Hybrid Stars with the Quark-Meson Model

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Master's thesis excerpt

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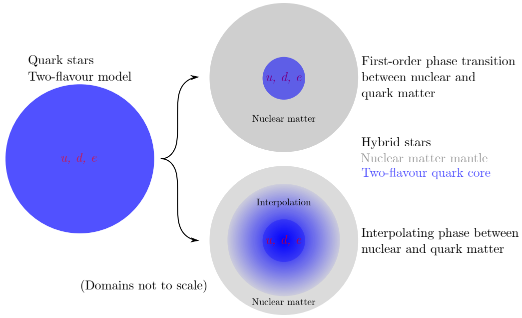
Outline

1. Spherically Symmetric Compact Stars
2. Two-Flavour Quark-Meson Model
3. Mass-Radius Relations for Quark Stars
4. Hybrid Equation of State
5. Hybrid Stars
6. Unified Hybrid Stars



Spherically Symmetric Compact Stars

We model compact stars with the TOV-equation,

$$\frac{dp}{dr} = -\frac{GM\epsilon}{r^2}\left[1+\frac{p}{\epsilon}\right]\left[1+\frac{4\pi r^3 p}{M}\right] \cdot \left[1-\frac{2GM}{r}\right]^{-1}, \quad [c=1]$$

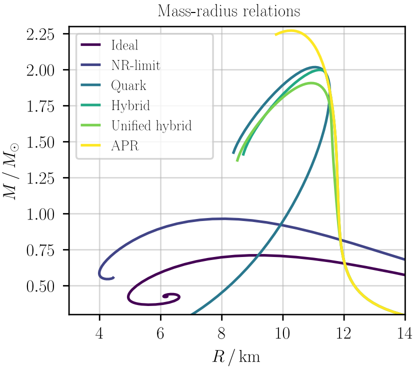
$$M = \int_0^r dr'\, 4\pi r'^2 \epsilon(r').$$

To close this system of equations, we need an equation of state (EoS),

$$\epsilon = \epsilon(p).$$

Integrating the TOV-equation yields mass-radius relations.

- Maximum mass of $\gtrsim 2M_\odot$ [1] (stiffness of EoS)
- Speed of sound $\frac{v_s}{c} = \left(\frac{\partial p}{\partial \epsilon}\right)^{\frac{1}{2}} < 1$



[1] R. W. Romani *et al.* (2022) *PSR J0952 0607: The Fastest and Heaviest Known Galactic Neutron Star*

Two-Flavour Quark-Meson Model

Two-flavour quark-meson (QM) Lagrangian

$$\mathcal{L}_{\mathrm{QM}} = \bar{q}\left[i\partial\!\!\!/ + \mu\gamma^0 - g(\sigma + i\gamma^5 \boldsymbol{\tau}\cdot\boldsymbol{\pi})\right] q + \frac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma + \partial_\mu\boldsymbol{\pi}\partial^\mu\boldsymbol{\pi}\right) - \frac{\lambda}{4}\left(\sigma^2 + \boldsymbol{\pi}^2 - v^2\right)^2 + h\sigma.$$

Mean field approximation in the mesonic sector leads to quark part of the Lagrangian

$$\mathcal{L}_q = \bar{q}\left[i\partial\!\!\!/ + \mu\gamma^0 - g\langle\sigma\rangle\right] q.$$

Grand canonical partition function for a fermion ξ

$$\Theta = \oint_{-} \mathcal{D}\xi\mathcal{D}\xi^\dagger \exp\left(\int_0^\beta d\beta \int d^3x \mu\mathcal{N} - \xi^\dagger\dot{\xi} - \mathcal{H}\right).$$

From Θ, we can find all we need[2]

$$p = \frac{\ln(\Theta)}{\beta V} = -\Omega,$$

$$n_i = -\frac{\partial\Omega}{\partial\mu_i},$$

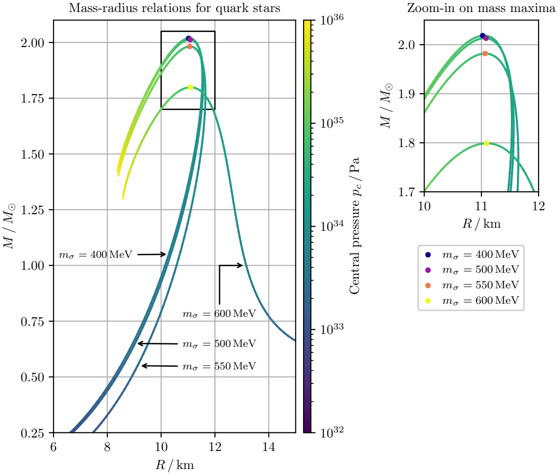
$$\epsilon = \frac{\partial(\beta\Omega)}{\partial\beta} + \sum_i \mu_i n_i.$$

[2] Additional constraints: $\sum_i q_i n_i(r) = 0$, $\mu_u + \mu_e = \mu_d$, and $\frac{\partial\Omega}{\partial\langle\sigma\rangle} = 0$.

Mass-Radius Relations for Quark Stars

Using the QM-model Lagrangian, we find $\epsilon_{\rm QM} = \epsilon_{\rm QM}(p)$. Integrating the TOV-equation, we find mass-radius relations. m_σ is taken as a parameter, $m_\sigma \in [400, 600]\,\mathrm{MeV}$.

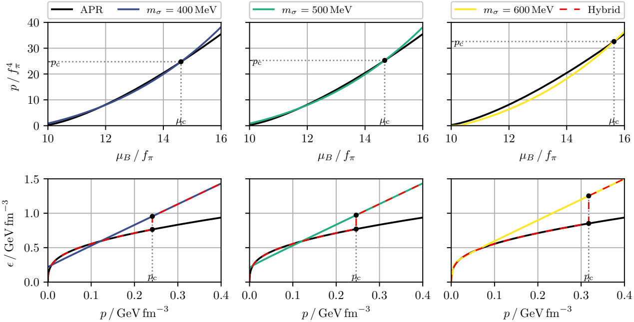
Quark stars assumes quark matter at relatively small energy densities.



Hybrid Equation of State

Combine nuclear matter EoS with a quark matter EoS. We use Akmal-Pandharipande-Ravenhall (APR) EoS[3].
Q: When to change from nuclear EoS to QM EoS?
Choose the EoS which *minimises the grand potential*, Ω.

$$\epsilon(p) = \begin{cases} \epsilon_{\mathrm{APR}}(p_{\mathrm{APR}}), & -p_{\mathrm{APR}}(\mu_B) < -p_{\mathrm{QM}}(\mu_B) \\ \epsilon_{\mathrm{QM}}(p_{\mathrm{QM}}), & -p_{\mathrm{APR}}(\mu_B) > -p_{\mathrm{QM}}(\mu_B). \end{cases}$$



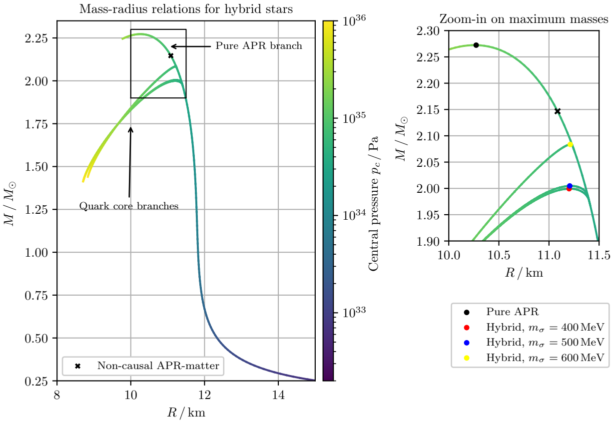
[3] A. Akmal *et al.* (1998) *Equation of state of nucleon matter and neutron star structure.*

Hybrid Stars

The hybrid EoS and the TOV-equation yields hybrid star mass-radius relations.

Small stable branches with QM-model core.

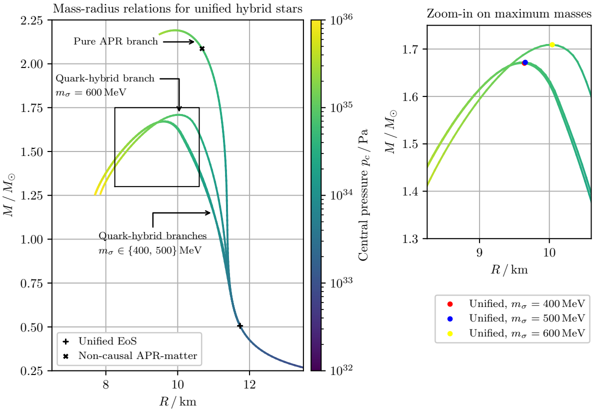
APR EoS yields a superluminar speed of sound at large enough energy densities.



Unified Hybrid Stars

Another approach: Interpolate APR and QM between number densities n_{lower} and n_{upper}.

Interpolation introduces a choice of when to stop using APR and when to start using the QM model.



Mass-radius relations with $n_{\mathrm{lower}} = 2n_0$, $n_{\mathrm{upper}} = 6n_0$

$$p(\mu_B) = \begin{cases} p_{\mathrm{APR}}(\mu_B), & n_{APR}(\mu) \le n_{\mathrm{lower}} \\ p_{\mathrm{inter}}(\mu_B), & n_{APR}(\mu_B) > n_{\mathrm{lower}} \quad \text{and} \quad n_{QM}(\mu_B) < n_{\mathrm{upper}} \\ p_{\mathrm{QM}}(\mu_B), & n_{QM}(\mu_B) \ge n_{\mathrm{upper}} \end{cases}$$

Outlook

A natural extension of this is to go beyond the QM two-flavour model. This gives rise to interesting phenomena *e.g.* modelling a colour superconducting phase.

This is up to Mathias to present – Stay tuned!