An effective theory of medium induced radiation

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Based on 2206.02811

Jet quenching

- Colliding two heavy nuclei creates quark-gluon plasma
- Hard collision makes highly virtual particle
  - Radiates and creates jet
- Medium interacts with jet and modifies it
  - This is called jet quenching

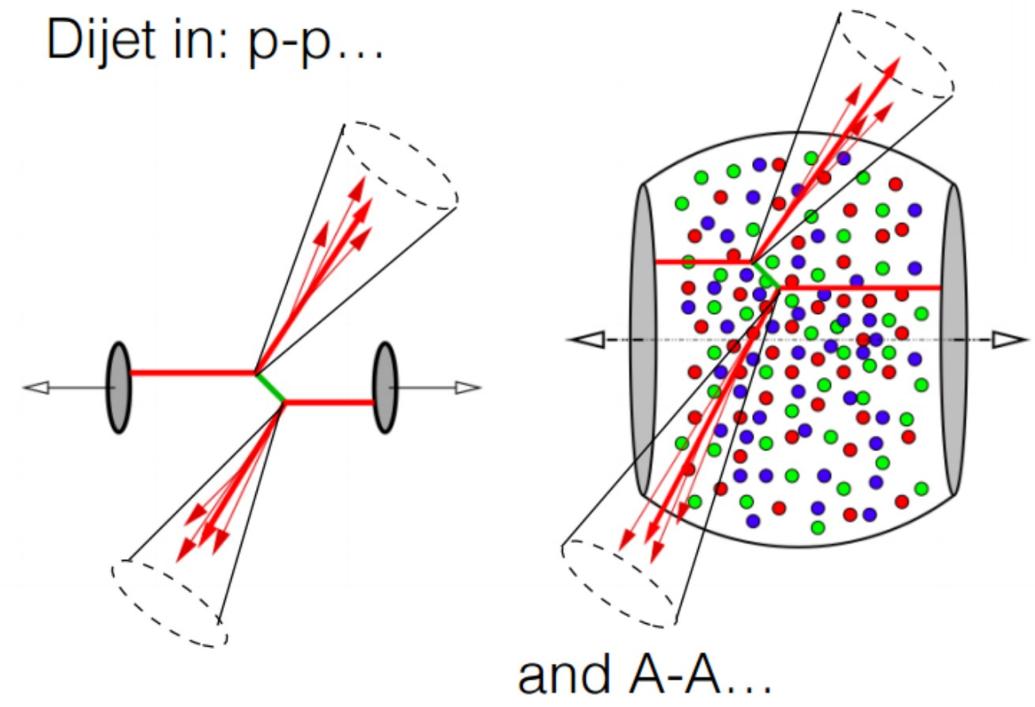

Illustration by C. Andres

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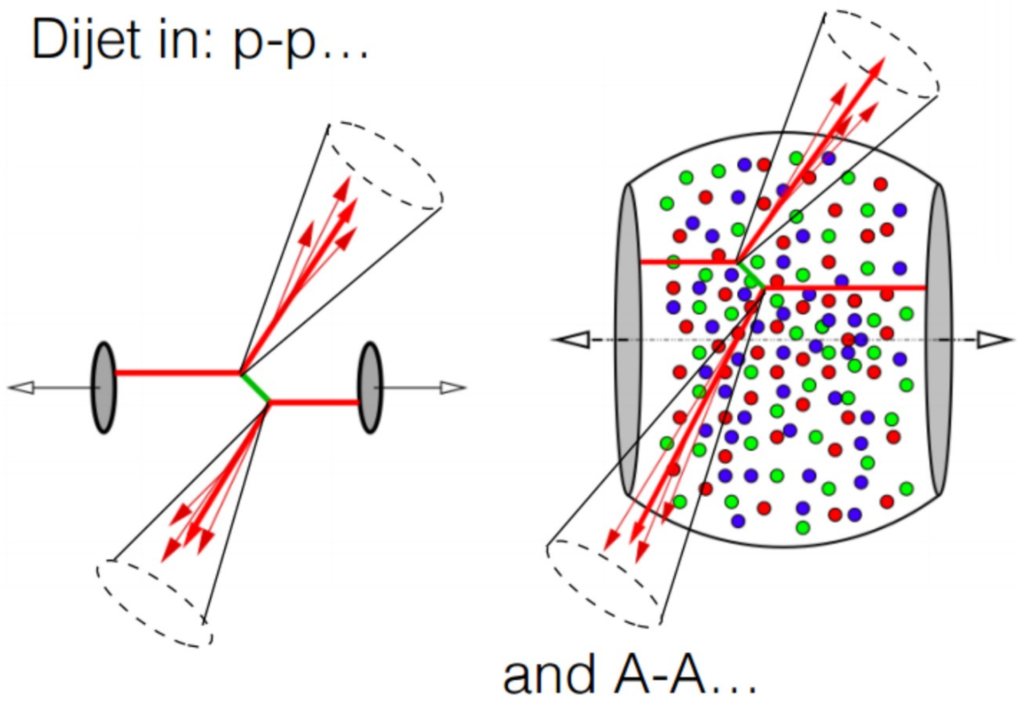
Partons going through the medium

- Transverse momentum broadening
- Elastic collisions with medium constituents
- Radiation
  - Vacuum-like
  - Medium induced



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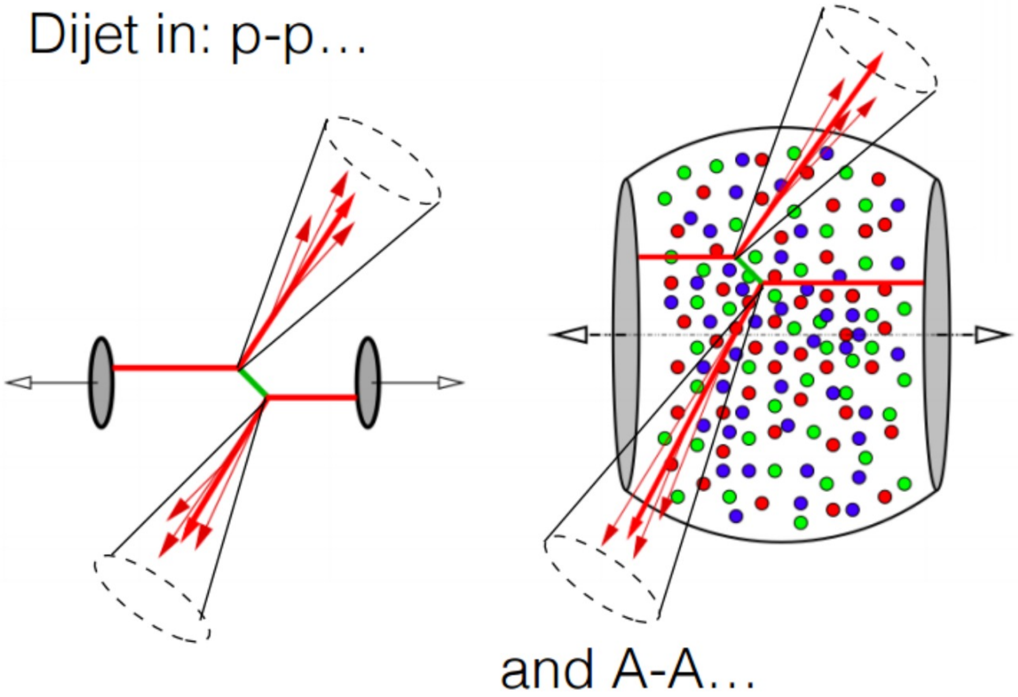

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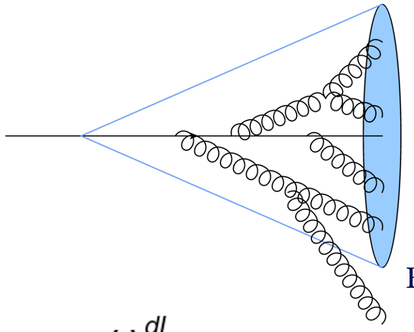
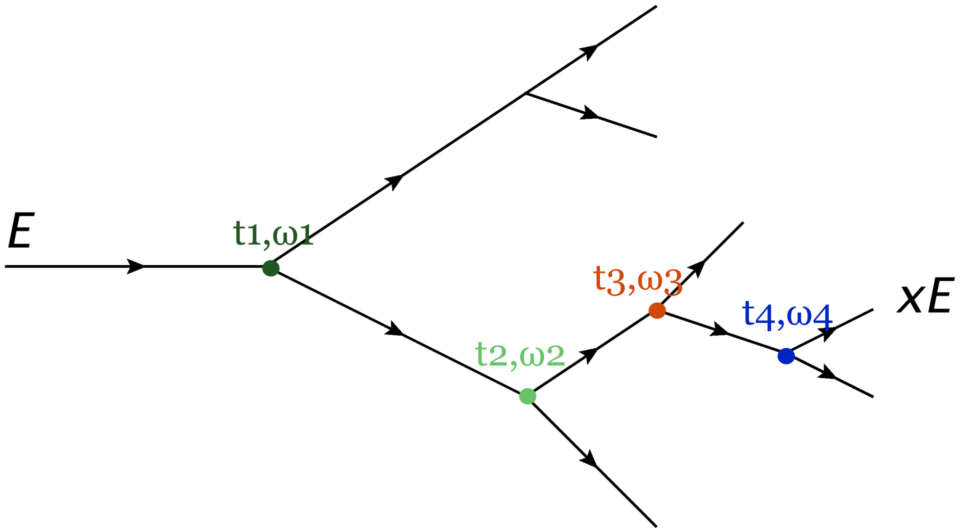
Partons going through the medium

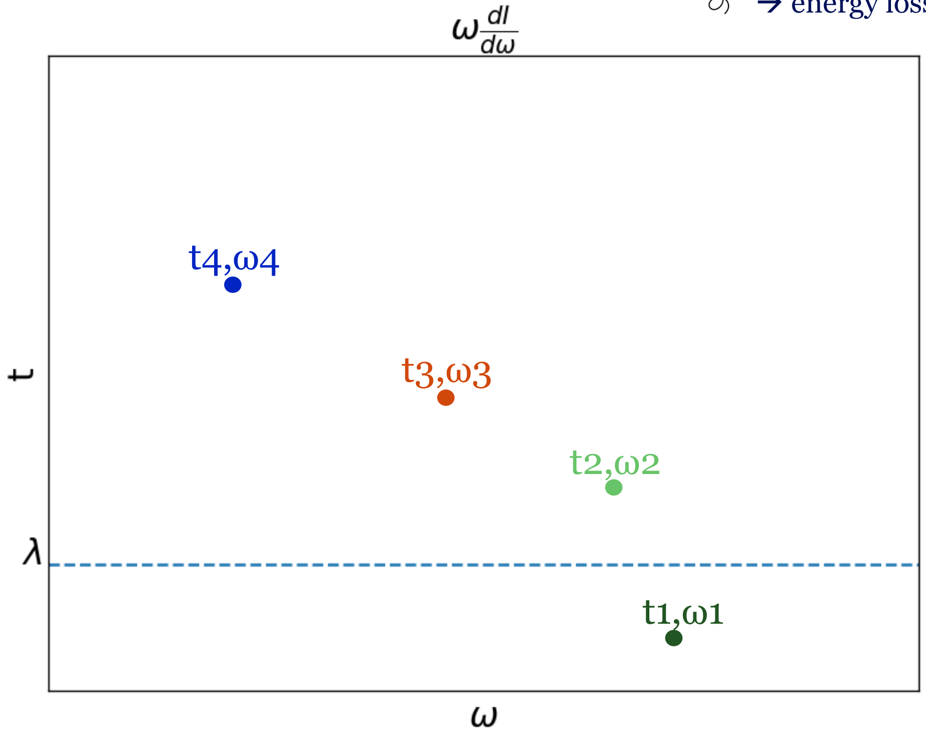
- Transverse momentum broadening
- Elastic collisions with medium constituents
- Radiation
  - Vacuum-like
  - Medium induced → This is what this talk is about

Energy loss in the QGP

- Partons going through the medium scatter with medium constituents
- Scatterings induce emissions
- Emissions lead to radiative energy loss
  - Dominant contribution to energy loss for light quarks and gluons







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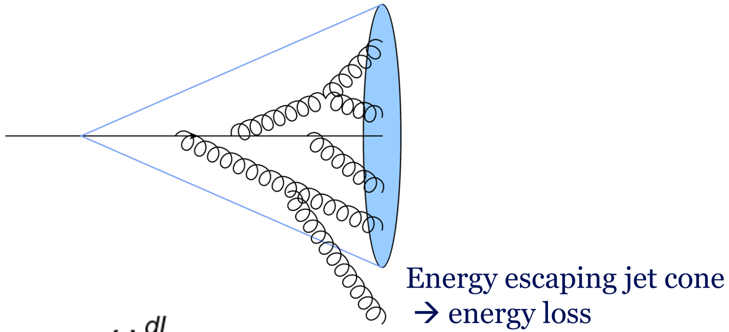
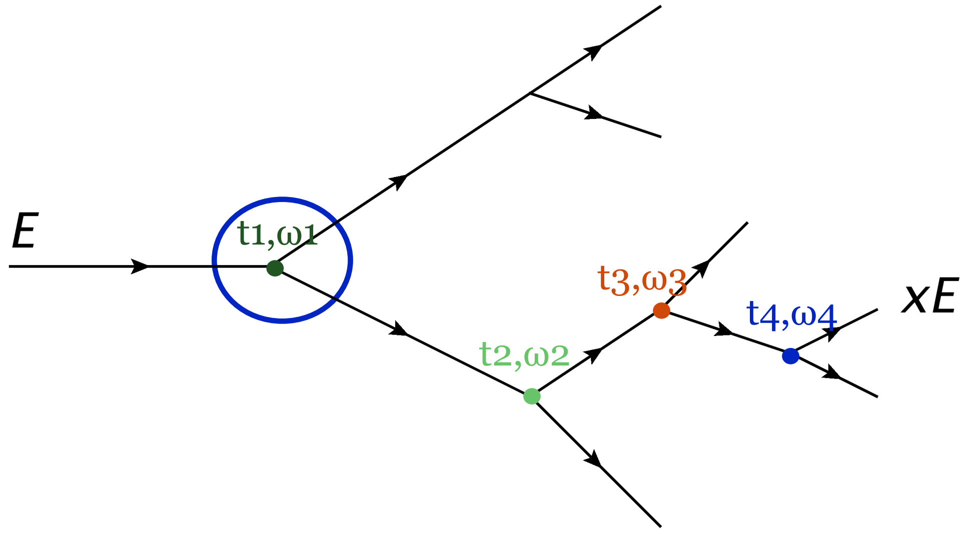
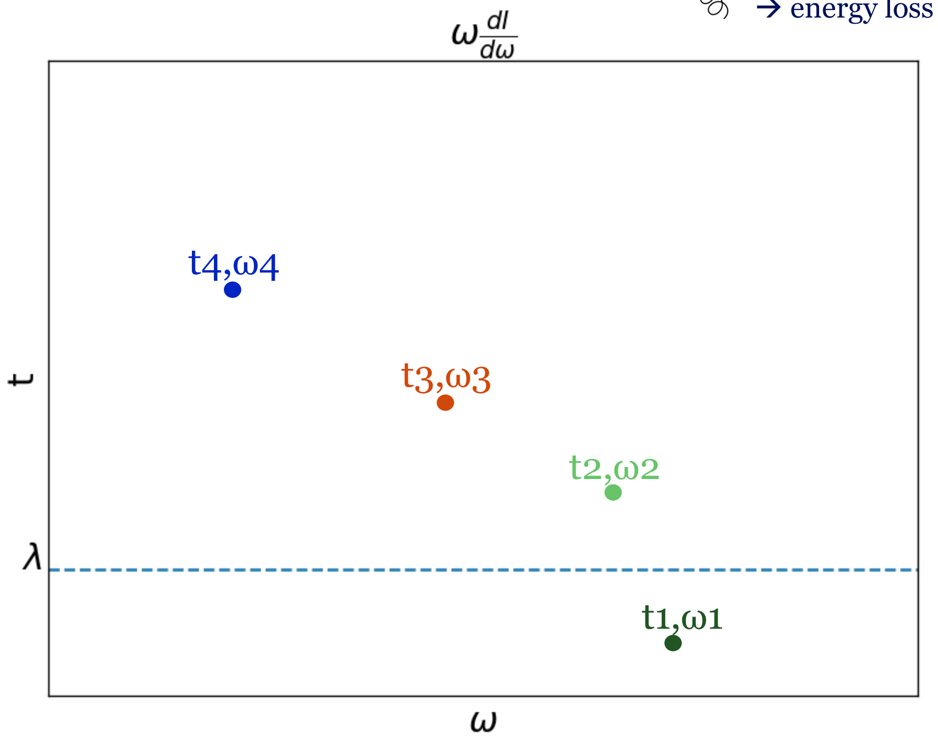






- To understand the process we need to zoom in and calculate the emission spectrum for each splitting

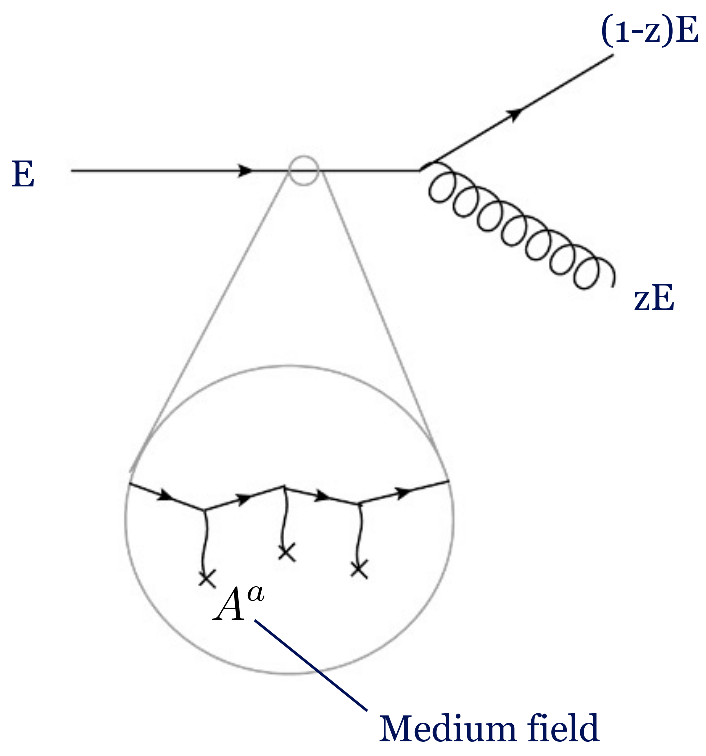
Medium induced emissions

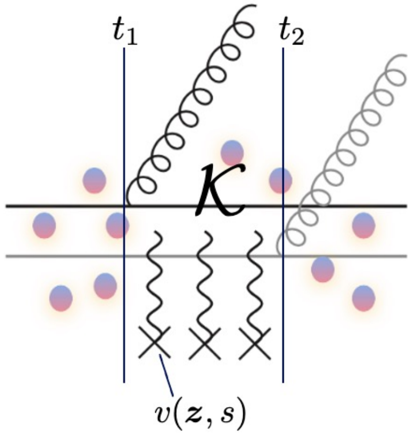
- The emission spectrum is given by

$$\omega\frac{\mathrm{d}I}{\mathrm{d}\omega} = \frac{2\alpha_s C_R}{\omega^2}\mathrm{Re}\int_0^\infty \mathrm{d}t_2 \int_0^{t_2} \mathrm{d}t_1\, \partial_{\boldsymbol{x}} \cdot \partial_{\boldsymbol{y}} \left[\mathcal{K}(\boldsymbol{x}, t_2; \boldsymbol{y}, t_1) - \mathcal{K}_0(\boldsymbol{x}, t_2; \boldsymbol{y}, t_1)\right]_{\boldsymbol{x}=\boldsymbol{y}=0}$$

- The three-point correlator $\mathcal{K}$ solves the Schrödinger equation

$$\left[i\partial_t + \frac{\partial_{\boldsymbol{x}}^2}{2\omega} + iv(\boldsymbol{x}, t)\right]\mathcal{K}(\boldsymbol{x}, t; \boldsymbol{y}, t_0) = i\delta(t - t_0)\delta(\boldsymbol{x} - \boldsymbol{y})$$





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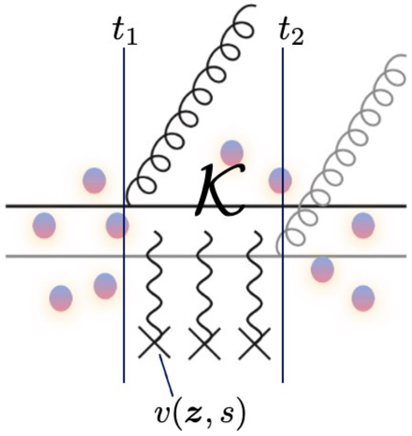
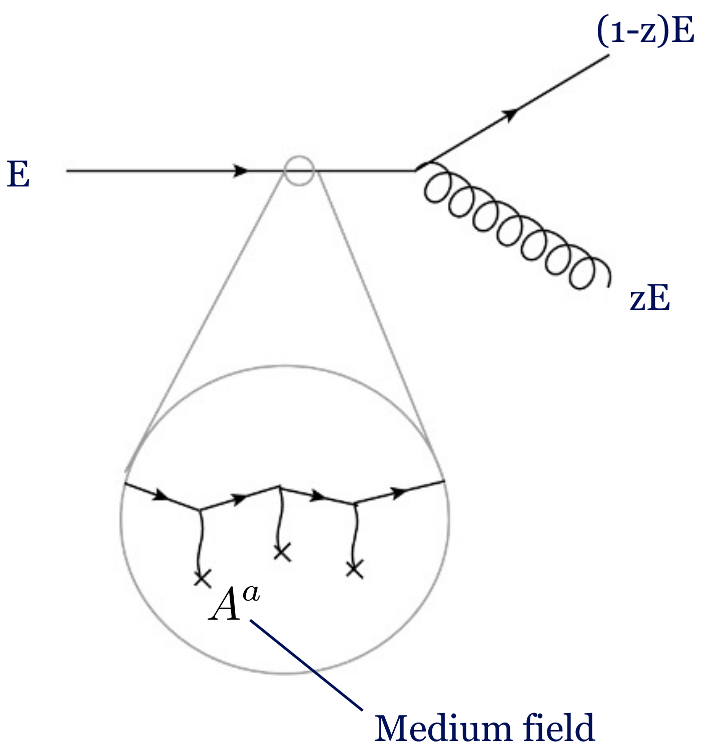
- Can in general only be evaluated numerically
- Analytical solutions of the spectrum are based on approximations





Medium induced emissions

Two well-known analytical solutions of the spectrum

- Opacity expansion
  - Expand $\mathcal{K}$ in the number of scatterings with the medium
  - Truncate at a finite order

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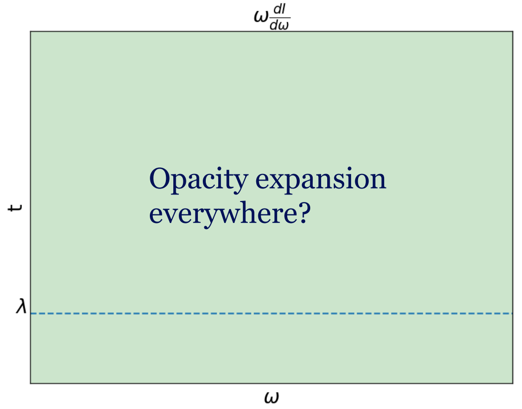
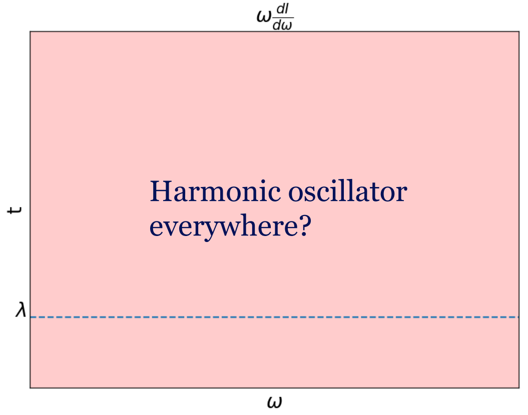
- Which is correct?





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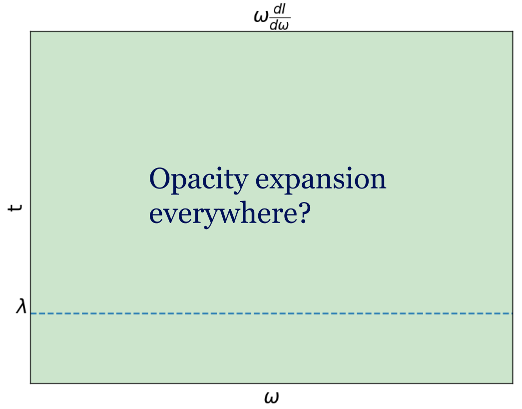
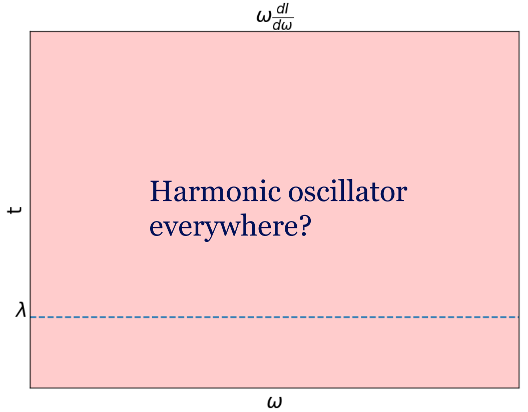
- Which is correct?

- None of these methods gives satisfying results in the whole phase space
- Combining three expansions gives a very good approximation
  - Opacity expansion (OE)*
  - Resummed opacity expansion (ROE)*
  - Improved opacity expansion (IOE)*





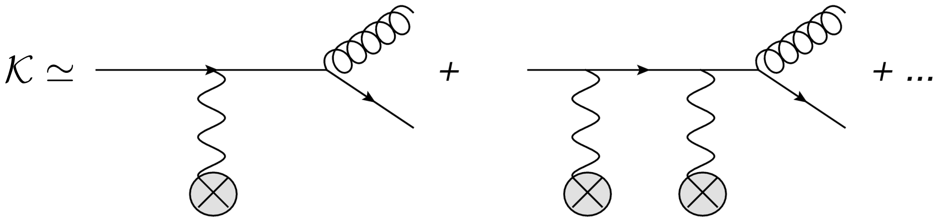
*Gyulassy et al. 9907461
Wiedemann 0005129

*Isaksen, Takacs, Tywoniuk 2206.02811
Schlicting, Soudi 2111.13731, Andres et al. 2011.06522

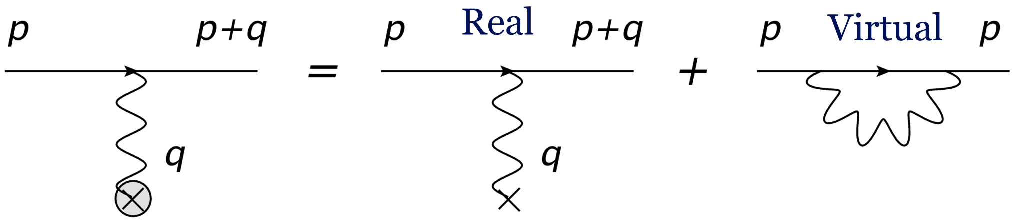
*Mehtar-Tani, Tywoniuk, Barata, Soto-Ontoso
1903.00506, 2106.07402

The opacity expansion

- Expansion in scatterings around the vacuum solution $\mathcal{K}_0$



- The scattering potential contains both a real and virtual part



- The emission spectrum depends on the energy scale $\bar{\omega}_c = \frac{\mu^2 L}{2}$

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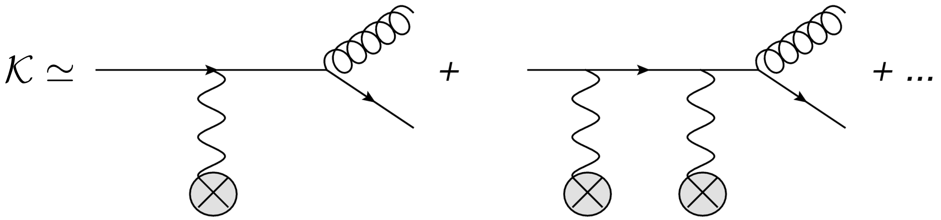

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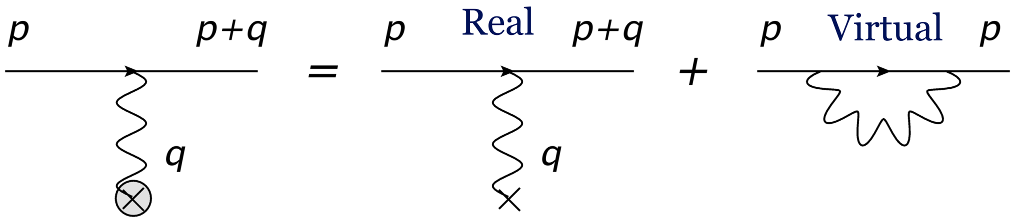
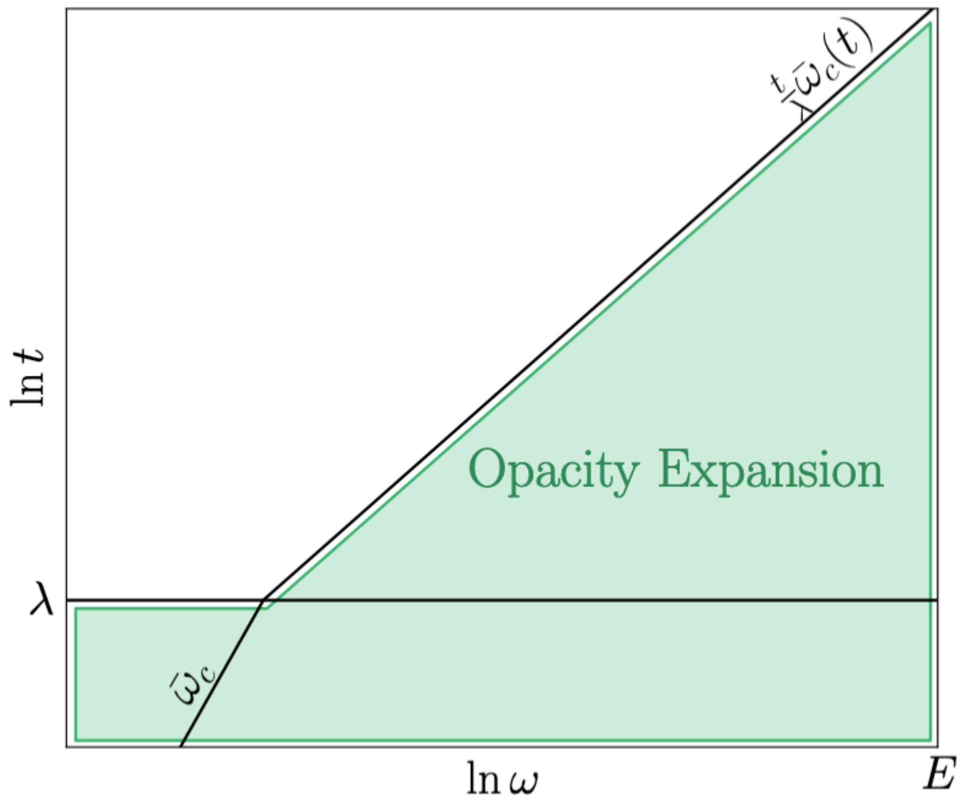


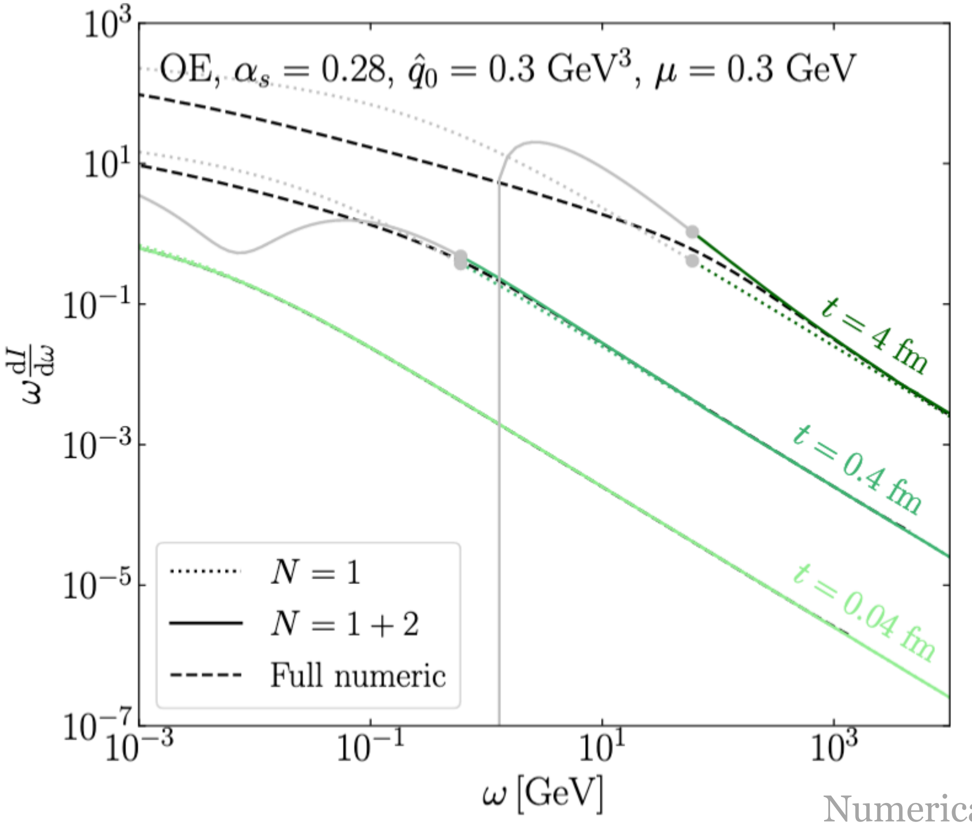
- The emission spectrum depends on the energy scale $\bar{\omega}_c = \frac{\mu^2 L}{2}$

- At low energy the spectrum goes as $\sim \left(\frac{L}{\lambda}\right)^n$ → convergence when opacity is small $\chi \equiv \frac{L}{\lambda} < 1$
- At high energy the spectrum goes as $\sim \left(\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}\right)^n = \left(\frac{\hat{q}_0 L^2}{2\omega}\right)^n$ → convergence when $\omega > \frac{\hat{q}_0 L^2}{2}$

The opacity expansion

- Valid for early times, but also late times if the energy is big
- Breaks down at later times for low energy





Numerical solution from
Andres, Dominguez, Gonzalez Martinez
2011.06522

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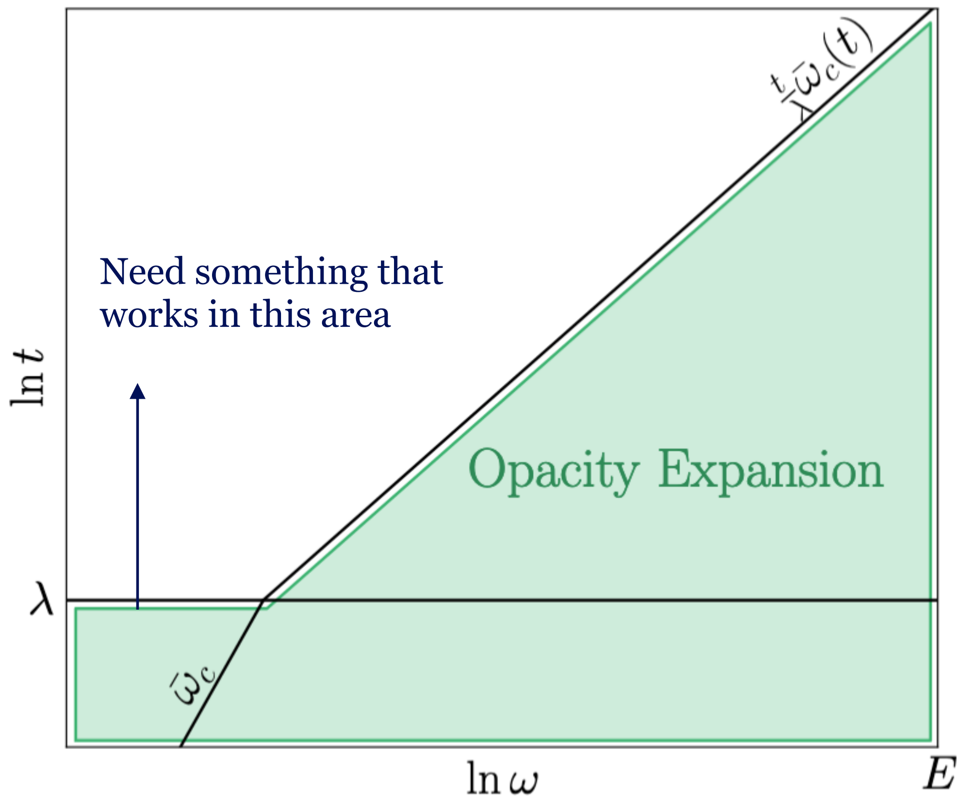

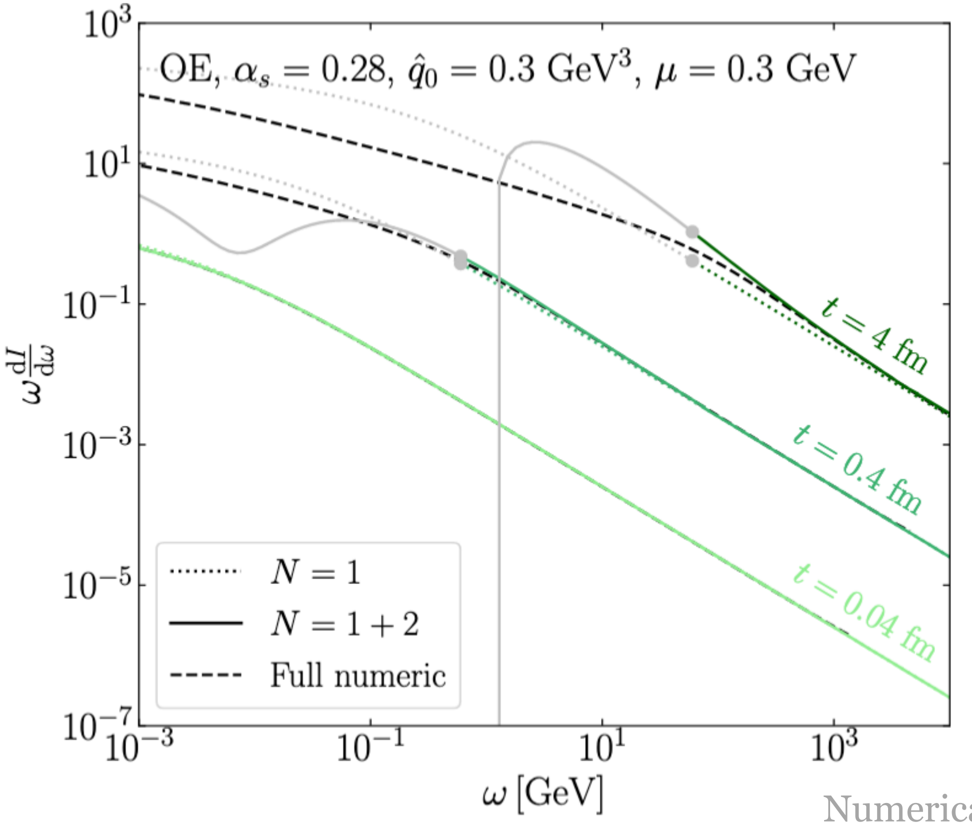

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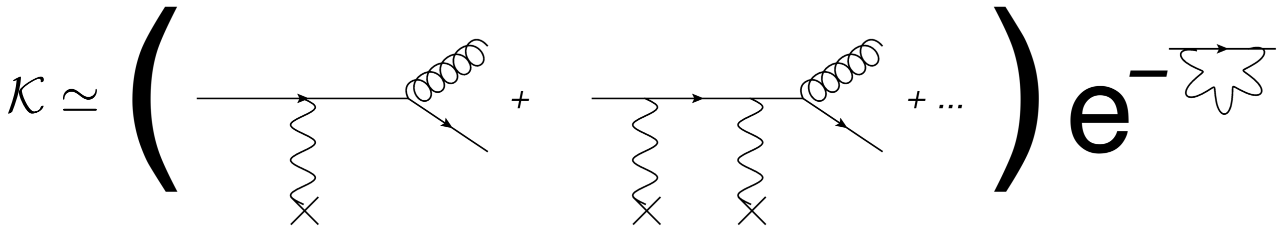
- To fill out more of the phase space another expansion is needed

The resummed opacity expansion

- Expand only in real scatterings
- All virtual scatterings are resummed in a Sudakov factor $\Delta(t, t_0) \equiv \mathrm{e}^{-\int_{t_0}^{t} \mathrm{d}s\, \Sigma(s)}$



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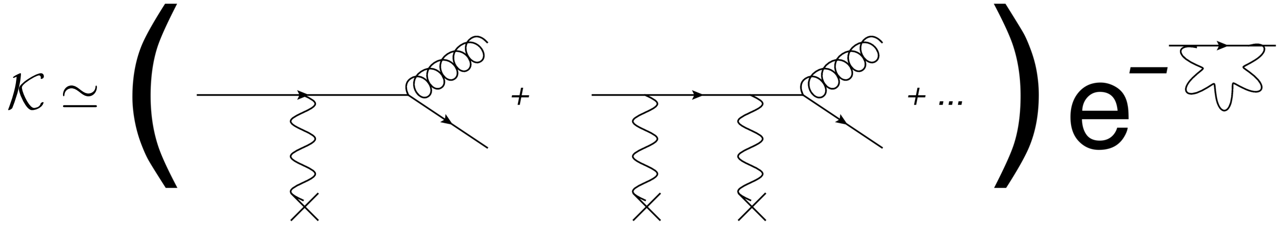

- For short media L < λ: resummed opacity expansion → opacity expansion

- However, also works for longer media L > λ:
  - New constant scale emerges: $\omega_{\mathrm{BH}} = \frac{\mu^2 \lambda}{2}$

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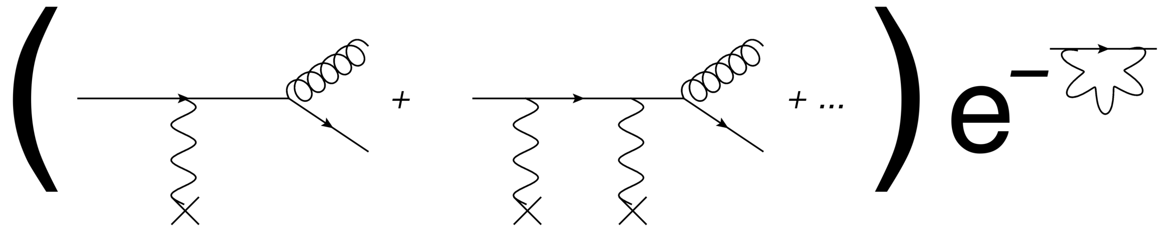
$\mathcal{K} \simeq$



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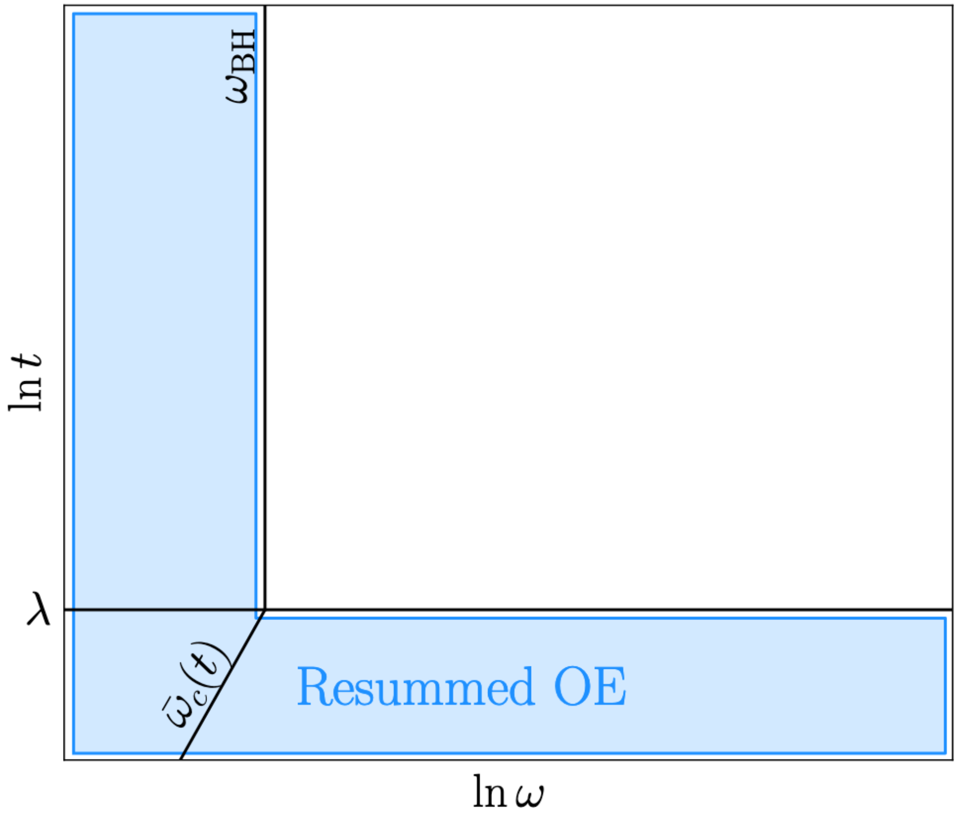
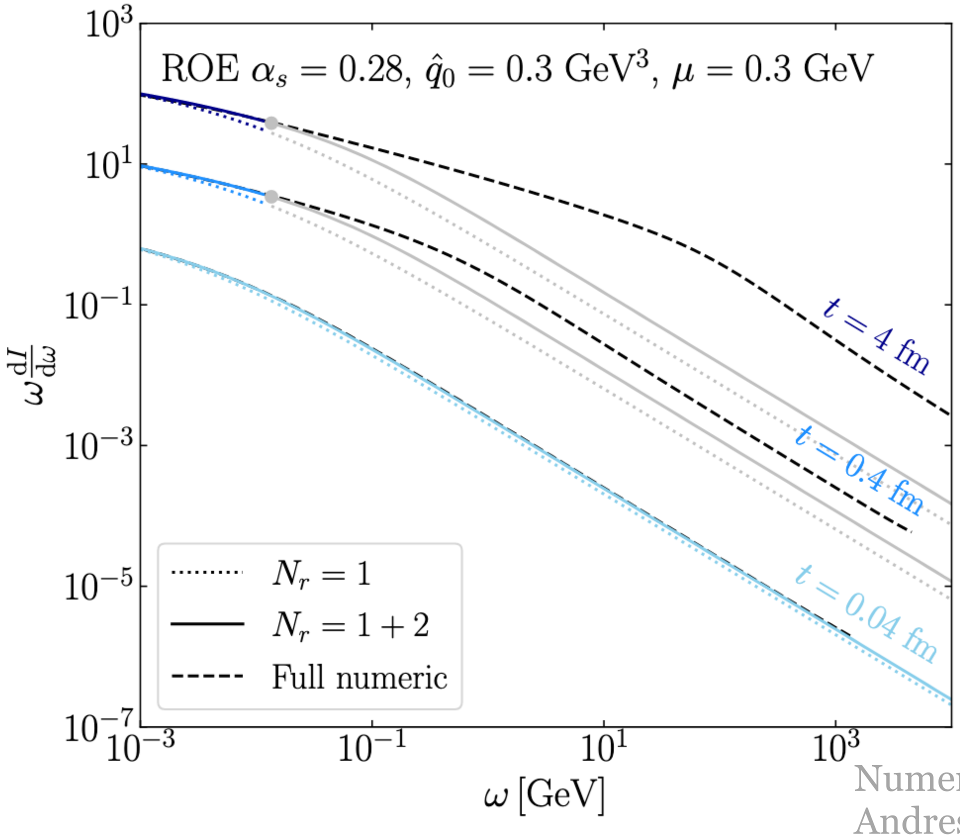
- First order contribution leading at low energy: convergence
- At high energy $\mathrm{d}I^{N_r=2} \sim \mathrm{d}I^{N_r=1}$: no sign of convergence

The resummed opacity expansion

- Valid for:
  - Early times (same as opacity expansion)
  - Late times at low energy (Bethe-Heitler region)





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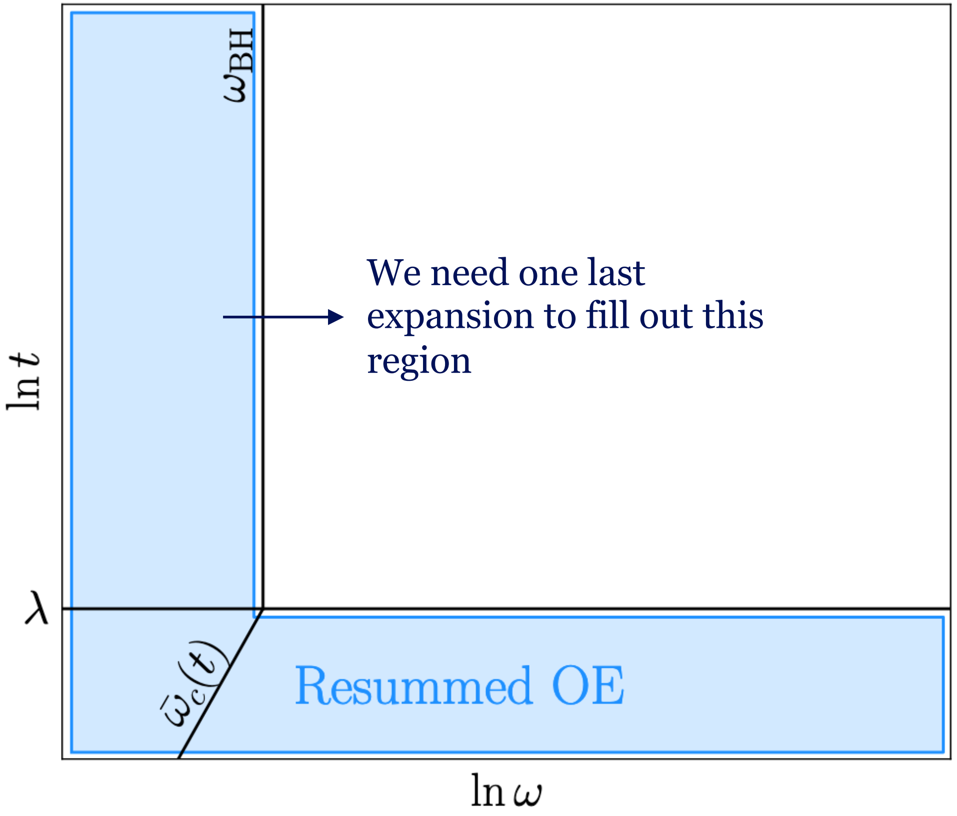
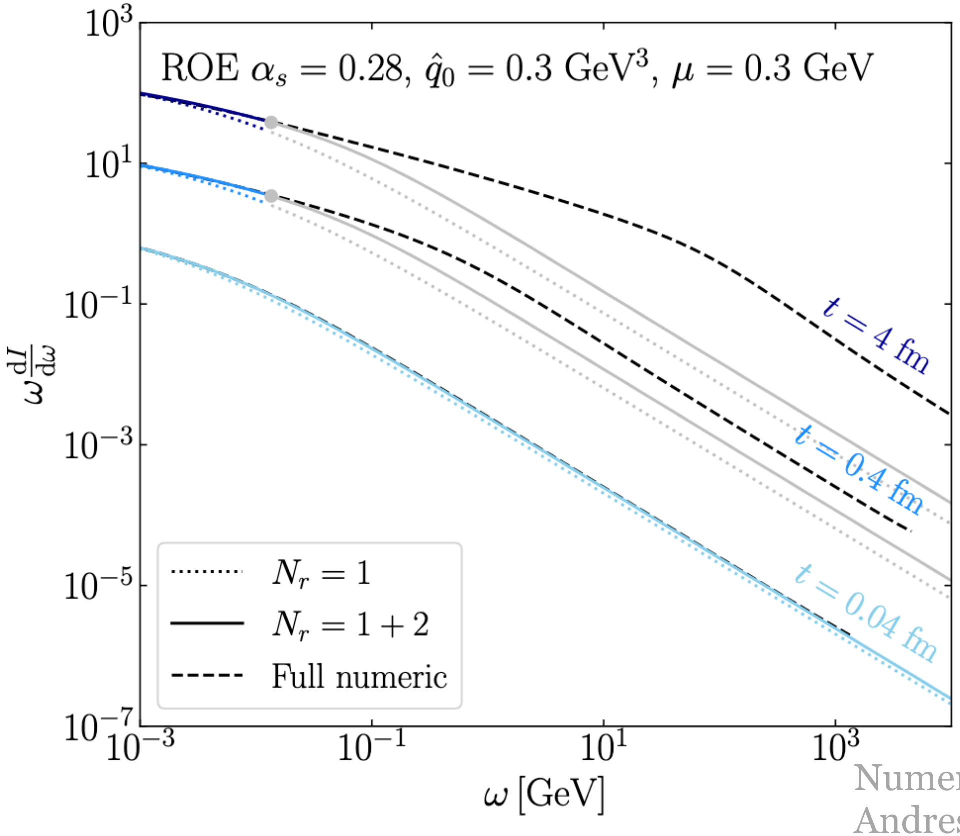




Numerical solution from Andres, Dominguez, Gonzalez Martinez 2011.06522

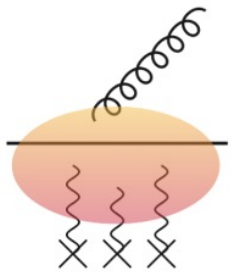
The improved opacity expansion

- Comes from manipulating the scattering potential $v(\boldsymbol{x},t) \simeq \frac{\hat{q}_0}{4}\boldsymbol{x}^2 \ln \frac{1}{\mu_*^2 \boldsymbol{x}^2}$

$$= \frac{\hat{q}}{4}\boldsymbol{x}^2 + \frac{\hat{q}_0}{4}\boldsymbol{x}^2 \ln \frac{1}{Q^2 \boldsymbol{x}^2}$$

$$\equiv v_{\rm HO}(\boldsymbol{x},t) + \delta v(\boldsymbol{x},t)$$

Harmonic oscillator



- The harmonic oscillator problem is an expansion in many soft scatterings
  - Solved exactly, resums an arbitrary number of scatterings
  - Can only create emissions with energy up to the emergent scale $\omega_c = \frac{\hat{q}L^2}{2}$
  - Emissions above this scale must be created by harder scatterings, leading to

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- The improved opacity expansion
  - Expansion in hard scatterings around the harmonic oscillator solution

$$\mathcal{K}(\boldsymbol{x},t_2;\boldsymbol{y},t_1) = \mathcal{K}_{\text{HO}}(\boldsymbol{x},t_2;\boldsymbol{y},t_1) - \int_{t_1}^{t_2}\mathrm{d}s\int_{\boldsymbol{z}}\mathcal{K}_{\text{HO}}(\boldsymbol{x},t_2;\boldsymbol{z},s)\delta v(\boldsymbol{z},s)\mathcal{K}(\boldsymbol{z},s;\boldsymbol{y},t_1)$$

- The improved opacity expansion makes it possible to go to higher energies than ω_c

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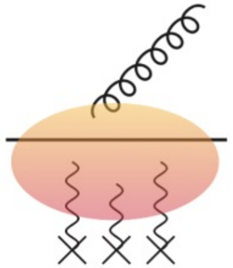
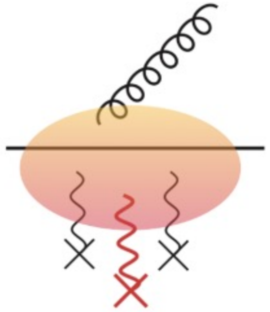


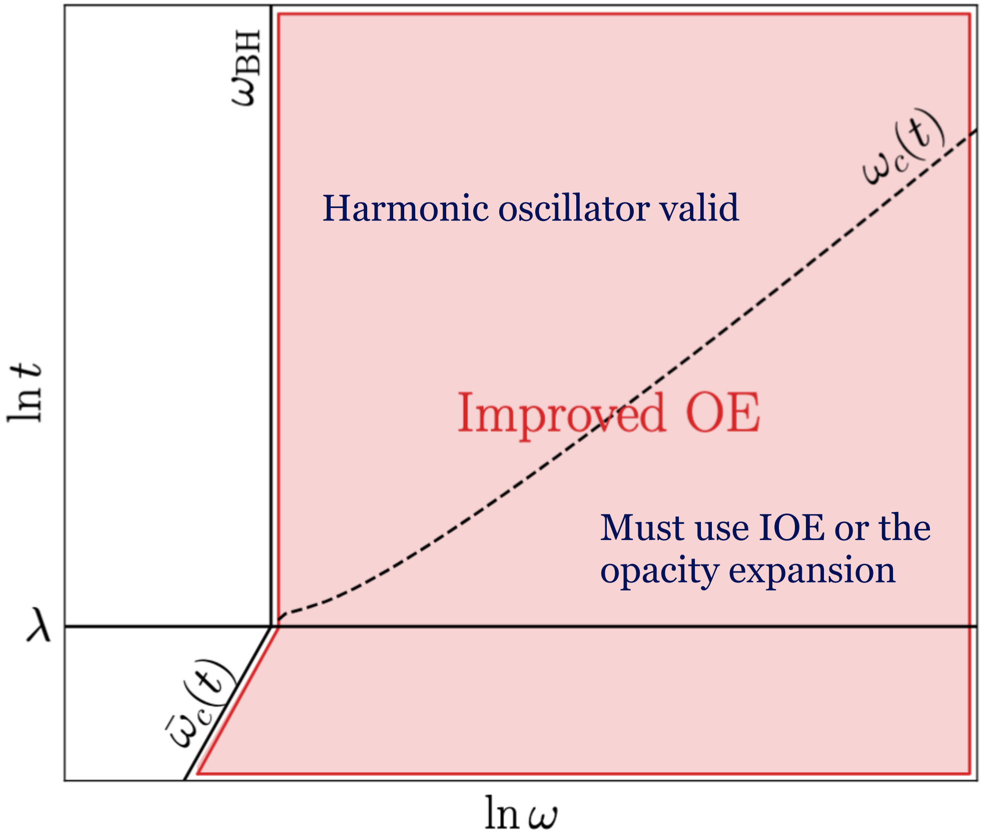
Next-to harmonic oscillator

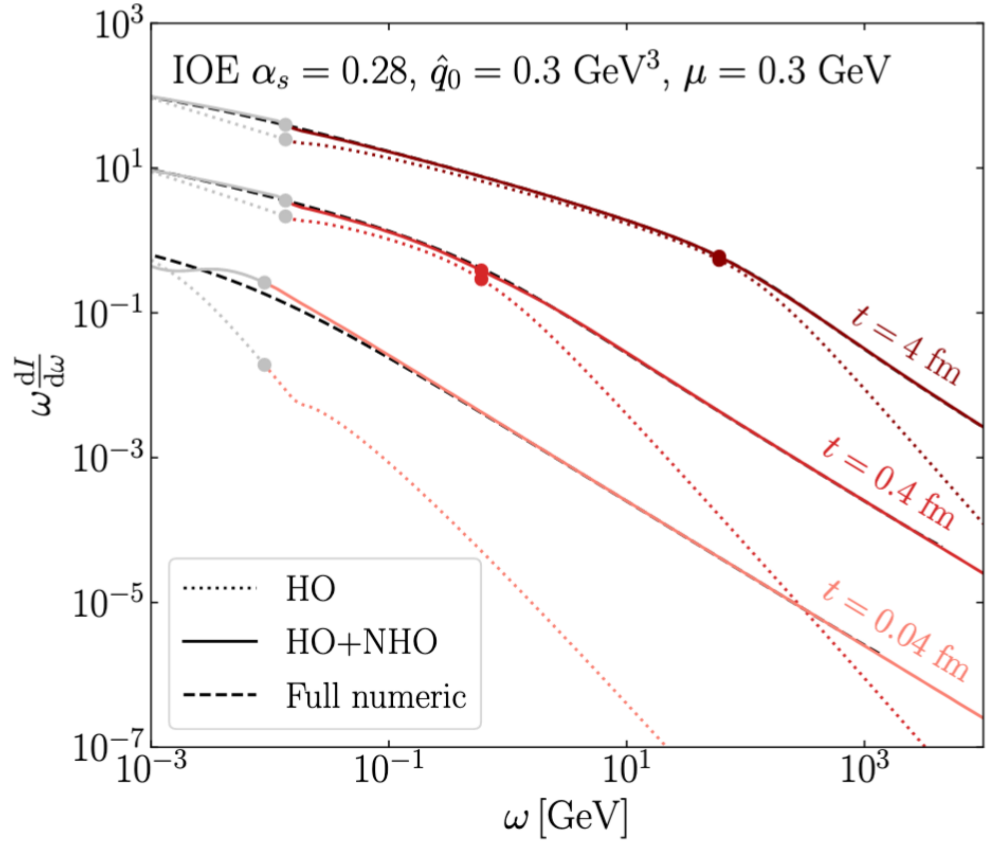


$\delta v(\boldsymbol{x},t)$

The improved opacity expansion

- Valid for energies over the Bethe-Heitler regime





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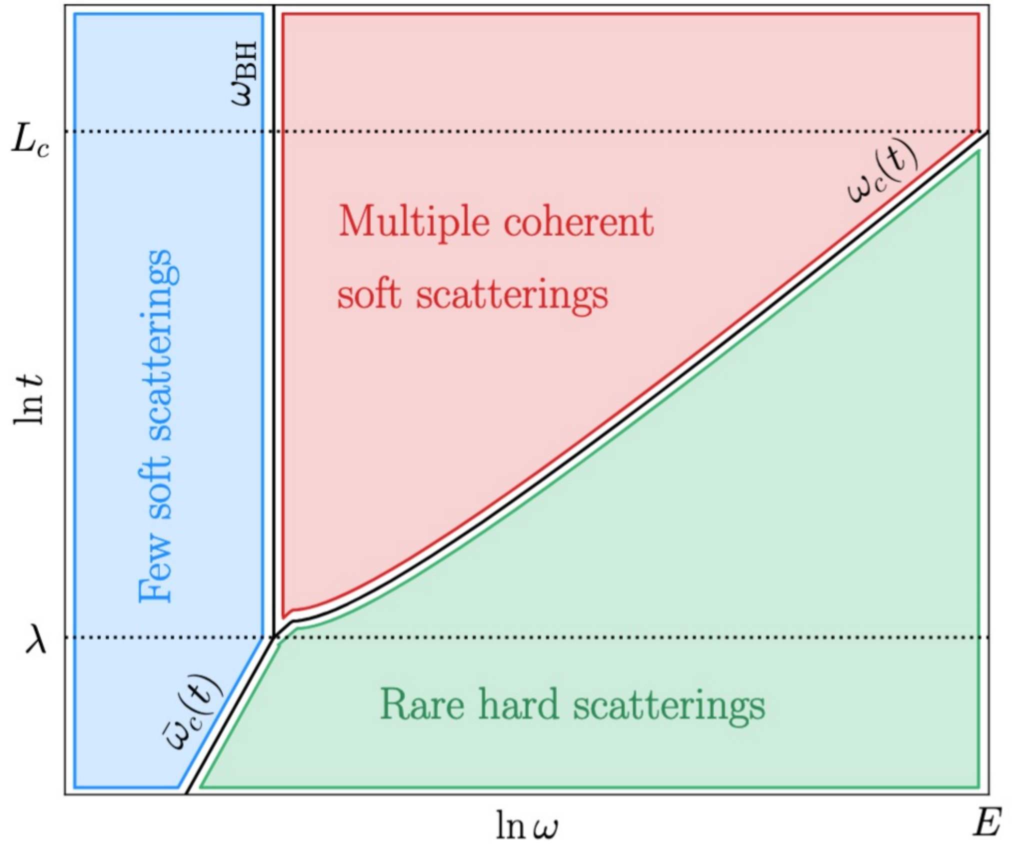
Summary of the expansions

- At early times $t < \lambda$ only few scatterings will induce emissions
  - Covered by both the opacity expansion and the resummed opacity expansion



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- At early times $t < \lambda$ only few scatterings will induce emissions
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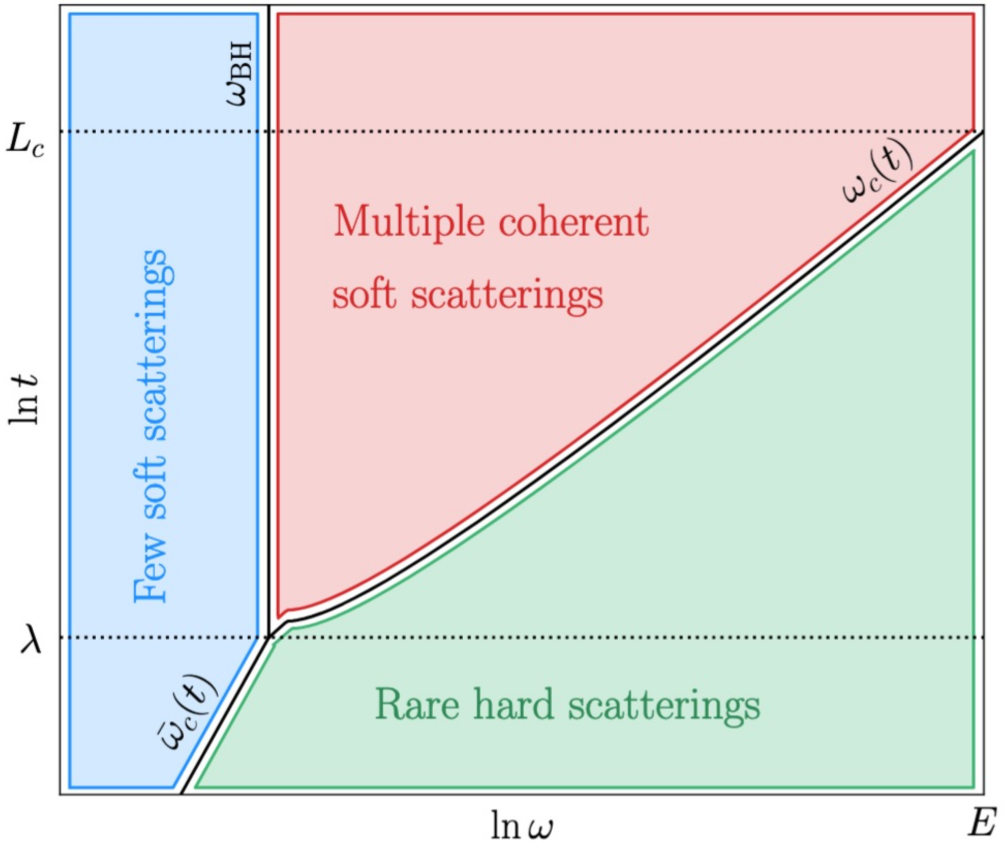
- At intermediate times $\lambda < t < L_c$ three different processes induce emissions
  - Few soft scatterings at low energy $\omega < \omega_{\text{BH}}$ (resummed opacity expansion)
  - Multiple coherent scatterings at intermediate energy $\omega_{\text{BH}} < \omega < \omega_c$ (improved opacity expansion)
  - Rare hard scatterings at $\omega_c < \omega$ (opacity expansion or improved opacity expansion)



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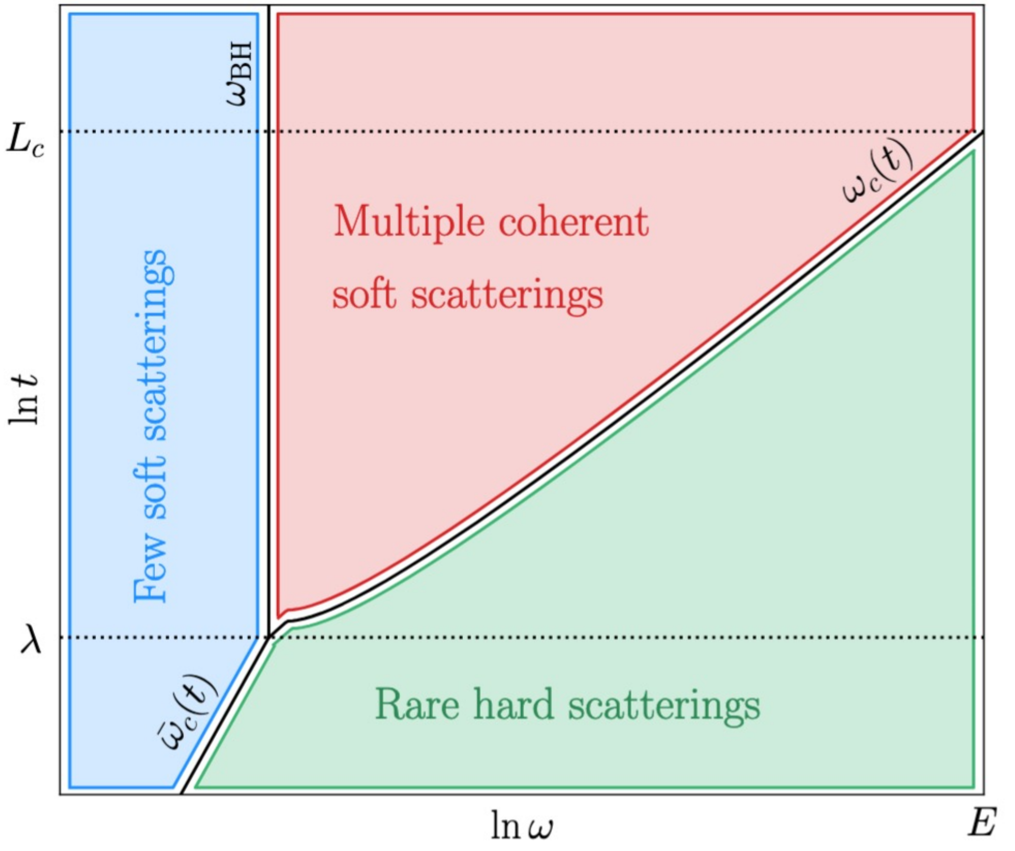

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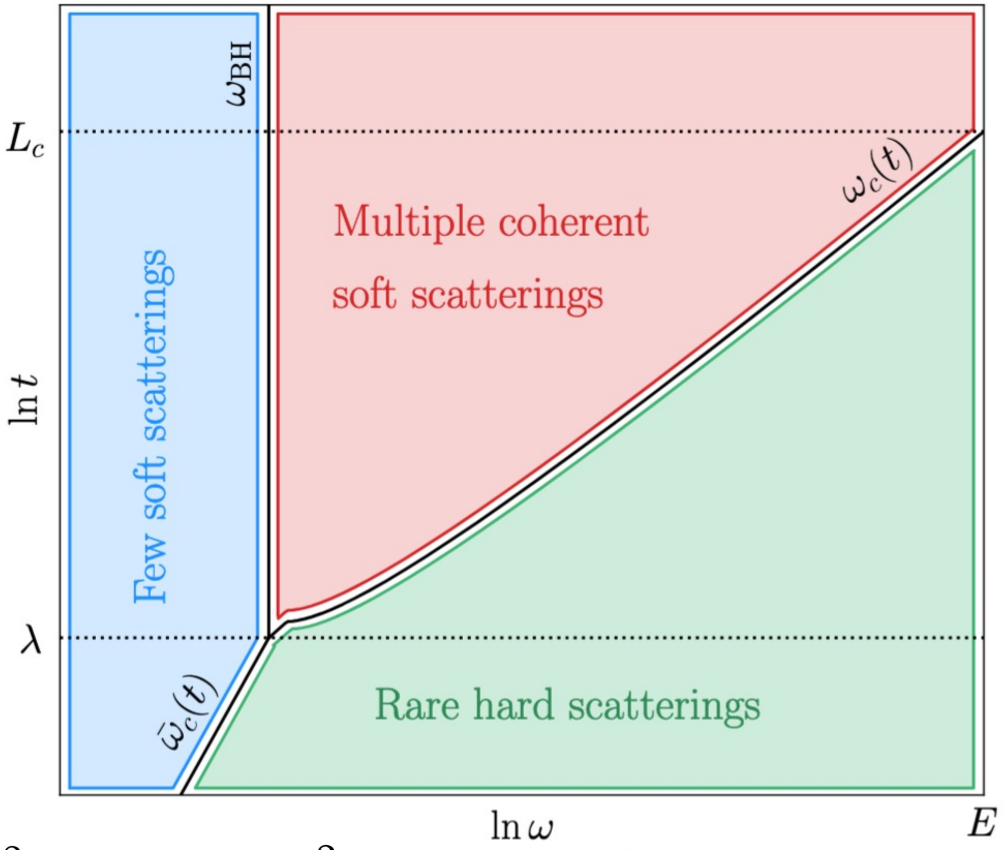

- Three energy scales naturally emerge from the expansions: $\bar{\omega}_c = \frac{\mu^2 L}{2}$ $\omega_{\mathrm{BH}} = \frac{\mu^2 \lambda}{2}$ $\omega_c = \frac{\hat{q} L^2}{2}$

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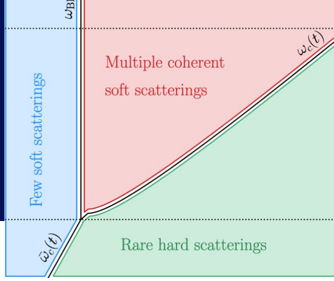
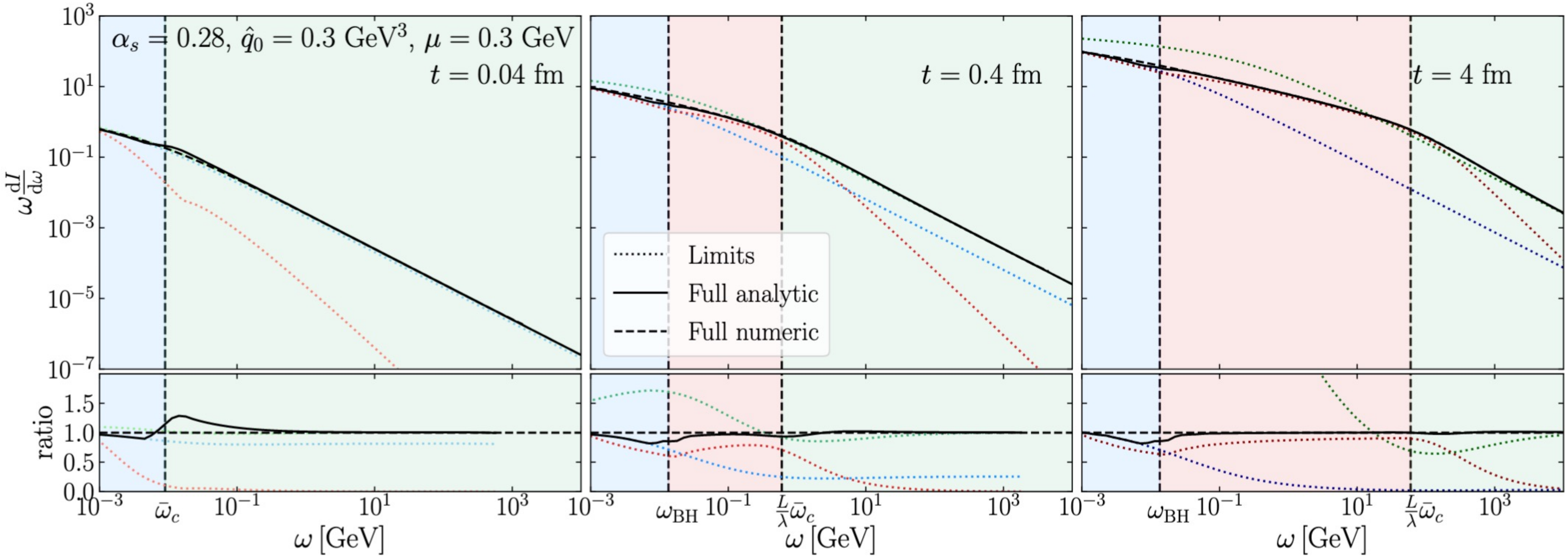


- For the full line we have used the ROE and IOE, with a smoothening transition function
- Error is biggest at the transitions between the areas, expect that higher orders make it smoother

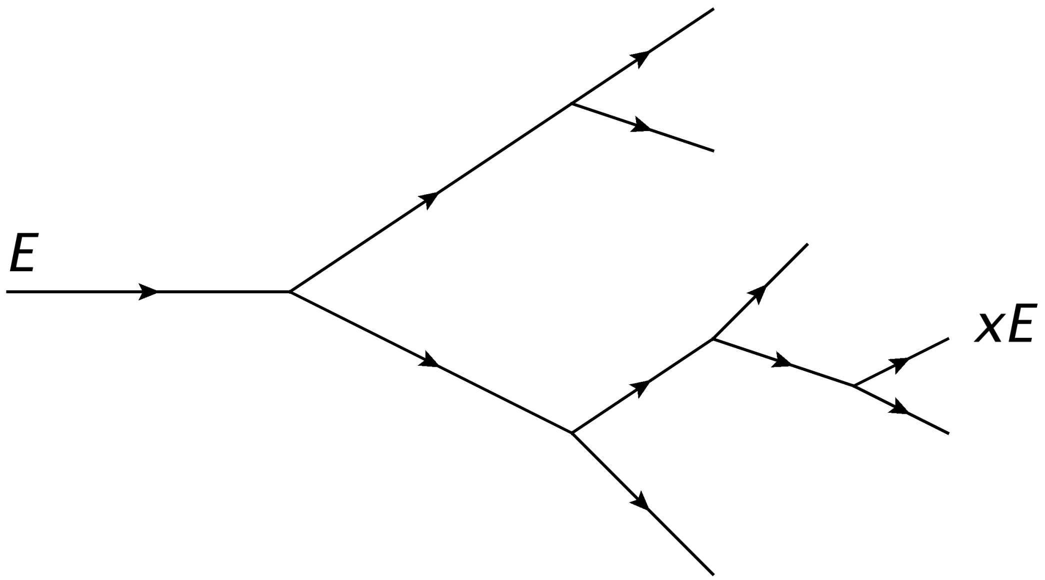


Numerical solution from Andres, Dominguez, Gonzalez Martinez 2011.06522

Multiple emissions

- Must take into account the possibility of multiple emissions
- Define the energy distribution of partons with energy xE after travelling time t in the medium:

$$D(x,t) \equiv x\frac{\mathrm{d}N}{\mathrm{d}x}$$



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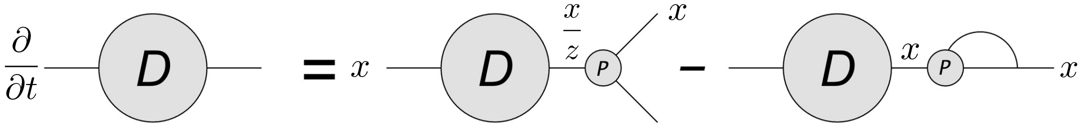
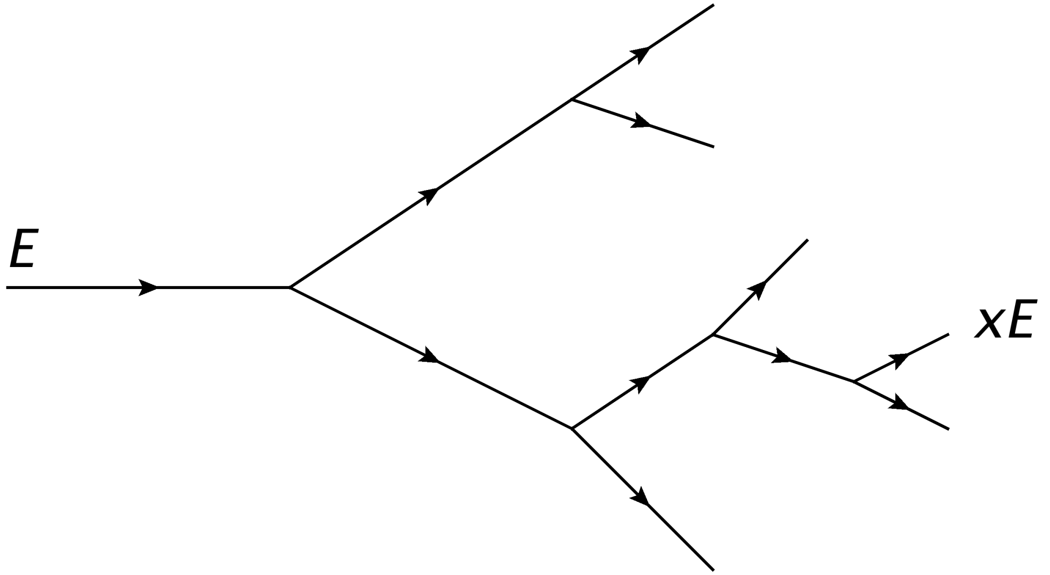
- Multiple emissions are resummed in a rate equation

$$\frac{\partial}{\partial t} D(x,t) = \int_x^1 dz\, 2P\left(z, \frac{x}{z}E, t\right) D\left(\frac{x}{z}, t\right) - \int_0^1 dz\, P\left(z, xE, t\right) D\left(x, t\right)$$



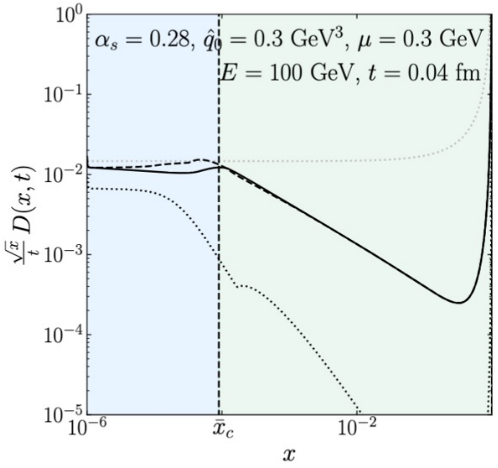


- The splitting rate is simply $P(z,E,t) = \left.\frac{\mathrm{d}I}{\mathrm{d}z\mathrm{d}t}\right|_E$
  - Follows directly from the emission spectrum

The energy distribution

- At early times $t < \lambda$ there are few emissions
  - Only small modification of the distribution

$t < \lambda$



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  - More effective transport of energy to soft modes
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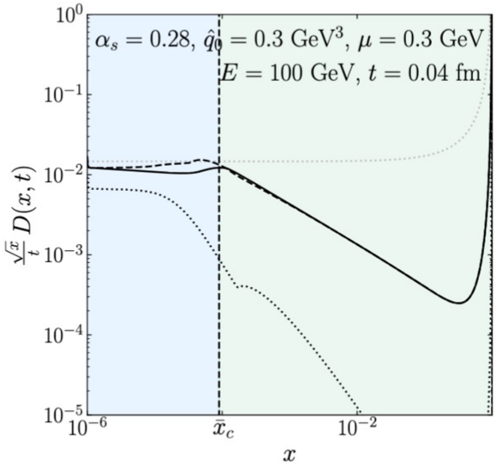

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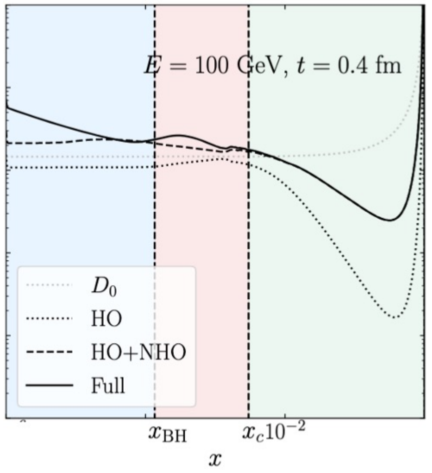

The energy distribution

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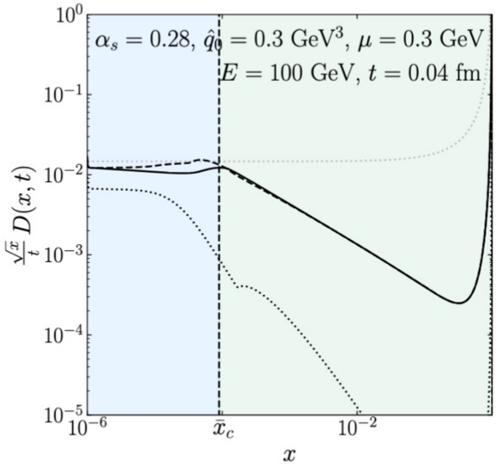
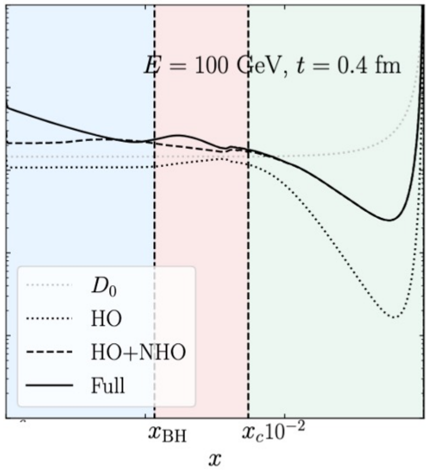
- At late times $t > L_c$ harmonic oscillator emissions dominate
  - Large modification to the distribution
  - Turbulent cascade of energy to soft modes

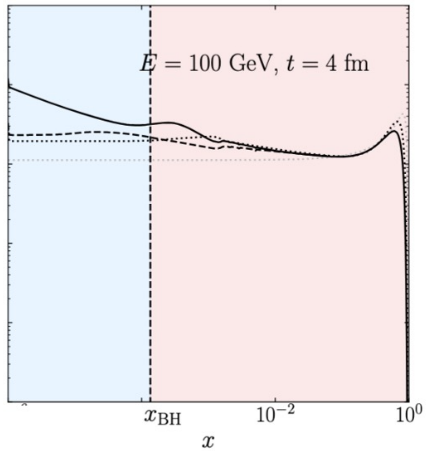
$t < \lambda$



$\lambda < t < L_c$



$t > L_c$



Conclusion and outlook

- Effective theory for the medium induced radiation
  - Good theoretical understanding and control in the different regimes
  - Systematically improvable order by order
- This is achieved through three expansions
  - The opacity expansion, the resummed opacity expansion and the improved opacity expansion
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  - Better understanding of the time evolution of a jet
- Outlook
  - Get closer to phenomenology by incorporating the vacuum
  - Rigorously define the accuracy of medium induced emissions
  - Incorporating quark masses

Thank you for your attention

Backup

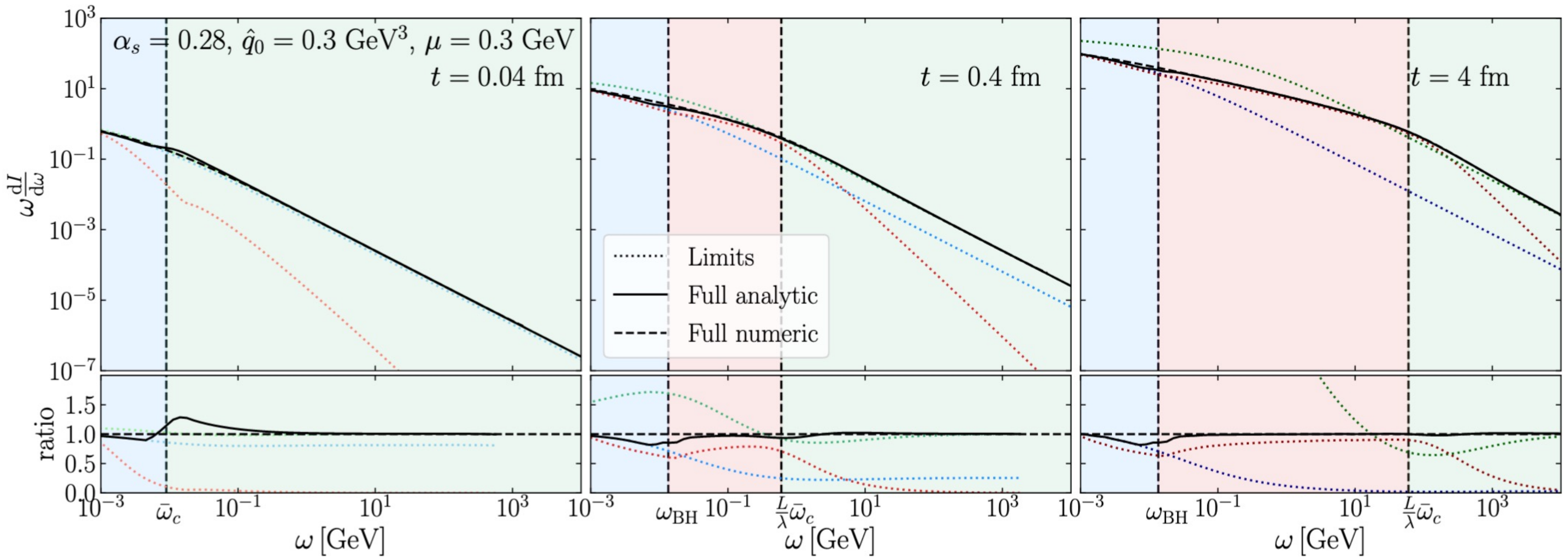
- The limits of the full emission spectrum are

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega}\bigg|_{t \ll \lambda} = \begin{cases} 2\bar{\alpha}\frac{L}{\lambda}\left(\ln\frac{\bar{\omega}_c}{\omega} - 1 + \gamma_E\right), & \text{for } \omega \ll \bar{\omega}_c(t) \quad ** \\ \frac{\pi}{2}\bar{\alpha}\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}, & \text{for } \bar{\omega}_c(t) \ll \omega \quad *** \end{cases}$$

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega}\bigg|_{t \gg \lambda} = \begin{cases} 2\bar{\alpha}\frac{L}{\lambda}\ln\left(\frac{\omega_{\mathrm{BH}}}{\omega}\right), & \text{for } \omega \ll \omega_{\mathrm{BH}}, \quad * \\ \bar{\alpha}\sqrt{\frac{2\omega_c}{\omega}}, & \text{for } \omega_{\mathrm{BH}} \ll \omega \ll \omega_c(t) \quad * \\ \frac{\pi}{2}\bar{\alpha}\frac{L}{\lambda}\frac{\bar{\omega}_c}{\omega}, & \text{for } \omega_c(t) \ll \omega, \quad ** \end{cases}$$

*Opacity expansion
*Resummed opacity expansion
*Improved opacity expansion

The union of IOE and ROE covers the whole phase space!



Backup

- Analytic energy distributions
  - Early time

$$D(x,t) \simeq \begin{cases} 2\bar{\alpha}\frac{t}{\lambda}\frac{1}{1-x}\ln\left(\frac{\bar{\omega}_c(t)}{x(1-x)E}\right) & \text{for } x \ll \bar{x}_c\,, \\ \frac{\pi\bar{\alpha}}{4}\frac{\hat{q}_0}{E}\frac{t^2}{x(1-x)^2} & \text{for } \bar{x}_c \ll x \ll 1-\bar{x}_c \end{cases}$$

  - Late time

$$D_0(x,\tau) = \frac{\tau}{\sqrt{x}(1-x)^{3/2}}\mathrm{e}^{-\pi\frac{\tau^2}{1-x}}$$

  - Intermediate time

$$D_1(x,\tau) = \int_0^\tau \mathrm{d}\sigma \int_x^1 \mathrm{d}\xi\, \delta P(x,\xi,\sigma) D_0(\xi,\sigma) - \int_0^\tau \mathrm{d}\sigma D_0(x,\sigma) \int_0^x \mathrm{d}\xi\, \delta P(\xi,x,\sigma)$$

$$\delta P(x,\xi,\tau) = (P_{\text{hard}}(x,\xi,\tau) - P_{\text{coh}}(x,\xi,\tau))\Theta(x-x_c)\Theta(\xi-x_c-x)$$