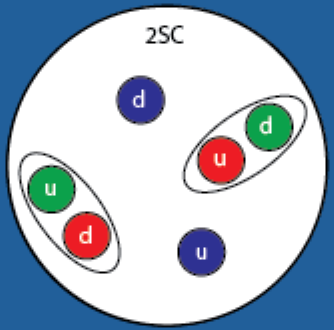

Color superconductivity in the quark-meson model

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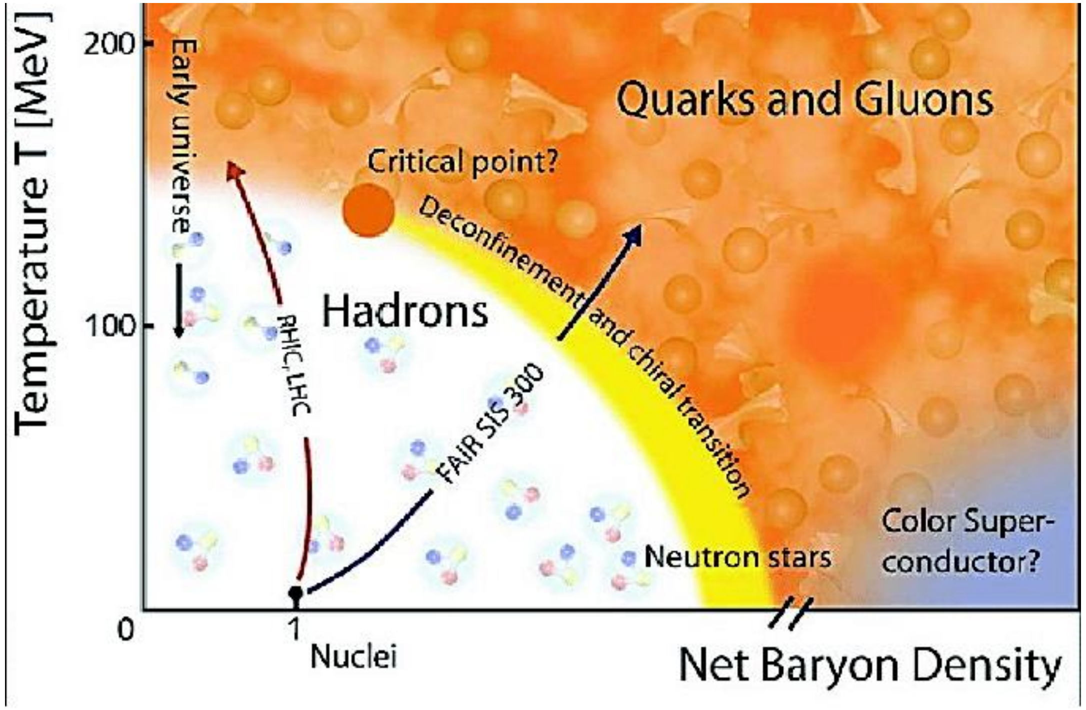
Supervisor: Jens O. Andersen

NTNU

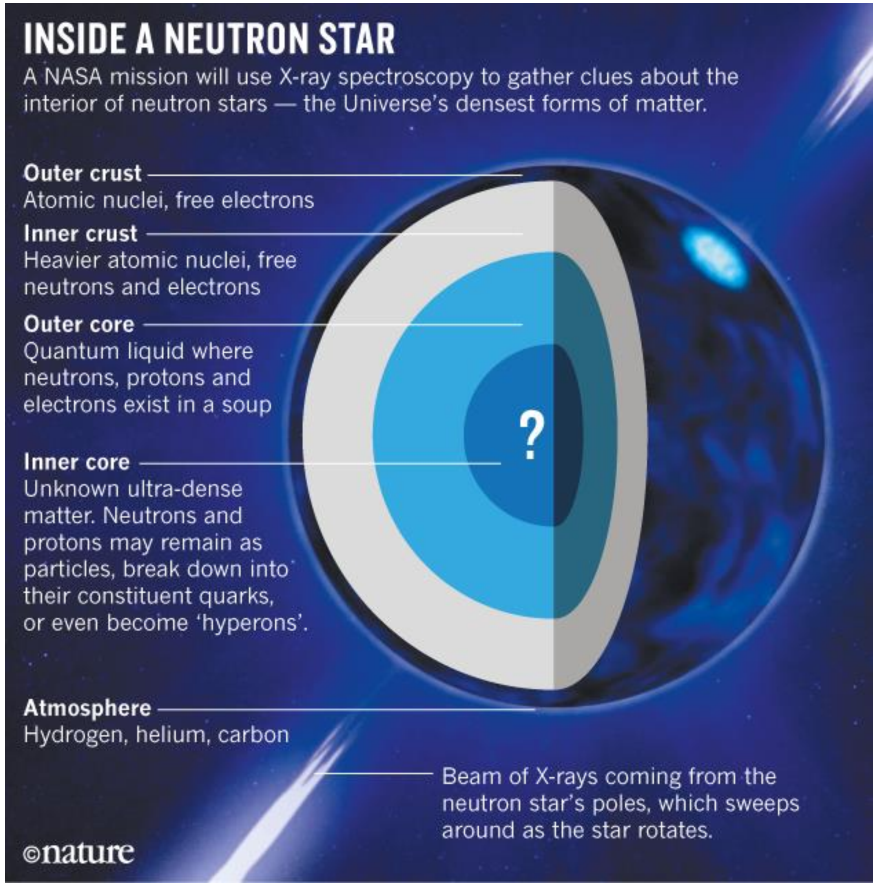
The QCD phase diagram

- Lattice for small chemical potentials
- Experimental results for large T
- Sign problem for small T and finite baryon chemical potential
- Neutron stars along small T at μ ≈ 400-500 MeV



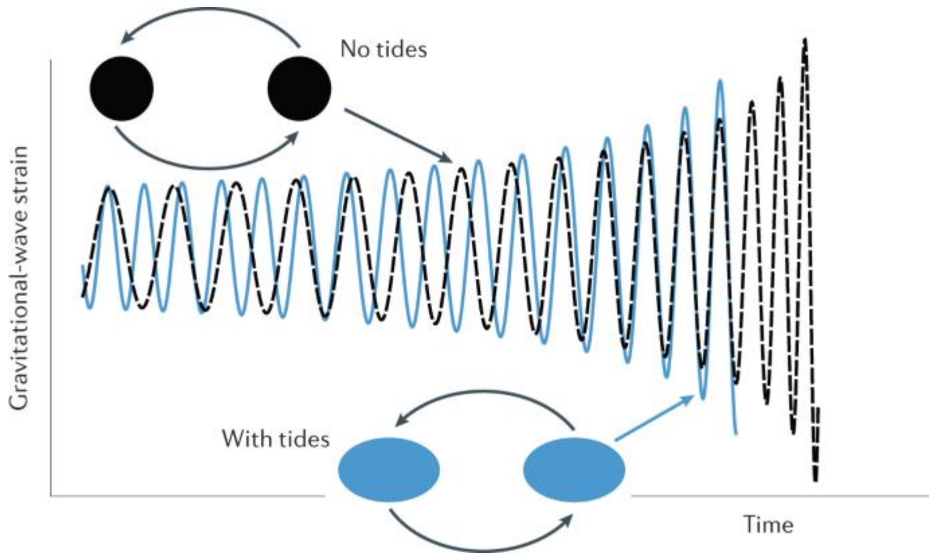
Neutron stars (NS) are great laboratories

- Possible phases of cold dense strongly interacting matter inside NSs
- NS are very cold (T approx 1MeV)
- Supernova and NS-NS mergers much higher temperature (Tens of MeV and up towards 100 MeV respectively)
- Interesting temperature-ranges for our theoretical calculations to compare to observations



How do we study neutron star models?

- The Equation of State (EoS) - pressure as function of energy density
- TOV = hydrostatic equilibrium
- Solving the TOV-equation -> mass-radius relationship (low T)
- Tidal deformability and other GW-observables (low T)
- NS-NS/BH merger simulations (high T)

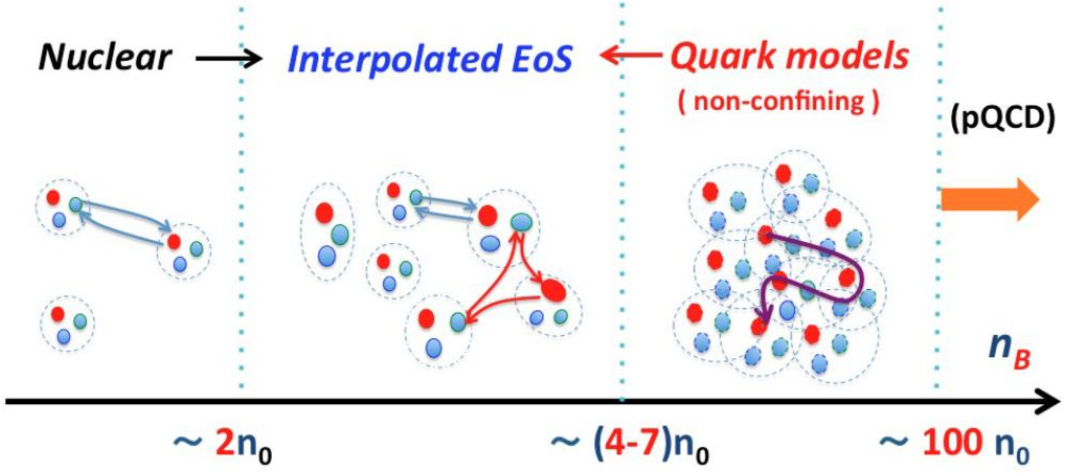


What do we expect for intermediate densities?

- Nuclear saturation density

$$n_0 = 0.16\text{fm}^{-3}$$

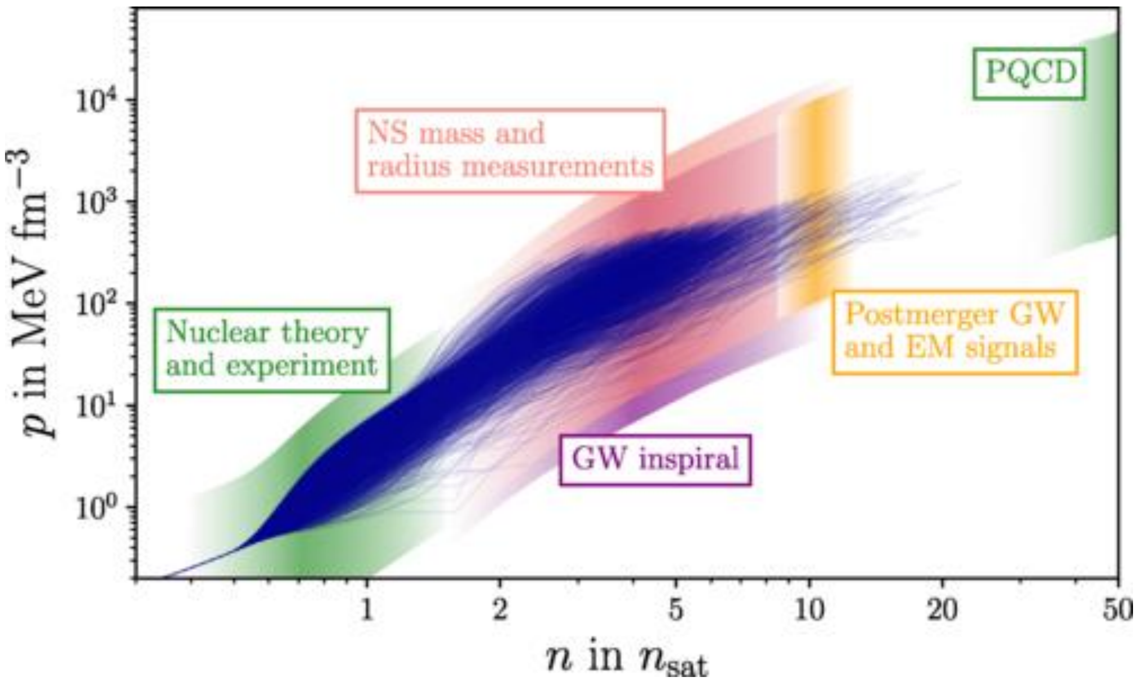
- NS core at approx 4-6 n_0
- QCD predicts some new phase at intermediate density



Baym *et al* 2018 *Rep. Prog. Phys.* **81** 056902

Equation of state

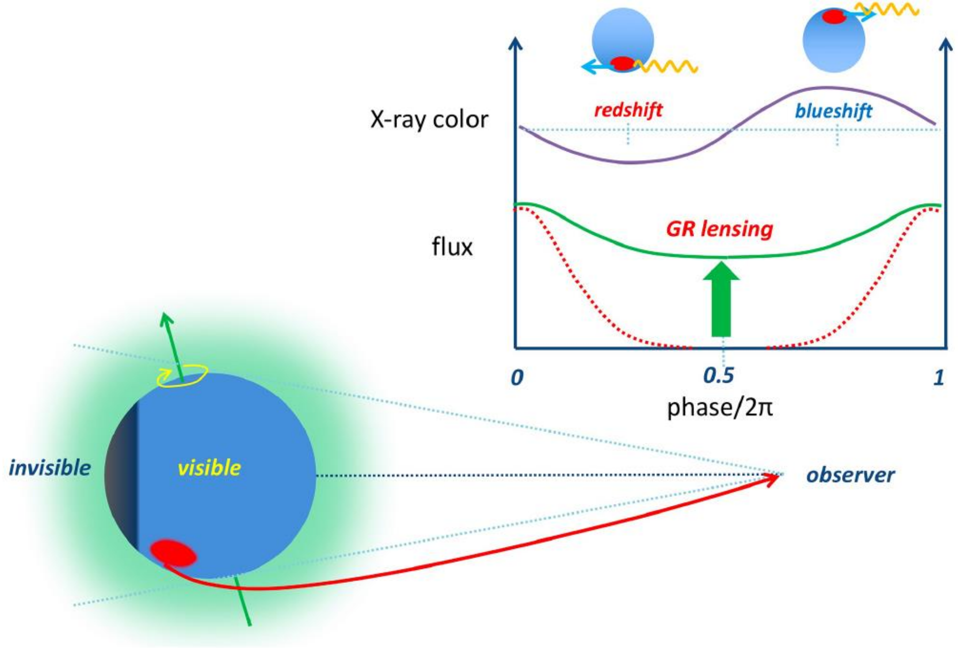
- From nuclear physics to pQCD
- Mass-Radius relations
- GW-constraints
- Speed of sound



Koehn, et. al. Phys. Rev. X 021014 (2025)

Determining the mass-radius relation observationally

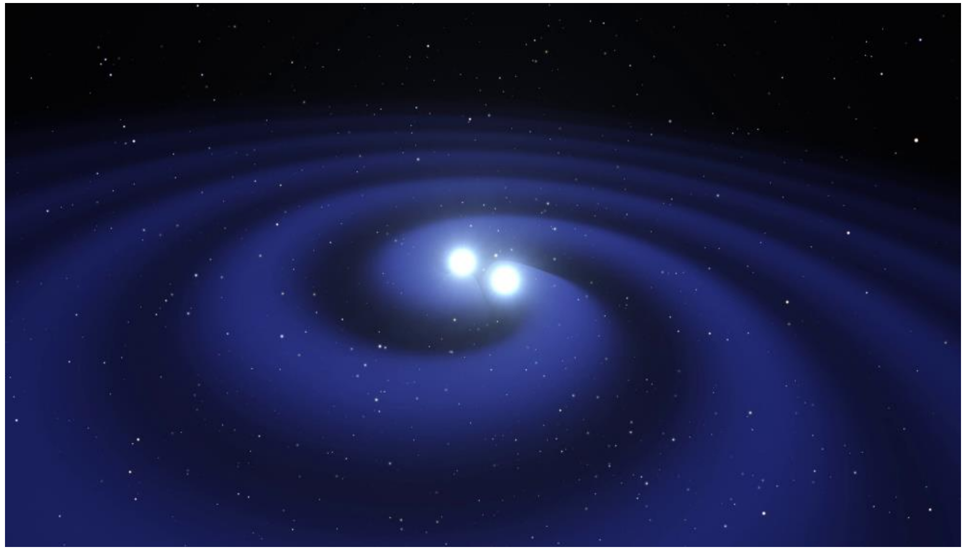
- Accurate M-R strongly constraints EoS
- Radius measurements are difficult
- NICER (Neutron star Interior Composition Explorer) X-ray telescope
- Obtaining M and R separately requires sufficiently rapidly rotating neutron stars (approx 300 Hz)



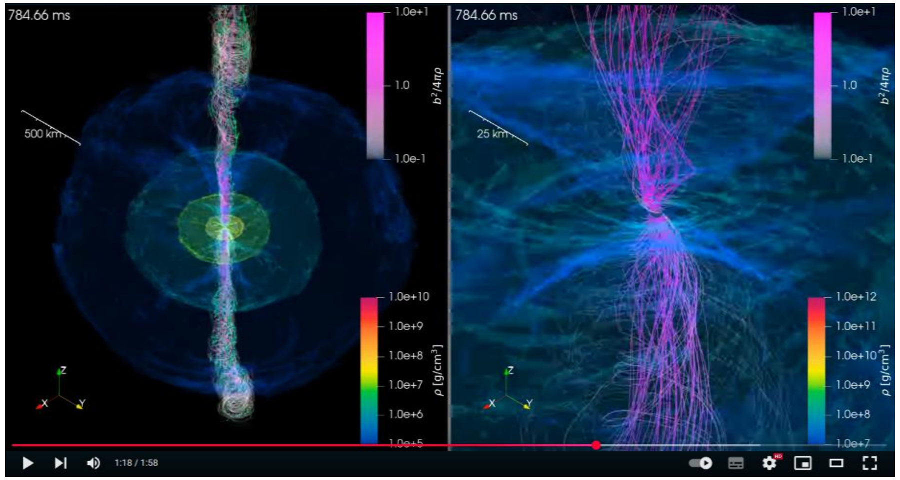
Baym *et al* 2018 *Rep. Prog. Phys.* **81** 056902

Binary neutron star mergers

- EoS for neutron star merger simulations
- Very interesting given future gravitatational wave observations (LIGO, LISA, etc.)
- Multi messenger astronomy
- However – this is hard – lots of work still remaining
- Max Planck Institute very recent video: https://www.youtube.com/watch?v=ehZTVPU04wE



Picture taken from ESA



Hayashi / Max Planck Institute for Gravitational Physics

Two Flavor QM-model with 2SC phase

- Most simple extension of QM-model is two flavors $\Delta_a \sim \langle \epsilon_{\alpha\beta a} \epsilon^{ij} \psi_i^\alpha \psi_j^\beta \rangle$
- Quark, mesons and diquarks are effective degrees of freedom
- Transforms as a singlet under $SU(2)_L \times SU(2)_R$
- Lagrangian density - now four unknown parameters introduced
- Goal: Renormalize the model and find some set of parameters giving reasonable results

$$
\begin{aligned}
\mathcal{L} \;=\; & \frac{1}{2}(\partial_\mu \sigma)(\partial^\mu \sigma) + \frac{1}{2}(\partial_\mu \pi_0)(\partial^\mu \pi_0) + (\partial_\mu + i\mu_I \delta_{\mu 0})\,\pi^+ \left(\partial^\mu - i\mu_I \delta^{\mu 0}\right)\pi^- - \frac{1}{2}m^2(\sigma^2 + \vec{\pi}^2) - \frac{\lambda}{24}(\sigma^2 + \vec{\pi}^2)^2 \\
& + h\sigma + (\partial_\mu + 2i\mu_a \delta_{\mu 0})\,\Delta_a^\dagger \left(\partial^\mu - 2i\mu_a \delta^{\mu 0}\right)\Delta_a - m_\Delta^2 \Delta_a^\dagger \Delta_a - \frac{\lambda_3}{12}(\sigma^2 + \vec{\pi}^2)\Delta_a^\dagger \Delta_a - \frac{\lambda_\Delta}{6}\left(\Delta_a^\dagger \Delta_a\right)^2 \\
& + \bar{\psi}(i\partial\!\!\!/ + \gamma^0 \hat{\mu})\psi - g\bar{\psi}[\sigma + i\gamma^5 \vec{\pi}\cdot\vec{\tau}]\psi + \frac{1}{2}g_\Delta \bar{\psi}_b^C \Delta_a \gamma_5 \tau_2 \epsilon_{abc} \psi_c + \frac{1}{2}g_\Delta \bar{\psi}_b \Delta_a^\dagger \gamma_5 \tau_2 \epsilon_{abc} \psi_c^C \;,
\end{aligned}
$$

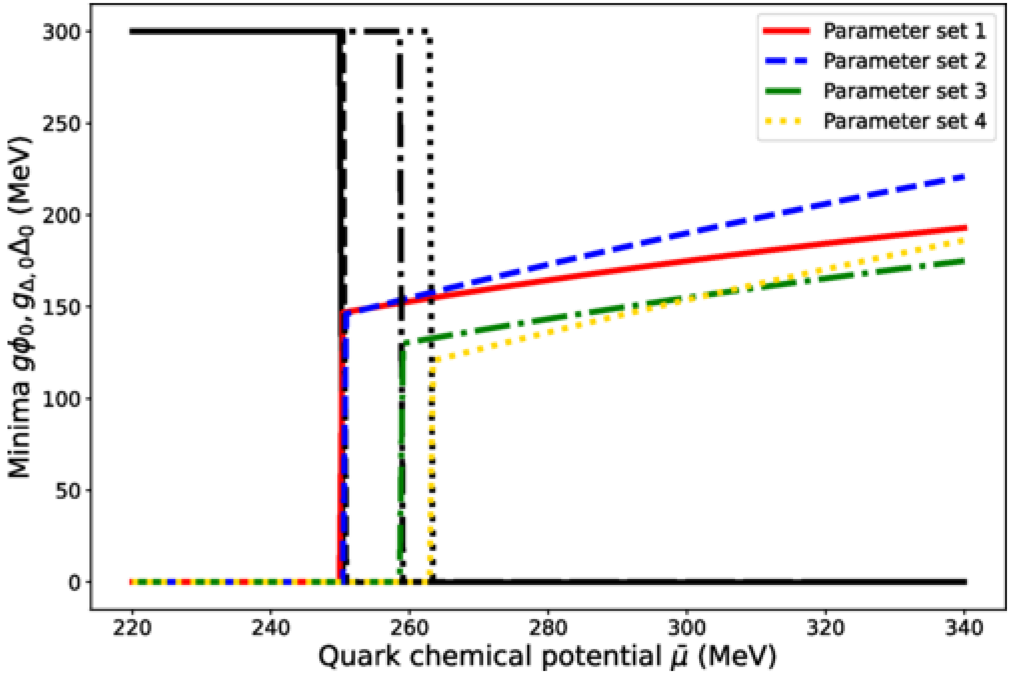
Some details

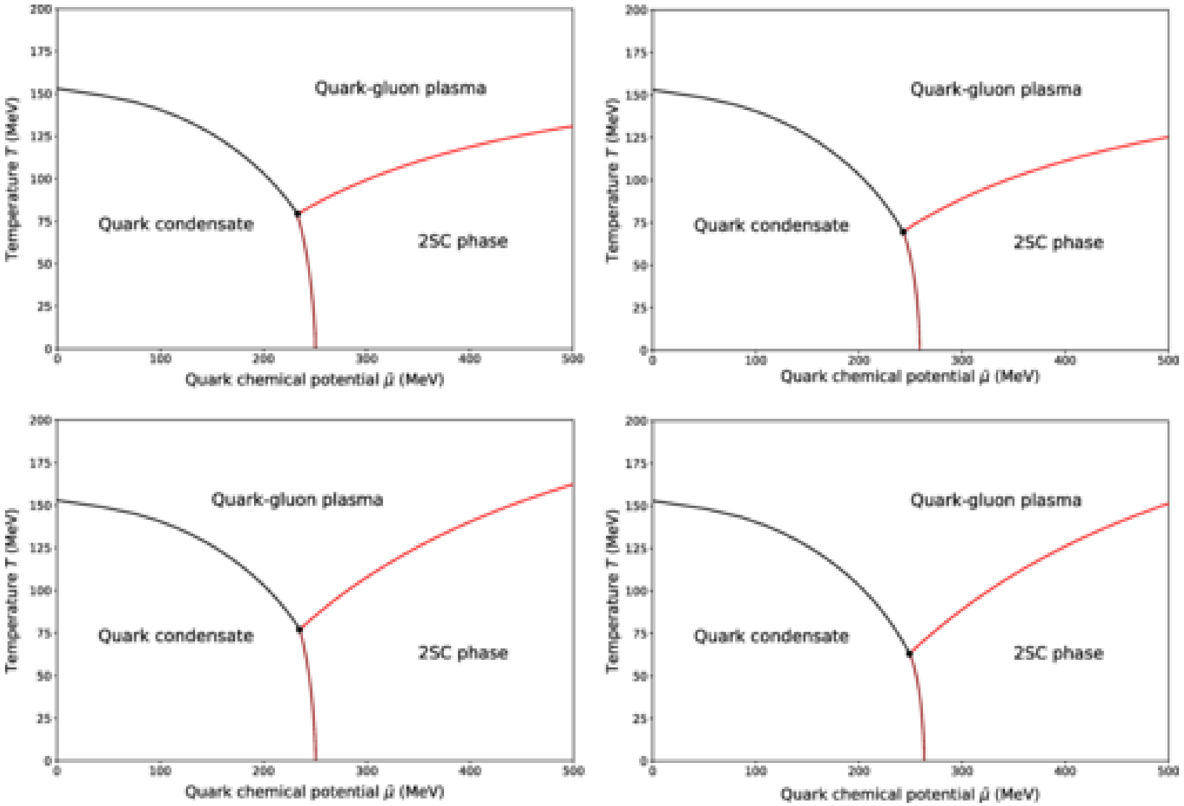
- Mean field $\sigma = \phi + \tilde{\sigma}$ $\Delta = \Delta_0 + \tilde{\Delta}$
- Calculate the quark determinant – introduces divergences
- Renormalize to one quark loop to cancel all divergences
- Minimize effective potential numerically

$$\frac{\partial \Omega}{\partial \phi} = 0 \quad \text{and} \quad \frac{\partial \Omega}{\partial \Delta_0} = 0$$

- Pressure and energy density calculated from potential

Results



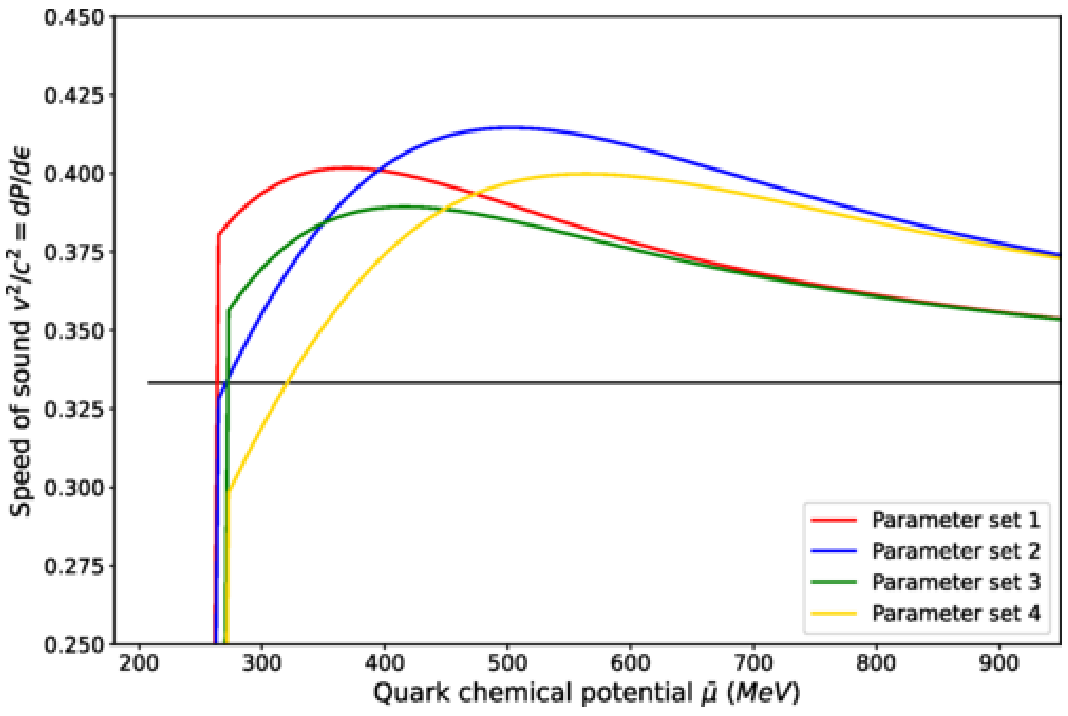


$$g_\Delta = 2g/1.5g \quad m_\Delta = 500/600MeV$$

$$\lambda_\Delta = \lambda/4 \quad \lambda_3 = \lambda$$

Results

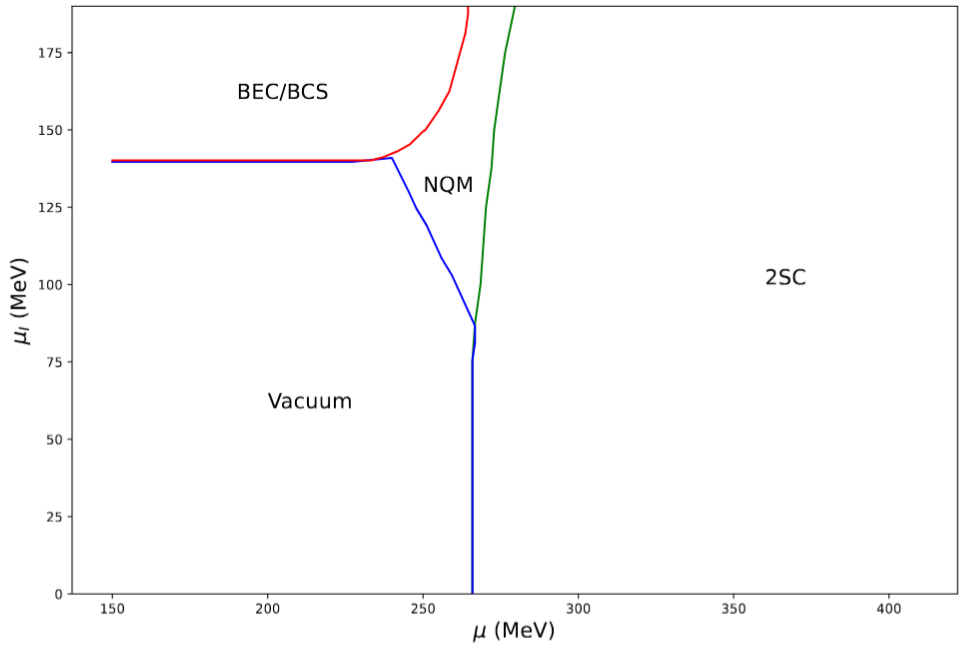
- Speed of sound (from EoS)
- At asymptotic chemical potential we can solve the system analytically – no numerics needed
- Gap goes to a constant – only dependent on quark-diquark coupling
- Speed of sound approaching the conformal limit from above – independent of our parameter set

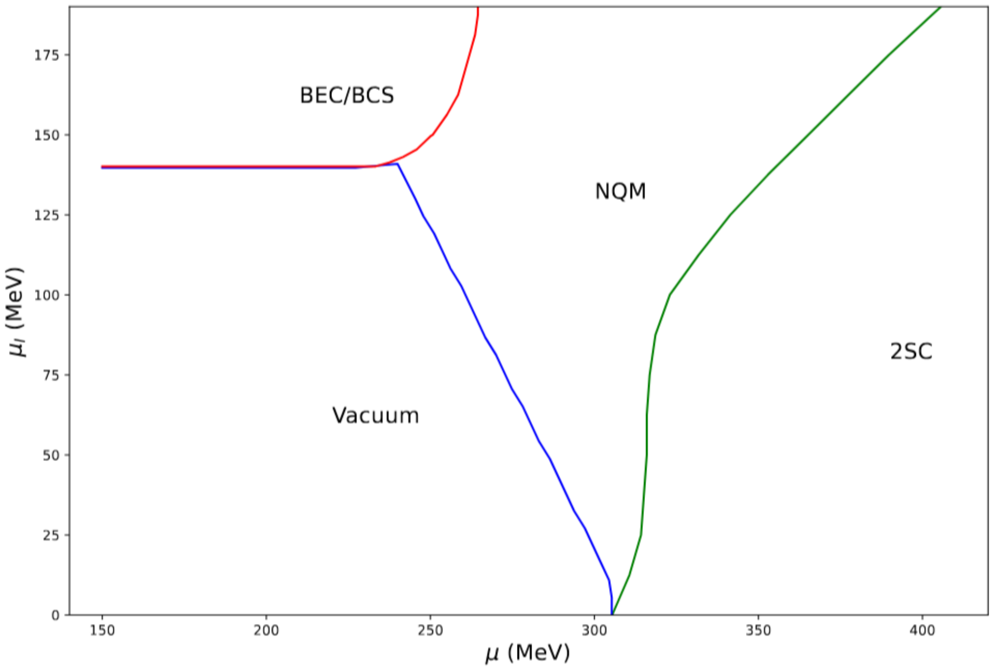


Pion condensation and neutrality conditions

- Pion condensation $\pi = \rho + \tilde{\pi}$
- Along isospin chemical potential we can compare to lattice!
- Still 2 flavors – no strange quark – but include neutrality conditions – closer to neutron star conditions

$$\frac{\partial \Omega}{\partial \mu_e} = 0 \quad \text{and} \quad \frac{\partial \Omega}{\partial \mu_8} = 0$$


BEC/BCS
NQM
2SC
Vacuum
μ_I (MeV)
μ (MeV)
0
25
50
75
100
125
150
175
150
200
250
300
350
400


BEC/BCS
NQM
2SC
Vacuum
μ_I (MeV)
μ (MeV)
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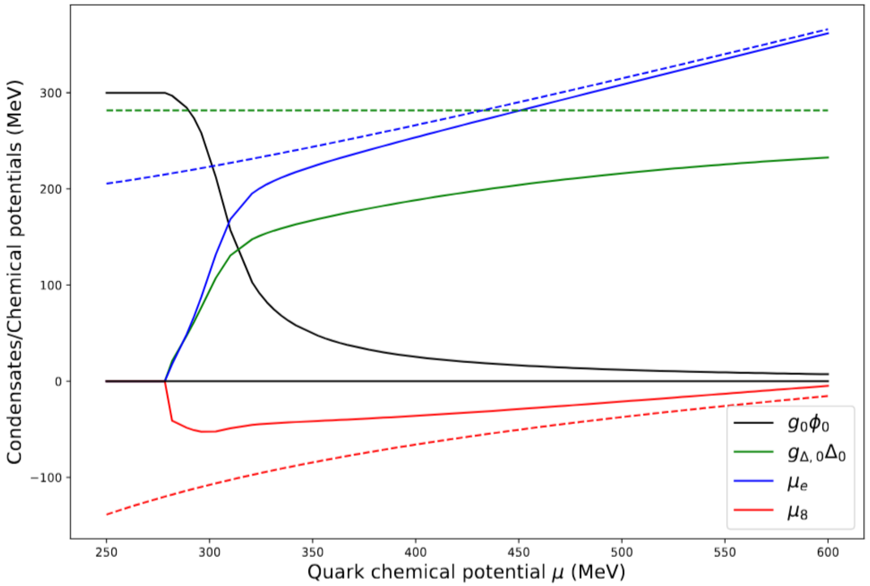
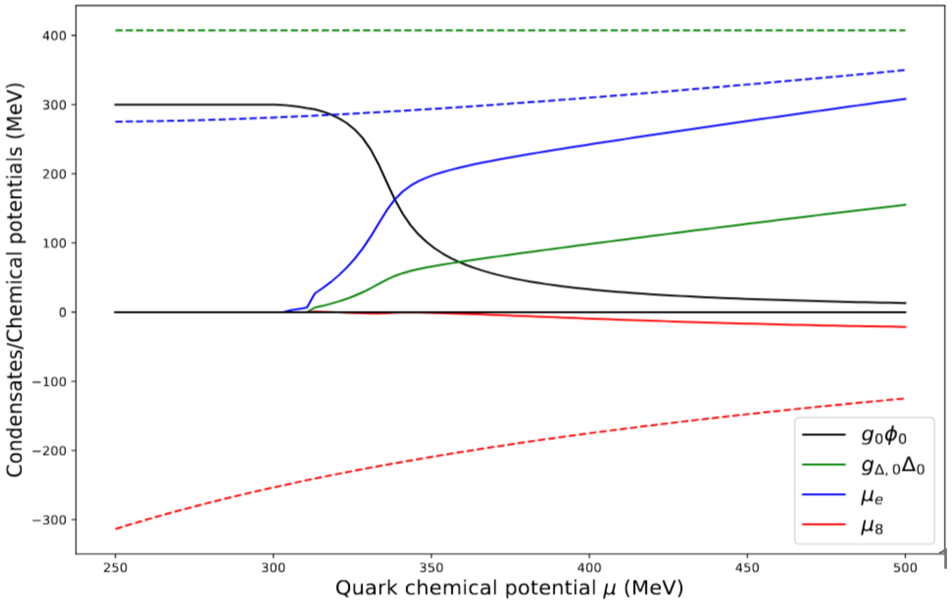

Condensates/Chemical potentials (MeV)
Quark chemical potential μ (MeV)
$g_0\phi_0$
$g_{\Delta,0}\Delta_0$
μ_e
μ_8
−100
0
100
200
300
250
300
350
400
450
500
550
600


Condensates/Chemical potentials (MeV)
Quark chemical potential μ (MeV)
$g_0\phi_0$
$g_{\Delta,0}\Delta_0$
μ_e
μ_8
−300
−200
−100
0
100
200
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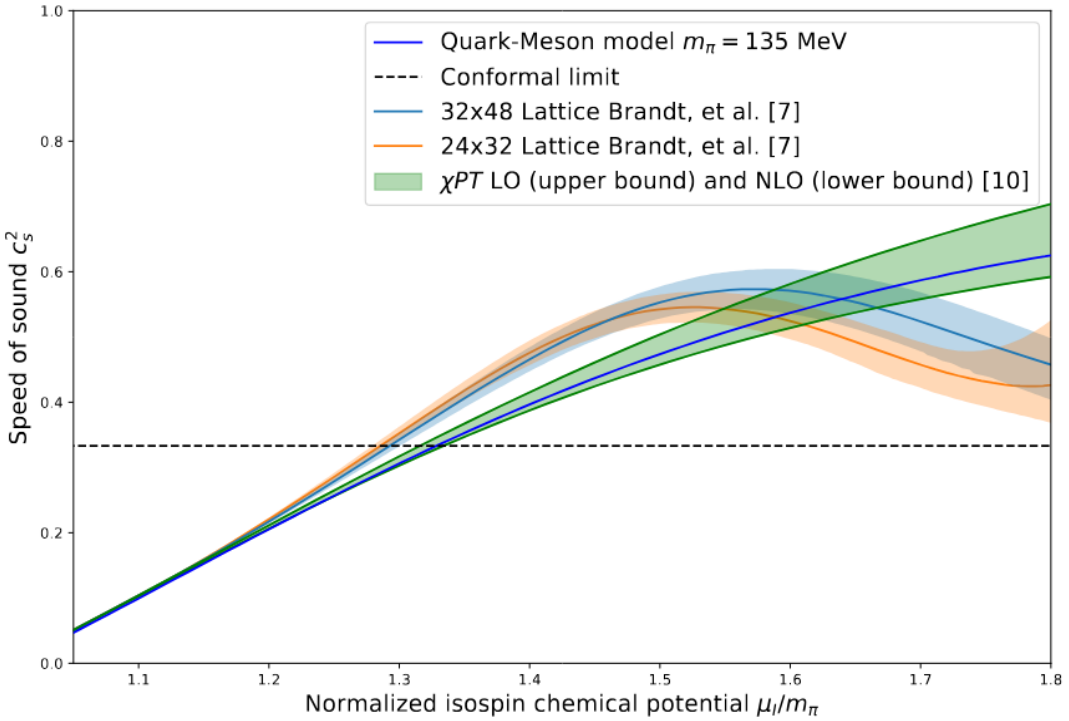
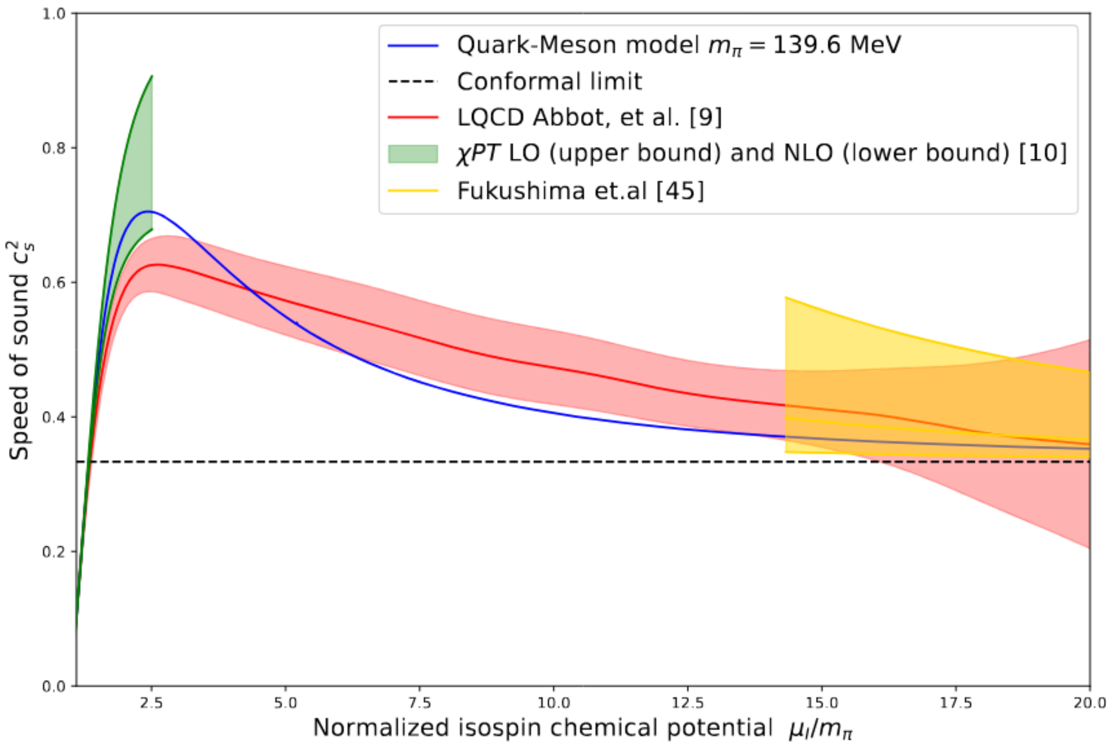
450
500

Comparing with lattice

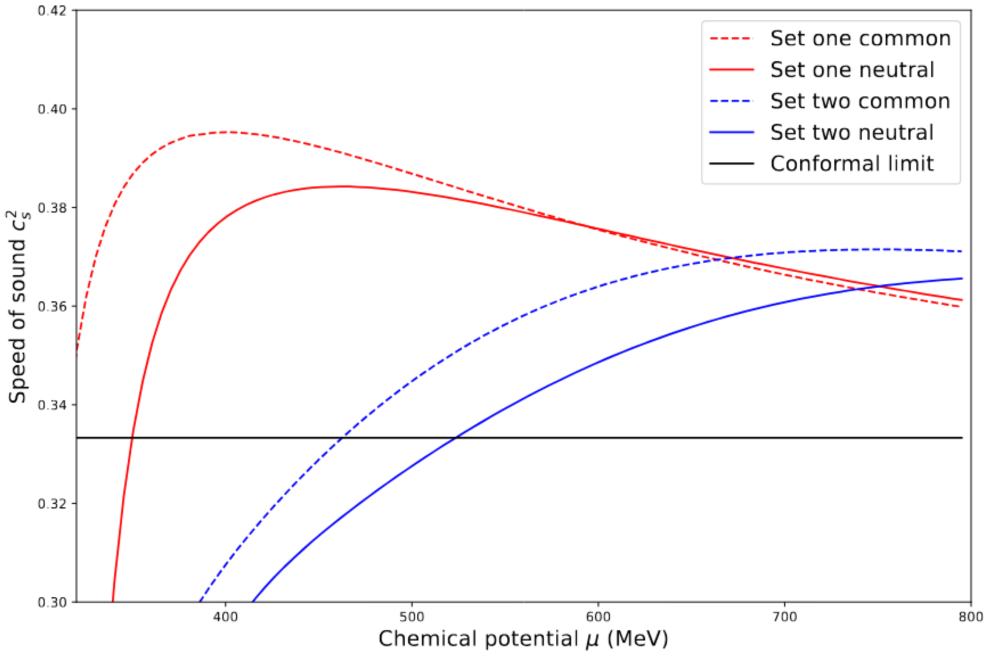




- We find good qualitative agreement with lattice calculations

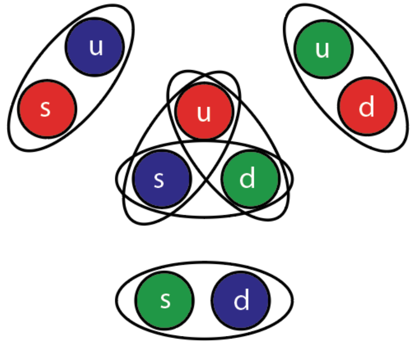
Speed of sound and asymptotic behavior

- The model can be solved analytically for large chemical potentials as before
- Observe noticeable difference when taking neutrality into account
- Neutral always higher than common asymptotically



Three flavors

Mesons: $\Sigma \to U_L \Sigma U_R^{-1}$



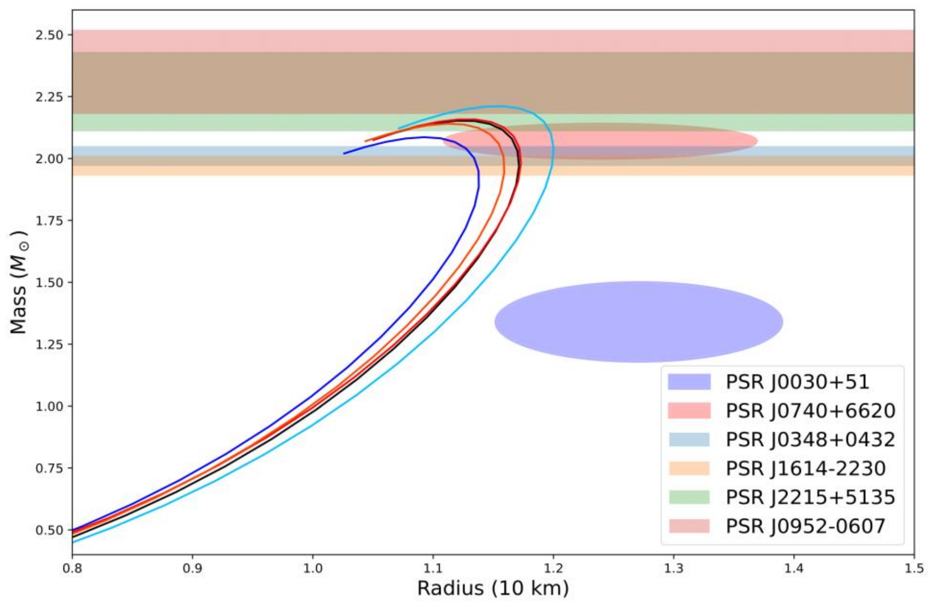
- The Lagrangian must respect the global symmetries

$$SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R$$

- Strategy :
- Construct diquark and meson fields with transformation properties
- Write down all relevant operators respecting the global symmetries
- Calculate the divergences from the quark-loop and renormalize the model
- For common chemical potential we can find dispersion relations analytically

Finishing remarks

- 4 (8) unknown parameters – not all are equally relevant – we wish to constrain them through observations
- Potential is large, but very symmetric
- Work in progress to solve gap equations numerically for three flavors
- "All" is included to begin to study cold NS's



Current attempt at matching to observations for quark star using the 2 flavor QMD model

Lagrangian

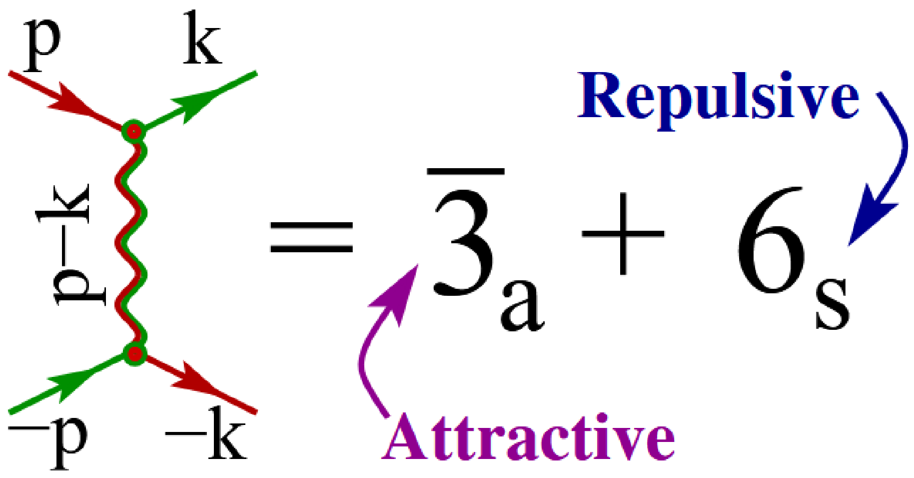
$$
\begin{aligned}
\mathcal{L}_{\text{scalar}} &= \langle D_\mu \Sigma^\dagger D^\mu \Sigma \rangle - m^2 \langle \Sigma^\dagger \Sigma \rangle - \lambda_1 \langle \Sigma^\dagger \Sigma \rangle^2 - \lambda_2 \langle (\Sigma^\dagger \Sigma)^2 \rangle + \langle H(\Sigma + \Sigma^\dagger) \rangle + c[\det \Sigma + \det \Sigma^\dagger] \,, \\
\mathcal{L}_{\text{quark}} &= \bar{\psi}(i\partial\!\!\!/ + \gamma^0 \hat{\mu})\psi \,, \\
\mathcal{L}_{\text{diquark}} &= D_\mu (\Delta^a_{L,i})^\dagger D^\mu \Delta^a_{L,i} - m^2_\Delta (\Delta^a_{L,i})^\dagger \Delta^a_{L,i} - \lambda^\Delta_1 [(\Delta^a_{L,i})^\dagger \Delta^a_{L,i}]^2 - \lambda^\Delta_2 (\Delta^a_{L,i})^\dagger \Delta^a_{L,k} (\Delta^b_{L,k})^\dagger \Delta^b_{L,i} \,, \\
&\quad + L \to R - \lambda^\Delta_3 [(\Delta^a_{L,i})^\dagger \Delta^a_{L,i}][(\Delta^b_{R,j})^\dagger \Delta^b_{R,j}] \\
\mathcal{L}_{\text{scalar}-\text{quark}} &= -g\bar{\psi} T_a (\sigma_a + i\gamma^5 \pi_a)\psi \,, \\
\mathcal{L}_{\text{scalar}-\text{diquark}} &= -\lambda_3 \langle \Sigma^\dagger \Sigma \rangle (\Delta^a_{L,i})^\dagger \Delta^a_{L,i} - \lambda_4 \Sigma^\dagger_{ij} \Delta^a_{L,k} (\Delta^a_{L,j})^\dagger \Sigma_{ki} + L \to R \,, \\
&\quad + \lambda_5 \Sigma_{ij} (\epsilon_{klj} \Delta^a_{R,k})^\dagger \Sigma_{ml} \epsilon_{nmi} \Delta^a_{L,n} + \lambda_5 \Sigma^\dagger_{ij} (\epsilon_{klj} \Delta^a_{L,k})^\dagger \Sigma^\dagger_{lm} \epsilon_{nmi} \Delta^a_{R,n} \\
\mathcal{L}_{\text{quark}-\text{diquark}} &= -\frac{1}{2\sqrt{2}} g_\Delta (\bar{\psi}^b_{L,j})^C \Delta^a_{L,i} \gamma^5 \epsilon_{abc} \epsilon_{ijk} \psi^c_{L,k} - \frac{1}{2\sqrt{2}} g_\Delta \bar{\psi}^b_{L,j} (\Delta^a_{L,i})^\dagger \gamma^5 \epsilon_{abc} \epsilon_{ijk} (\psi^c_{L,k})^C - L \to R \,,
\end{aligned}
$$

Lagrangian

$$
\begin{aligned}
\mathcal{L}_{\rm scalar} &= \langle D_\mu \Sigma^\dagger D^\mu \Sigma\rangle - m^2\langle\Sigma^\dagger\Sigma\rangle - \lambda_1\langle\Sigma^\dagger\Sigma\rangle^2 - \lambda_2\langle(\Sigma^\dagger\Sigma)^2\rangle + \langle H(\Sigma+\Sigma^\dagger)\rangle + c[\det\Sigma + \det\Sigma^\dagger]\ , \\
\mathcal{L}_{\rm quark} &= \bar\psi(i\not{\partial} + \gamma^0\hat\mu)\psi\ , \\
\mathcal{L}_{\rm diquark} &= D_\mu(\Delta^a_{L,i})^\dagger D^\mu\Delta^a_{L,i} - m_\Delta^2(\Delta^a_{L,i})^\dagger\Delta^a_{L,i} - \lambda_1^\Delta[(\Delta^a_{L,i})^\dagger\Delta^a_{L,i}]^2 - \lambda_2^\Delta(\Delta^a_{L,i})^\dagger\Delta^a_{L,k}(\Delta^b_{L,k})^\dagger\Delta^b_{L,i}\ , \\
&\quad + L\to R - \lambda_3^\Delta[(\Delta^a_{L,i})^\dagger\Delta^a_{L,i}][(\Delta^b_{R,j})^\dagger\Delta^b_{R,j}] \\
\mathcal{L}_{\rm scalar-quark} &= -g\bar\psi T_a(\sigma_a + i\gamma^5\pi_a)\psi\ , \\
\mathcal{L}_{\rm scalar-diquark} &= -\lambda_3\langle\Sigma^\dagger\Sigma\rangle(\Delta^a_{L,i})^\dagger\Delta^a_{L,i} - \lambda_4\Sigma^\dagger_{ij}\Delta^a_{L,k}(\Delta^a_{L,j})^\dagger\Sigma_{ki} + L\to R\ , \\
&\quad + \lambda_5\Sigma_{ij}(\epsilon_{klj}\Delta^a_{R,k})^\dagger\Sigma_{ml}\epsilon_{nmi}\Delta^a_{L,n} + \lambda_5\Sigma^\dagger_{ij}(\epsilon_{klj}\Delta^a_{L,k})^\dagger\Sigma^\dagger_{lm}\epsilon_{nmi}\Delta^a_{R,n} \\
\mathcal{L}_{\rm quark-diquark} &= -\frac{1}{2\sqrt{2}}g_\Delta(\bar\psi^b_{L,j})^C\Delta^a_{L,i}\gamma^5\epsilon_{abc}\epsilon_{ijk}\psi^c_{L,k} - \frac{1}{2\sqrt{2}}g_\Delta\bar\psi^b_{L,j}(\Delta^a_{L,i})^\dagger\gamma^5\epsilon_{abc}\epsilon_{ijk}(\psi^c_{L,k})^C - L\to R\ ,
\end{aligned}
$$

Changing gears - Color superconductivity

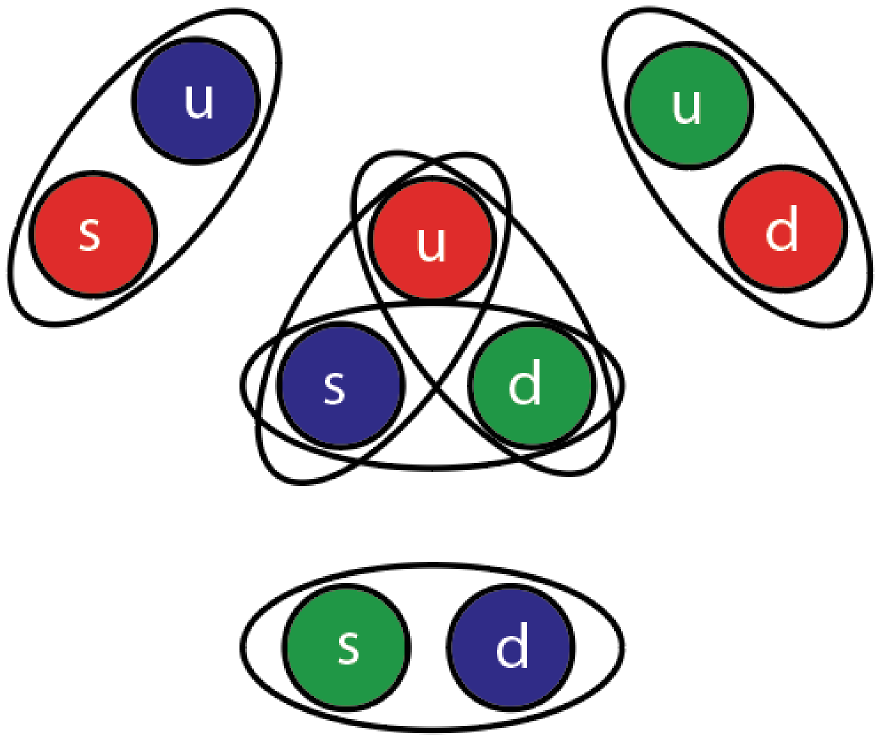
- Degenerate Fermi sphere – attractive interaction
- Quarks form very symmetric cooper pairs at asymptotic density
- All gluons acquire mass through the Meissner-effect
- Asymptotic freedom -> perturbative treatment and low coupling g
- Key difference from familiar BCS superconductivity – gluon exchange



Shovkovy Found Phys 35 (2005)

Ideal three flavor color superconductivity

- At asymptotic μ we have control - g is small and quarks are effectivly massless
- Color flavor locked phase (CFL)
- Nature is not ideal $m_s > m_d \sim m_u$
- Two flavor superconductivity (2SC phase)
- Electromagnetism changes – but no massive photon

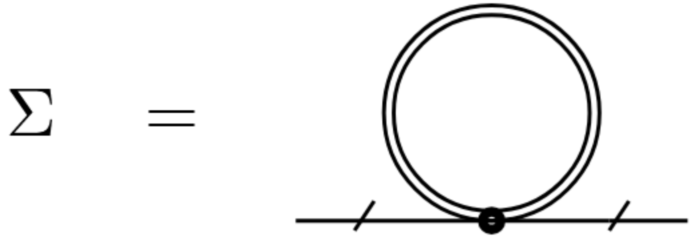


$$SU(3)_{\text{color}} \times SU(3)_L \times SU(3)_R \times U(1)_B \to SU(3)_{\text{color}+L+R} \times Z_2$$

Diagrammatic methods at asymptotic density

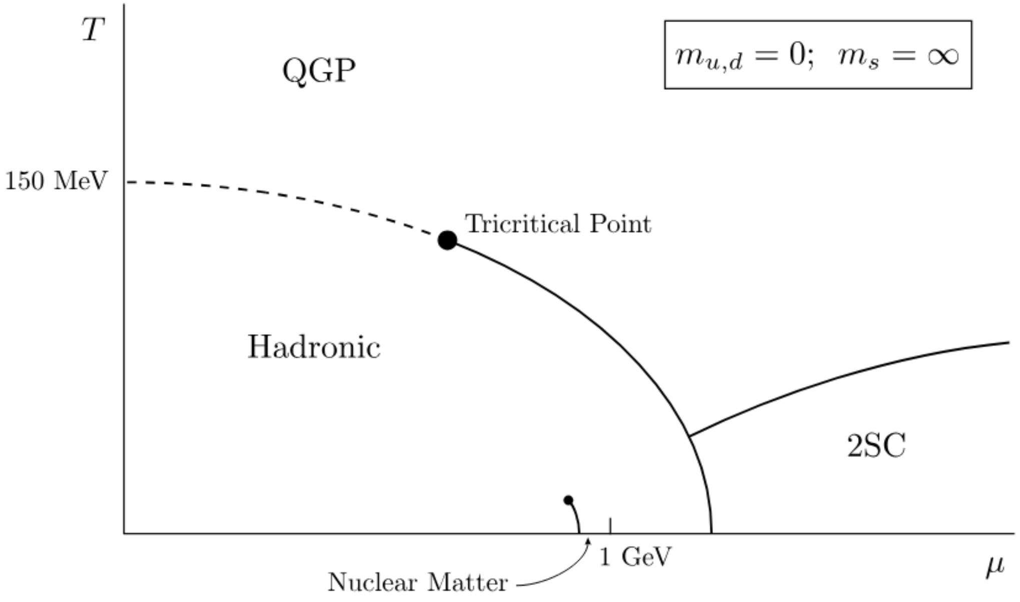
- The Dyson-Schwinger equations
- Solution for the gap for short range interactions and for one gluon exchange
- The gap is different when gluon is taken into account
- Asymptoticly: Interaction is dominated by certain gluon exchange

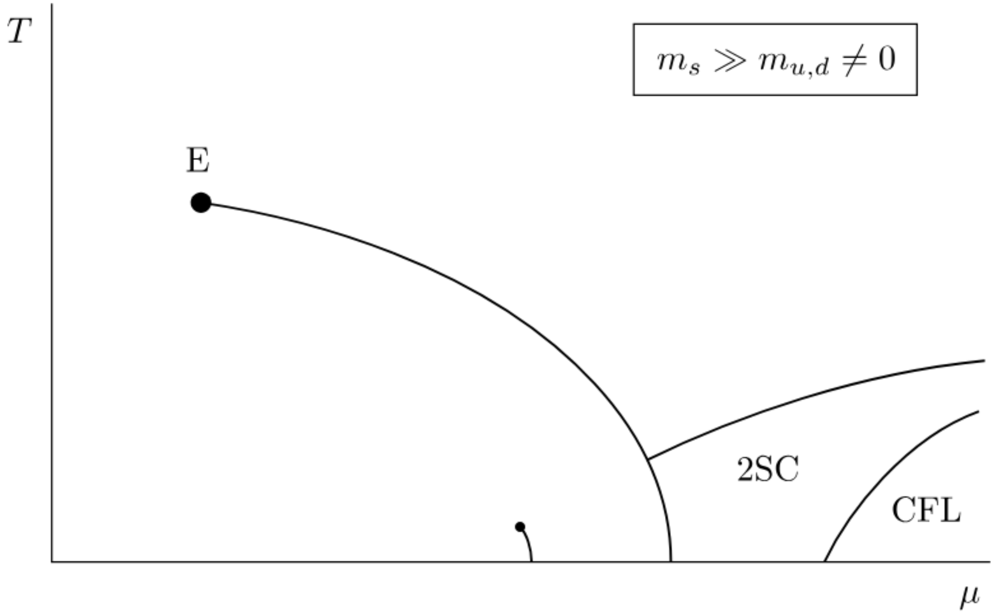
$$\Delta \sim e^{-\mathrm{const}/g^2} \qquad \Delta \sim e^{-\mathrm{const}/g}$$



K. Rajagopal and F. Wilczek
hep-ph/0011333

The phase diagram





K. Rajagopal and F. Wilczek hep-ph/0011333

Color superconductivity in compact stars

- It seems probable that a color superconductive state of quark matter may exist in the core of neutron stars
- What are the possible effects? Are any measurable?
- Equation of state
- GW - constraints
- Cooling by neutrino emission
- Magnetic field evolution
- Glitches

The three flavor QMD model (recent work)

- Three-flavor QM-model is more complicated than its two-flavor counterpart

$$\Delta_i^a \sim \psi_j^b \epsilon_{abc} \epsilon_{ijk} \gamma^5 (\psi_k^c)^C$$

- We encounter two main problems:
- No longer possible to calculate the dispersion relations analytically – how to extract divergences from determinant? Solution:

$$\Omega_1 = -\frac{1}{\beta} \sum_n \int_p \log \det S^{-1} \qquad \frac{1}{\beta} \sum_n = \int_{-i\infty}^{i\infty} \frac{dp_0}{2\pi i}$$

- What kind of terms need to be included to get a renormalizable model free from infinities? Gap no longer transform as a singlet

$$\Delta_i^a \to U_L \Delta_i^a U_c^T$$

Asymptotic results in three flavors

- For asymptotically large chemical potential we can solve the model analytically
- All gaps are equal

Same relations is found in asymptotic analysis

$$\Delta_0 = \frac{\Lambda_0}{g_{\Delta,0}} 2^{-\frac{1}{3}} e^{\frac{(4\pi)^2}{8 g_{\Delta,0}^2} - \frac{1}{2}} = 2^{-\frac{1}{3}} \Delta_0^{2\mathrm{SC}} ,$$

- Speed of sound - still approaches conformal limit from above

$$c_s^2 = \frac{1}{3}\left[1 + \frac{4}{3}\frac{g_\Delta^2 \Delta_0^2}{\bar{\mu}^2}\right]$$