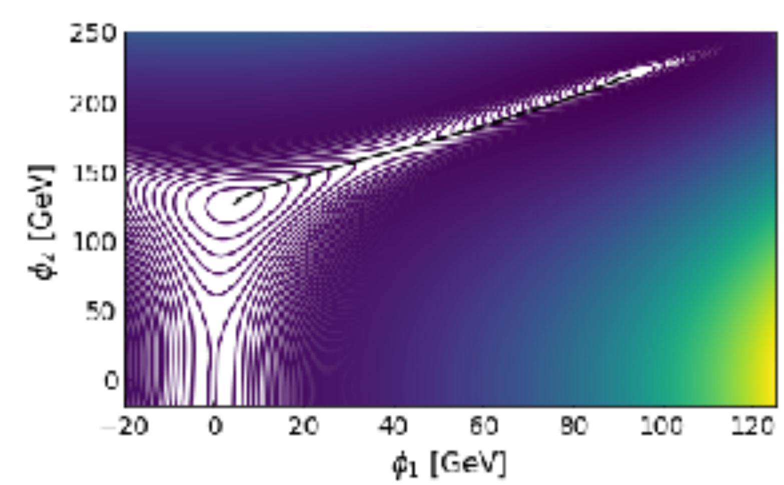


2 step

MA, Komatsu, Shibuya, PTEP (2022)







Two Higgs Doublet Model

❖ **The one-loop corrected effective potential at the finite temperature :**

$$V = V_0 + V_{\rm CW} + V_{\rm CT} + V_T \qquad \Phi_i = \begin{pmatrix} 0 \\ \frac{\phi_i}{\sqrt{2}} \end{pmatrix}$$

* The tree-level scalar potential :

$$V_0(\phi_1, \phi_2) = \frac{m_1^2}{2}\phi_1^2 + \frac{m_2^2}{2}\phi_2^2 - m_3^2\phi_1\phi_2 + \frac{\lambda_1}{8}\phi_1^4 + \frac{\lambda_2}{8}\phi_2^4 + \frac{1}{4}(\lambda_3 + \lambda_4 + \lambda_5)(\phi_1\phi_2)^2$$

* The Coleman-Weinberg potential :

$$V_{\rm CW}(\phi_1, \phi_2) = \pm\frac{1}{64\pi^2}\sum_k n_k m_k^4(\phi_1, \phi_2)\left[\log\frac{m_k^2(\phi_1, \phi_2)}{\mu^2} - c_k\right]$$

+(-) for boson (fermion) $\quad k = H^\pm, H, h, A, W, Z, \gamma, t, b, \tau$

* The counter term potential : $V_{\rm CT}$

* The one-loop thermal contributions : V_T

Two Higgs Doublet Model

- **Parameters in the Higgs potential**

$$m_1^2,\ m_2^2,\ m_3^2,\ \lambda_{1-5}$$

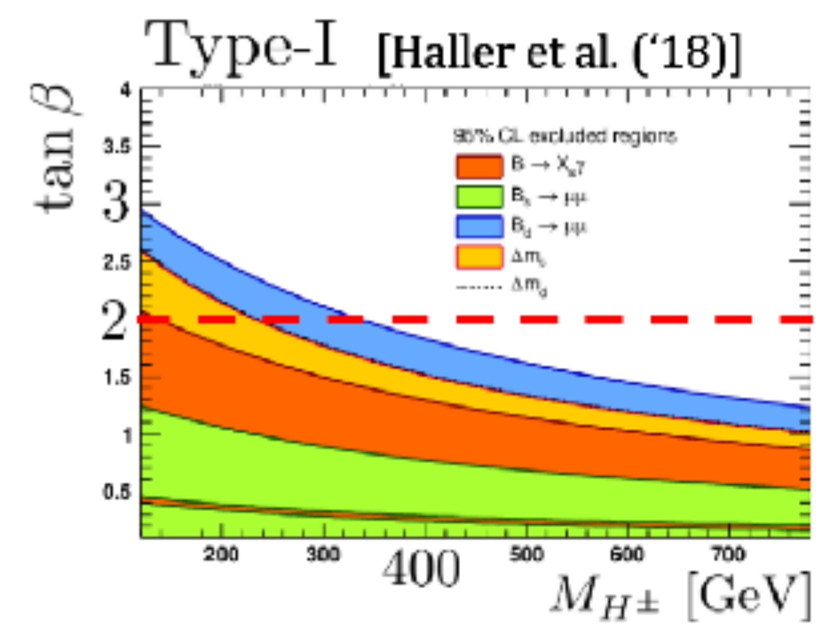
$$\rightarrow \boxed{\tan\beta,\ \cos(\beta-\alpha),\ m_3^2,\ v,\ m_h,\ m_H,\ m_A,\ m_{H^\pm}}$$

Constraints

❖ Theoretical constraints :

- Stability $\lambda_1 > 0, \quad \lambda_2 > 0, \quad -\sqrt{\lambda_1\lambda_2} < \lambda_3, \quad -\sqrt{\lambda_1 + \lambda_2} < \lambda_3 + \lambda_4 - \lambda_5$

- Perturbativity $|\lambda_n| < 4\pi \ (n = 1, 2, \cdots 5)$



❖ Experimental constraints

- Electroweak precision data

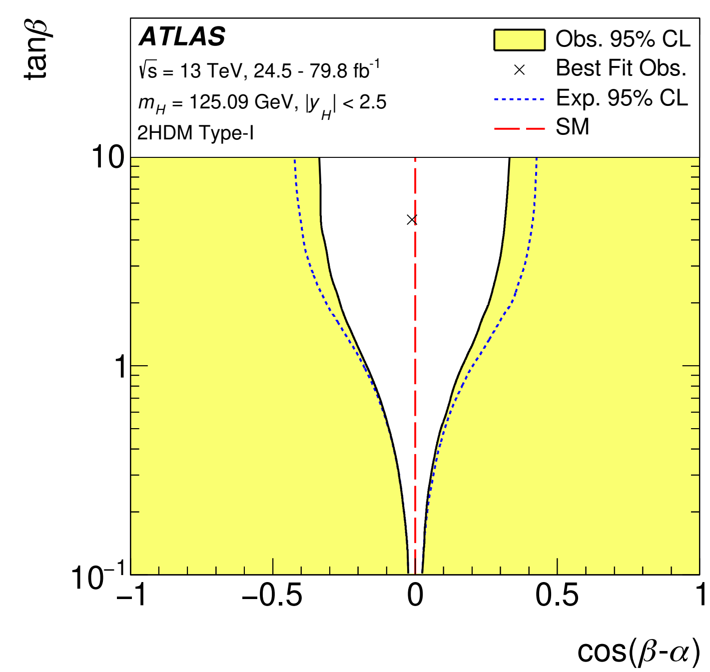
$$m_{H^\pm} = m_A \text{ or } m_H$$

- Flavor experiments

$$B_d \to \mu\mu \quad \tan\beta \gtrsim 2$$

- Higgs couplings strength

$$|\cos(\beta - \alpha)| \lesssim 0.25$$



Multistep Phase Transition

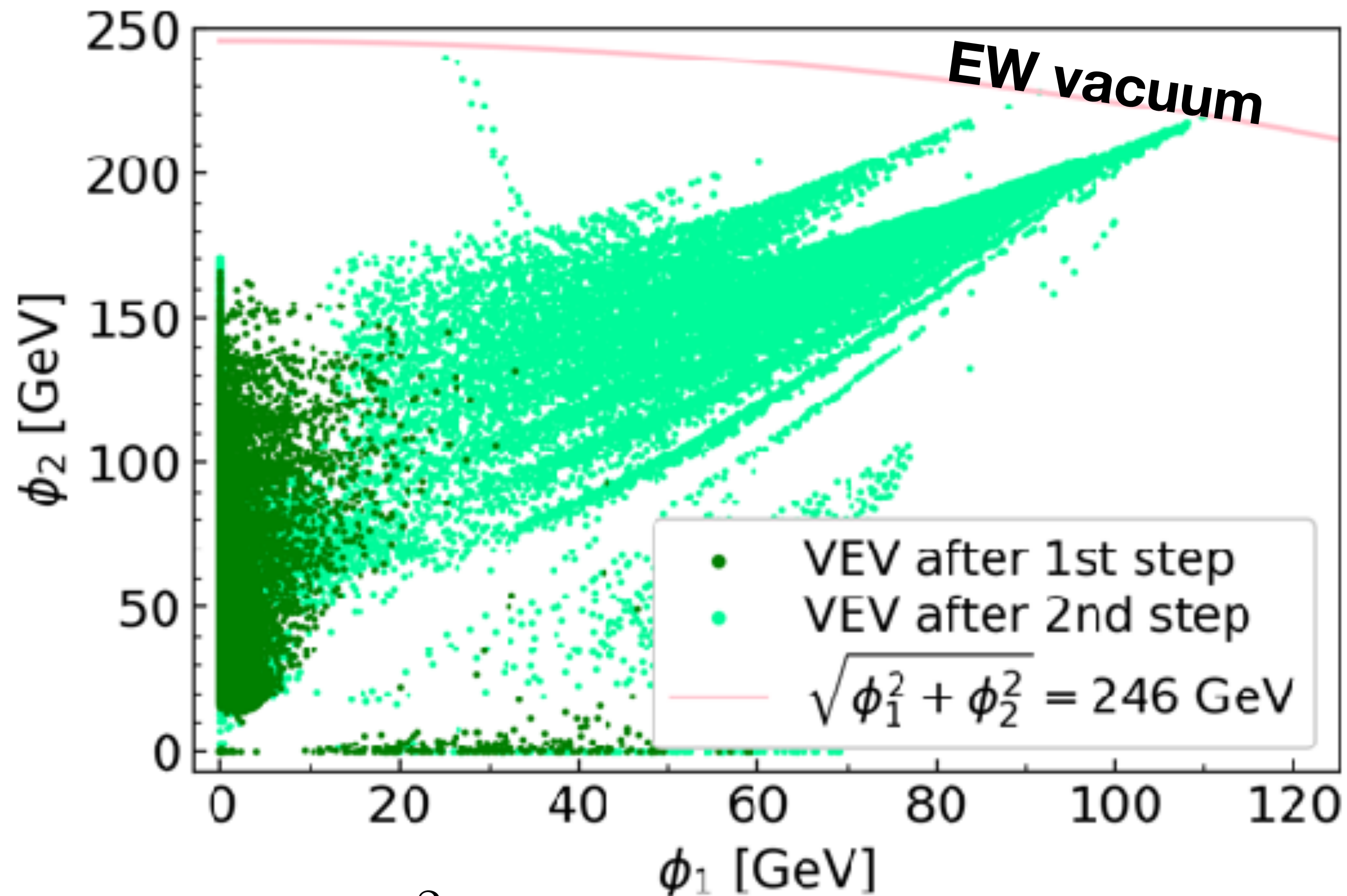
Type-I $(m_A = m_{H^\pm})$

MA, Komatsu, Shibuya, PTEP (2022)

m_A [GeV]	m_H [GeV]	$\tan\beta$	$\cos(\beta-\alpha)$	m_3 [GeV]
180–1000(/10)	130–1000(/10)	2–10(/0.5)	−0.25–0.25(/0.05)	0–100(/5)

- **2-step PT**

The VEVs after each step of the 2-step PTs



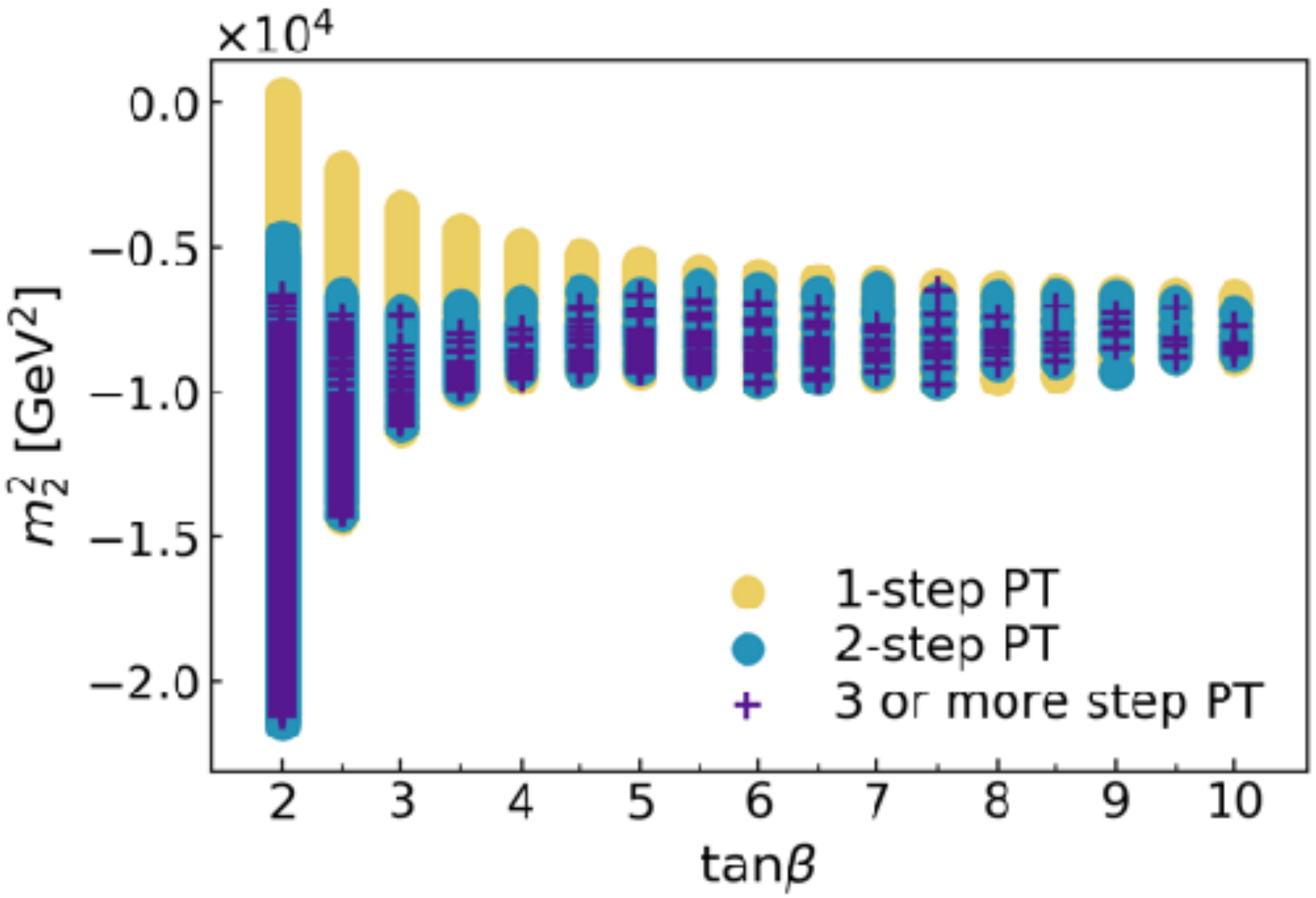
- $m_2^2 < 0$ with large $|m_2^2|$

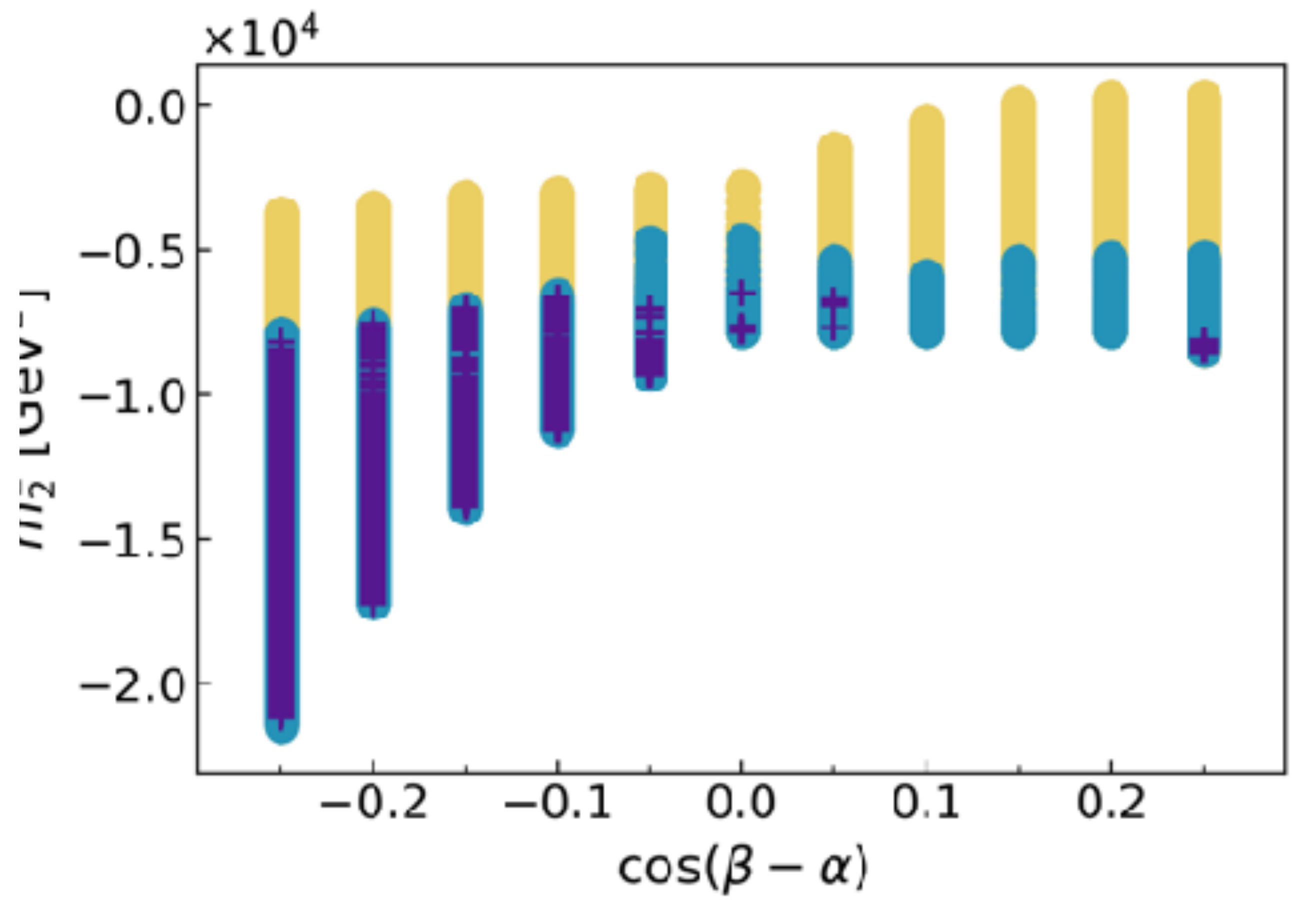
→ **The first step in the 2-step PT tends to occur along the ϕ_2 axis.**

Type I (mA=mH+)

Minimum conditions :

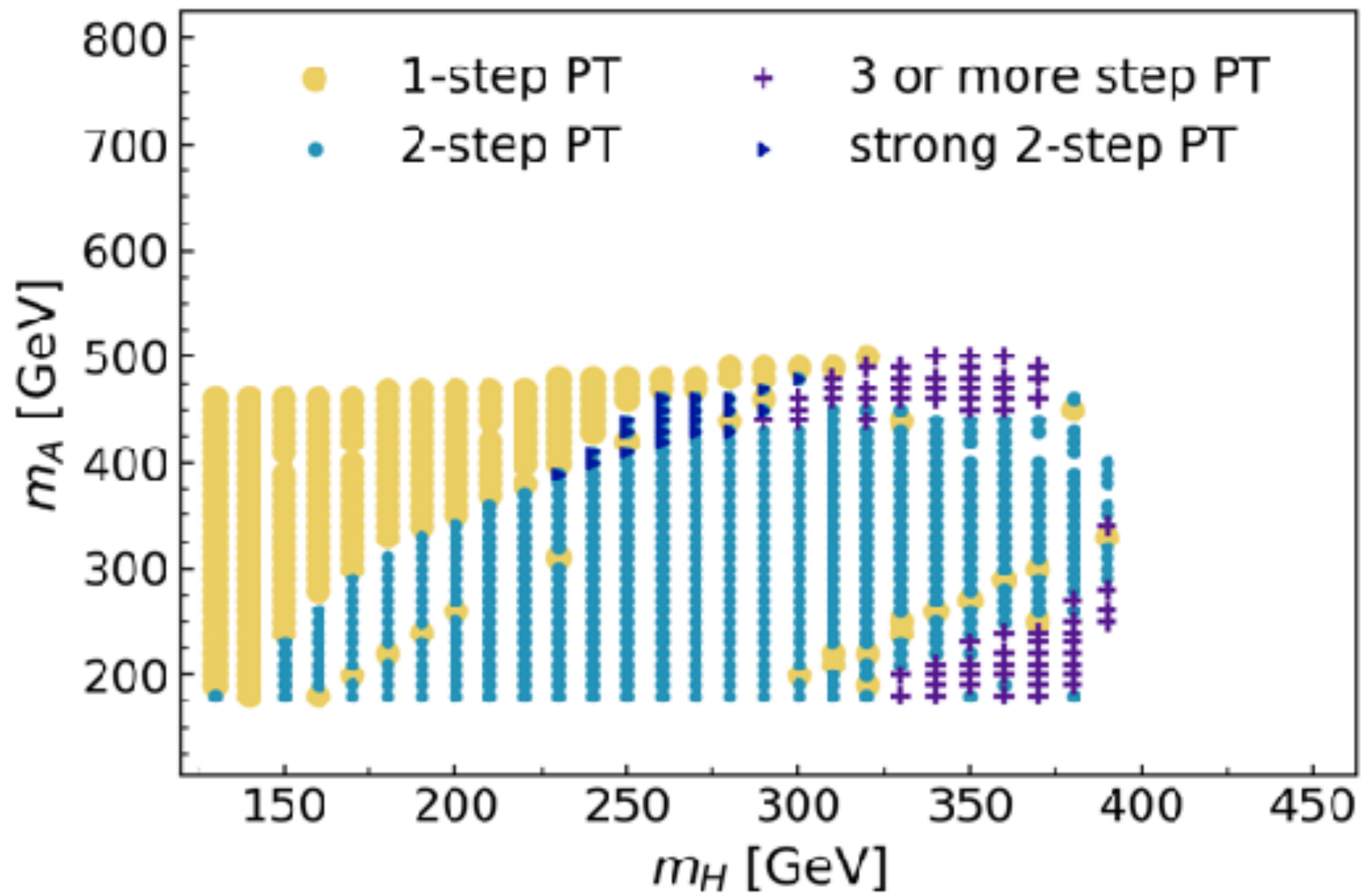
$$m_2^2 = \frac{1}{\tan\beta}\left[m_3^2 - \frac{1}{2}(m_H^2 - m_h^2)\cos\alpha\sin\alpha\right] - \frac{1}{2}(m_h^2\cos^2\alpha + m_H^2\sin^2\alpha).$$





Type I (mA=mH+)

$$\tan\beta = 2, \quad \cos(\beta - \alpha) = -0.2, \quad m_3 = 0$$



- **strong 2-step PTs : The first step PTs of the 2-step PTs are strongly first order.**
- **The strong 2-step PTs occur only with the mass hierarchy** $m_A > m_H$.

Exotic intermediate phases?

such as charge-breaking ones

MA, Biermann, Borschensky, Ivanov, Mühlleitner, Shibuya [2308.04141]

CP-conserving 2HDM with softly broken Z2 symmetry

- **Potential:**

$$V_{\text{tree}} = m_{11}^2 \Phi_1^\dagger \Phi_1 + m_{22}^2 \Phi_2^\dagger \Phi_2 - m_{12}^2 \left(\Phi_1^\dagger \Phi_2 + \text{h.c.}\right) + \frac{\lambda_1}{2}\left(\Phi_1^\dagger \Phi_1\right)^2 + \frac{\lambda_2}{2}\left(\Phi_2^\dagger \Phi_2\right)^2$$
$$+ \lambda_3 \left(\Phi_1^\dagger \Phi_1\right)\left(\Phi_2^\dagger \Phi_2\right) + \lambda_4 \left(\Phi_1^\dagger \Phi_2\right)\left(\Phi_2^\dagger \Phi_1\right) + \frac{\lambda_5}{2}\left[\left(\Phi_1^\dagger \Phi_2\right)^2 + \text{h.c.}\right],$$

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_1 + i\eta_1 \\ \zeta_1 + \bar{\omega}_1 + i\psi_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \rho_2 + \bar{\omega}_{\text{CB}} + i\eta_2 \\ \zeta_2 + \bar{\omega}_2 + i\left(\psi_2 + \bar{\omega}_{\text{CP}}\right) \end{pmatrix}.$$

- **charge-breaking (CB) vacuum:**

$$\Phi_1 = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ \bar{\omega}_1 \end{pmatrix}, \quad \Phi_2 = \frac{1}{\sqrt{2}}\begin{pmatrix} \bar{\omega}_{\text{CB}} \\ \bar{\omega}_2 \end{pmatrix}.$$

- Conditions for a CB vacuum:

$$\sqrt{\lambda_1 \lambda_2} - \lambda_3 > 0, \quad \lambda_4 > |\lambda_5|. \rightarrow \quad |\lambda_3| < \sqrt{\lambda_1 \lambda_2}$$

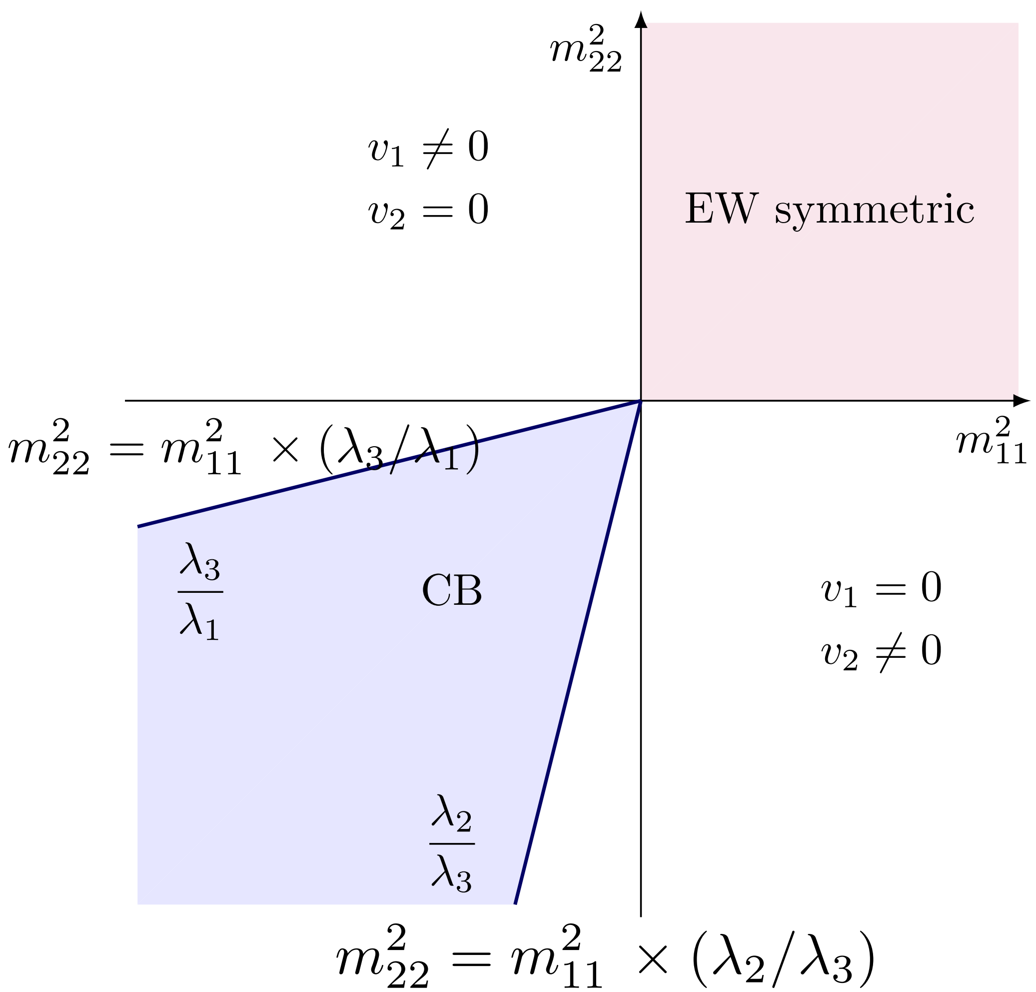
The sign of λ3 is not fixed.

$$m_{11}^2 \sqrt{\lambda_2} + m_{22}^2 \sqrt{\lambda_1} < 0, \quad m_{11}^2 < m_{22}^2 \frac{\lambda_3}{\lambda_2}, \quad m_{22}^2 < m_{11}^2 \frac{\lambda_3}{\lambda_1}.$$

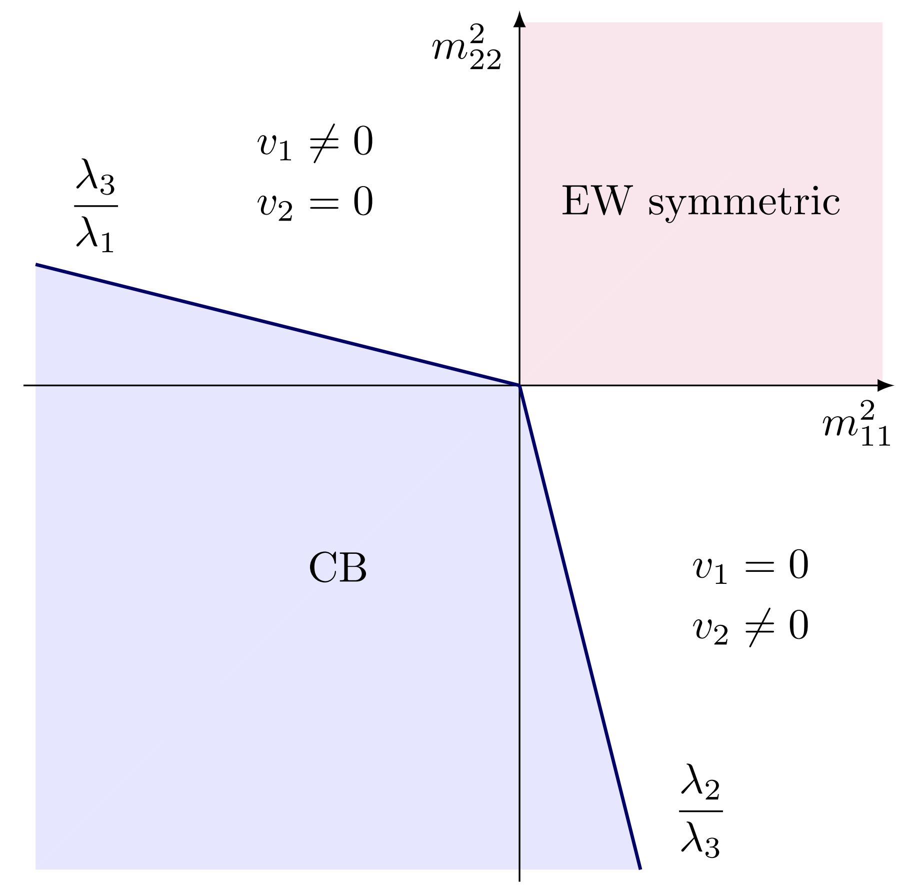
A toy model (m_{12}^2=0)

T = 0

$\lambda_3 > 0$



$\lambda_3 < 0$



Type -I $\quad v^2 = v_2^2 = \frac{2|m_{22}^2|}{\lambda_2}\,, \quad \frac{m_{h_{\rm SM}}^2}{v^2} = \lambda_2\,, \quad \frac{m_{H^\pm}^2}{v^2} = \frac{1}{2}\lambda_3\left(1 - \frac{m_{11}^2}{m_{22}^2}\frac{\lambda_2}{\lambda_3}\right)\,.$

Temperature evolution

Type-I

$$v_1 = 0,\ v_2 \neq 0$$

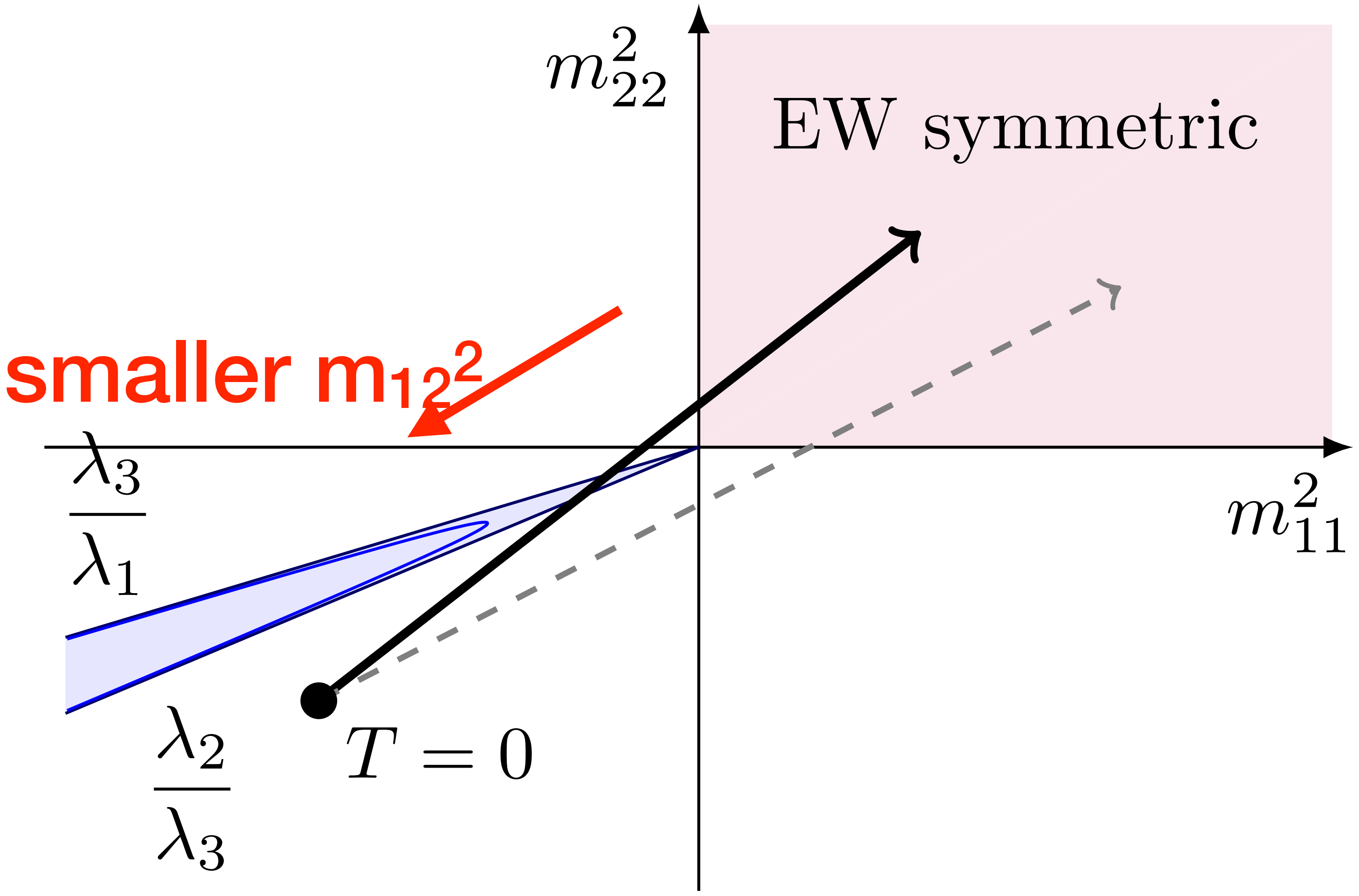
- **one-loop high-T corrections:**

$$m_{11}^2(T) = m_{11}^2 + c_1 T^2\,, \quad m_{22}^2(T) = m_{22}^2 + c_2 T^2\,,$$

with

$$c_1 = \frac{1}{12}\left(3\lambda_1 + 2\lambda_3 + \lambda_4\right) + \frac{1}{16}\left(3g^2 + g'^2\right)\,,$$
$$c_2 = \frac{1}{12}\left(3\lambda_2 + 2\lambda_3 + \lambda_4\right) + \frac{1}{16}\left(3g^2 + g'^2\right) + \frac{1}{12}\left(y_\tau^2 + 3y_b^2 + 3y_t^2\right)\,,$$

High-T limit



m_{22}^2
EW symmetric
smaller m_{12}^2
$\frac{\lambda_3}{\lambda_1}$
m_{11}^2
$\frac{\lambda_2}{\lambda_3}$
$T = 0$

Symmetry (non-)restoration

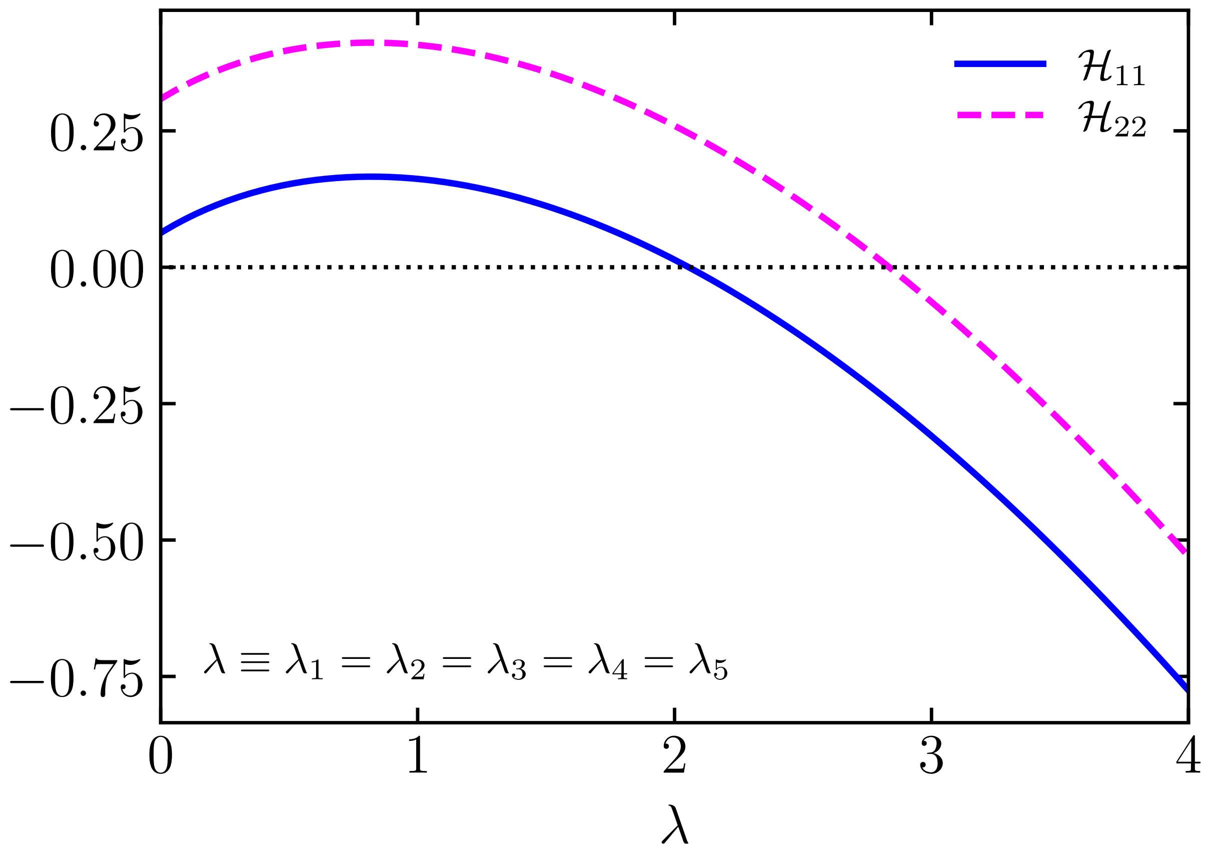
$$H_{ij} \equiv \left.\frac{\partial^2 V_T}{\partial\bar{\omega}_i \partial\bar{\omega}_j}\right|_{\bar{\omega}_{i,j}=0}, \quad i,j = 1,2.$$

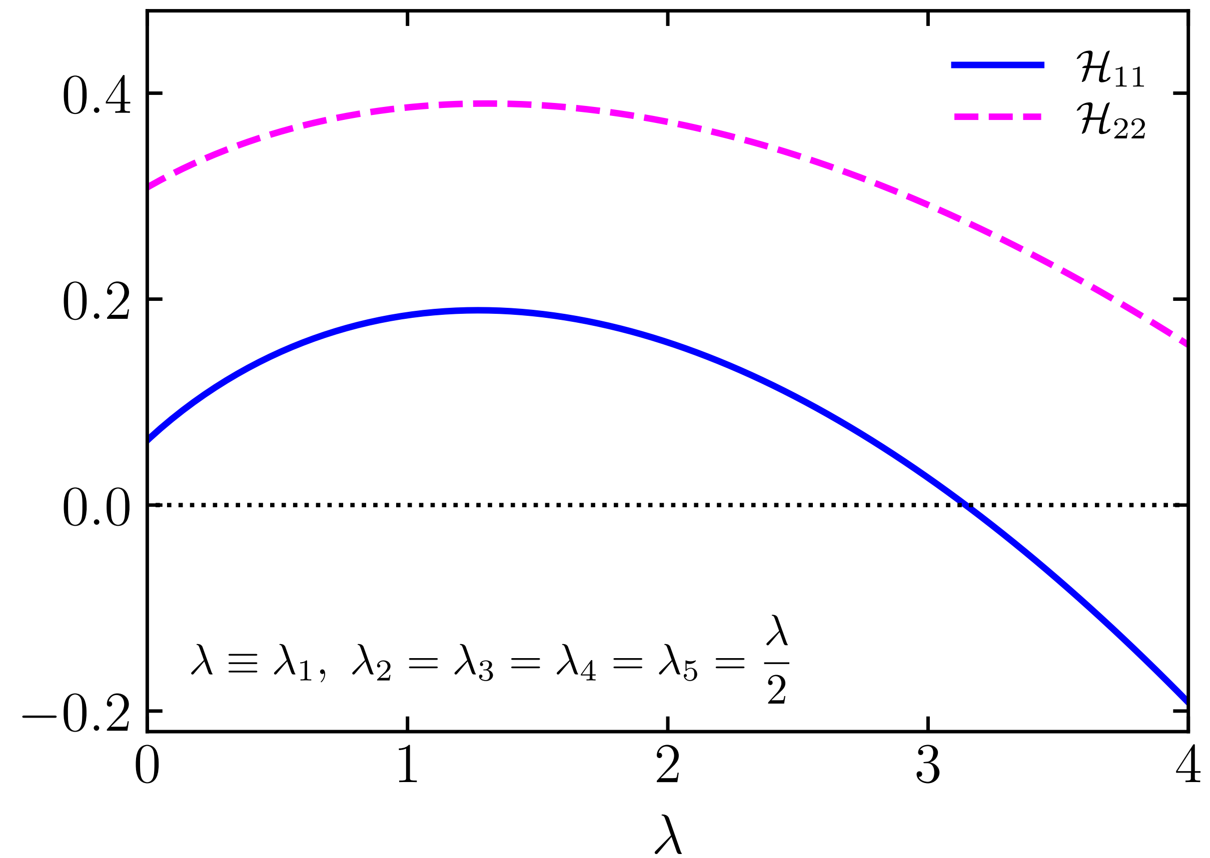
$$\mathcal{H} \equiv \lim_{T\to\infty} \frac{H}{T^2} = \lim_{T\to\infty} \begin{pmatrix} \frac{H_{11}}{T^2} & \frac{H_{12}}{T^2} \\ \frac{H_{21}}{T^2} & \frac{H_{22}}{T^2} \end{pmatrix} = \begin{pmatrix} \mathcal{H}_{11} & 0 \\ 0 & \mathcal{H}_{22} \end{pmatrix},$$

- **In order for the stationary point at the origin to be a minimum, all eigenvalues of the Hessian matrix are required to be positive.**

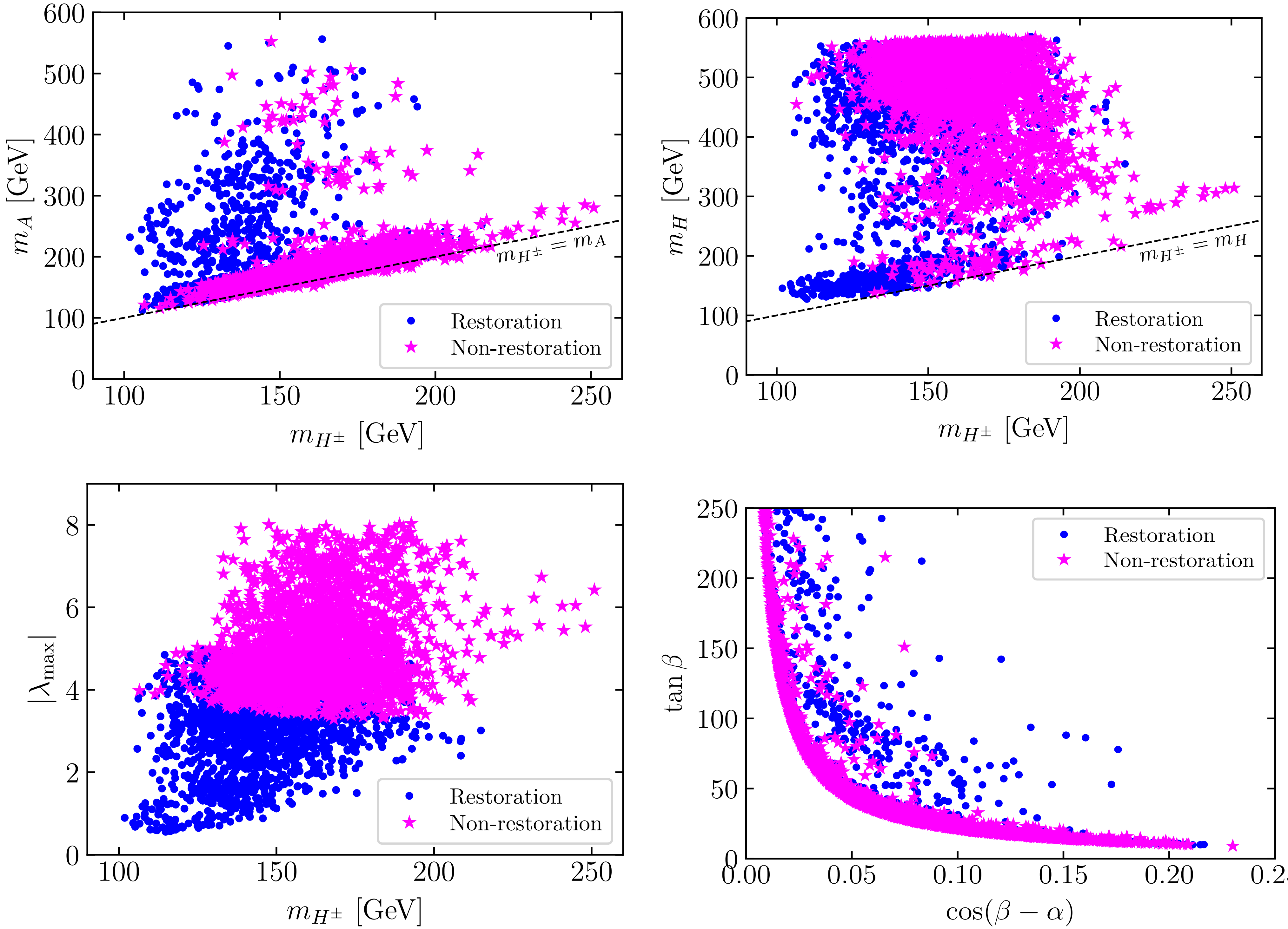
$$\mathcal{H}_{11} > 0 \qquad \text{and} \qquad \mathcal{H}_{22} > 0\,.$$

$\mathcal{H}_{11},\ \mathcal{H}_{22}$





CB phase not required



CB phase required