Conformal Field Theories at large charge sector & Heavy-ion Collisions

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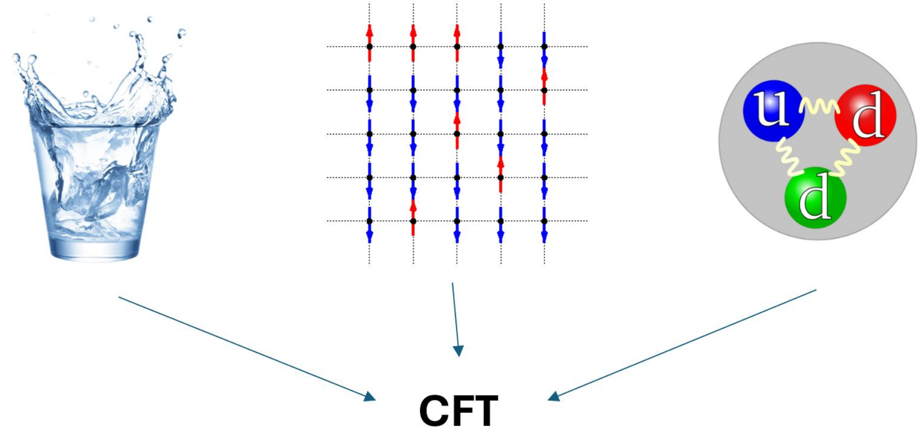
1. Conformal Field Theories (Master Thesis)
2. Heavy Ion Collisions (current field)

Why CFTs



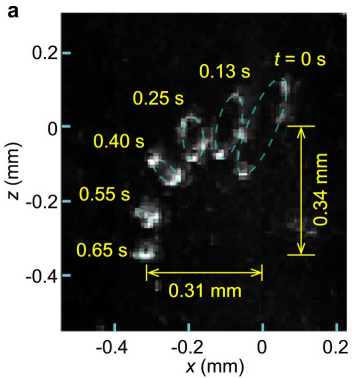
Motivation & Objectives

- Study Lagrangian with a Global Charge:

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + c|\phi|^2 + g^2|\phi|^4$$

where $\vec{\phi}(x)$ is an $N-$ vector

- Compute Scaling Dimensions (Δ)
- Specialize to $O(N)$ models for $N = 2, 4$
- Experimental observation in rotating trapped superfluids



Y. Tang et al 2023

2-pt & 3-pt functions

- Scaling Dimension

$$x' \to \lambda x \qquad \tilde{\phi}(x') = b(x)^{-\Delta}\phi(x)$$

- General structure

$$2pt = \frac{\delta_{ij}}{(x-y)^{2\Delta}} \;\; ; \;\; 3pt = \frac{C_{123}}{|x_{12}|^{2\alpha_1}|x_{13}|^{2\alpha_2}|x_{12}|^{2\alpha_3}}$$

- Operator Product Expansion (OPE)

$$\phi_1(x)\phi_2(0)|0\rangle = \sum_{\mathcal{O}} C_{\mathcal{O}}\left(x, \partial_y\right)\mathcal{O}(y)|_{y=0}|0\rangle$$

State-Operator/Radial Quantization

- Evolution Operator

$$U = e^{i\mathcal{H}\Delta t} \to e^{iD\Delta\tau} \;\; ; \;\; \tau \sim \log(r)$$

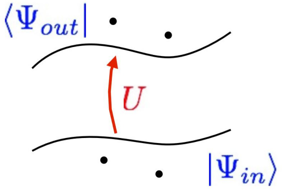
- Manifold foliation

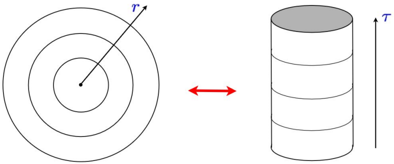
  Euclidean: $\mathbb{R}^D \to$ Cylinder: $S^{D-1} \times \mathbb{R}$

$$E = \frac{\Delta}{R}$$

- Insertion of states

$$[D, \mathcal{O}(0)] = -i\Delta\mathcal{O}(0)$$





S.Rychkov 2016

EFT/Superfluids

- Renormalization Group

$$\int \mathcal{D}\phi e^{-i\int \mathcal{L}[\phi]} \to \int \mathcal{D}\phi' e^{-i\int \mathcal{L}_{eff}[\phi']}$$

- Scale Separation *A.Monin et al 2016*
  - Fixing Global $U(1)$ charge $\to$ New scale: $\mu = \frac{\sqrt{Q}}{R}$
  - Large Charge implies $\mu \gg 1/R$
- Allows for Effective Lagrangian description for light modes
- Most simple realization: $U(n) \to U(n-1)$ spontaneous breaking pattern
- Admits Superfluid Lagrangian description in terms of Goldstones

$$\mathcal{L} \propto |\partial\chi|^{3/2}$$

O(2) model

- Symmetry Breaking pattern: $SO(3,1) \times U(1) \to SO(2) \times D'$
- Ground state scaling dimension *G.Cuomo & Z.Komargodskia 2023*

$$\Delta = c_{3/2} Q^{3/2} + c_{1/2} Q^{1/2} + \mathcal{O}(Q^{-1/2})$$

- Study contributions of fluctuations $\chi = \mu t + \varphi$
- Introducing angular momentum
  - Find the spectrum $\Delta(Q, J)$

Phases O(2) model

G.Cuomo & Z.Komargodski 2023

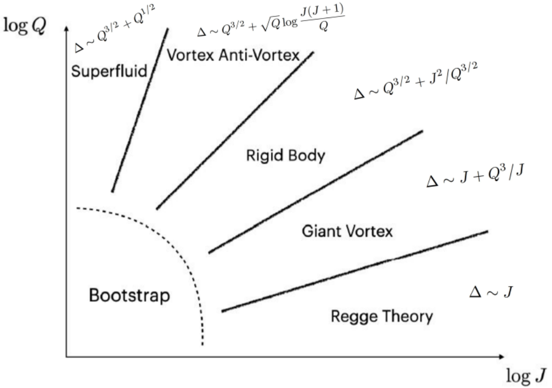


Figure.: Phase diagram for the O(2) model

O(2n) model

- Effective Lagrangian from RG flow *L.Alvarez-Gaumé et al 2021*

$$\mathcal{L}_{eff} = \sqrt{g}\left[\frac{2c_1}{3}||d\psi||^3 - \frac{4c_2}{R^2}||d\psi|| + \frac{3}{4}(\lambda - 1)\frac{\kappa^2}{||d\psi||}\right]$$

- Subtelties due to Non-Abelian Noether current
- Parametrization of the fields $\psi_{1,2,3,4} \sim \psi_{1,2,3,4}(\alpha, \beta, \gamma)$
- More available DoF allow for Type-II Goldstone bosons and Inhomogeneous states

Charge Distribution

L.A.Gaumé et al 2021

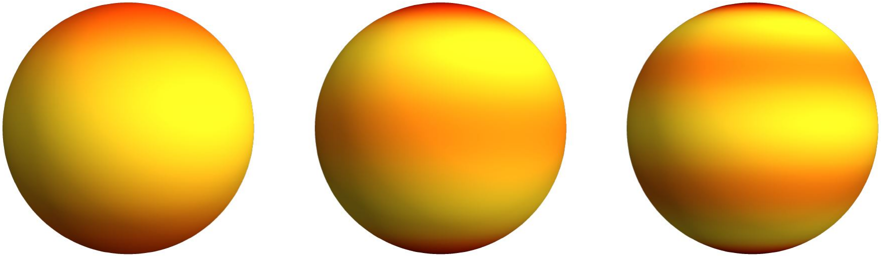
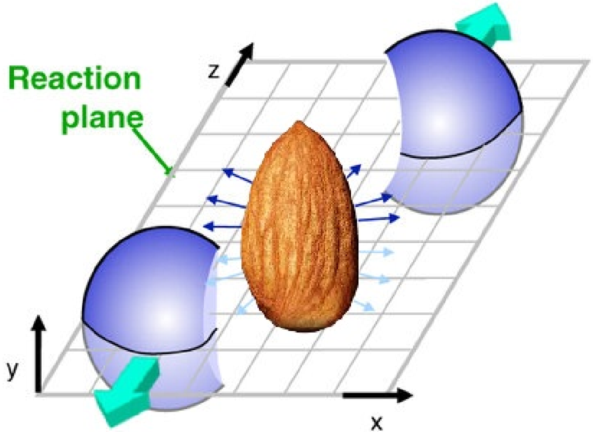


Figure.: Charge distribution of inhomogeneous Ground states

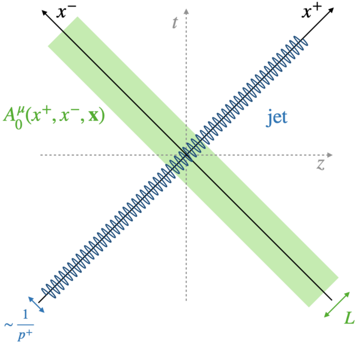
- Large-Charge approximation has proven to be a fruitful approach for CFTs
- Combined with SSB leads to analytically solvable systems
- Further research can be conducted to reproduce $\Delta(Q, J_1, J_2)$ diagram for the $O(4)$ model

Heavy Ion Collisions



Light-front Perturbation Theory

- Light cone coordinates
  $x^\mu = (x^+, x^-, \mathbf{x}_\perp)$
  $x^\pm \equiv \frac{x^0 \pm x^3}{\sqrt{2}}; \mathbf{x}^i = (x^1, x^2)$
- Adequate for boosted objects (jets)
  - $p^+ \gg \mathbf{p}_\perp, p^-$
  - First non-Eikonal corrections
    $p^+ > \mathbf{p}_\perp \gg p^-$
  - p^+ is always conserved



Dressing Propagators

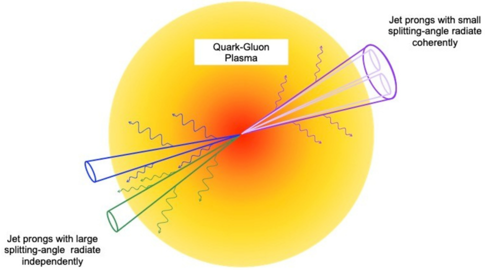
- Modify in-medium Propagators/Feynman rules

$$(p, i|D|p_0, j) = \frac{\not{p}\gamma^+ \not{p}_0}{2p^+}(p, i|G_{\text{scal}}|p_0, j), \qquad (p, a|G^{\mu\nu}|p_0, b) = d^{\mu k}(p)\, d^{k\nu}(p_0)(p, a|G_{\text{scal}}|p_0, b)$$

- Vertex will be dressed and dynamical Dof will now be projected free quarks $u(p) \to \mathbb{P}_+ u(p) \equiv \xi(p)$

Quark Gluon Plasma (QGP)

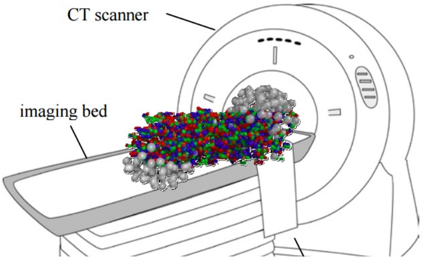
- Hard parton collision (quark-quark)
- Medium interaction
  - Momentum broadening
  - Medium induced emissions (quenching)
  - (De-)Coherence
- Types of observables
  - Nuclear Modification factor
  - Event shapes (semi-/inclusive observables)
  - Rare emissions



Ron Soltz and Aaron Angerami, Lawrence Livermore National Laboratory

Our approach

- Tomography with photon production
  - Clean angular distribution ($\alpha_{EM}/\alpha_s \ll 1$)
  - 'Simple' analytic computation
- Multiple-scale problem
  - $Q_0 \sim \Lambda_{\rm QCD}, gT$ (thermal gluons), Θ_c (Coherence angle)...



- Non-perturbative effects
  - SCET introduces a controlled hierarchy: $A^- > \mathbf{A}_\perp > A^+$
  - Interactions with $\mathbf{A}_\perp$?
  - Large angle Energy-Energy correlators (EECs)?

Questions/suggestions?

Inhomogeneous state

- Breaking of spatial symmetries
  - Implemented via inhomogeneous ansatz $\mu_1 \neq \mu_2$ and $\gamma = \epsilon \mathcal{F}(\phi, \theta)$
- Solving EOM in the limit $\mu_2 - \mu_1 \ll \mu_2 + \mu_1$ and $\epsilon \ll 1$ *L.Alvarez-Gaumé et al 2021*

$$\begin{aligned} \mathcal{F}(\phi, \theta) &= P_l(\cos\theta) \\ \mu_2^2 - \mu_1^2 &= \lambda l(l+1) \end{aligned}$$

- Linearizing ($\epsilon \to 0$) implies quantized values for μ_2, μ_1

Novel Solution

- Relaxing condition $\mu_2 - \mu_1 \ll \mu_2 + \mu_1$ and imposing only $\epsilon \to 0$
- Solving EOM:

$$\begin{aligned} \mathcal{F}(\phi, \theta) &= P_l(\cos\theta) \\ l(l+1) &= (\tilde{\mu}_2^2 - \tilde{\mu}_1^2)\left(\frac{1 - A\tilde{\mu}_1^2}{1 - A\lambda\tilde{\mu}_1^2}\right) \equiv (\tilde{\mu}_2^2 - \tilde{\mu}_1^2)\mathcal{C}(\tilde{\mu}_1; \lambda) \\ A &= \frac{c_1 R^2}{2c_2} \end{aligned}$$

- Leads to a difference in the total charge

$$\Delta Q = -2\pi c_1 \frac{l(l+1)}{\mathcal{C}(\mu_1; \lambda)}\left[2 + \frac{(\mu_2 - \mu_1)}{(\mu_2 + \mu_1)}\right]$$