Higgs-Mediated Neutralino Pair Production in the Next-to-Minimal Supersymmetric Standard Model

Hadronic cross sections with real emission and virtual contributions

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Overview

A bullet-point abstract

- The Next-to-Minimal Supersymmetric Standard Model (NMSSM)
  - A gauge-singlet superfield scalar-fermion pair
  - Three CP-even and two CP-odd Higgs bosons
  - Five neutral fermions (neutralinos) $\tilde{\chi}_i^0$, $i = 1, \ldots 5$.
- Higgs-mediated neutralino pair production in proton–proton collisons
  - $\sigma(PP \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 + X)$
  - LO in Higgs-Yukawa and $O(\alpha_s)$ in QCD coupling
  - $q\overline{q}$, qg and $\overline{q}g$ incoming states
  - Summing all interferences

Overview

Motivation: The Higgs Hierarchy problem

A large di□erence, or *hierarchy*, between the electroweak scale and the Plack scale

$$\Lambda_{\text{EW}} \quad 100\ \text{GeV} \quad \Lambda_{\text{Planck}} \quad 10^{18}\ \text{GeV}\,, \qquad (m_{\text{proton}} \approx 1\ \text{GeV})$$

The Higgs boson discovered by ATLAS [1] and CMS [2] collaborations in 2012 has a mass $m_H \approx 125$ GeV.

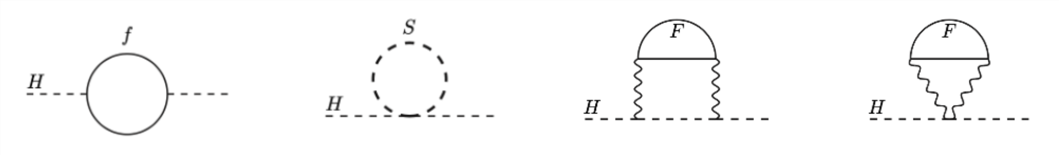


Figure 1: Perturbative corrections to the Higgs boson's mass.
Diagrams from S. P. Martin, Adv. Ser. Direct. High Energy Phys. 18 (1998) 1

[1] [hep-ex/1207.7214], [2] [hep-ex/1207.7235]

Motivation: The Higgs Hierarchy problem

A priori nothing in the Standard Model (SM) that prohibits m_H $O(\Lambda_{\text{Planck}})$. Possible solutions:

- An exact cancellation mechanism
- Fine-tuning between SM parameters
- New physics at some scale Λ_{new}, where $\Lambda_{\text{EW}} < \Lambda_{\text{new}} < \Lambda_{\text{Planck}}$

Motivation: The Higgs Hierarchy problem

A priori nothing in the Standard Model (SM) that prohibits $m_H \quad O(\Lambda_{\text{Planck}})$. Possible solutions:

- **An exact cancellation mechanism** $\quad =$ **Supersymmetry**
- Fine-tuning between SM parameters
- New physics at some scale Λ_{new}, $\Lambda_{\text{EW}} < \Lambda_{\text{new}} < \Lambda_{\text{Planck}}$

Prerequisite: $m = \tilde{m}$, however, superpartners are have not been observed experimentally.

Phenomenologically, supersymmetry must be **broken** at some scale $\Lambda_{\text{EW}} \quad \Lambda_{\text{SUSY}} < \Lambda_{\text{Planck}}$.

Review of the Hierarchy problem [hep-ph/9709356] (MSSM) and [hep-ph/0906.0777] (NMSSM)

Motivation: A Dark Matter candidate

Dark Matter - an umbrella term for matter hypothesized to exist from observations of anomalies in the dynamics of large astrophysical systems.

- Motion of stars and gas in galaxies
  [Begeman *et al.* Mon. Not. Roy. Astron. Soc. **249** (1991) 523.]
- Gravitational lensing of distant galaxies by foreground structures
  [Hoekstra *et al.* New Astronomy Reviews **46** (2002) 767]

No consensus on the exact nature of it, could be a collection of particles.
Non-baryonic, electromagnetically neutral particles are a promising class of candidates.

Supersymmetry has a candidate: the *neutral fermion* — a **neutralino**.

Motivation: A collider observable

The exploration of fundamental physics requires **high energy** and the ability to produce a large amount of **detectable interactions**.

Modern particle colliders serve as a means of achieving that, for instance, the Large Hadron Collider (LHC) at CERN.

The data manifests as the number of **events** N_{exp}

$$N_{exp} = \sigma_{exp} \int l(t) \mathrm{d}t \,. \tag{1}$$

The σ_{exp} is a **collider observable.**

LHC design report: Bruning *et al.* CERN Yellow Reports: Monographs. CERN, Geneva, June, 2004

Motivation: A collider observable

The σ_{exp} ID document
Name: Scattering cross section **Units**: [Area] **Occupation**: Relates theory to experiment **Family**: Related to the probability for a certain event and the inner product of quantum states

The $l(t)$ ID document
Name: Instantaneous luminosity **Units**: $[\text{Area}]^{-1}[\text{time}]^{-1}$ **Occupation**: Tells us how many particles pass through a certain unit of area per second **Family**: Related to the experimental set-up

Motivation: A collider observable

Let us talk a bit more about $\sigma_{exp} \equiv \sigma$.

Quantum mechanics speaks the language of **probability densities**.
These are, in turn, related to **inner product of quantum states**.

Modeling assumptions, motivated by the experimental set-up:

- A set of two incoming particles $\{i\}$, $i = 1, 2$.
- A set of n outgoing particles $\{f_n\}$.
- $\{i\}$ start out as free and non-interacting.
- The transition $\{i\} \to \{f_n\}$ takes a finite amount of time during the collision.

We expect σ to be related to the **time evolution** between the $\{i\}$ and $\{f_n\}$ states:

$$\sigma \equiv \sigma(\{i\} \to \{f_n\}) \, . \tag{2}$$

Motivation: A collider observable

With these assumptions, the quantum field theoretical formulation of σ is

$$\mathrm{d}\sigma = \frac{1}{4\left[(k_1 \cdot k_2)^2 - m_1^2 m_2^2\right]^{1/2}} |\ i|M|f_n\ |^2 \mathrm{d}\Pi_{\mathrm{LIPS}}\,. \tag{3}$$

The Lorentz invariant phase space $\mathrm{d}\Pi_{\mathrm{LIPS}}$:

$$\mathrm{d}\Pi_{\mathrm{LIPS}} \equiv (2\pi)^d \delta^d \left(\sum_i k_i - \sum_j p_j \right) \prod_{j=1}^{n} \frac{\mathrm{d}^{d-1}\boldsymbol{p}_j}{(2\pi)^{d-1}} \frac{1}{2E_j}\,. \tag{4}$$

Motivation: A collider observable

Switching back from theory to the **experimental set-up**.

Key components of a particle collider:

- **Triggers** — gatekeeping devices.
  - Criteria from *physics priorities*
- **Trackers** — particle trajectories
  - Only for long-lived particles
- **Calorimeters** — the energy of a particle
  - Jets and missing transverse energy
- **Reconstruction techniques**
  - A statement of *detection*

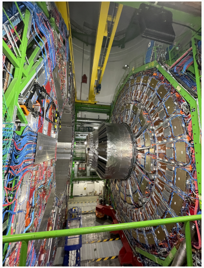


Figure 2: The CMS detector.
Photo by S. Sianiuta.

This sets the stage for many of the **choices** and **priorities** we make from the **theoretical side**.

Motivation: A collider observable

Our goal: $\sigma(PP \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 + X)$, LO in the Yukawa and $O(\alpha_s)$ in the QCD coupling.

What we care about in our results is:

- The physical and probabilistic interpretation
  - Reliability
- The magnitude of the cross section
  - Observability
- The uncertainties from neglecting higher order contributions
  - Accuracy
- The relationship between theoretical uncertainties and other systematic uncertainties
  - The pay-off

Overview

The calculation: Factorization

Our goal: $\sigma(PP \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 + X)$, LO in the Yukawa and $O(\alpha_s)$ in the QCD coupling.

$$\frac{\mathrm{d}\sigma}{\mathrm{d}Q^2} \equiv \frac{\mathrm{d}\sigma^{\alpha_s^0}}{\mathrm{d}Q^2}(P_1 P_2 \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 + X_0) + \frac{\mathrm{d}\sigma^{\alpha_s}}{\mathrm{d}Q^2}(P_1 P_2 \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 + X_1) + O\left(\alpha_s^2\right) . \qquad (5)$$

Assuming **factorization**:

$$\sigma = \sum_{p_1, p_2} f_{p_1} \quad f_{p_2} \quad \hat{\sigma} + O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right) . \qquad (6)$$

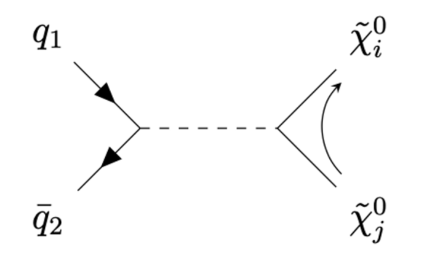
The calculation: Factorization

The hands-on expression:

$$\frac{d\sigma}{dQ^2}\left(P_1 P_2 \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 + X\right) = \sum_{p_1 p_2} \int_0^1 dx_1 \int_0^1 dx_2 \Bigg[\theta\left(\hat{s} - Q^2\right) f_{p_1}(x_1) f_{p_2}(x_2) \\ \times \frac{d\hat{\sigma}}{dQ^2}\left(p_1 p_2 \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 + X\right) \Bigg]. \qquad (7)$$

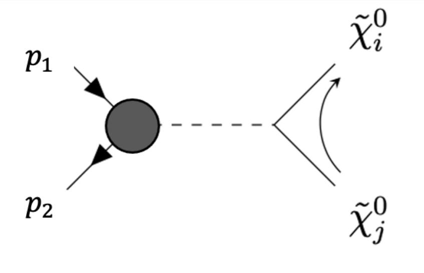
The calculation: A different type of factorization

The LO diagram for the **partonic cross section**: $\frac{d\hat{\sigma}}{dQ^2}\left(p_1 p_2 \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 + X\right)$:



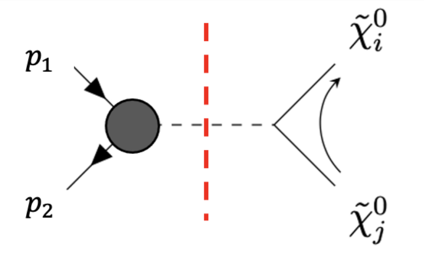
The calculation: A different type of factorization

The NLO corrections for **partonic cross section**: $\frac{d\hat{\sigma}}{dQ^2}\left(p_1 p_2 \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 + X\right)$:



The calculation: A different type of factorization

LO **partonic cross section**: $\frac{d\hat{\sigma}}{dQ^2}\left(p_1 p_2 \to \tilde{\chi}_i^0 \tilde{\chi}_j^0 + X\right)$:



The calculation: All interferences aboard

We consider all possible interferences.

- Different processes
- Different coupling multiplicities
- Different Higgs bosons

$$M\left(p_1 p_2 \rightarrow h^{x}_{n_x} \rightarrow \tilde{\chi}^0_i \tilde{\chi}^0_j + X_{C_1}\right) M \left(\tilde{\chi}^0_i \tilde{\chi}^0_j + X_{C_2} \rightarrow h^{y}_{m_y} \rightarrow p_1 p_2\right) \tag{8}$$

The calculation: The processes

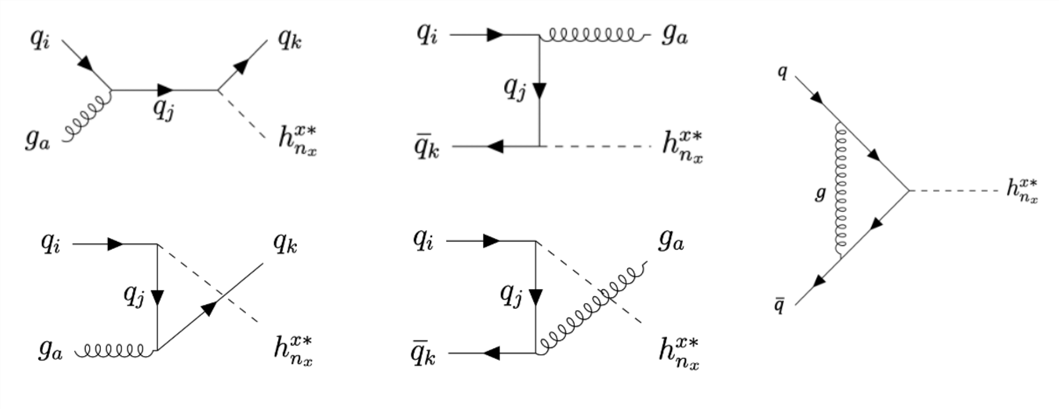


Figure 3: Feynman diagrams at $O(\alpha_s)$.

Divergences in our calculation: “Yay” or “Nay”?

The calculation: Physical observables

- Only physical observables (σ) are required to be finite

$$\sigma = \sum_{p_1, p_2} f_{p_1} \quad f_{p_2} \quad \hat{\sigma} \tag{9}$$

- $\hat{\sigma}$ and $f_{p_i}(x)$ are not physical
  - Only their convolution is

$$\hat{\sigma} \equiv \hat{\sigma}(\mu)\,, \quad f_{p_i} \equiv f_{p_i}(\mu) \tag{10}$$

- $\hat{\sigma}$ and $f_{p_i}(x)$ are only defined at a certain scale

$$\frac{d\sigma}{d\mu} \stackrel{!}{=} 0 \tag{11}$$

Overview

Numerical results: Physical and probabilistic interpretation

- Constrained NMSSM
- 13.6 TeV CM energy at the LHC
- Only b-quark included

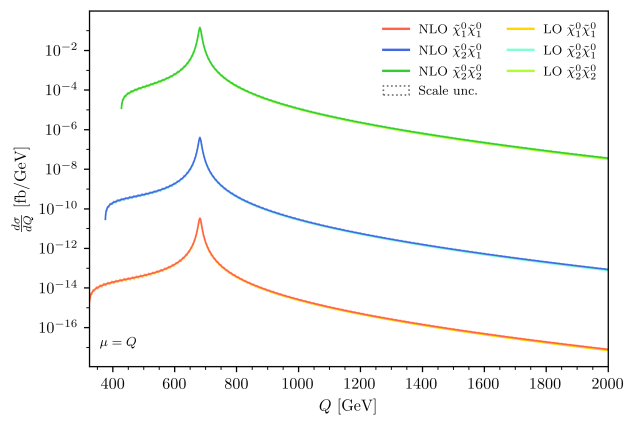


Figure 4: The differential cross section

Numerical results: The magnitude of the cross section

- Varying the mass of the lightest neutralino
- Resonant enhancement
- A one-event threshold estimate

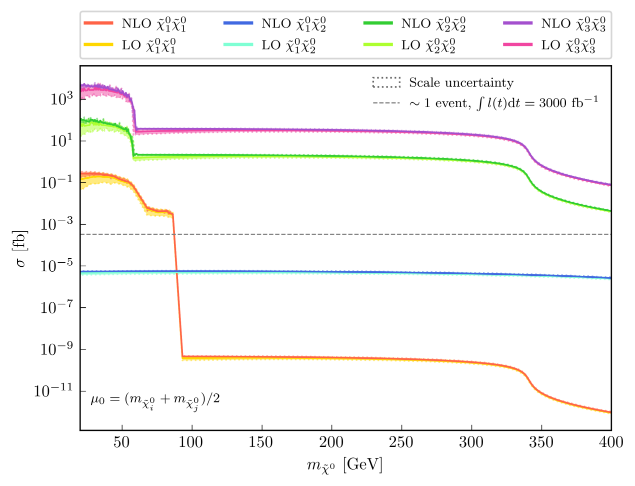


Figure 5: The total cross section.

Numerical results: Theoretical uncertainties

- Varying μ_R and μ_F independently
- Central scale is $\mu_0 = (m_{\tilde{\chi}_i^0} + m_{\tilde{\chi}_j^0})/2$
- Comparing LO and NLO

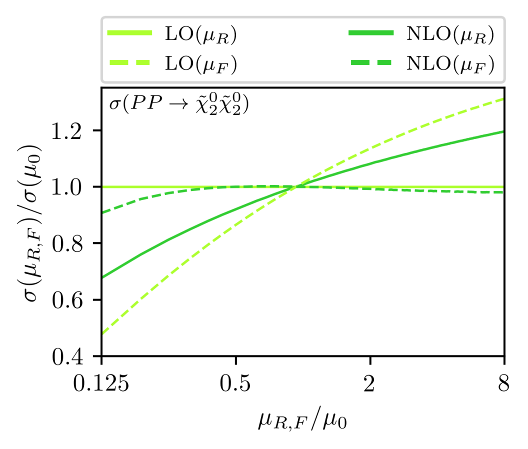


Figure 6: Scale uncertainty estimates in the total cross section.

Numerical results: Comparing to PDF and α_s uncertainties

- `PDF4LHC21_40_pdfas` set from PDF4LHC Working Group hep-ph/2203.05506
- δ_{PDF} calculated by prescription in hep-ph/2203.05506
- $\alpha_s(M_Z^2) = 0.1180 \pm 0.0010$

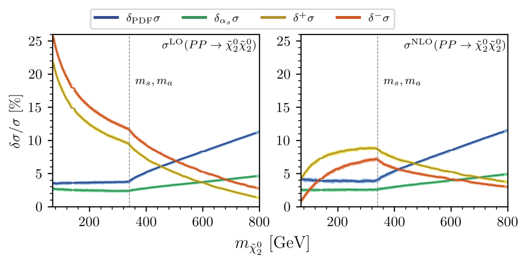


Figure 7: Scale uncertainty estimates VS PDF and α_s uncertainty estimates.

Thank you for your attention!

Motivation: The NMSSM spectrum

Names		spin–0	spin–1/2	$SU(3)_C, SU(2)_L, U(1)_Y$
squarks, quarks	$\hat{Q}$	$(\tilde{u}_L\ \tilde{d}_L)$	$(u_L\ d_L)$	$(\mathbf{3},\ \mathbf{2},\ \frac{1}{6})$
(×3 families)	$\hat{\bar{U}}$	$\tilde{u}_R^*$	$u_R^\dagger$	$(\bar{\mathbf{3}},\ \mathbf{1},\ -\frac{2}{3})$
	$\hat{\bar{D}}$	$\tilde{d}_R^*$	$u_R^\dagger$	$(\bar{\mathbf{3}},\ \mathbf{1},\ \frac{1}{3})$
sleptons, leptons	$\hat{L}$	$(\tilde{\nu}\ \tilde{e}_L)$	$(\nu\ e_L)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$
(×3 families)	$\hat{\bar{E}}$	$\tilde{e}_R^*$	$e_R^\dagger$	$(\mathbf{1},\ \mathbf{1},\ 1)$
Higgs, higgsinos	$\hat{H}_u$	$(H_u^+\ H_u^0)$	$(\psi_u^+\ \psi_u)$	$(\mathbf{1},\ \mathbf{2},\ +\frac{1}{2})$
	$\hat{H}_d$	$(H_d^0\ H_d^-)$	$(\psi_d\ \psi_d^-)$	$(\mathbf{1},\ \mathbf{2},\ -\frac{1}{2})$
scalar singlet, singlino	$\hat{S}$	S	ψ_S	$(\mathbf{1},\ \mathbf{1},\ 0)$

Figure 8: The SM fermions and their supersymmetric counterparts in the NMSSM, in addition to the gauge-singlet field. Adapted from S. P. Martin, Adv. Ser. Direct. High Energy Phys. 18 (1998) 1

The NMSSM Lagrangian

$$\mathit{L}_{\text{NMSSM}}(x, \theta, \theta^{\dagger}) = \mathit{L}_{\text{kin-gauge}} + \left(W_{\text{Higgs}} + W_{\text{Yukawa}} + \mathit{L}_{\text{Soft}} + \text{c.c.}\right). \tag{12}$$

The NMSSM Lagrangian

$$
\begin{aligned}
\mathsf{L}_{\text{kin–gauge}} =& \hat{H}_u^\dagger e^{g_1 \hat{B} + 2 g_2 \hat{W}} \hat{H}_u + \hat{H}_d^\dagger e^{-g_1 \hat{B} + 2 g_2 \hat{W}} \hat{H}_d + \hat{S}^\dagger \hat{S} \\
&+ \hat{Q}_i^\dagger e^{\frac{1}{3} g_1 \hat{B} + 2 g_2 \hat{W} + 2 g_s \hat{C}} \hat{Q}_i + \hat{U}_i^\dagger e^{-\frac{4}{3} g_1 \hat{B} + 2 g_s \hat{C}} \hat{U}_i + \hat{D}_i^\dagger e^{\frac{2}{3} g_1 \hat{B} + 2 g_s \hat{C}} \hat{D}_i \\
&+ \hat{L}_i^\dagger e^{-g_1 \hat{B} + 2 g_2 \hat{W}} \hat{L}_i + \hat{E}_i^\dagger e^{2 g_1 \hat{B}} \hat{E}_i \\
&+ \frac{1}{4 g_1^2} \operatorname{Tr}\left[\hat{B}^A \hat{B}_A\right] + \frac{1}{4 g_2^2} \operatorname{Tr}\left[\hat{W}^A \hat{W}_A\right] + \frac{1}{4 g_s^2} \operatorname{Tr}\left[\hat{C}^A \hat{C}_A\right] ,
\end{aligned}
\tag{13}
$$

$$
W_{\text{Higgs}} = (\mu + \lambda \hat{S}) \hat{H}_u \cdot \hat{H}_d + \xi_F \hat{S} + \frac{1}{2} \mu \, \hat{S}^2 + \frac{\kappa}{3} \hat{S}^3 , \tag{14}
$$

$$
W_{\text{Yukawa}} = h_u \hat{U} \hat{Q} \cdot \hat{H}_u + h_d \hat{D} \hat{Q} \cdot \hat{H}_d + h_e \hat{E} \hat{L} \cdot \hat{H}_d . \tag{15}
$$

The NMSSM Lagrangian

$$
\begin{aligned}
-\mathcal{L}_{\text{soft-Higgs}} &= m_{H_u}^2 |H_u|^2 + m_{H_d}^2 |H_d|^2 + m_S^2 |S|^2 + m_Q^2 |Q^2| + m_U^2 |U_R^2| \\
&+ m_D^2 |D_R^2| + m_L^2 |L^2| + m_E^2 |E_R^2| \\
&+ (\lambda A_\lambda \, H_u \cdot H_d \, S + \frac{1}{3}\kappa A_\kappa \, S^3 + m_3^2 H_u \cdot H_d + \frac{1}{2} m_S'^2 S^2 + \xi_S S + \text{h.c.}). \qquad (16)
\end{aligned}
$$

$$
-\mathcal{L}_{\text{soft-gauge}} = \frac{1}{2} M_1 + \frac{1}{2} M_2 + \text{h.c.}\,, \qquad (17)
$$

The NMSSM Scalar potential

$$
\begin{aligned}
V_{\text{Higgs}} = & \left|\lambda\left(H_u^+ H_d^- - H_u^0 H_d^0\right) + \kappa S^2 + \mu\, S + \xi_F\right|^2 \\
& + \left(m_{H_u}^2 + |\mu + \lambda S|^2\right)\left(|H_u^0|^2 + |H_u^+|^2\right) + \left(m_{H_d}^2 + |\mu + \lambda S|^2\right)\left(|H_d^0|^2 + |H_d^-|^2\right) \\
& + \frac{g_1^2 + g_2^2}{8}\left(|H_u^0|^2 + |H_u^+|^2 - |H_d^0|^2 - |H_d^-|^2\right)^2 + \frac{g_2^2}{2}\left|H_u^+ H_d^0 + H_u^0 H_d^-\right|^2 \\
& + m_S^2 |S|^2 + \Bigg(\lambda A_\lambda\left(H_u^+ H_d^- - H_u^0 H_d^0\right) S + \frac{1}{3}\kappa A_k\, S^3 + m_3^2\left(H_u^+ H_d^- - H_u^0 H_d^0\right) \\
& + \frac{1}{2} m_S'^2\, S^2 + \xi_S S + \text{h.c.}\Bigg)\,. \qquad (18)
\end{aligned}
$$

The results

Defining $z = Q^2/\hat{s}$, Q is the invariant neutralino mass.

$$\omega_{qq}^{\delta}(1) = 1 + \frac{\alpha_s C_F}{\pi}\left(\frac{3}{2}\ln\left(\frac{\mu_R^2}{Q^2}\right) - 1 + \frac{\pi^2}{3}\right) - \frac{\alpha_s C_F}{\pi}\frac{3}{2}\ln\left(\frac{\mu_F^2}{Q^2}\right), \quad (19)$$

$$\omega_{qq}^{+,1}(z) = -\frac{\alpha_s C_F}{\pi}(1+z^2)\ln\left(\frac{\mu_F^2}{Q^2}\right), \quad (20)$$

$$\omega_{qq}^{+,2}(z) = \frac{2\alpha_s C_F}{\pi}(1+z), \quad (21)$$

$$\omega_{qq}^{F}(z) = \frac{\alpha_s C_F}{\pi}\left((1-z) + \frac{(1+z^2)\ln z}{(1-z)}\right), \quad (22)$$