N-PACT Meeting 2025

PT2GWFinder

A Handy Tool for Cosmological PTs and GWs

LIP-Minho | U. Aveiro | U. Stavanger

2025-06-19

Marco Finetti

Support

PRT/BD/154730/2023

Bolsas de Investigação para Doutoramento FCT-ECIU

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Universidade de Aveiro
theoria poiesis praxis

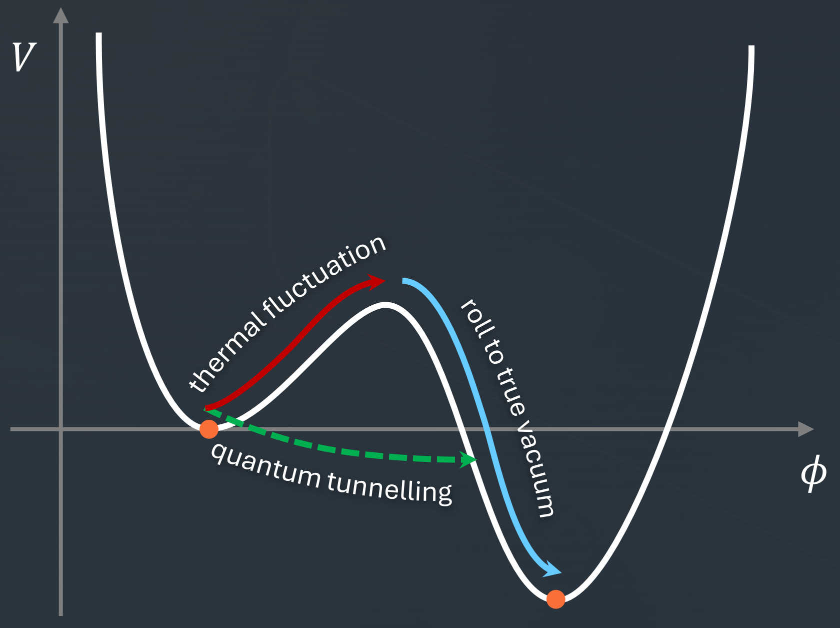
Universitetet i Stavanger

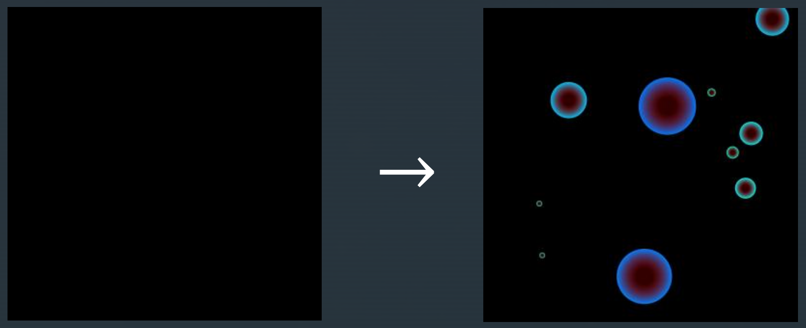
ECIU university

Fundação para a Ciência e a Tecnologia

Cosmological Phase Transitions
& PP motivation

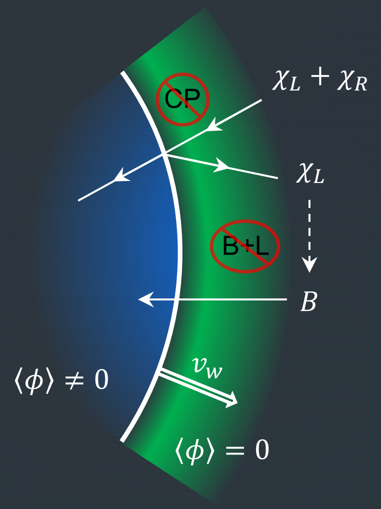
I order phase transitions (FOPTs)



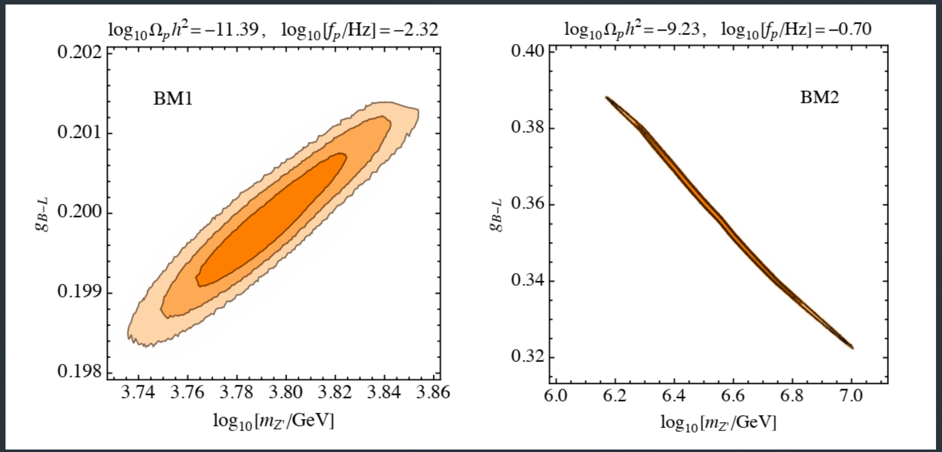


Motivation

↑ EW baryogenesis

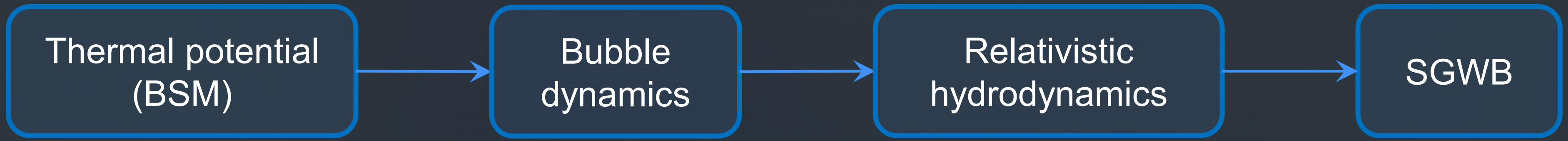


↓ Constraints on BSM models



C. Caprini et al. (JCAP10(2024)020)

Thermal potential → GW
analytics



EFT $\overset{?}{\to}$ SGWB

1. Analytics
   - bounce action: $\frac{S_3}{T}(T)$
     $\frac{d^2\phi}{dr^2} + \frac{2}{r}\frac{d\phi}{dr} = \partial_\phi V(\phi, T)$
   - nucleation: $\frac{S_3}{T} \approx k$ (EW: $k \approx 140$)
   - transition strength: $\alpha = \frac{1}{\rho_\gamma}\Delta\left[V - \frac{T}{4}\partial_T V\right]$
   - inverse duration: $\frac{\beta}{H} = T\frac{d}{dT}\frac{S_3}{T}$

$\longrightarrow$

decay rate: $\Gamma = A(T)\left(\frac{S_3}{2\pi T}\right)^{\frac{3}{2}} e^{-\frac{S_3}{T}}$

T_n: $\int_{T_n}^{T_c} \frac{dT}{T}\frac{\Gamma}{H^4} \sim \mathcal{O}(1)$

T_p: $P_{FV}(T) \approx 0.71$

$$\log P_{FV} = \frac{4\pi}{3} v_w^3 \int_T^{T_p} \frac{dT'}{T'^4}\frac{\Gamma(T')}{H^4(T')}\left(\int_T^{T'} \frac{d\tilde{T}}{H(\tilde{T})}\right)^3$$

2. Numerical methods

Thermal potential → GW
tool comparison

Similar packages

Package	Language	Maintained	Dimensional reduction	PT+GW parameters
CosmoTransitions	`Python`	X	X	X
BSMPTScanner	`C++`	✓	X	✓
PhaseTracer	`C++`	✓	~	✓

Thermal potential → GW

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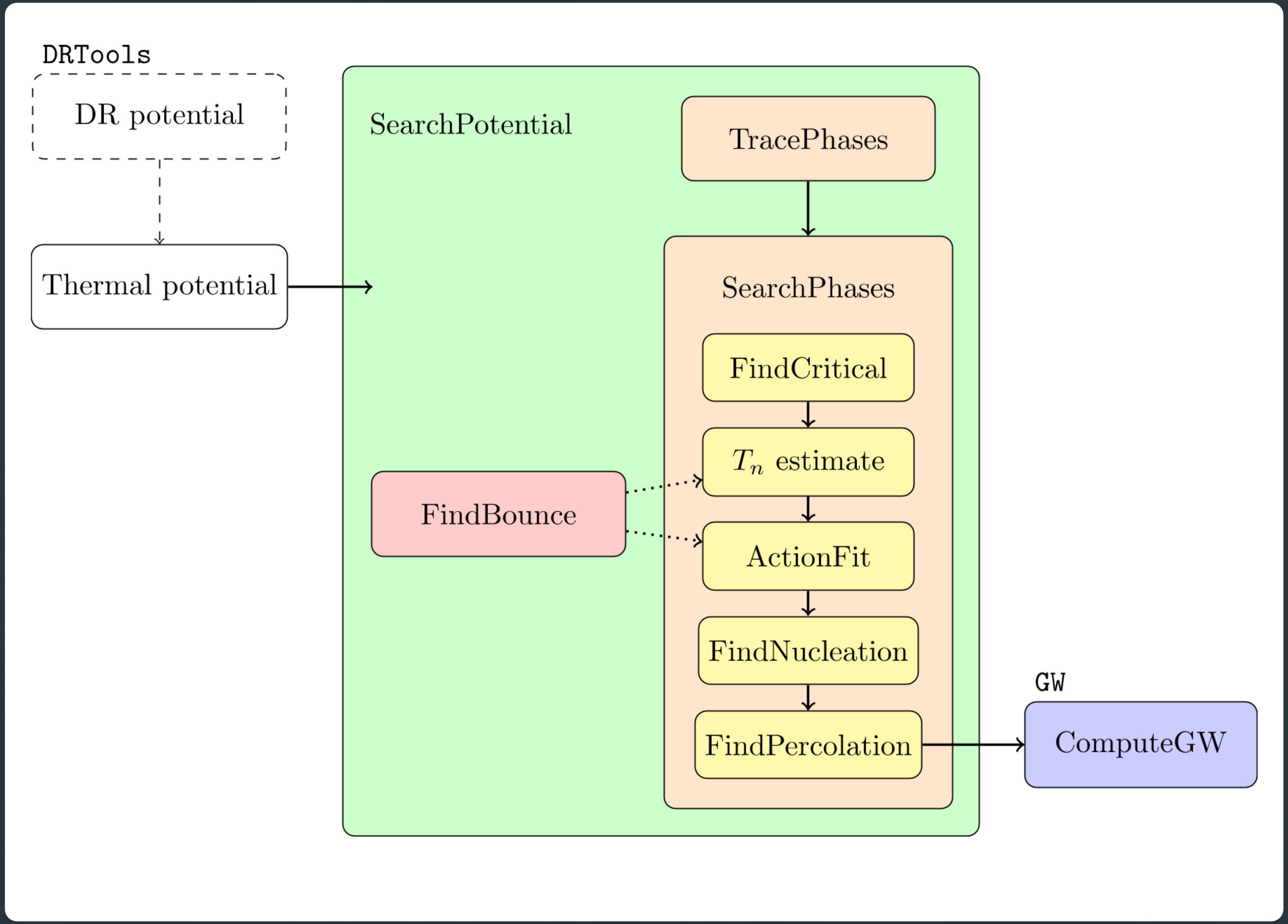
PT2GW	`Mathematica`	$\geq t_{graduation}$	✓	✓

arXiv 2505.04744

Strengths

- User-friendly
- Freedom to construct effective potential
  - interface with DRalgo
- Polygonal bounce method (FindBounce)
- Modular

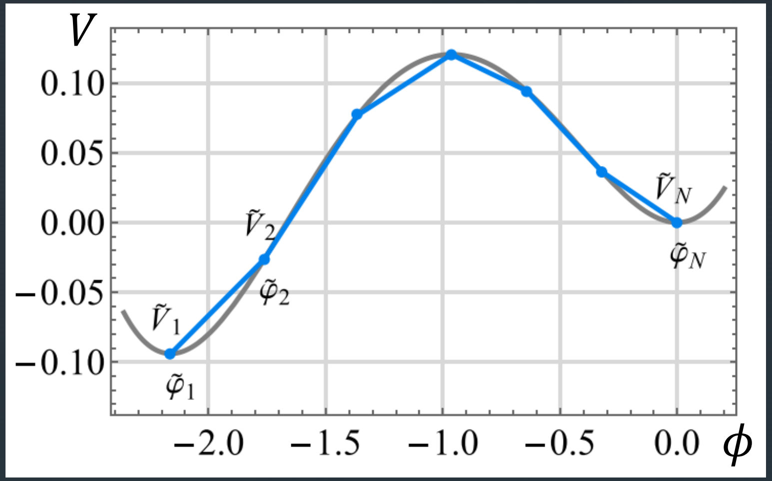
PT2GWFinder
schematics



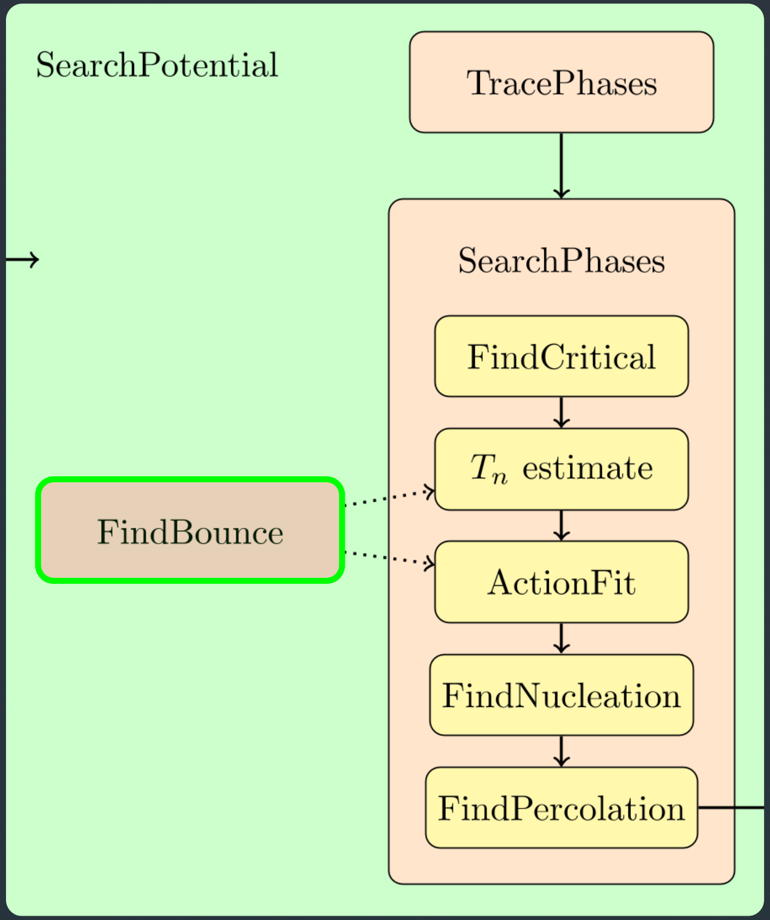
PT2GWFinder
method

- $\frac{S_3}{T}(T)$ numerical estimation: FindBounce
  - ✓ thin-wall regime
  - efficient: $t \sim \mathcal{O}(\#\text{ fields}),\ \mathcal{O}(\#\text{ segments})$
  - $4d\ (S_4)$ or $3d\ (S_3)$

polygonal bounce



Guada, Nemevšek, Pintar (CPC 256 (2020) 10748)



PT2GWFinder
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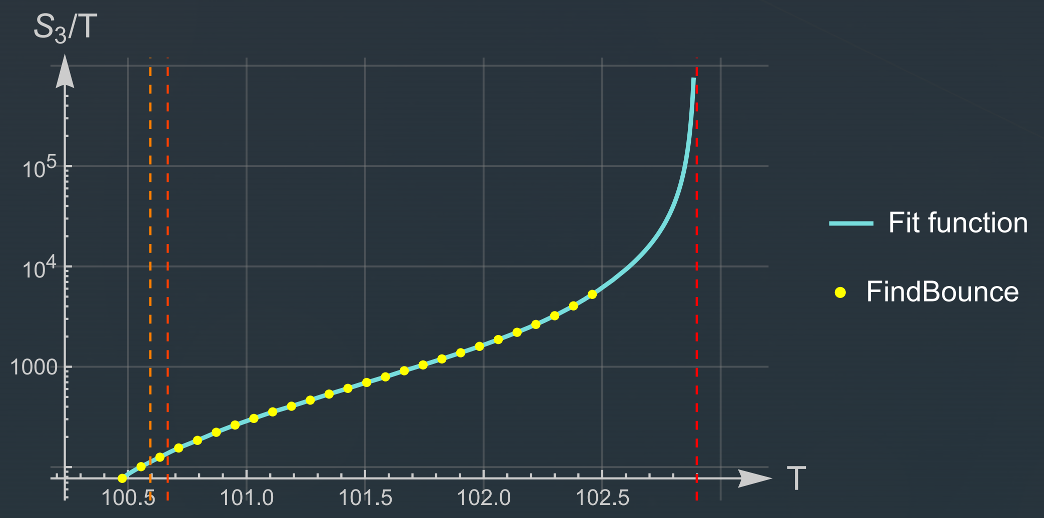
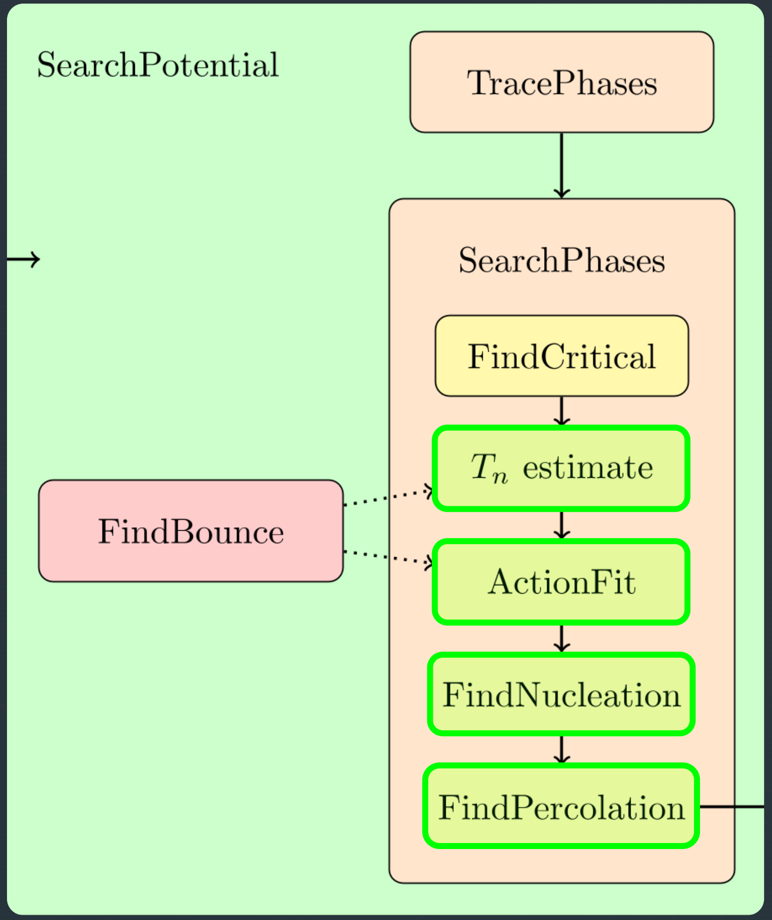
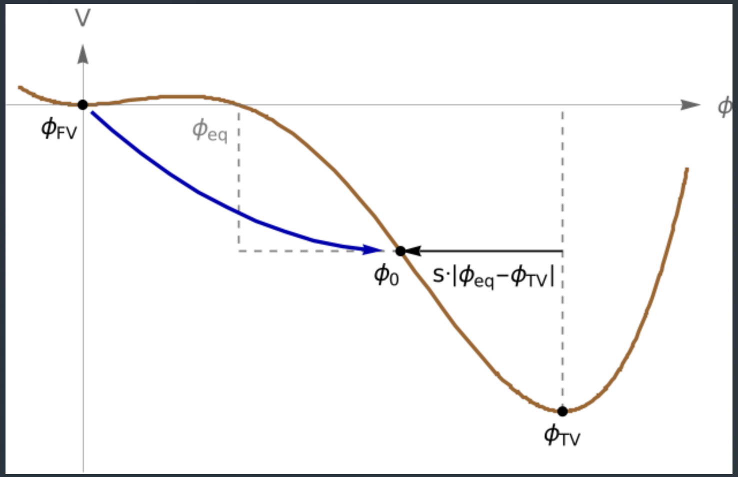
  - ✓ thin-wall regime
  - efficient: $t \sim \mathcal{O}(\text{\# fields}),\ \mathcal{O}(\text{\# segments})$
  - $4d\ (S_4)$ or $3d\ (S_3)$
- Method
  1. $\tilde{T}_n$ estimate via $\Gamma/H^4 \sim 1$
  2. S_3/T fit/interpolation about $(\sim \tilde{T}_n, T_c)$
  3. T_n, T_p via above integrals

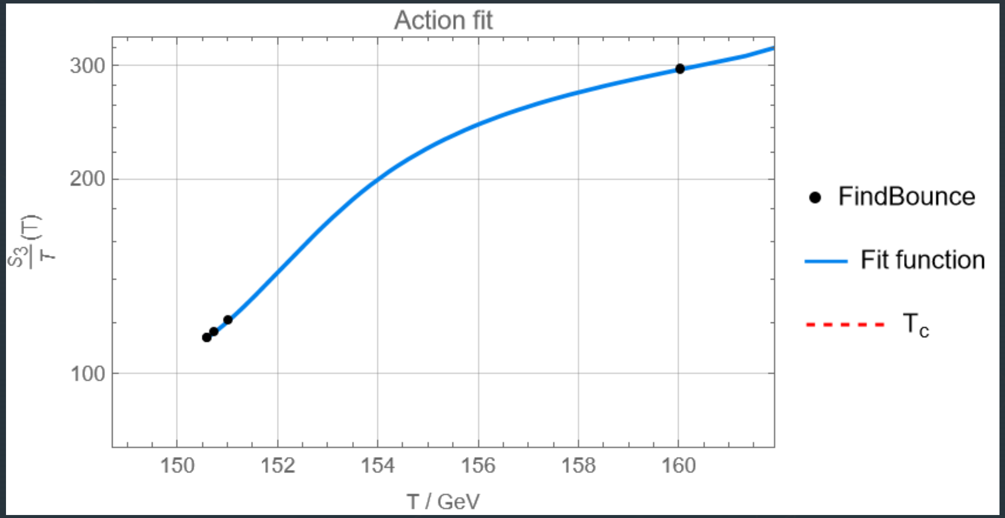




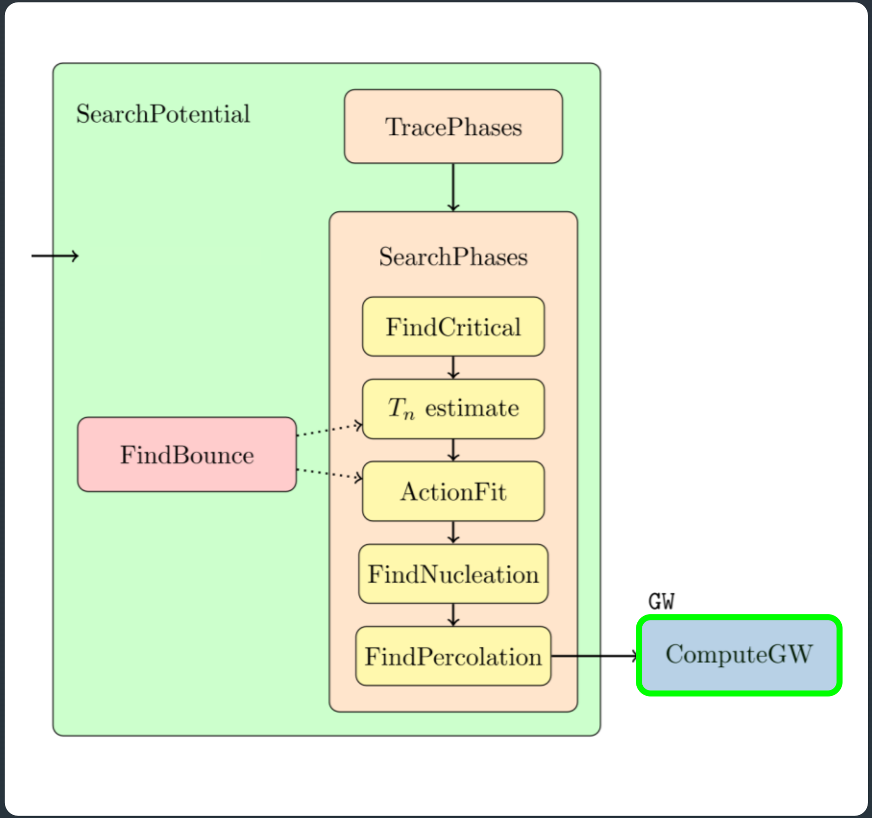
- Several options to refine the bounce solution
  - Check profile
    - Shift of exit point
    - Lower action tolerance



  - Refine around inflection
    - Bisection method to identify the failure temperature



PT2GWFinder
GWs



Soundwaves & MHD turbulence:
→ double broken power law

$$h^2\Omega_{sw} \sim K^2 \frac{f}{f_1^{sw}} \left[1+\left(\frac{f}{f_1^{sw}}\right)^2\right]^{-1} \left[1+\left(\frac{f}{f_2^{sw}}\right)^4\right]^{-1}$$

$$h^2\Omega_{tur} \sim \varepsilon^2 K^2 \frac{f_1^{tur}}{f_2^{tur}} \left(\frac{f}{f_1^{tur}}\right)^3 \left[1+\left(\frac{f}{f_1^{tur}}\right)^4\right]^{-\frac{1}{2}} \left[1+\left(\frac{f}{f_2^{tur}}\right)^{2.15}\right]^{-1.7}$$

Bubble wall collisions:

→ broken power law

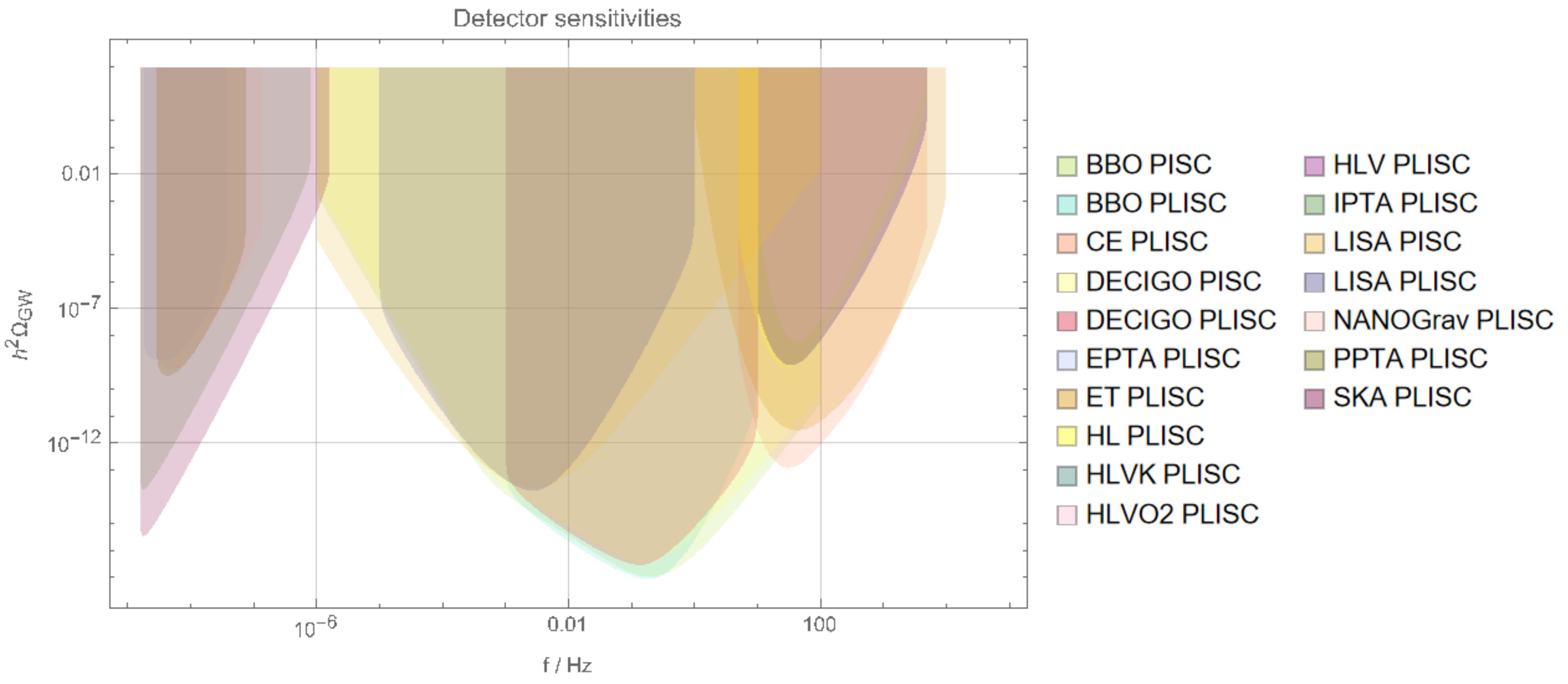
$$h^2\Omega_{sw} \sim \tilde{K}^2 \left(\frac{f}{f_p}\right)^{2.4} \left[1+\left(\frac{f}{f_p}\right)^{1.2}\right]^{-4}$$

C. Caprini et al. (JCAP10(2024)020)

PT2GWFinder
GWs

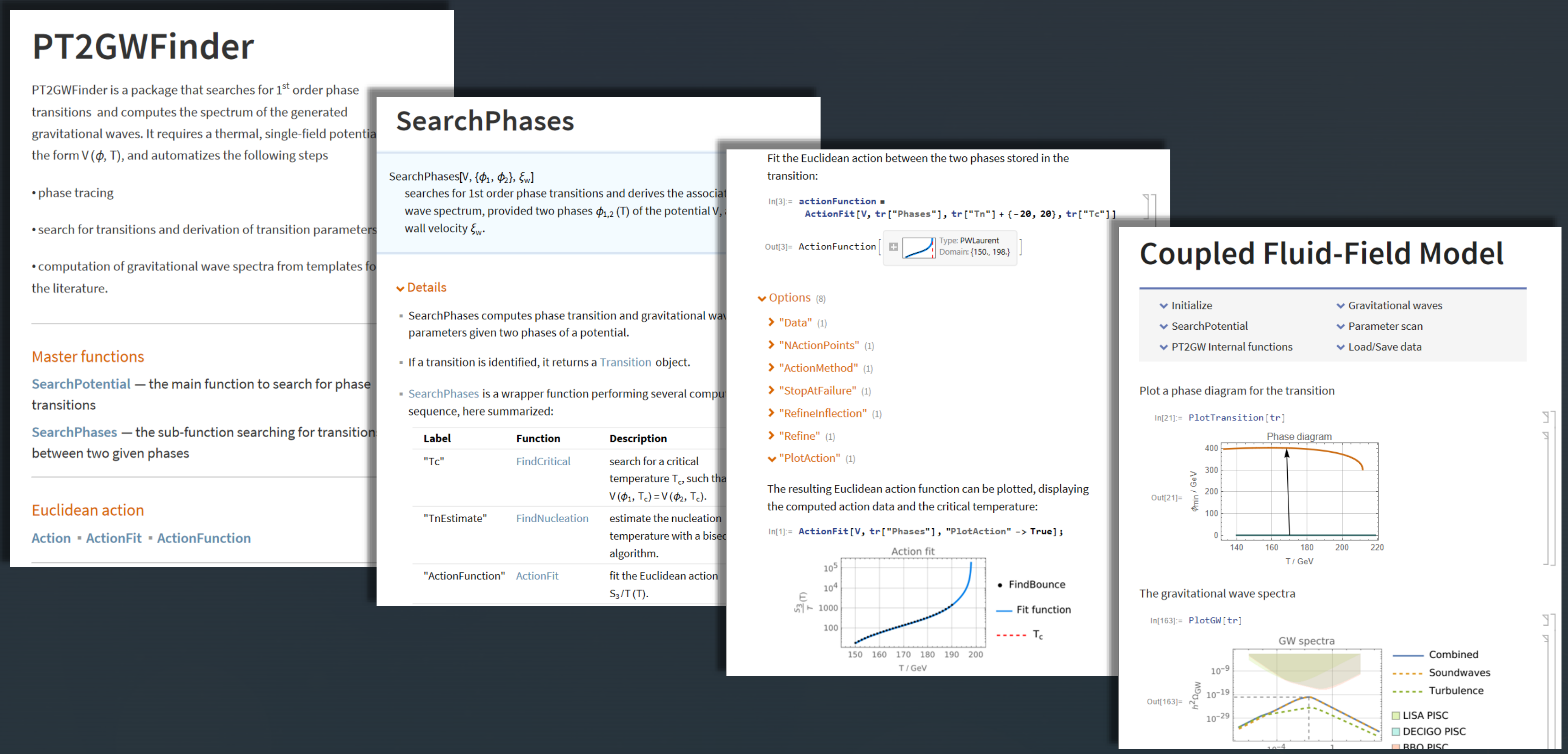
```
In[2]:= PlotGWSensitivities[10^{-9, 5}, All]
```

Out[2]=



data adapted from K. Schmitz (JHEP01(2021)097)

PT2GWFinder
fully documented



PT2GWFinder

PT2GWFinder is a package that searches for 1st order phase transitions and computes the spectrum of the generated gravitational waves. It requires a thermal, single-field potentia[l] the form V (ϕ, T), and automatizes the following steps

- phase tracing
- search for transitions and derivation of transition parameters
- computation of gravitational wave spectra from templates fo[r] the literature.

Master functions

SearchPotential — the main function to search for phase transitions

SearchPhases — the sub-function searching for transition[s] between two given phases

Euclidean action

Action ▪ ActionFit ▪ ActionFunction

SearchPhases

SearchPhases[V, {ϕ_1, ϕ_2}, ξ_w]
searches for 1st order phase transitions and derives the associa[ted] wave spectrum, provided two phases $\phi_{1,2}$ (T) of the potential V, wall velocity ξ_w.

Details

- SearchPhases computes phase transition and gravitational wa[ve] parameters given two phases of a potential.
- If a transition is identified, it returns a Transition object.
- SearchPhases is a wrapper function performing several compu[tations in] sequence, here summarized:

Label	Function	Description
"Tc"	FindCritical	search for a critical temperature T_c, such tha[t] $V(\phi_1, T_c) = V(\phi_2, T_c)$.
"TnEstimate"	FindNucleation	estimate the nucleation temperature with a bise[ction] algorithm.
"ActionFunction"	ActionFit	fit the Euclidean action S_3/T (T).

Fit the Euclidean action between the two phases stored in the transition:

```
In[3]:= actionFunction =
  ActionFit[V, tr["Phases"], tr["Tn"] + {-20, 20}, tr["Tc"]]
Out[3]= ActionFunction[ Type: PWLaurent Domain: {150., 198.} ]
```

Options (8)
"Data" (1)
"NActionPoints" (1)
"ActionMethod" (1)
"StopAtFailure" (1)
"RefineInflection" (1)
"Refine" (1)
"PlotAction" (1)

The resulting Euclidean action function can be plotted, displaying the computed action data and the critical temperature:

```
In[1]:= ActionFit[V, tr["Phases"], "PlotAction" -> True];
```


Coupled Fluid-Field Model

Initialize
SearchPotential
PT2GW Internal functions
Gravitational waves
Parameter scan
Load/Save data

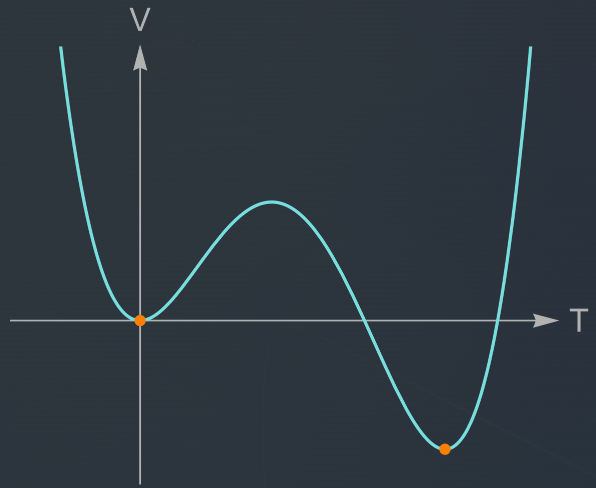
Plot a phase diagram for the transition

```
In[21]:= PlotTransition[tr]
```


The gravitational wave spectra

```
In[163]:= PlotGW[tr]
```

Example
Fluid-field model



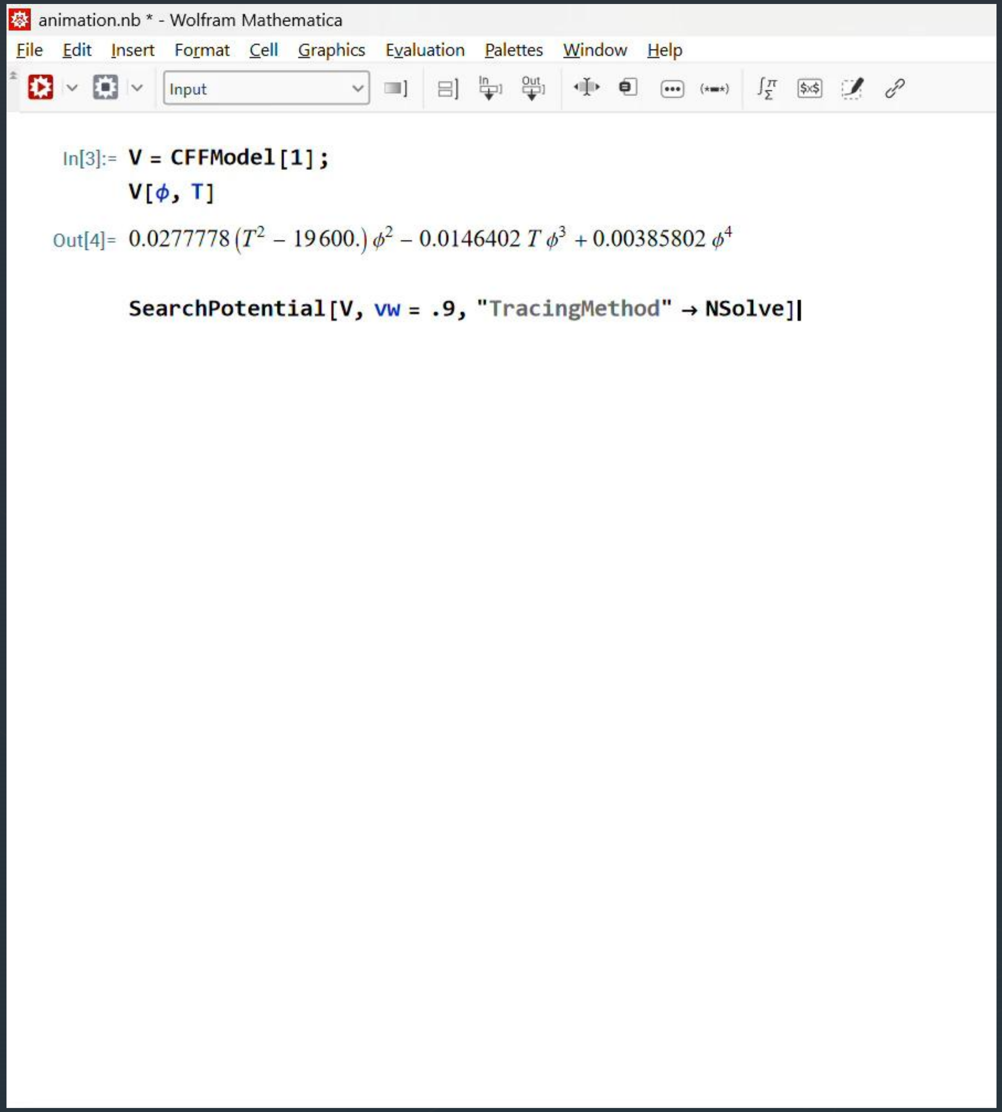
- Scalar potential

$$V(\phi,T) = \frac{c_2}{2}(T^2 - T_0^2)\phi^2 - \frac{c_3}{3}T\phi^3 + \frac{c_4}{4}\phi^4$$

- Analytic derivation of the action
  - in thin/thick wall regimes
  - intermediate interpolation

Matteini, Nemevšek, Shoji, Ubaldi (2024, 2404.17632)

Linde (1983, NPB 216. 2)
Hindmarsh, Huber, Rummukainen, Weir (PRD.92.123009)



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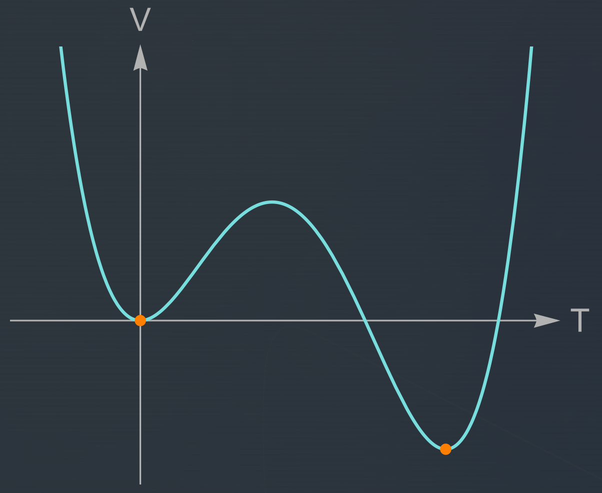

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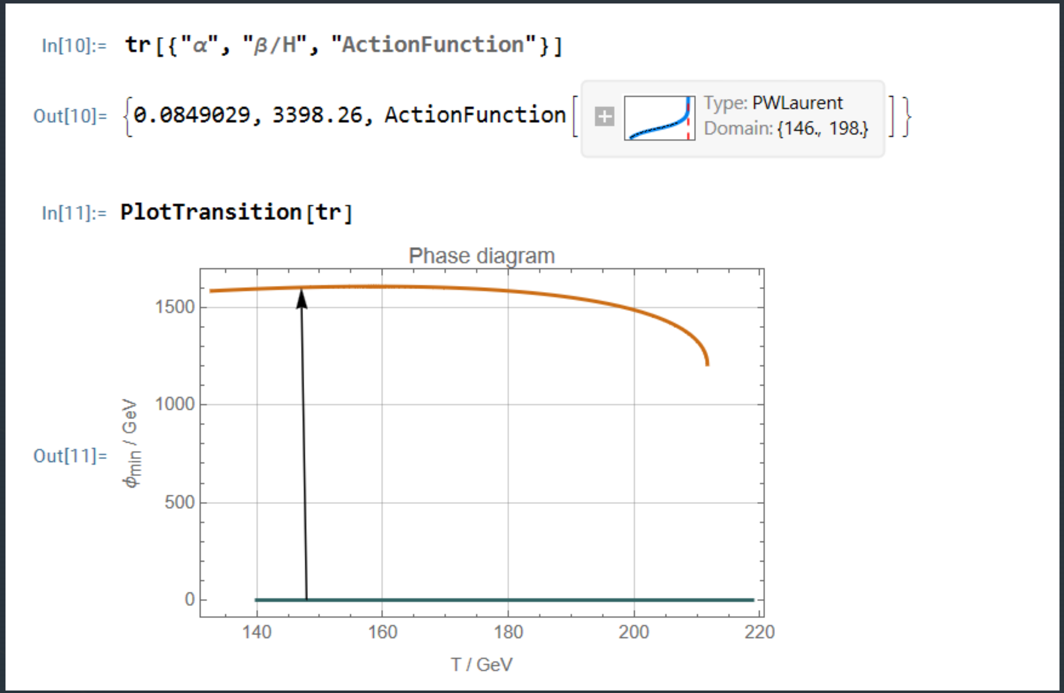

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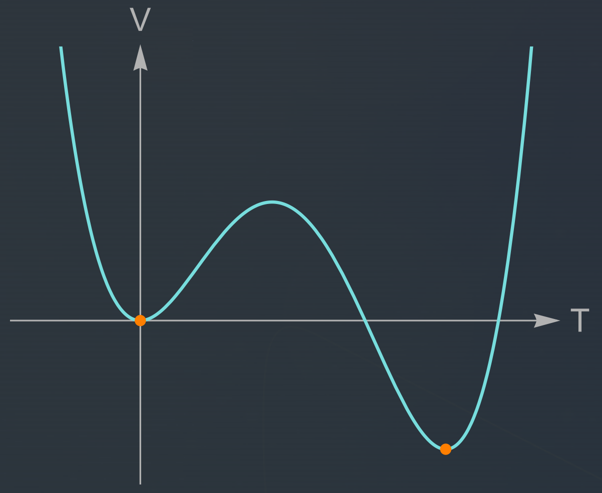

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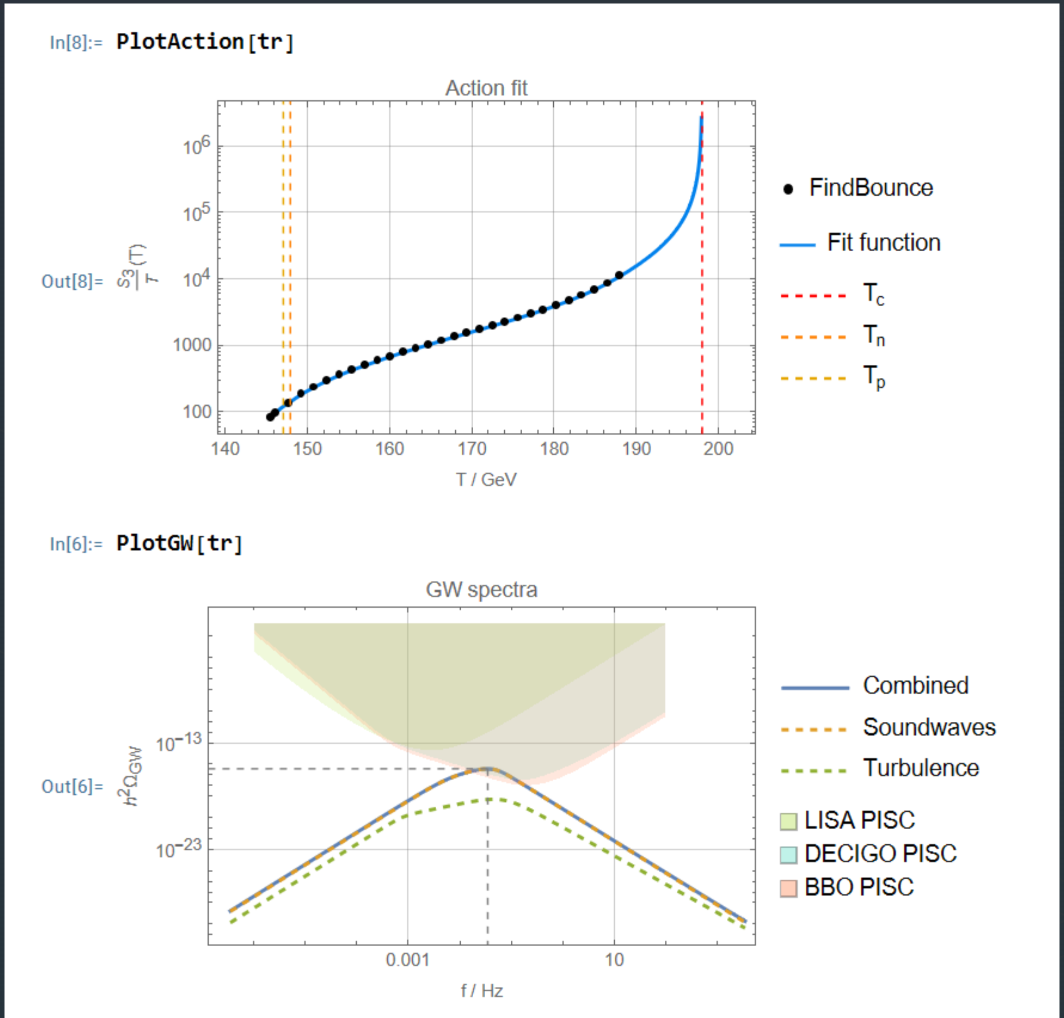

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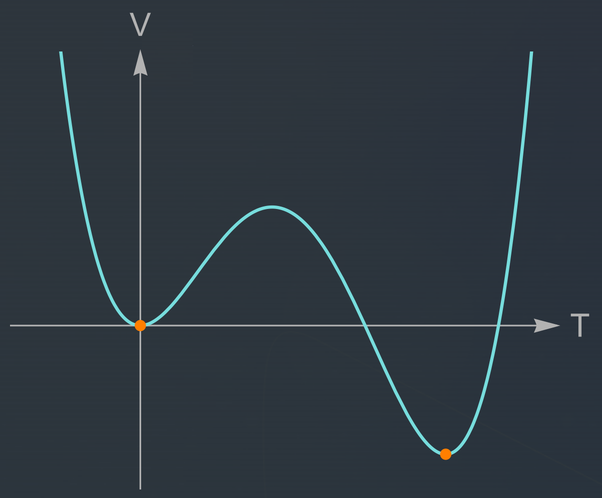

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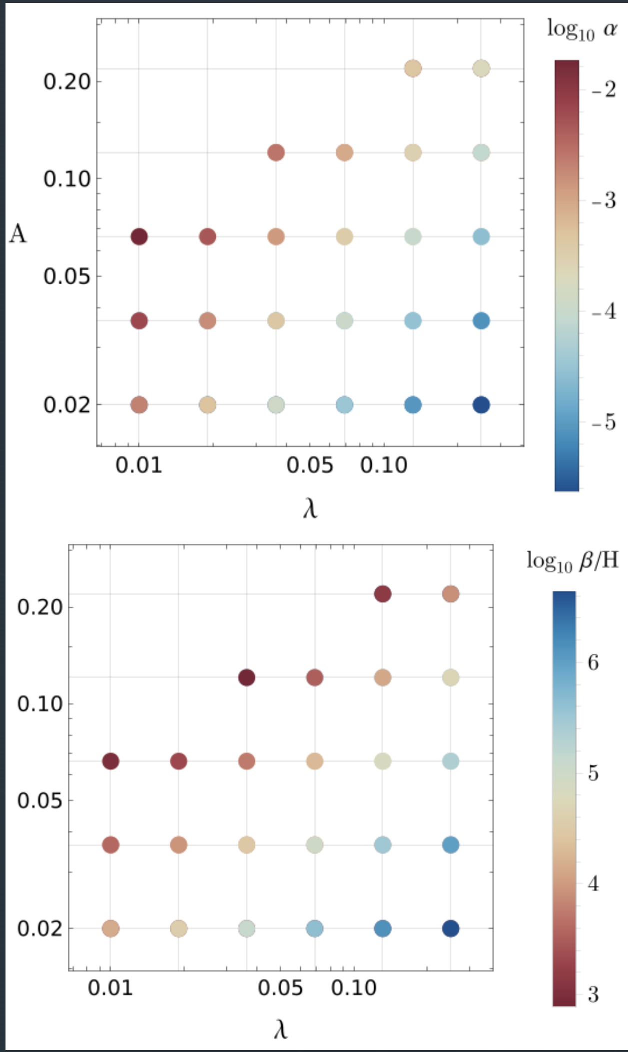

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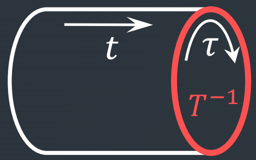
Dimensional Reduction
An improved recipe for thermal EFTs

- Dimensional reduction (DR)
  - time → temperature ⇒ high-T approach
  - include systematically higher-order resummations

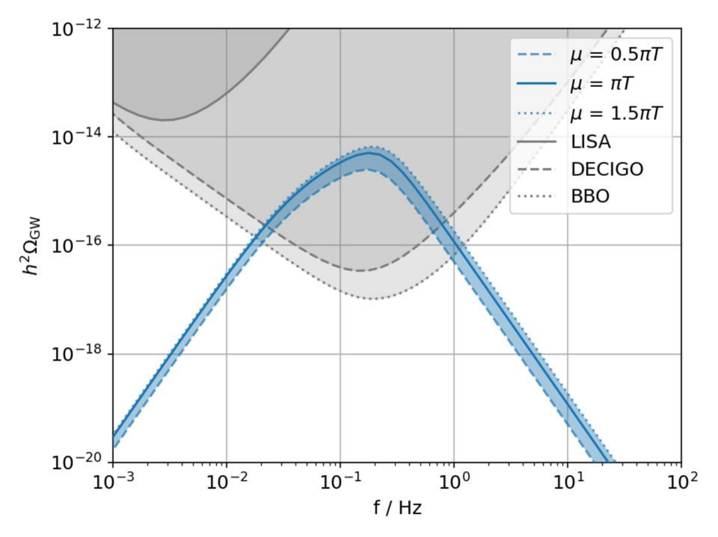
$$4d \xrightarrow{t \to t + i/T} 3d \text{ EFT}$$



- Narrower theoretical uncertainties

  ⇒ narrower GWB uncertainties

- DR implementation
  Automated extraction `Dralgo` → V_{eff} ,
  including
  - export of DR quantities
  - RG resolution
  - closed-form $V_{\text{eff}}(\phi, T)$



M. Bertenstam, MF, A.P. Morais, R. Pasechnik, J. Rathsman (2501.01286)

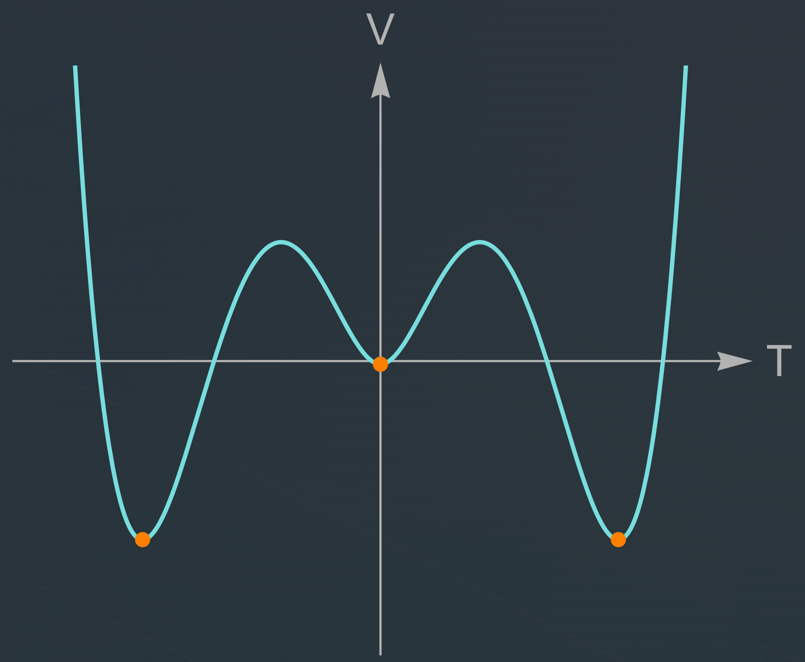
Dimensional Reduction
Dark Abelian Higgs

- Dark $U(1)$ gauge sector
- Scalar content:

  $V(\phi, T) = \mu^2\phi^2 + \lambda\phi^4$

  + fermions
- V_{eff} @ NLO

→ Paclet



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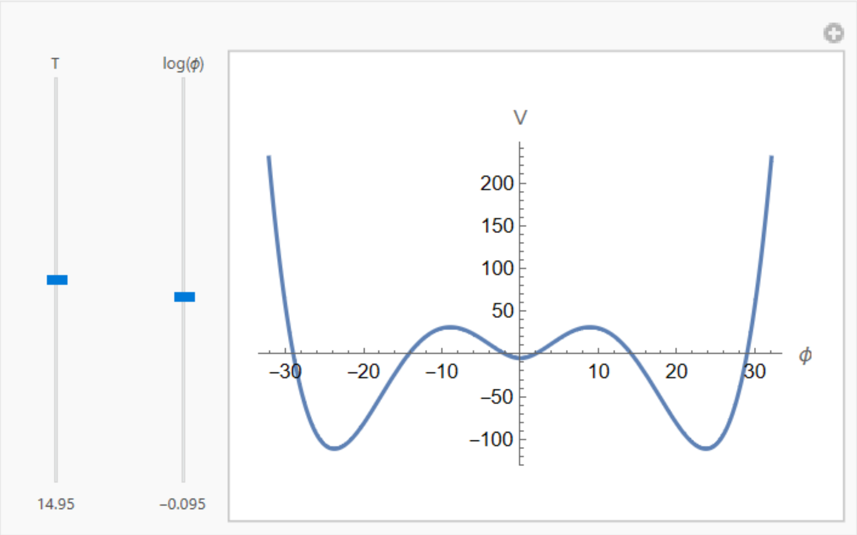
→ Paclet

```
In[84]:= <<PT2GW/DRTools.m
✓ Imported DRTools`

In[85]:= V=ComputeDRPotential[{"gsq","λ","μsq"}/.bp,{Λ0,Λ0/10,100 Λ0},
      "SubRules"→subRules,
      "OrderDR"→"NLO",
      "OrderVeff"→"NLO",
      "US"→False,
      "NumericRules"→{Y→1.}
      ];

In[86]:= PlotPotential[Re@*V,Λ0{-1,1},3/8 Λ0,"LogϕRange"→1,"LogTRange"→1]
```

Out[86]=



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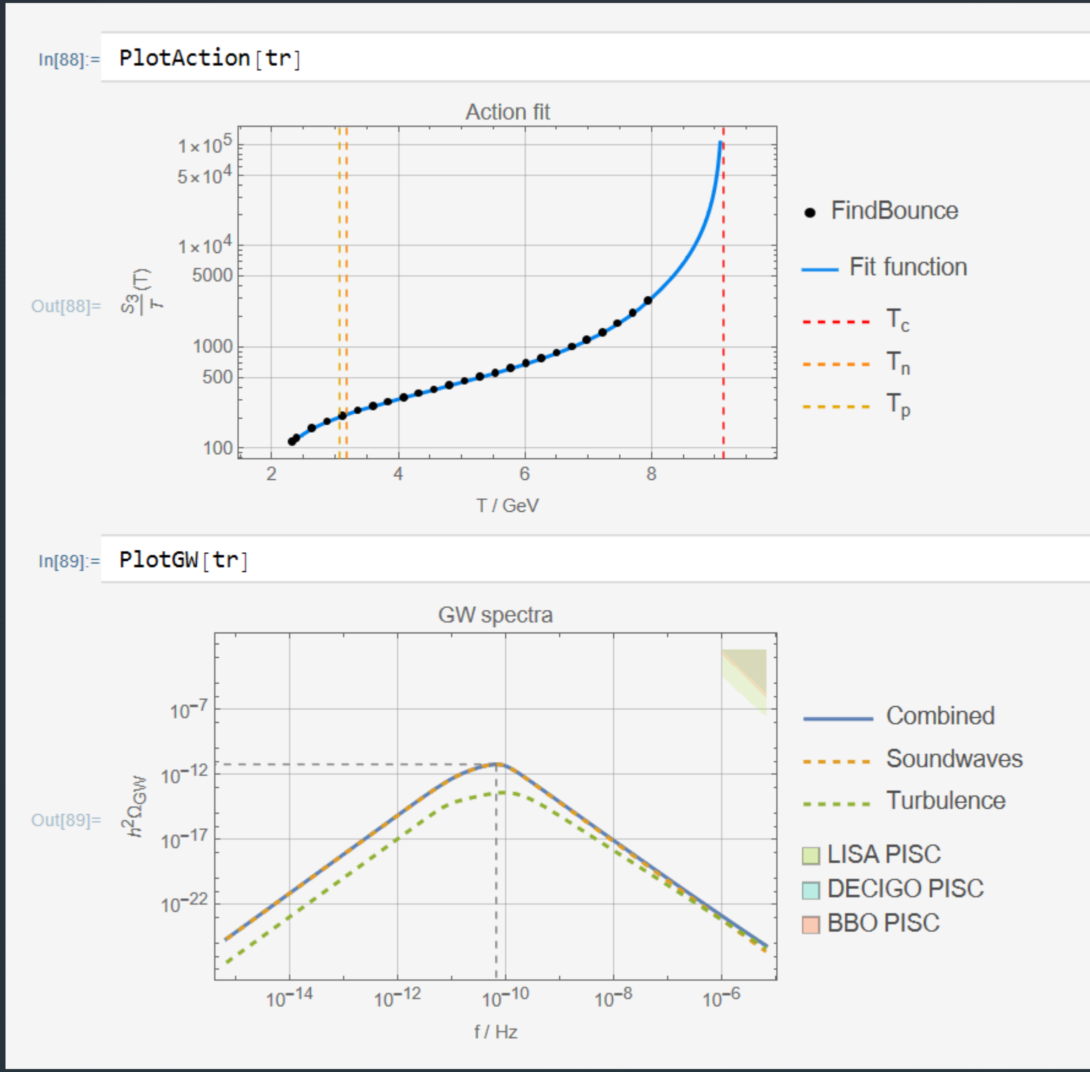

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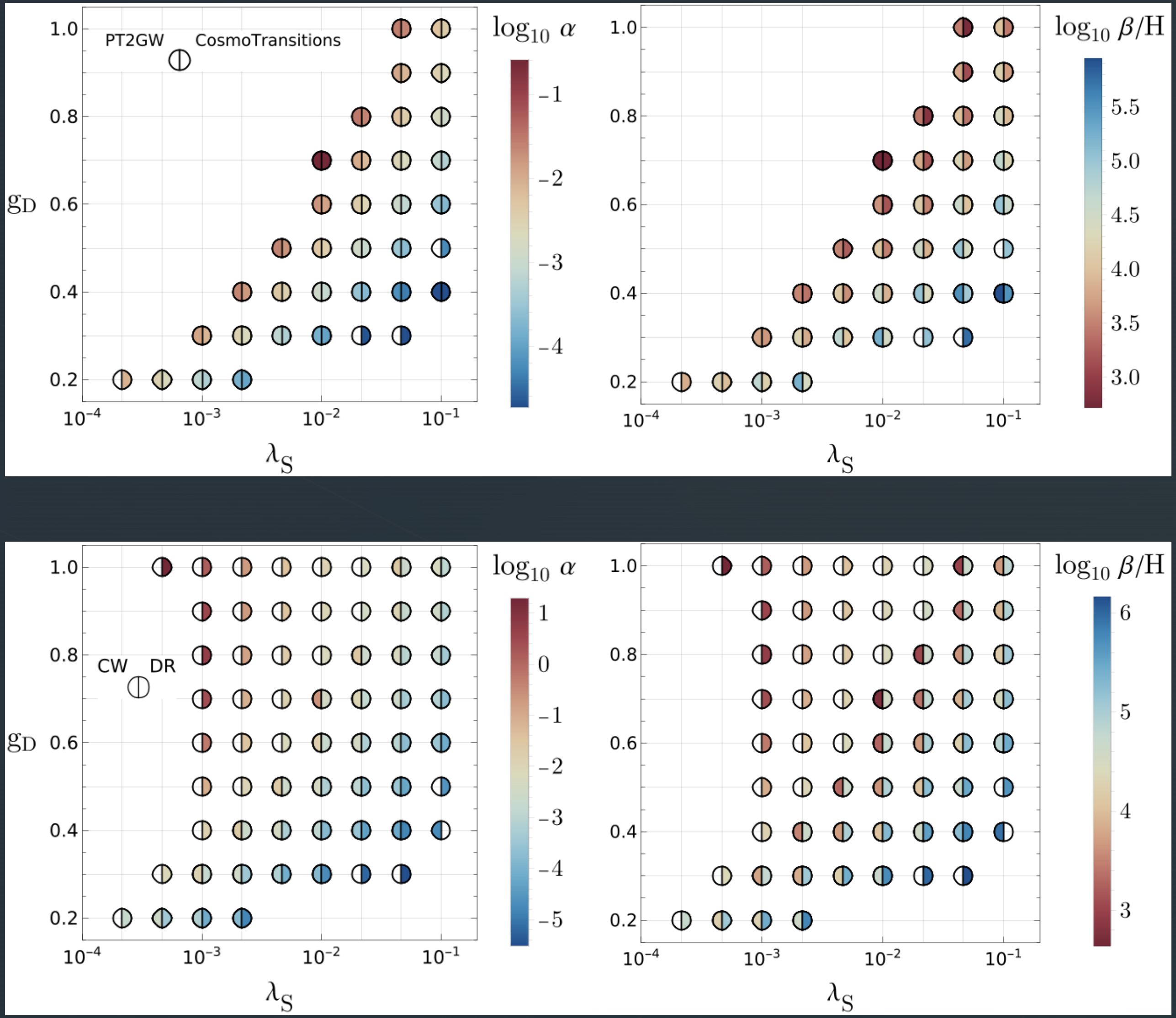
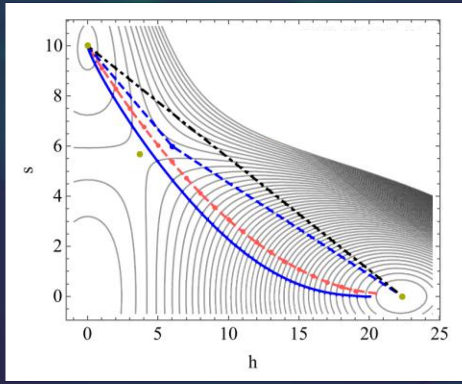


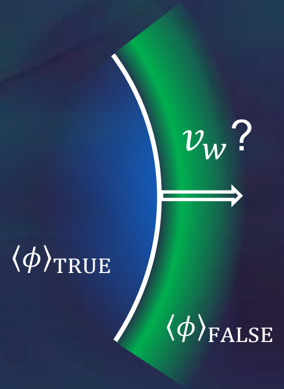
Outcome
& Future Endeavours

- Paclet current status
  - ✓ characterization of FOPTs and GWB
    - of single-field models
  - ✓ S_3/T via *polygonal bounce* (FindBounce)
  - ✓ optional, user-friendly interface with DRalgo



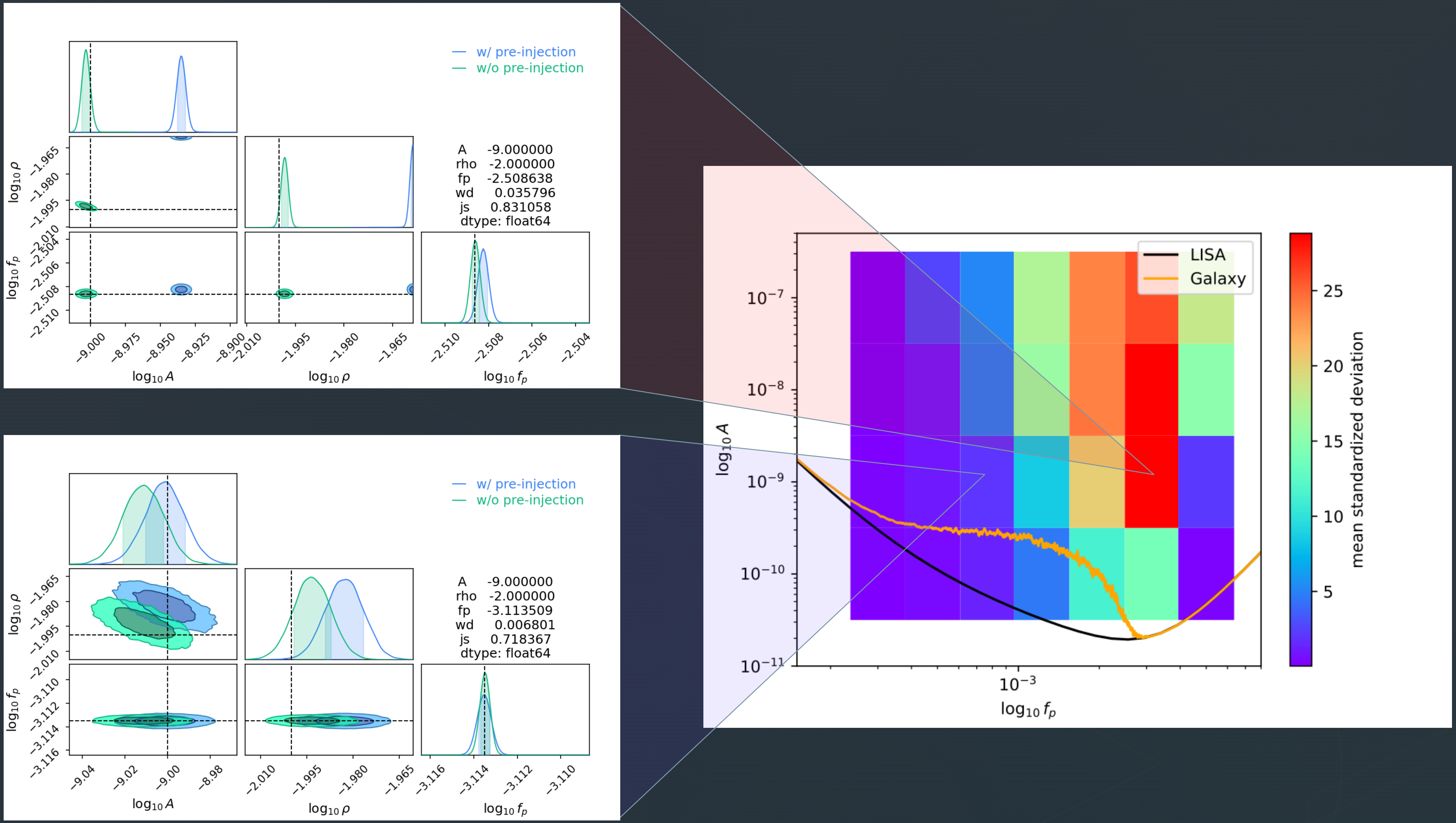
CPC 256 (2020) 10748

- Upcoming developments
  - ➢ improved phase-tracing routine
  - ➢ multi-field
- Potential developments
  - ? v_w estimation in LTE (WallGo)
  - ? Decay rate prefactor $\Gamma = A\, e^{-S_3/T}$
  - ? higher order in β
  - ? GW uncertainties



Suggestions are welcome!

LISA Project
Filtering out the Astrophysical Foreground



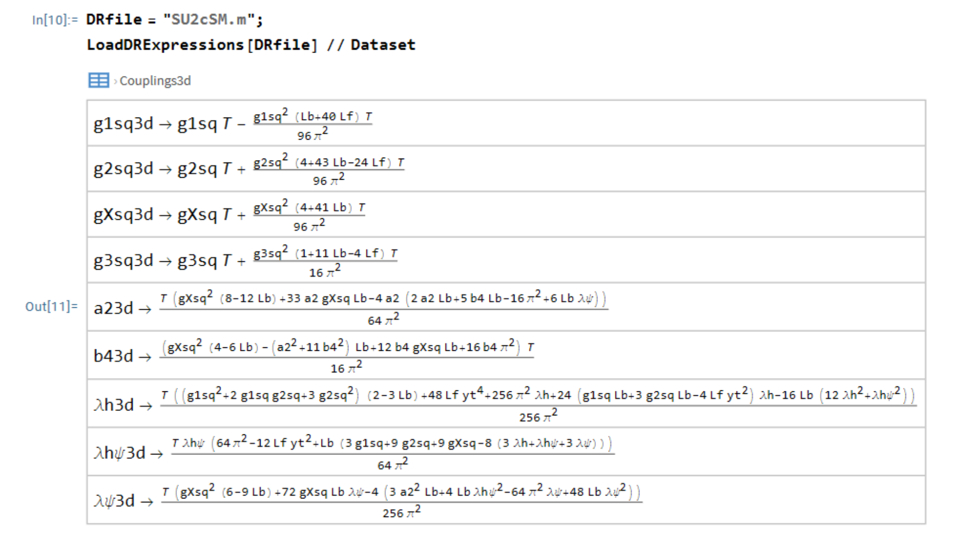
```
In[5]:= SetDirectory[NotebookDirectory[]];
        LoadDRExpressions["ahDRExpressions.m"]

        ComputeEffectivePotential[{gsq0,λ0,msq0},{μ0,μ0/10,100 μ0},
            subRules,"OrderVeff"→"NLO","LoadDRFrom"→"ahDRExpressions.m"]

                   gsq → InterpolatingFunction[ Domain: (10.  1.00·10^4) Output: scalar ]
» RG solutions     λ → InterpolatingFunction[ Domain: (10.  1.00·10^4) Output: scalar ]
                   msq → InterpolatingFunction[ Domain: (10.  1.00·10^4) Output: scalar ]

In[18]:= V[ϕ,μ0]
```

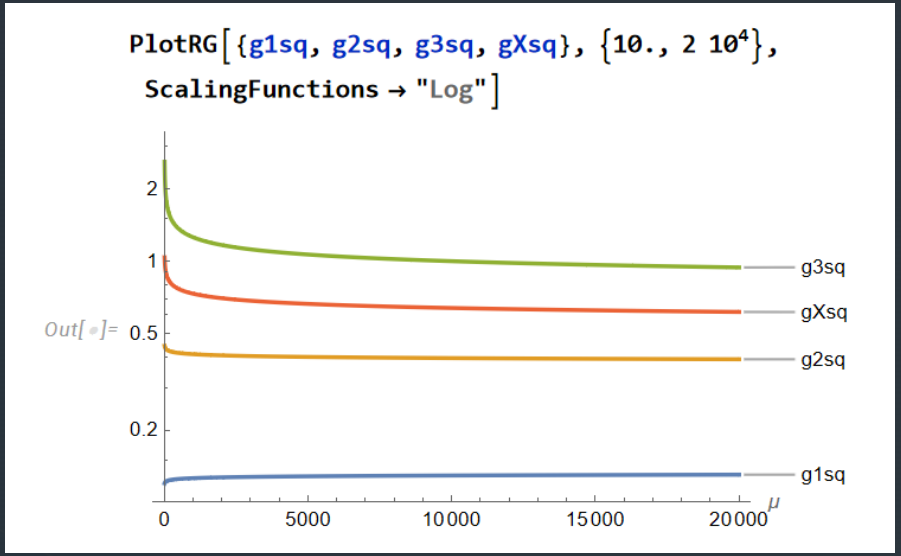
Out[18]=

$$-1.06103\left((53.3507-0.00338267\,\phi^2)^{3/2}+(53.3507-0.00112756\,\phi^2)^{3/2}+0.000265675\,\phi^4-25.1409\,\phi^2+0.118862\,(\phi^2)^{3/2}\right)$$

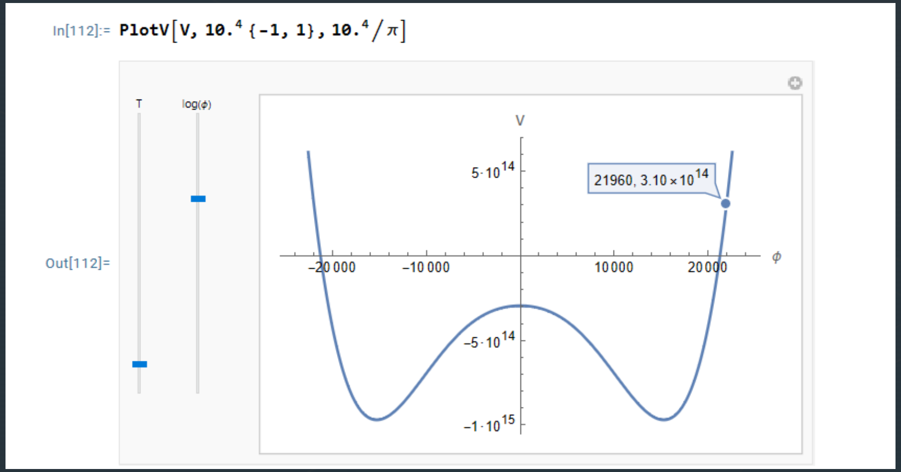
Extras

Dimensional Reduction
Paclet Tool





1. Extract & store DR expressions (⇐ `DRalgo`)
2. Benchmark-specific check functions
3. Construct effective potential



Extras