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# Modeling Graviational Waves via Post-Newtonian Expansion and Geometric Optics

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19.juni 2025

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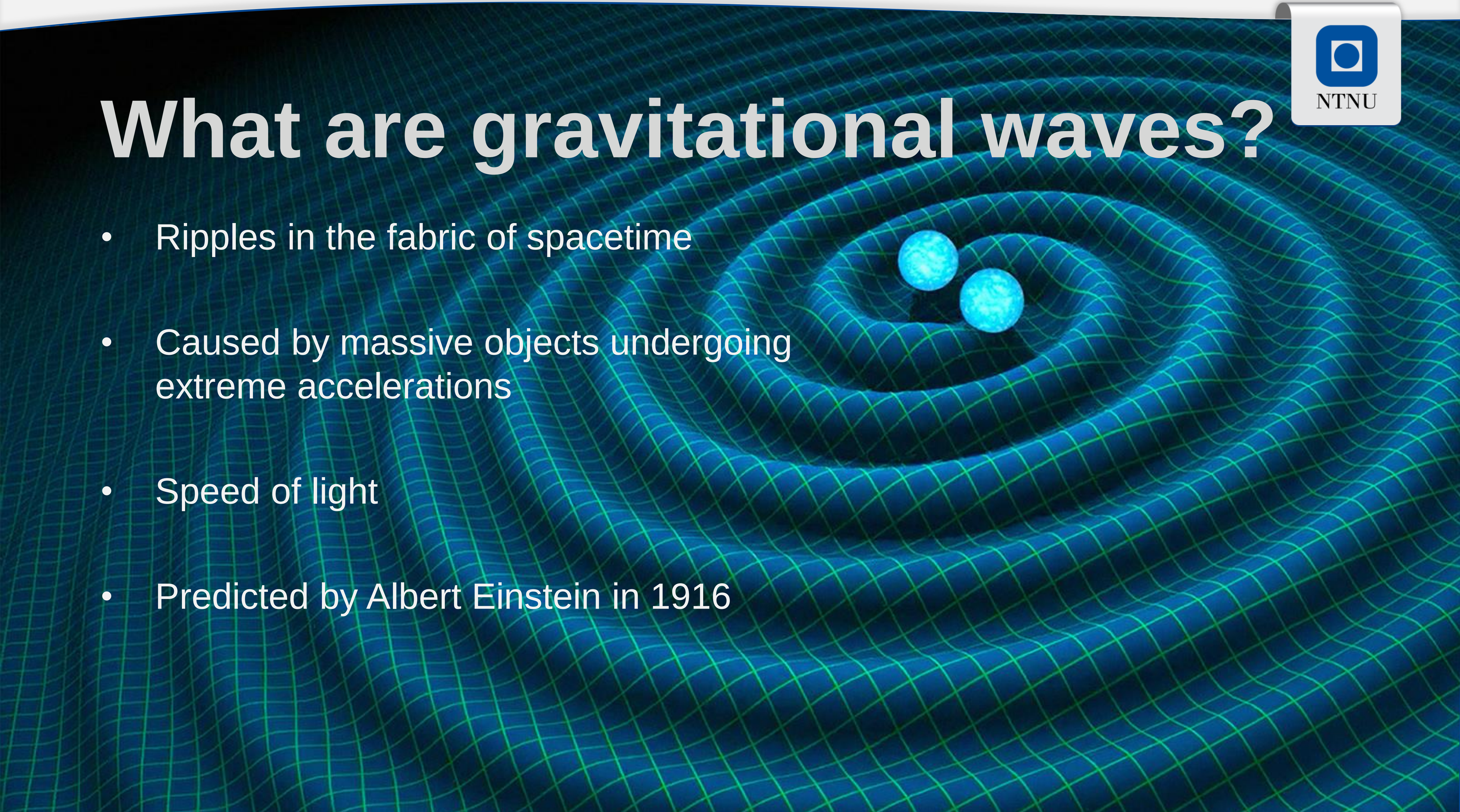
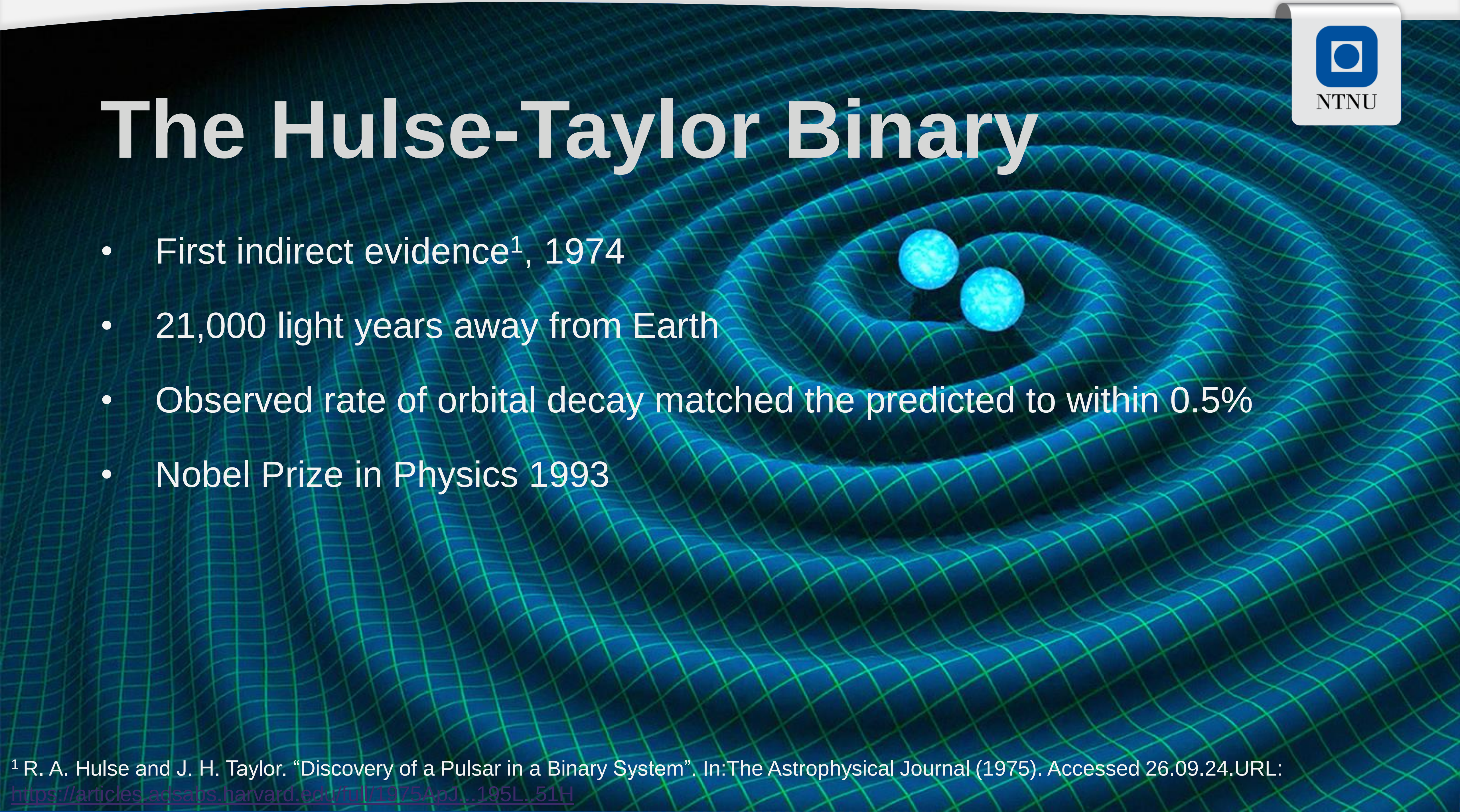
# What are gravitational waves?



- Ripples in the fabric of spacetime
- Caused by massive objects undergoing extreme accelerations
- Speed of light
- Predicted by Albert Einstein in 1916

# The Hulse-Taylor Binary



- First indirect evidence[1], 1974
- 21,000 light years away from Earth
- Observed rate of orbital decay matched the predicted to within 0.5%
- Nobel Prize in Physics 1993

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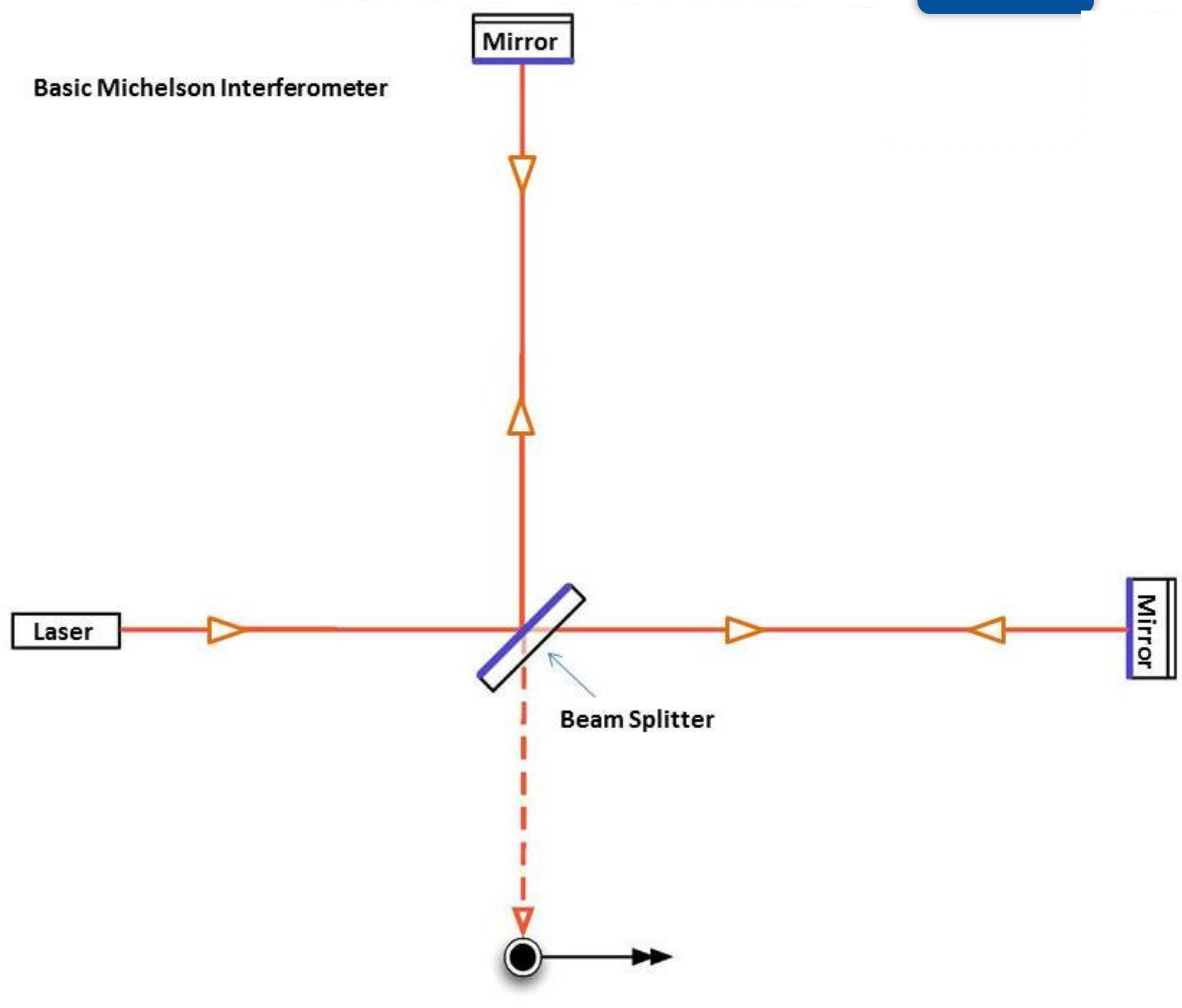
[1] R. A. Hulse and J. H. Taylor. “Discovery of a Pulsar in a Binary System”. In:The Astrophysical Journal (1975). Accessed 26.09.24.URL: https://articles.adsabs.harvard.edu/full/1975ApJ...195L..51H

# Detection of GWs

- September 14 2015
- LIGO[2]
- Collision of two black holes, 1.3 billion light-years away
- Laser Interferometry



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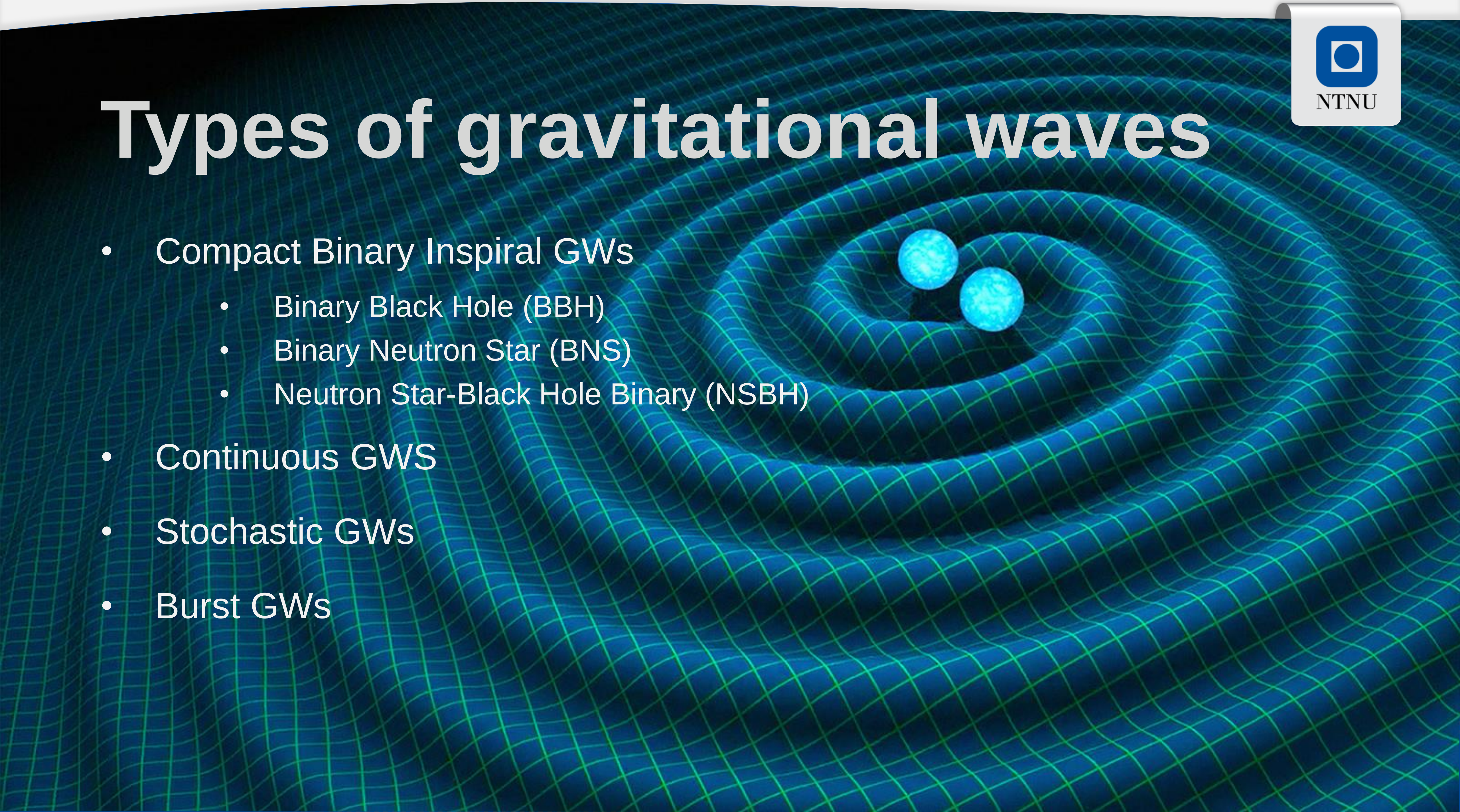
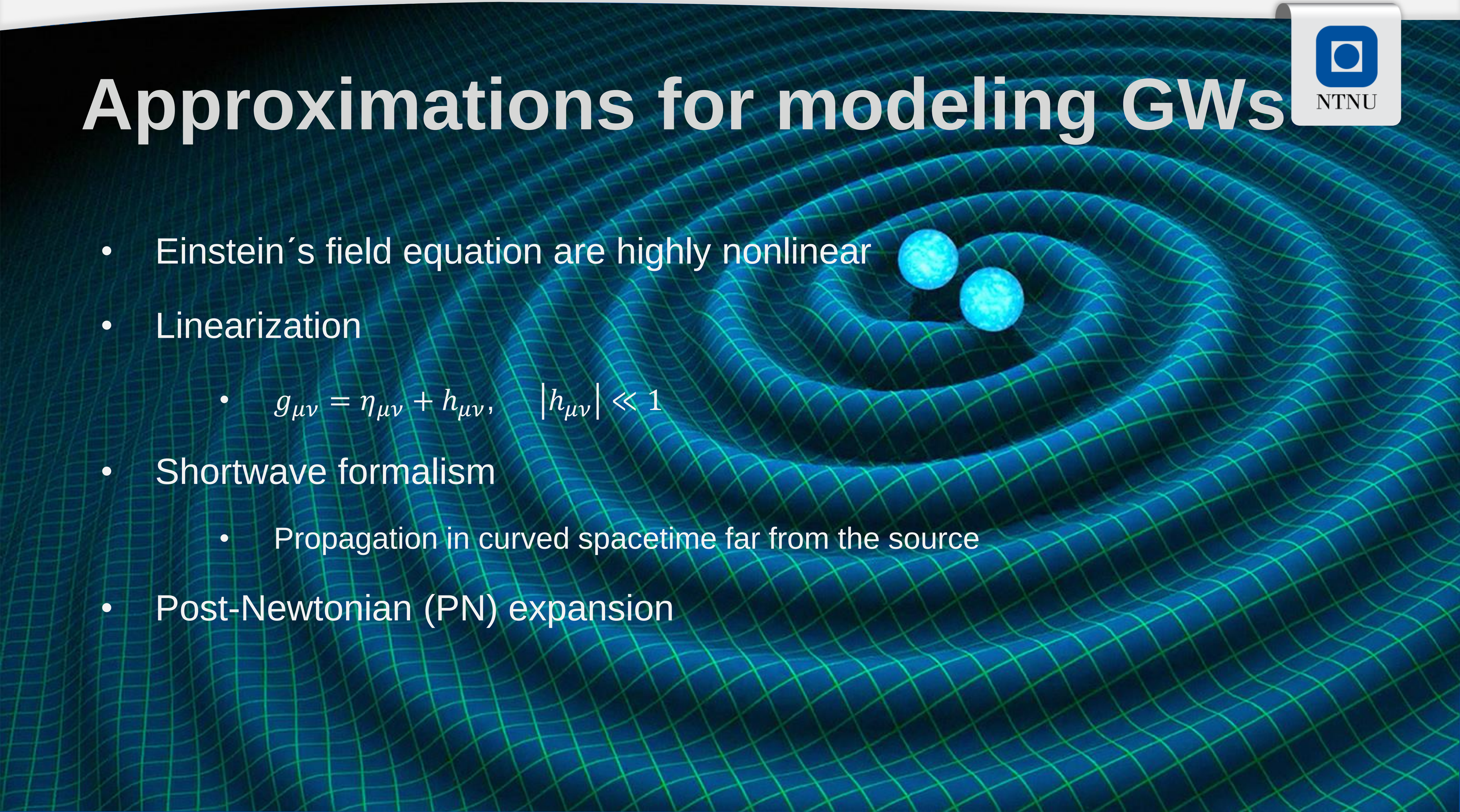
[2] LIGO - Laser Interferometer Gravitational-Wave Observatory. Accessed 24.09.24.(2024). URL:https://www.ligo.caltech.edu/

# Types of gravitational waves



- Compact Binary Inspiral GWs
  - Binary Black Hole (BBH)
  - Binary Neutron Star (BNS)
  - Neutron Star-Black Hole Binary (NSBH)
- Continuous GWS
- Stochastic GWs
- Burst GWs

# Approximations for modeling GWs



- Einstein´s field equation are highly nonlinear
- Linearization
  - $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}, \quad |h_{\mu\nu}| \ll 1$
- Shortwave formalism
  - Propagation in curved spacetime far from the source
- Post-Newtonian (PN) expansion

# Post-Newtonian Expansion

- Velocities small compared to the speed of light, but high enough for relativistic corrections to become significant
- Systematic expansion in powers of v/c
- 1PN level
- Applying the framework to physical system

# Periastron shift PSR B1913+16

- Expression for the general relativistic periastron advance per orbit:

$$\delta\phi = \frac{6\pi G(m_1 + m_2)}{a(1 - e^2)c^2}$$

- Angular shift per year:

$$\dot{\omega}_{GR} = [\text{numerical factor}] \cdot \left(\frac{M}{kg}\right)^{\frac{2}{3}} \cdot \left(\frac{1}{P_b[s]}\right)^{\frac{5}{3}}$$

$$\dot{\omega}_{GR} \approx 4.22597 \frac{\text{deg}}{yr}$$

$\dot{\omega}_{OBS}$[3] $\approx 4.226585 \frac{\text{deg}}{yr}$

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[3] J. M. Weisberg and Y. Huang. “Relativistic measurements from timing the binary pulsar PSR B1913+16”. In: arXiv (2016). Accessed 18.11.24. url: https://arxiv.org/pdf/1606.02744

# Orbital period decay PSR B1913+16

- 1PN-corrected flux-based expression for the orbital period derivative $\dot{P}_b$:

$$\dot{P}_b = -\frac{192\pi}{5c^5}\frac{\eta(GMn)^{5/3}}{(1-e_t^2)^{7/2}}\left\{1+\frac{73}{24}e_t^2+\frac{37}{96}e_t^4+\frac{(GMn)^{2/3}}{336c^2(1-e_t^2)}\left[1273+\frac{16495}{2}e_t^2+\frac{42231}{8}e_t^4 + \frac{3947}{16}e_t^6 - \left(924+3381e_t^2+\frac{1659}{4}e_t^4-\frac{259}{4}e_t^6\right)\eta + \left(3297e_t^2+4221e_t^4+\frac{2331}{8}e_t^6\right)\frac{\delta m}{M}\right]\right\},$$

$$\dot{P}_b^{\mathrm{GR}} \approx -2.4023 \times 10^{-12}\,\mathrm{s/s}.$$

$$\dot{P}_b^{\mathrm{obs}} = -2.423 \times 10^{-12}\,\mathrm{s/s}.$$ [3]

- Relative difference of only 0.85%

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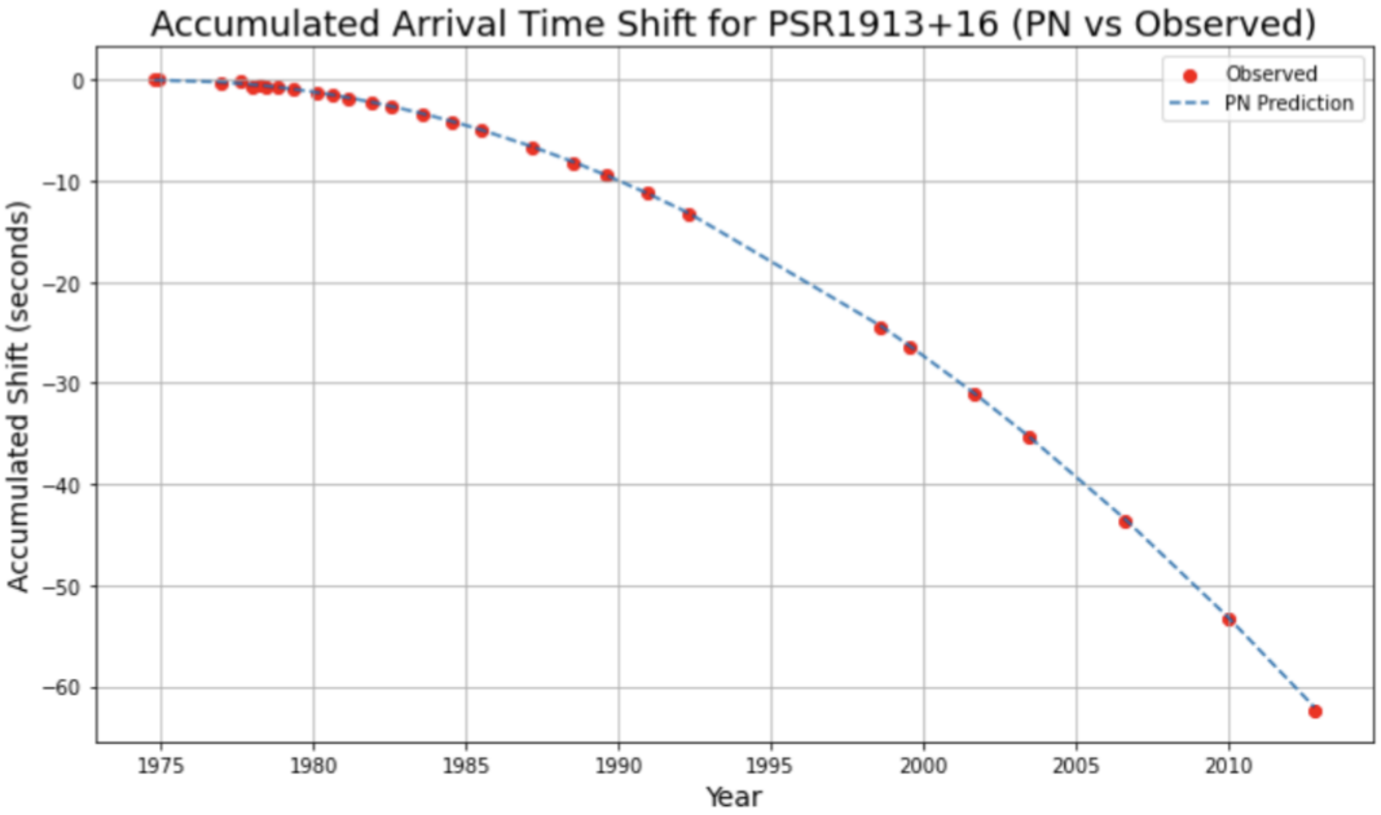
[3] J. M. Weisberg and Y. Huang. “Relativistic measurements from timing the binary pulsar PSR B1913+16”. In: arXiv (2016). Accessed 18.11.24. url: https://arxiv.org/pdf/1606.02744

# Accumulated Arrival Time Shift - PSR B1913+16

- The periastron arrives earlier over time due to gravitational wave emission
- These shifts accumulate over decades, measurable with pulsar timing
- 1PN prediction (blue curve) closely matches timing observations (red dots)[3]

$$\left\langle \frac{da_r}{dt} \right\rangle = -\frac{2}{15}\eta c \frac{\xi^3}{(1-e_r^2)^{11/2}}\Big\{(1-e_r^2)^2(96+292e_r^2+37e_r^4) - \frac{1}{56}\xi(1-e_r^2)$$
$$\Big[(28016+9408\eta)+(160248+43120\eta)e_r^2+(34650+20916\eta)e_r^4$$
$$-(5501-1036\eta)e_r^6)\Big]\Big\},$$

$$\left\langle \frac{de_r}{dt} \right\rangle = -\frac{1}{15}\frac{c^3}{G}\frac{\eta}{m}\frac{\xi^4 e_r}{(1-e_r^2)^{9/2}}\Big\{(304+121e_r^2)(1-e_r^2)^2$$
$$-\frac{1}{56}\xi(1-e_r^2)\Big[(133640+37408\eta)+(108984+33684\eta)e_r^2$$
$$-(35211-3388\eta)e_r^2\Big]\Big\},$$



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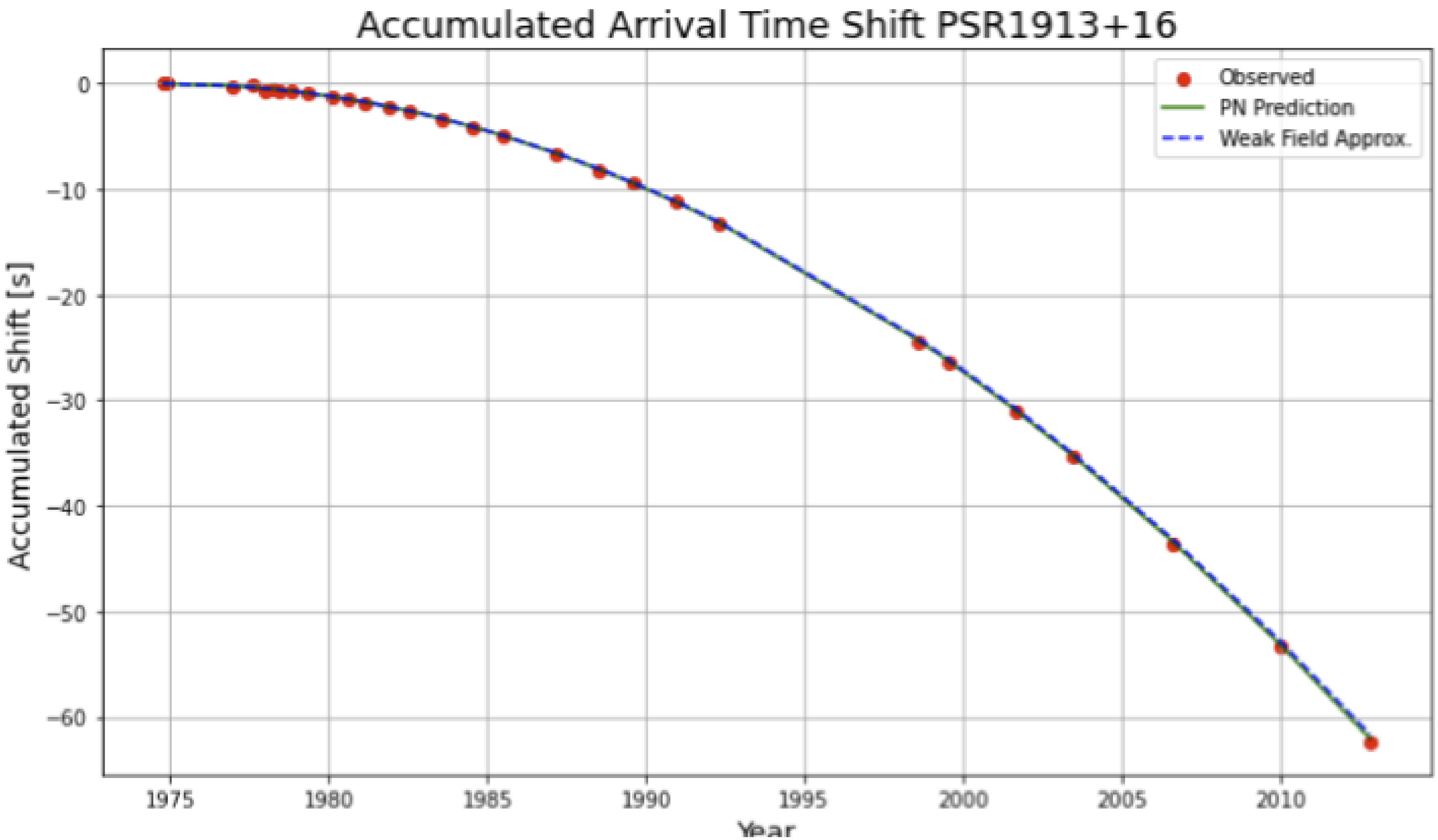
[3] J. M. Weisberg and Y. Huang. “Relativistic measurements from timing the binary pulsar PSR B1913+16”. In: arXiv (2016). Accessed 18.11.24. url: [https://arxiv.org/pdf/1606.02744](https://arxiv.org/pdf/1606.02744)

# Comparison with Linearized Weak-Field Predictions

- Weak-field model uses the quadrupole formula and energy conservation
- Indirectly verifying the existence of gravitational waves
- PSR B1913+16: ideal system for testing the energy loss due to gravitational waves



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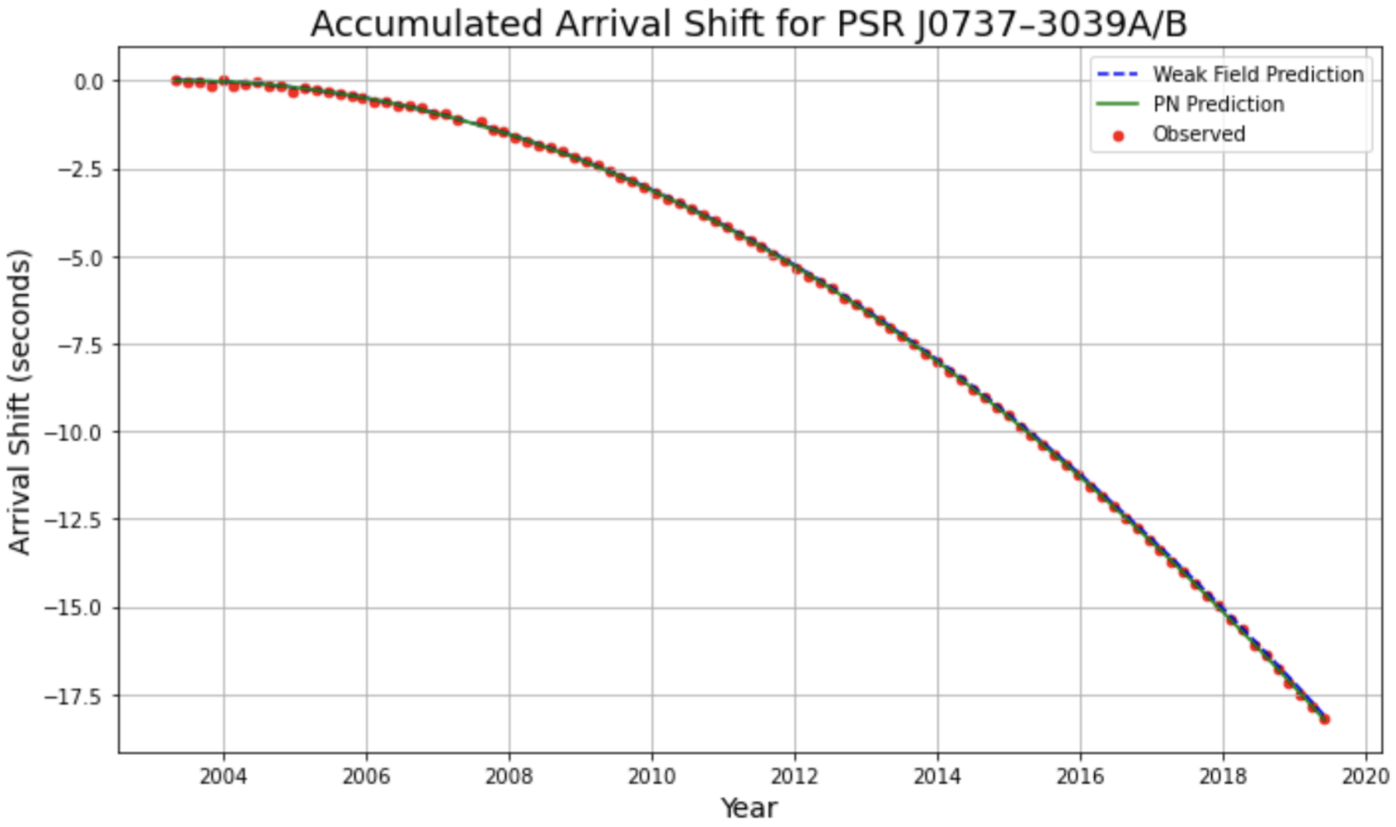
[3] J. M. Weisberg and Y. Huang. “Relativistic measurements from timing the binary pulsar PSR B1913+16”. In: arXiv (2016). Accessed 18.11.24. url: [https://arxiv.org/pdf/1606.02744](https://arxiv.org/pdf/1606.02744)

# Double Pulsar PSR J0737-3039A/B

- Orbital period of just 2.45 hours
- Both the 1PN and the weak-field prediction match the observed data[4] well



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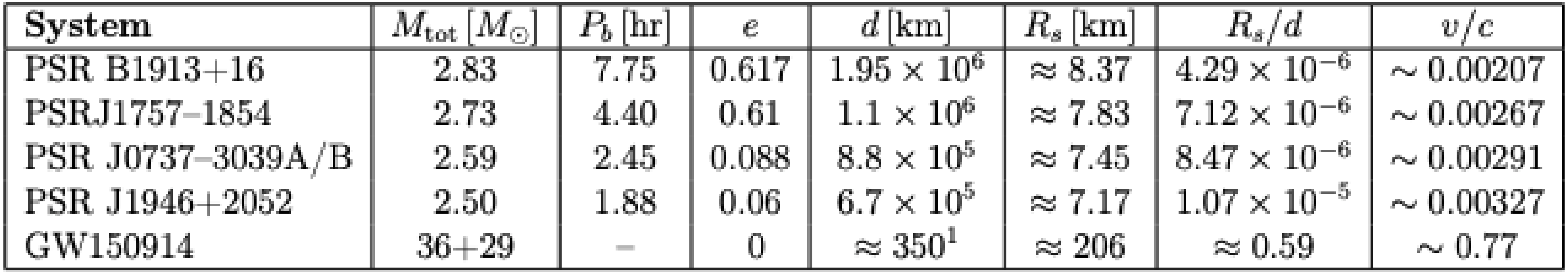
[4] M. Kramer et al. “Strong-field Gravity Tests with the Double Pulsar”. In: Physical Review X 11.4 (2021). Accessed 22.03.25. DOI: https://doi.org/10.1103/PhysRevX.11.041050

# When are 1PN Corrections Necessary?



| **System** | $M_{\text{tot}}\,[M_\odot]$ | $P_b$ [hr] | $e$ | $d$ [km] | $R_s$ [km] | $R_s/d$ | $v/c$ |
|---|---|---|---|---|---|---|---|
| PSR B1913+16 | 2.83 | 7.75 | 0.617 | $1.95 \times 10^6$ | $\approx 8.37$ | $4.29 \times 10^{-6}$ | $\sim 0.00207$ |
| PSRJ1757–1854 | 2.73 | 4.40 | 0.61 | $1.1 \times 10^6$ | $\approx 7.83$ | $7.12 \times 10^{-6}$ | $\sim 0.00267$ |
| PSR J0737–3039A/B | 2.59 | 2.45 | 0.088 | $8.8 \times 10^5$ | $\approx 7.45$ | $8.47 \times 10^{-6}$ | $\sim 0.00291$ |
| PSR J1946+2052 | 2.50 | 1.88 | 0.06 | $6.7 \times 10^5$ | $\approx 7.17$ | $1.07 \times 10^{-5}$ | $\sim 0.00327$ |
| GW150914 | 36+29 | – | 0 | $\approx 350^1$ | $\approx 206$ | $\approx 0.59$ | $\sim 0.77$ |

**PSR B1913+16:** J. M. Weisberg and Y. Huang. “Relativistic measurements from timing the binary pulsar PSR B1913+16”. (2016).

**PSRJ1757-1854**: A. D. Cameron et al. “New constraints on the kinematic, relativistic and evolutionary properties of the PSR J17571854 double neutron star system”. (2023)

**PSR J0737-3039A/B:** M. Kramer et al. “Strong-field Gravity Tests with the Double Pulsar”. In: Physical Review X 11.4 (2021)

**PSR J1946+2052:** K. Stovall et al. “PALFA Discovery of a Highly Relativistic Double Neutron Star Binary”. In: arXiv preprint arXiv:1802.01707 (2018).

**GW150914:** B. P. Abbott et al. “GW150914: First results from the search for binary black holecoalescence with Advanced LIGO”. In: Physical Review D 93.12 (2016).

# Conclusions and outlook

- Two complementary approximation methods
- In the near zone: leading order post-Newtonian expansion
- The 1PN model tracks the observed arrival shift closely for both PSR B1913+16 and PSR J0737-3039A/B
- In the far zone: the shortwave approximation
- Together: PN theory describes how gravitational waves are generated, while the shortwave approximation describes how they propagate
- Outlook:
  - Including higher order of the PN expansion
  - Including corrections beyond geometric optics
  - Bridging the two regimes

# For more information:

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