# 3 things in Xtreme QCD

1. What is the (time-like) Wilson loop if $A_0 = 0$ ?

2. Fractional topological charge ($\bar{c}$ V.P. Nair)

3. Solution of cold, dense QCD @ small $N_c$, 1+1 dim's ($\bar{c}$ R. Konik, M. Lajer, A. Tsvelik)

# Wilson loops in Hamiltonian form

2202.11122 Inspired by T. Cohen, 2202.08745

Timelike Wilson line $W \sim \mathrm{tr}\, e^{ig \int A_0 dx}$

But $\bar{c}$ (with) $\mathcal{H}$, $A_0 = 0 \Rightarrow W$?

Answer: $W \sim$ "defect"; need to extend Hilbert space

"Holonomous" potential @ constant $A_0$

$\Rightarrow$ "twisted" partition function

$\Rightarrow$ can extend $W$ to real time

# SU(N) gauge

No dynamical qks (for now). Gauge transf. $\Omega$:

$$A_\mu \to \frac{1}{-ig}\,\Omega^\dagger D_\mu \Omega$$

$$D_\mu \to \partial_\mu - igA_\mu$$

$\Omega \in SU(N) \Rightarrow \det \Omega = \mathbb{1}$.

Gauge transf.'s:

Local: $\Omega = \Omega(x) \longrightarrow$ const. as $x \to \infty$

and

Global: $\Omega(x) = \Omega_\infty =$ const.

Meh. Usually don't worry about global $\Omega$'s

# Global $\Omega$

Under arbitrary global gauge transf.,

$$A_\mu^\Omega = \Omega^\dagger A_\mu \Omega \neq A_\mu$$

So what. Special subset

$$\omega_j = e^{2\pi i j/N} \mathbb{1}_N \longrightarrow \det \omega_j = 1$$

$$A_\mu^{\omega_j} = A_\mu$$

$\omega_j$'s = global $Z(N)$ sym., center of local $SU(N)$)

# Add test quark

With mass $M \to \infty$.

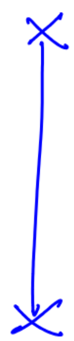
$$\mathcal{L}_{test} = \bar{\chi} D_0 \chi$$

Choose $\chi$ = particle, going forward in time
$\Rightarrow$ eliminate anti-particles

Can neglect spin, as effects $\sim 1/M$

$\Rightarrow$ $\chi$ = single component, only carries color

$$\langle \bar{\chi}(t)\chi(0)\rangle \sim \frac{1}{D_0} = \underbrace{\mathbb{P}\, e^{ig\int_0^x A_0\, dx}}_{\mathbb{L}(\bar{x};\, 0,\, x)} \theta(x)$$



= Wilson line

# Symmetries

Temperature $T \neq 0$. Gluons = bosons $\Rightarrow$

$$A_\mu(\bar{x}, 1/T) = + A_\mu(\bar{x}, 0)$$

Choose $\Omega(\bar{x}, 1/T) = \omega_j \, \Omega(\bar{x}, 0)$.

$A_\mu$'s still periodic. Thermal Wilson line not:

$$\mathbb{L}(\bar{x}; 0, 1/T) \to \Omega^\dagger(\bar{x}, 0) \, \mathbb{P} e^{ig \int_0^{1/T} A_0} \, \Omega(\bar{x}, 1/T)$$

$$= \omega_j \, \mathbb{L}$$

1-form sym.: Gaiotto, Kapustin, Seiberg, Willett
for $\mathbb{L}$ 1412.5148

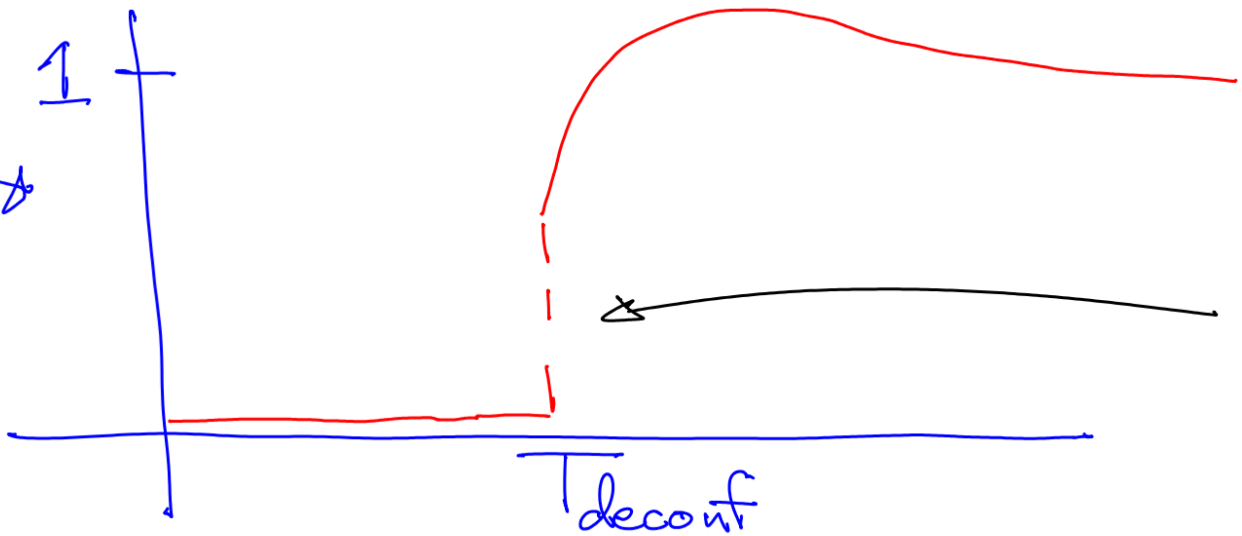
# Polyakov loop

Wilson line $\mathbb{L} \sim$ propagator of test quark

Polyakov loop $P(\bar{x}) = \mathrm{tr}_{color}\, \mathbb{L}(\bar{x})$

$$\int \frac{d^3x}{V} \langle P \rangle \sim \omega_j \left( 1 + \# g^3 + \dots \right), \quad T > T_{deconf}$$

$$= 0 \qquad \text{"} < \text{"}$$




Deconfining transition (NO qks)

1st order, $N \geq 3$

# Hamiltonian

Lagrangian:

$$\mathcal{L} = \frac{1}{2} \operatorname{tr} G_{\mu\nu}^2 + \overline{\chi} D_0 \chi(y)$$

@ one point, y

Go to $A_0 = 0$ gauge

$$\mathcal{H} = \operatorname{tr}(\vec{E}^2 + \vec{B}^2) + \underset{q}{0}$$

Nothing from gks. $\overline{\chi} D_0 \chi \Rightarrow \overline{\chi} \partial_0 \chi$.

Momenta conjugate to $\chi$ is $\overline{\chi}$, $\chi$'s completely drop out of $\mathcal{H}$. Like coupling to external current $\sim \int j_0 A_0 = 0$ for $A_0 = 0$

# Gauss' law

Sure, need to impose conservation of electric charge

$$\mathcal{H}_{Gauss} = i \, \mathrm{tr}_{color}\left( \chi \left( D \cdot E - g Q \right) \right)$$

$$Q^a = \bar{\psi} t^a \psi \quad = \text{adjoint field @ 1 point}$$

$\chi$ = constraint field
in adjoint representation;
on lattice, lives on sites

Also: need to fix residual gauge symmetry
under time-independent gauge transf.'s

# Partition function

As usual,

$$Z = \sum_{\text{states}} \int \mathcal{D}\chi \; e^{-(\mathcal{H} + \mathcal{H}_{Gauss})/T}$$

$A_i$, $E_i$ <u>and</u> $\psi$, $\overline{\psi}$ $\Leftarrow$ only carry color @ $y$

But:

$$Z = \sum \ldots e^{-\mathrm{tr}_{color}\, ig\chi Q}$$

$$\chi \sim A_0$$

$$W = \mathrm{tr}_{color}\, e^{ig\int A_0 dx}$$

$\chi \sim A_0$. Sure. But how does $\mathrm{tr}_{color}$ "upstairs", $\bar{c}$ $\chi$, go "downstairs", $\bar{c}$ $A_0$?

# Test states

With test charges, need to expand the Hilbert space

Easy to do! Can always choose $\chi$'s to be diagonal by gauge rotation

$$\bar\chi, \chi = \text{color } 1 \Rightarrow \bar\chi t^a \chi = t^a_{11}$$

$$\vdots \qquad\qquad \vdots \qquad\qquad \vdots$$

$$\text{"} \qquad \text{"} \quad N \Rightarrow \bar\chi t^a \chi = t^a_{NN}$$

$$\Rightarrow \sum_{\chi, \bar\chi} e^{-ig \int \mathrm{tr}_{\text{color}} XQ/T} = \underbrace{\mathrm{tr}_{\text{color}}\, e^{-igX(y)/T}}_{\text{Polyakov loop}}$$

N.B.: <u>need</u> to include states for $\chi, \bar\chi$

In all

Partition fnc.:

$$Z = \sum_{\{A,E\}} \int \mathcal{D}X \; e^{-\frac{1}{T}\int \mathcal{H} + ig\,\mathrm{tr}\, X D\cdot E} \; \mathrm{tr}\, e^{-igX(y)}$$

Test chg like "defect" @ $y$: Gaiotto + . . . .

In $\mathcal{H}$-form, need to add extra states

↳ Gervais & Sakita PRD 18 '78

Character expansion in group theory

Susskind PRD 20 '79

W. Greiner & B. Muller Sym.'s of Quantum Mechanics

# Gauge transf.'s

Int. by parts:

$$\int \mathrm{tr}\, \chi D\cdot E = \int_{\infty} \mathrm{tr}\, \chi E_i - \int \mathrm{tr}\, \underbrace{D_i \cdot \chi}_{\text{generates } \Omega = e^{i\chi} \text{ on } A_i} E_i$$

spatial $\infty$

$\chi$ = local ($\to 0$ @ $\infty$) + global

Local $\chi$:

$$P_\chi = \int \mathcal{D}\chi \; e^{ig\int \mathrm{tr}\, \chi D\cdot E} = \delta(D\cdot E)$$

= projector, $\quad P_\chi^2 = P_\chi$

Sure...

## Global transf.'s

Total color charge $= \int_\infty E^a$

$\Rightarrow$ integrating over all constant $\chi^a$

imposes total color chg $= 0$

Ok in confined phase, <u>not</u> necessary in deconfined

For $\chi_\infty =$ const.,

$$\underbrace{e^{ig\int \mathrm{tr}\, \chi_\infty E/T}}_{} |A\rangle = |A^{\Omega_\infty}\rangle \qquad e^{ig\chi_\infty/T}$$

<u>not</u> a projector,

but a global gauge rotation

# Twisting

For const. $\Omega_\infty$

$$Z(\Omega_\infty) = \sum \langle A_i | e^{-\mathcal{H}/T} \delta(D \cdot E) \underbrace{e^{ig\int \chi_\infty E/T} | A_i \rangle}$$

$$= \sum \langle A_i | e^{-\mathcal{H}/T} \delta(D \cdot E) | A_i^{\Omega_\infty} \rangle$$

If $\Omega_\infty = \omega_j$, $|A_i^{\omega_j}\rangle = |A_i\rangle \Rightarrow$

$$Z(\omega_j) = Z(1) \qquad \text{else} \qquad Z(\Omega_\infty) \neq Z(1)$$

$Z(\Omega_\infty)$ = "twisted" partition function

# Polyakov loop

Need path in group space

$$\chi^j = \frac{2\pi T}{gN}\, j\, t_N \,, \quad t_N = \begin{pmatrix} \mathbb{1}_{N-1} & \\ & -(N-1) \end{pmatrix} \quad \omega_j = e^{ig\chi^j/T}$$

$$\langle \mathrm{tr}\, \mathbb{1} \rangle = \sum \langle A_i | e^{-H/T} S(D\cdot E)\, e^{ig\chi/T} | A_i \rangle$$

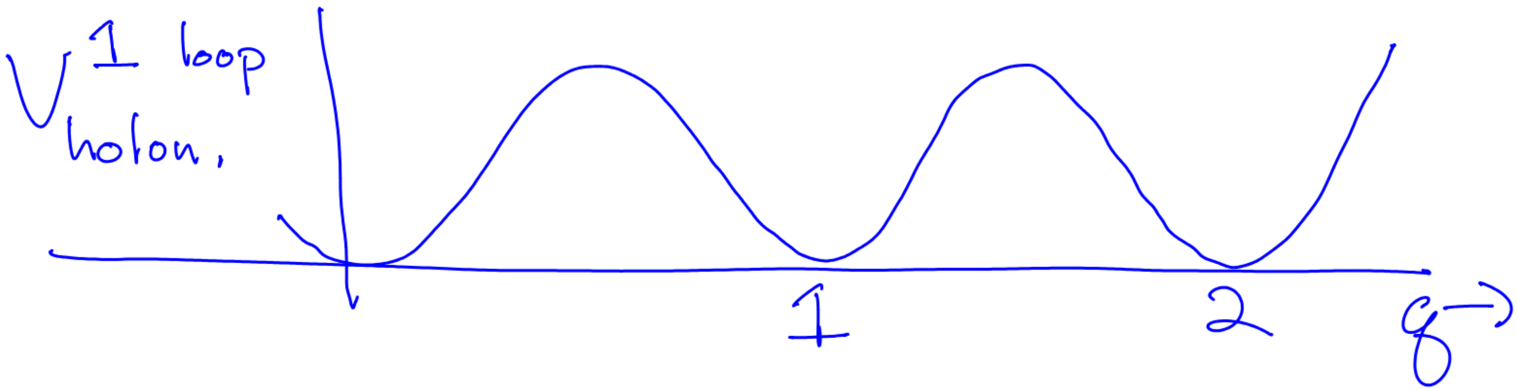
$$\xrightarrow{\chi + \chi^j} \sum \langle A_i | e^{-H/T} S(D\cdot E)\, e^{ig\chi/T} | A_i^{\omega_j} \rangle\, \omega_j$$

In $H$ form, no 1-form sym., only usual (0-form) sym. for defect. Gaiotto + ...

# Holonomous potential

With $\mathcal{L}$ form, compute potential for const. $A_0$:

$$A_0 = \chi' q \Rightarrow V^{1\ \text{loop}}_{\text{holon.}} \sim T^4 q^2 (1-q)^2 \underbrace{V}_{\text{volume}}$$




$q = n \Rightarrow \omega_n$

Degeneracy of $V_{\text{holon.}}$ @ $q = n$, $\mathbb{1} = \omega_n$

= Degeneracy of twisted partition function

+ Quarks

$\bar{c}$ qks, $\psi(\bar{x}, 1/T) = -\psi(\bar{x}, 0)$

$\Rightarrow \psi^{\Omega}(1/T) \Rightarrow -\Omega\,\psi(1/T) \neq -\psi(0) \quad \forall\, \Omega$

$Z(N)$ sym. lost. In $\mathcal{H}$ form:

$$Z(\omega_j) = \sum \langle A_i \psi | e^{-\mathcal{H}/T} \delta(D \cdot E)\, \underbrace{e^{ig\chi^j/T} | A_i, \psi \rangle}_{|A_i^{\omega_j}, \omega_j \psi\rangle = |A_i, \omega_j \psi\rangle}$$

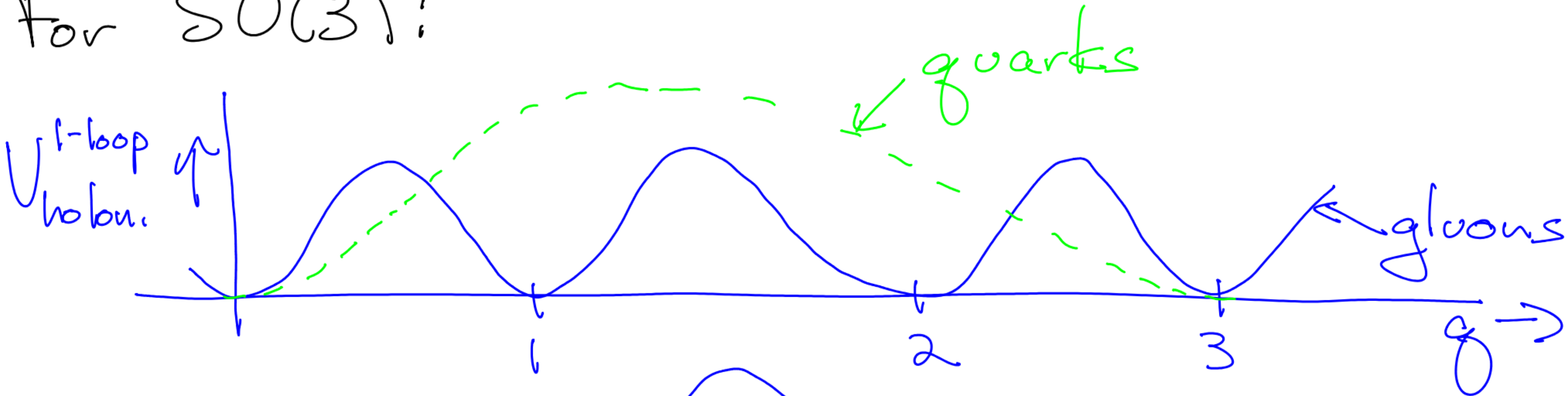
$\neq Z(1)$

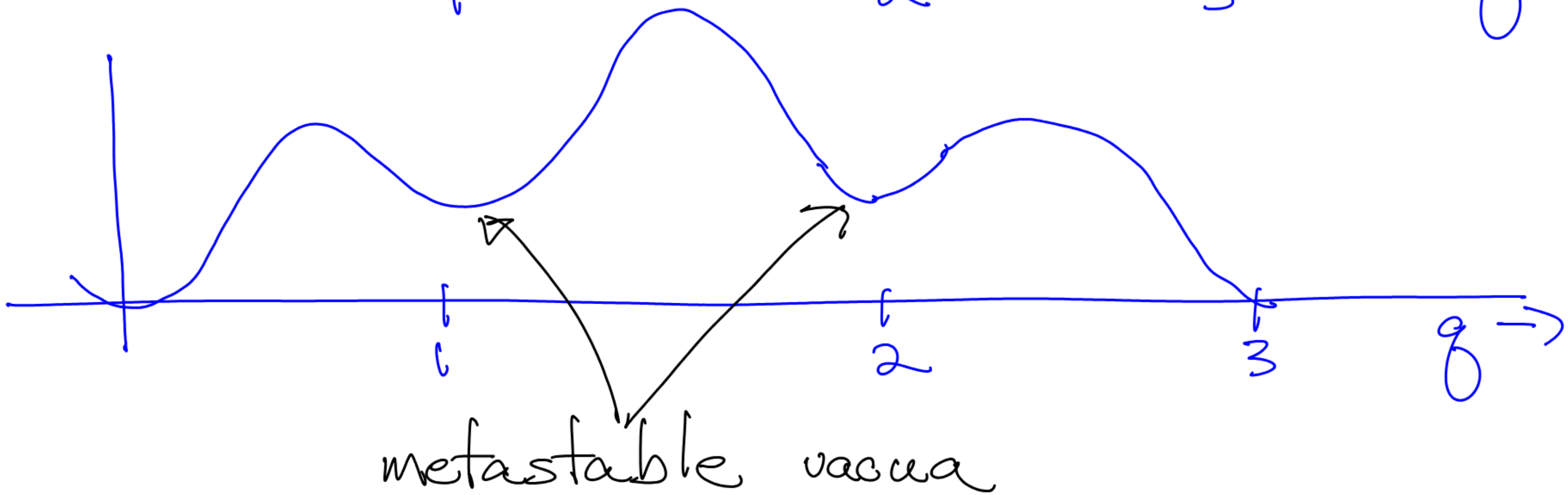
Can compute $Z(\Omega_\infty)$ directly: $\chi^j$ acts imaginary chemical potential for color

# Holonomous pot. + $q$ks

In $\mathcal{L}$ form, computing @ const. $A_0$

For SU(3):



# In all

The Polyakov loop is not a figment of an overly active (Euclidean) imagination

Const. $A_0$ (holonomy) in $\mathcal{L}$ form

$\sim$ twisted partition fnc. in $\mathcal{H}$

For QED in 1+1 dim.'s, are there <u>stable</u> thermal kinks (Smilga '94)?

Implications for domain walls in 3+1 dim.'s

Relevant for QED in 3+1 dim.'s & cosmology?

# Fractional Topological Charge

RDP & V.P. Nair, 2206.11284

In SU(N) gauge theories withOUT dynamical quarks,
at large N, need objects not $\bar{c}$ $Q_{top} = \pm 1, \pm 2 \ldots$,
but

$$Q_{top} = \pm \frac{1}{N}, \pm \frac{2}{N} \ldots$$

Arise immediately $\bar{c}$ $Z(N)$ twisted b.c.'s
't Hooft '80 + ...

On "femto-slab": width $L \ll \Lambda^{-1}_{QCD}$
Unsal 2007.03880; Poppitz 2111.10423

Today: "quantum" instantons $\bar{c}$ size $\sim 1/\Lambda^{-1}_{QCD}$
measurable on lattice with<u>out</u> cooling

# Instantons

Topological charge $Q \sim \int d^4x \, \mathrm{tr}\, G_{\mu\nu} \tilde{G}^{\mu\nu}$ $\quad \epsilon^{\mu\nu\alpha\beta} G^{\alpha\beta}$

Instantons – solutions to classical equations of motion

self-dual, $G_{\mu\nu} = \pm \tilde{G}_{\mu\nu}$

As classical eqs. scale invariant, instantons come in all scale sizes, $\rho : 0 \to \infty$

For $Q = n =$ integer, $S = \int \mathrm{tr}\, G_{\mu\nu}^2 = \frac{8\pi^2}{g^2} n$

Construction of all instantons known – collective coor.'s (moduli space) involved

Atiyah, Hitchin, Drinfeld, Manin Phys. Lett. A65 '78

# Instantons$^2$

Inst.'s valid semi-classically, when $g^2 \ll 1$.

E.g., temperature $T \gg \Lambda_{QCD}$.

By asymptotic freedom, $g^2(T) \sim \# / \ln T$

$$\Rightarrow Z_{inst.} \sim e^{-8\pi^2/g^2(T)} \sim \#' / T^c \qquad c = \frac{11N_c - 2N_f}{3}$$

$\underline{T \gg \Lambda_{QCD}}$

Lattice, $N_c = 3$:

Instantons dominant for $T > 300$ MeV $\sim \Lambda_{QCD}$ !

$\#' \sim 10 * $ 1-loop result - need 2-loop!

Borsanyi + ... 1606.07494 ... Jahn, Junnarkar, Moore + Robaina 2103.01069

Petreczky + ... 1606.03145

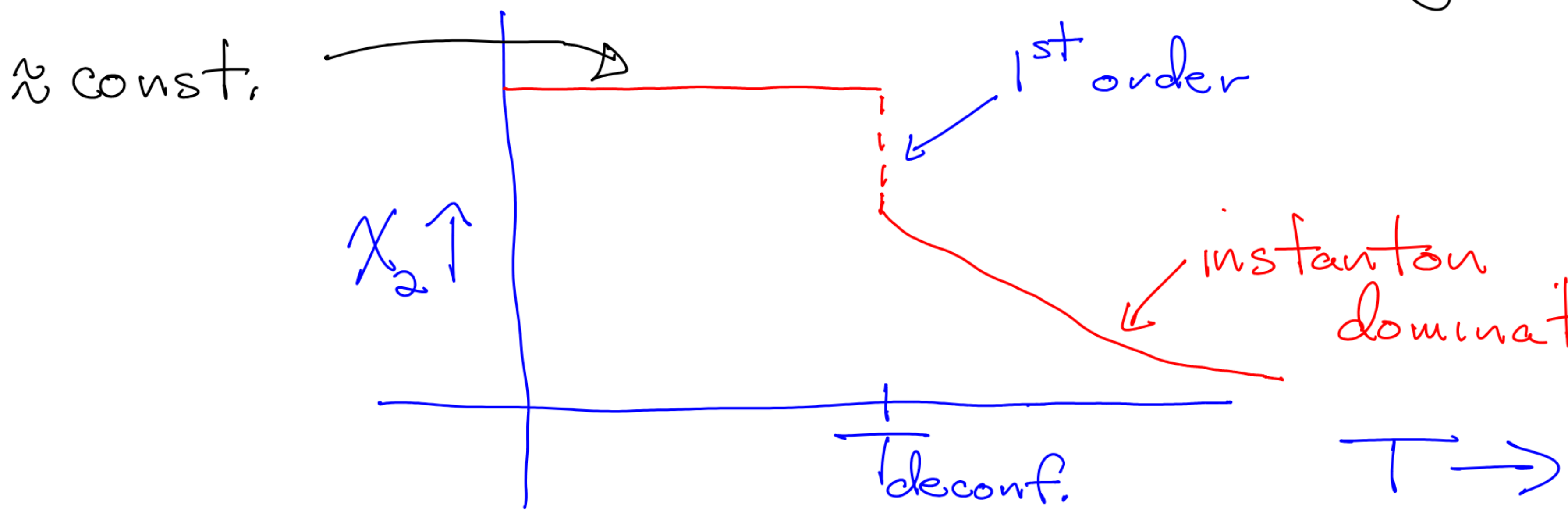
# Lattice - pure glue

Compute top. susceptibility:

$$S_\theta = S_0 + i\theta Q \qquad \chi_2 = \frac{\partial^2 \ln Z}{\partial \theta^2} \sim \langle Q^2 \rangle$$

For pure SU(N) glue, for $N \geq 3$, weak dependence on N. First order transition @ $T_{deconfinement} \sim 270$ MeV

E.g., $T_c/\sqrt{\sigma} \sim$ const. $\approx N$ ($\sigma$ = T=0 string tension)



# Lattice – $\bar{c}$ quarks

Three regimes: $T > 300$ MeV – instanton dominated

QCD – $T_\chi \sim 155$ MeV (crossover)

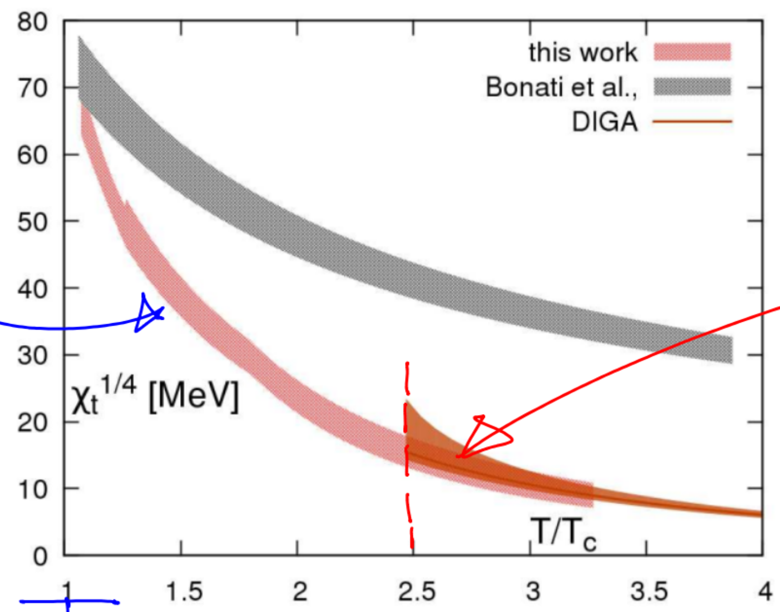
Intermediate T: $300 > T > 155$ MeV

<u>not</u> instanton – slower fall off for $\chi_2$

Low T: $\chi_2 \approx$ const, $T < 155$ MeV

$\chi_2^{1/4} \uparrow$

<u>not</u> instantons




instanton dominated

$T_\chi$

$T \longrightarrow$

Petreczky + ...
1606.03145

# Instantons @ large $N$

Veneziano '79, Witten '79

As $N \to \infty$, hold $g^2 N$ fixed as $N \to \infty$

$$\chi_2 \sim e^{-8\pi^2/g^2} \sim e^{\left(-8\pi^2/g^2 N\right) N}$$

Even so, argued $\chi_2 \sim 1$ @ $T = 0$, $N = \infty$

# Instanton liquid?

At $N=\infty$, instantons of <u>one</u> scale will dominate

$$S(g^2) = 8\pi^2 N \left( \frac{1}{g^2} + n \ln g^2 + \ldots \right)$$

1 loop    2 loop

$$\left.\frac{\partial S}{\partial g^2}\right|_{g_*^2} = 0 \Rightarrow g_*^2 = \frac{1}{n}$$

Instantons of one size dominate as $N \to \infty$,

But $S(g_*^2) \sim N \neq 0$

but <u>need</u> the action $\sim N = O$!

N.B. - Liu, Shuryak, Zahed 1802.00540

# "Fractional" instantons

IF there are objects $\bar{c}$ $Q_{top} \sim 1/N$, then no problem!

$$\chi_2 \sim e^{-8\pi^2/(g^2 N)} \sim 1 \quad \text{as} \quad N \to \infty$$

't Hooft '80, van Baal '82, Sedlacek CMP 86 '82

$Q_{top} \sim 1/N$ $\bar{c}$ $Z(N)$ twisted boundary conditions

manifestly finite volume

only analytic soln's for finite box

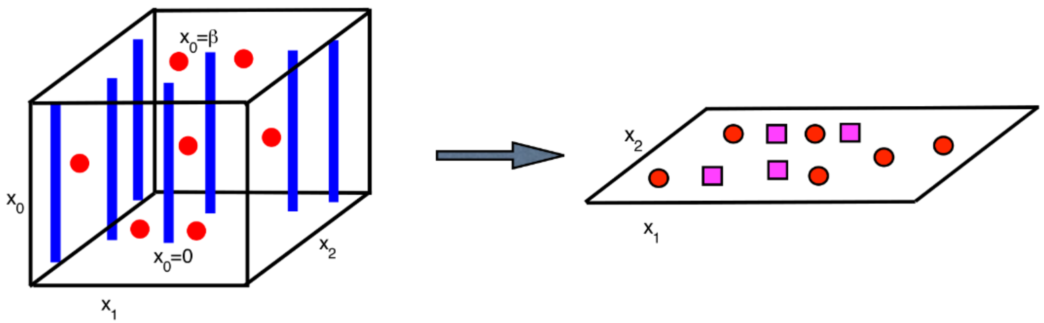
$\bar{c}$ sizes in certain ratio, etc.

Do they persist in infinite volume?

# Femto-slab

Unsal, Poppitz, Anber ...

Take one spatial dimension, $L$, $\ll \Lambda^{-1}_{QCD}$, so $g^2(L\Lambda_{QCD}) \ll 1$. Using results in 2+1 dim.'s, over large distances, confining thy $\bar{c}$ "monopole-instantons", $Q_{top} \sim 1/N$




Poppitz, 2111.10423

Size of monopole-inst.'s $\sim L$

What happens as $L \sim \Lambda^{-1}_{QCD}$ ?

# Punchline

Size of monopole-instanton gets stuck at $L \sim \Lambda_{QCD}^{-1}$

Dominant configurations $\approx$ one size

Measurable on lattice $\bar{c}$ <u>adjoint</u> (not fund.) quark prop. as <u>external</u> probe

<u>not</u> dynamical

# $CP^{N-1}$ model

In 1+1 dim.'s: $N$ component field $z^i$, $\bar{z}^i z^i = 1$, inv.:

$$z^i(x) \to e^{i\alpha(x)} z^i(x)$$

local $U(1)$

$$\mathcal{L} = \frac{1}{g^2}\int d^2x \, |D_\mu z^i|^2 \qquad D_\mu = \partial_\mu - iA_\mu$$

Global sym.: $z^i \to U^{ij} z^j \Rightarrow SU(N)$

But: if $U = \omega_1 = e^{2\pi i/N}$, $z^i \to \underbrace{e^{2\pi i/N}}_{\text{part of } U(1)} z^i$

$\Rightarrow$ global sym. $SU(N)/Z(N)$

$g^2$ asymptotically free, soluble as $N \to \infty$

Witten '79 d'Adda, Lüscher, Di Vecchia '78

# $CP^{N-1}$ inst.'s

Topological chg.

$$Q_{top} = \frac{1}{2\pi}\int d^2x \; \varepsilon^{\mu\nu}\partial_\mu A_\nu$$

<u>All</u> classical soln's known; self-dual,

$$D_\mu z = \varepsilon^{\mu\nu} D_\nu z \qquad z^i \sim \frac{(x+iy)\, v^i}{\sqrt{x^2+y^2+\rho^2}}$$

Can compute 1-loop fluc's about all instantons

$$S_{top} \sim \frac{1}{g^2} \sim \left(\frac{1}{\underbrace{g^2 N}_{\text{fixed}}}\right) N \;\Rightarrow\; e^{-S_{top}} \sim e^{-\# N}$$

But: large $N$ soln. shows that

$$\chi \sim \langle Q_{top}^2 \rangle \sim \frac{1}{N} \quad \underline{\text{not}} \quad e^{-N} \;!$$

# Frac. inst.'s in $CP^{N-1}$

Consider

$$z^1(r,\theta) = e^{i\varphi/N}\, h(r) \qquad z^{2,\ldots} = 0$$

Not single-valued:

$$z^1(r,2\pi) = e^{2\pi i/N}\, h(r) \sim z^1(r,0)$$

by $Z(N)$ sym. Then the corresponding

$$A_\varphi \sim \frac{1}{rN}, \quad r\to\infty \quad \Rightarrow \quad Q_{top} = \frac{1}{N}$$

To obtain frac. $Q$, require multi-valued soln.'s allowed by $Z(N)$

# $CP^N$ @ $N \to \infty$

Introduce a constraint field $\lambda(|z|^2 - 1)$, integrate out $z$'s

$$S_{eff} = N \,\mathrm{tr} \ln(-D_\mu^2 + i\lambda) - i \int \frac{\lambda}{g^2} d^2x$$

Vacuum: $i\lambda = m^2$ (dim. trans.), $A_\mu = 0$

Frac. inst.: action non-local, only limiting behavior

But: scale sym. of classical action lost

frac. inst. has <u>one</u> size $\sim 1/m$ = confinement distance

SU(N) gauge, NO quarks

Back to 3+1 dim.'s, <u>no</u> quarks, $A_0 = 0$ gauge

Parametrize gauge field as function of arbitrary parameter $\xi$,

$$A_i(\bar{x}, \xi) = (1-\xi) A_i(\bar{x}) + \xi A_i^{\Omega}(\bar{x})$$

gauge transf. of $A_i$

If $\Omega \to \mathbb{1}$ @ $\bar{x} \to \infty$,

$$Q_{top} = \frac{1}{8\pi^2} \int \mathrm{tr}\, G_{\mu\nu} \tilde{G}^{\mu\nu} = \frac{1}{24\pi^2} \int d^3x \; \mathrm{tr} \left( \Omega^{\dagger} \partial_i \Omega \right)^3$$

$$= \text{integer}$$

Above doesn't give soln. $\bar{c}$ minimal action, but gets $Q_{top}$ right

# $Z(N)$ vacua

But alternately, we can choose $\Omega_\infty = \omega_j = e^{2\pi i j/N} \mathbb{1}$

$$\Omega_\infty(\xi) = e^{i\chi^j \xi} \qquad \chi^j = \frac{2\pi}{N} j\, t_N, \quad t_N = \begin{pmatrix} \mathbb{1}_{N-1} & \\ & -(N-1) \end{pmatrix}$$

For finite $r = \sqrt{x^2+y^2+z^2}$, need more involved ansatz

$$Y_{ij} = \frac{\sigma\cdot\hat{x}}{2} + \frac{1}{N} - \frac{1}{2} \quad i,j = 1,2 \qquad Y_{ij} = \frac{\delta^{ij}}{N} \quad i,j = 3\ldots N$$

$$\Omega(\bar{x}, \xi) = e^{iY\theta(r,\xi)}$$

$\theta = 0$ @ $r = 0$, $\xi = 0$

$= 2\pi\xi$ @ $r = \infty$

Only illustrative construction; not minimal action

# Topological Chg.

$$G_{\mu\nu} \to \Omega^{+} G\, \Omega + \frac{\partial}{\partial \xi} A^{\Omega} = \Omega^{+} (G - Da)\, \Omega$$

Variation in $\xi$ like "time" $\qquad a = \frac{d\Omega}{d\xi}\, \Omega^{-1}$

"a" analogous to $A_0$

$$Q_{top} = \frac{1}{8\pi^2} \int tr (G - Da)^2\, d^4x$$

$$= \frac{1}{4\pi^2} \int_{x=\infty} d^2 s^i \int dx\; tr (a B^i)$$

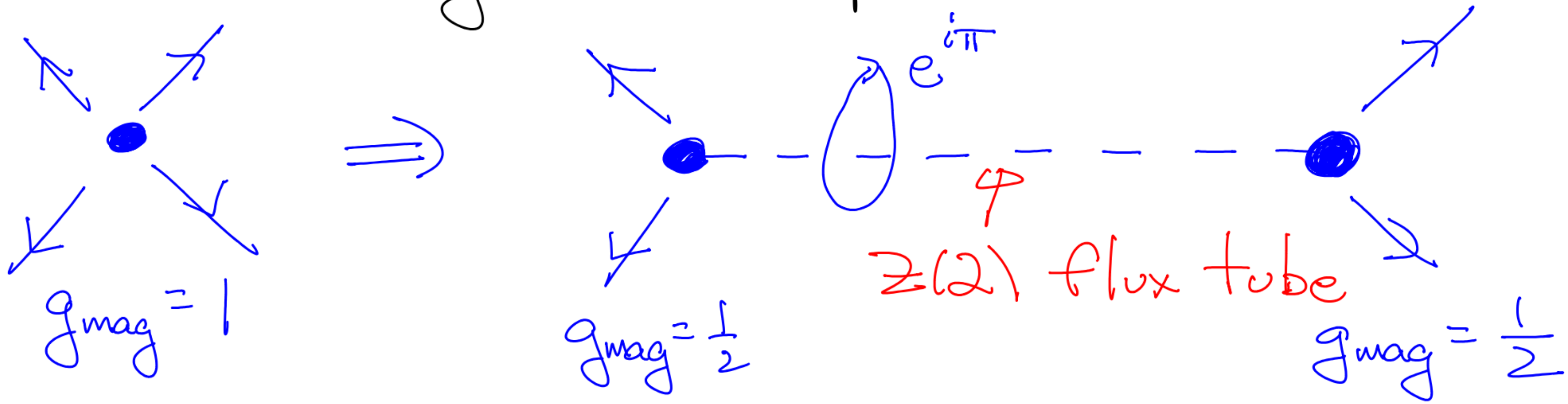
Need magnetic chg. For $SU(2)$:

$$G_{ij} \sim -\frac{i}{2} \vec{\sigma} \cdot \hat{x}\; \varepsilon_{ijk} \frac{\hat{x}^k}{r^2} \quad \Rightarrow \quad Q_{top} = 1$$

$S_0$?

# "Split" $Z_2$ monopole

For a SU(2) magnetic monopole



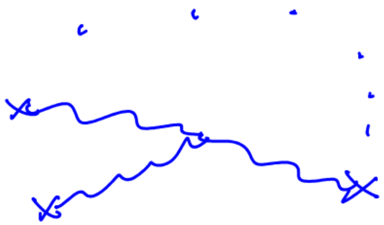

Without quarks, $Z(2)$ flux tube invisible

Each end has $Q_{top} = +\frac{1}{2}$.

To observe each end seperately, do need $Z(2)$ twisted boundary conditions

# Vacuum of frac. inst.'s

For $SU(N)$, instanton $\bar{c}$ $Q_{top} = 1$ $= N$ frac.'s



Natural length scale of $\mathbb{Z}(N)$ flux tubes $\sim \Lambda_{QCD}^{-1}$

Configurations manifestly non-perturbative.

So what? Vacuum = sum of $I$'s & $\bar{I}$'s

or - better described as sum of $Q_{top} = +\frac{1}{N}$ & $-\frac{1}{N}$?

Unsal: on femto-slab, $\theta$-dependence <u>is</u> $f(\theta/N)$, not $f(\theta)$. Presumably carries over

2007.03880

# $Z(N)$ dyon

A explicit construction, @ $T > T_{deconf}$. Now $A_0 \neq 0$

$$A_0^\infty = \frac{2\pi T}{gN} k, \qquad k = t_N = \begin{pmatrix} \mathbb{1}_{N-1} & \\ & -(N-1) \end{pmatrix}$$

$$\mathbb{L} = e^{ig\int_0^{1/T} A_0 dx} = e^{\frac{2\pi i}{N} k} \qquad \text{or} \qquad t_N' = \begin{pmatrix} -(N-1) & \\ & \mathbb{1}_{N-1} \end{pmatrix}$$

$k$ = nontrivial holonomy. Above value is a minimum of the holonomous potential

$$A_0 = \frac{2\pi T}{gN} q k \Rightarrow V_{holonomous} \sim T^4 q^2 (1-q)^2$$

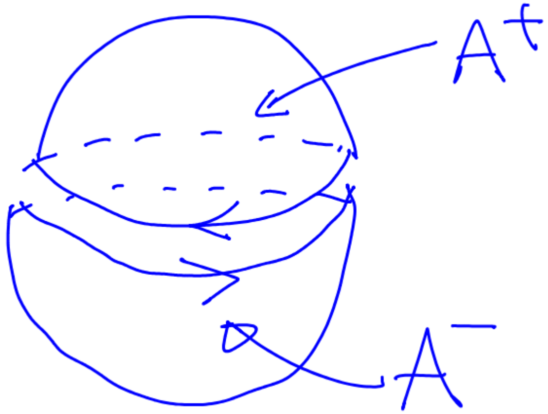
$|q| \bmod 1$

# Z(N) monopole

At spatial $\infty$,

$$A_\varphi^{\pm} = \frac{1}{N\,2r}\, m\, \frac{(\pm 1 - \cos\theta)}{\sin\theta} = \frac{1}{N} * \text{Dirac monopole}$$

$m$ = magnetic chg, $\sim k_1$ or $k_2$




$$e^{i\oint A^+} e^{-i\oint A^-} = e^{\frac{2\pi i}{N} m}$$

Above $A_\varphi$ at spatial $\infty$. For $A_0$

$$A_0(r) \underset{r\to\infty}{\sim} \frac{2\pi T}{N} k - \frac{1}{2Nr} m + \ldots$$

# $Z(N)$ dyon

Assume there is a regular solution for all $r$, esp. $r=0$. For simplicity, take it to be static

$$Q_{top} = \frac{1}{4\pi^2}\int \partial_i \, tr \, A_0 B_i = \frac{1}{N^2} tr(mk)$$

$m=k$: $\quad Q_{top} = \frac{N-1}{N}$ $\qquad\qquad$ $m \neq k$: $\quad Q_{top} = \frac{1}{N}$

Identical to 't Hooft, but $Z(N)$ twist not in a box, but in radial direction

# Dyons vs calorons

Lee & Lu: th/9802108
Kraan & van Baal: th/9805168 } KvBLL

Show instanton @ $T \neq 0$ composed of
N constituents $\bar{c}$ $Q_{top} = 1/N$

| | KvBLL | Z(N) dyon |
|---|---|---|
| magnetic chg | integer | $1/N$ |
| $V_{hol}(q)$ | maximum $\sim 1/N$ | minimum ⇒ integer |

But max. of $V_{hol.}$ ⇒ @ 1-loop, free energy
$\sim +$ volume of space!

## Z(N) dyons

$T > T_{deconf}$ : Z(N) electric chg unconfined
but Z(N) magnetic chg confined
$\Rightarrow$ Z(N) dyons bound $> T_d$ $\Rightarrow$ instantons soon dominate

$T < T_{deconf}$ : Z(N) dyons propagate freely
Geometry not obvious – world lines of dyons twisted

Presumably size $\sim \Lambda_{QCD}^{-1}$

# Lattice - pure glue

Edwards, Heller, Narayanan lat/9806011

To measure $Q_{top} \sim 1/N$, use $\chi$-symmetric gt. prop.
in adjoint, not fund., representation

Fund. rep.: 2 zero modes for $Q_{top} = 1$

Adj. rep.: $2N$ " " " "

2 zero modes for $Q_{top} = 1/N$

From eigenvector, estimate size

If $Q_{top} \sim 1/N$, are they dilute or $\underbrace{\text{densely packed}}_{\text{probably}}$?

## Lattice simulations

Fodor + ... 0905.3586 - SU(3) $\bar{c}$ sextet rep.

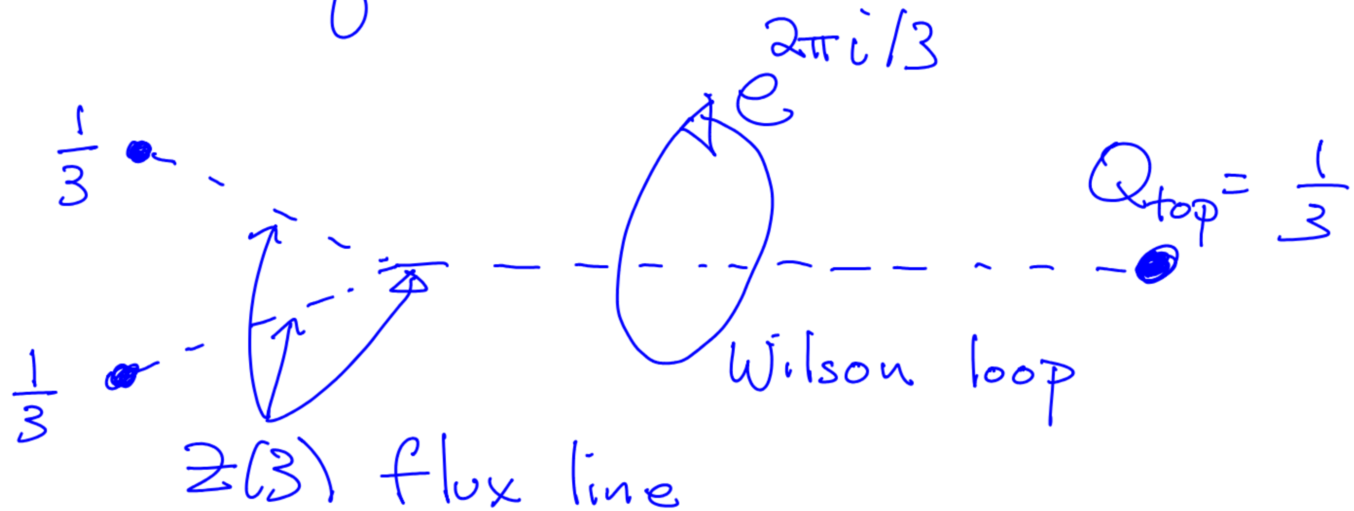
No evidence for frac. $Q_{top}$

But sextet only sensitive to $Q_{top} = \frac{1}{5}$, not $\frac{1}{3}$

$$\left.\begin{array}{l}\text{S. Sharma: } N_c = 2, \text{ near } T_{deconf} \\ \text{N. Karthik \& R. Narayanan: large } N_c\end{array}\right\} \text{ in progress}$$

## + Dynamical quarks

"Split" monopoles in SU(3):




Z(3) flux tube invisible to gluons, but Wilson loop in fund. rep., <u>or</u> dynamical quarks, see it.

$\Rightarrow$ Dynamical quarks <u>confine</u> objects $\bar{c}$ $Q_{top} = \frac{1}{N}$

Yes, quarks confine. Clearly complicated int.'s between qks & Z(N) dyons

SU(3) ; $T \neq 0$, $\mu = 0$

Lattice : c/o qks, $T_{deconf} \sim 270$ MeV

2+1 flavors, $T_{\chi} \sim 155$ "

Perhaps three regimes

$T > 300$ : instanton dominated

(Z(3) dyons confined)

$300 > T > T_{\chi}$ : Z(3) dyons + ≈ massless qks

$T_{\chi} > T$ : Z(3) dyons + massive qks

Quantitative tests of dynamical qks $\bar{c}$ dyons?

# Three regimes @ $\mu \neq 0$, $T \approx 0$?

Because of # d.o.f., instantons suppressed @ $T \neq 0$ much more than $T = 0$, $\mu \neq 0$

$$m^2_{Debye} = g^2 \left( \frac{1}{3} \left( N_c + \frac{N_f}{2} \right) T^2 + 2 N_f \left( \frac{\mu_{qk}}{2\pi} \right)^2 \right)$$

RDP & Rennecke: instantons don't dominate until $\mu_{qk} > 2$ GeV (or more)

Perhaps:

$\mu_{qk} > 2$ GeV – instantons

$2 \text{ GeV} > \mu_{qk} > \mu_\chi$ – $Z(N)$ dyons + massless qks

$\mu_\chi > \mu > 313$ MeV – " " + massive qks

?

Clearly base speculation

# Summary

For pure gauge, two sources of fluc's in topological charge

instantons in weak coupling – all sizes

Z(N) dyons " strong " – one size

→ confine? world line of Z(N) dyon ~ Z(N) vortex

With quarks: much more complicated

$\mu = 0$, $T > 300$ MeV – dyons confined into inst.'s

$T <$ " " – dyons & quarks int.'g

$T = 0$, $\mu \neq 0$: dyons & qks relevant for all densities in neutron stars

instantons never matter

# Nuclar matter in 1+1 dim.'s

't Hooft : $SU(N_c)$ soluble as $N_c \to \infty$

Gauge coupling $g \sim$ mass. Need $m_{qk} > g$

Konik, Lajer, RDP & Tsvelik, 2112.10238 - small $N_c$

$m_{qk} \ll g$ : solve near Fermi surface - simple

$N_f = 1$ - just "Luttinger liquid"

$N_f \geq 2$ - & Wess-Zumino-Witten

Soluble using Conformal Field Theory

# Abelian bosonization, $N_f = 1$

Coleman '74: $j_\mu = \bar{\psi} \gamma_\mu \psi = \varepsilon_{\mu\nu} \partial^\nu \phi \Rightarrow j_0 = \partial_1 \phi$

In 1+1 dim's, can choose $A_0 = 0$. Leaves $A_1 \equiv A$

Baluni '80: clever gauge. As color matrices

$$A = \text{off-diagonal} \qquad E = \partial_0 A = \text{diagonal}$$

Gauss' Law:

$$\partial_1 E^a = j_0^{aa} \qquad ig(E^a - E^b) A^{ab} = j_0^{ab} \qquad a \neq b$$

$$E^{ab} = E^a \delta^{ab} \qquad a, b = 1 \ldots N_c$$

# Sine-Gordons

Integrating A out, end up $\bar{c}$ $N_c - 1$ sine-Gordons:

$$\mathcal{H} = \frac{1}{2}\sum_{a=1}^{N_c} \pi_a^2 + \underbrace{\tilde{m}}_{\sim m_q k}\left(1 - \cos\left(2\sqrt{\pi}\, E^a\right)\right)$$

$$\mathcal{H}_{gauge} = \sum_{a,b=1}^{N_c} \frac{g^2}{8\pi N_c}\left(E^a - E^b\right)^2 + \Lambda^2 \frac{\sin\left(2\sqrt{\pi}\left(E^a - E^b\right)\right)}{E^a - E^b}$$

$N_c - 1$ sine-Gordon models: solitons, anti-solitons, breathers

<u>Plus</u> $\mathcal{H}_{gauge}$! <u>Not</u> simple

$$N_f = 1, \quad \mu \neq 0$$

Define $\quad \varphi = \frac{1}{\sqrt{N_c}} \sum_{a=1}^{N_c} E^a$

At $\mu \neq 0$, only $\varphi$ matters; $E^a - E^b$ drop out

$j_0 \sim \partial_z \varphi \Rightarrow$ const. $j_0 \Rightarrow \varphi \sim \mu z$

$$\mathcal{L}_{eff} = \frac{1}{2}(\partial_\mu \varphi)^2 - \frac{\tilde{m}}{2\pi} \cos\left(\sqrt{\frac{4\pi}{N_c}}\, \varphi + 2k_0 z\right)$$

$\hookrightarrow \mu / N_c$

Instead of $N_c - 1$ sG models, left $\bar{c}$ <u>one</u>.

N.B.: only for low energy excitations, $< g$

# Solution

$\mu$ = baryon (soliton) $\mu$ $\Rightarrow$ $\mu > m_{\text{soliton}}$

$$\mathcal{L}_{eff} = \frac{K(\mu)}{2}\left(\frac{1}{v_F}(\partial_0\varphi)^2 + v_F(\partial_z\varphi)^2\right)$$

Just <u>free</u>, massless boson (Luttinger liquid)

$\varphi$ = angular variable $\Rightarrow$ $K$ = Luttinger parameter, physical

$v_F$ = Fermi velocity

non-Fermi liquid - no baryons near Fermi surface, just $\varphi$

# Luttinger liquid

Both $K$ & $v_F$ are functions of $\mu$.

Can solve $\bar{c}$ Thermodynamic Bethe Ansatz

$\mu \to m_{soliton}$ : $K \to 1$ , $v_F \to 0$

$\Rightarrow \varphi$ doesn't propagate

$\mu \gg m_{soliton}$ : $K \to \frac{1}{N_c}$ , $v_F \to 1$

$\Rightarrow \varphi$ sub-leading @ large $N_c$