

All order resummed leading and next-to-leading logarithms for soft modes of dense QCD pressure

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July 28, 2022, XQCD conference, Trondheim

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Motivations

- High and mid range of μ_B hardly accessible by Lattice Simulations (with $T = 0$)
- Infrared divergences spoil naive perturbation theory
- Large scale dependence uncertainty in high loop HTLpt calculations
- Renormalization Group properties can address these points

pQCD State-of-the-art at $T = 0, \mu \neq 0$

pQCD pressure for $N_f = 3, T = 0, \mu_B \neq 0$

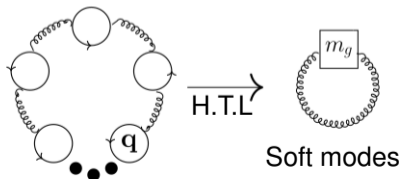
$$\mathcal{P}^{\text{C.Q.M}} = \mathcal{P}_f \left(1 - \frac{2}{\pi} \alpha_s - \frac{N_f}{\pi^2} \alpha_s^2 \ln \alpha_s - 0.874355 \alpha_s^2 - 2d_A \alpha_s^2 \frac{(11N_c - 2N_f)}{3(4\pi)^2} \ln \left(\frac{M_h}{\mu} \right) \right)$$

Ring (Soft)

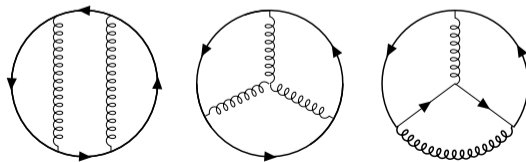
Hard scale: $\mu, M_h \sim$ renormalization scale

Quarks (Hard)

Soft scale: $m_E \sim \sqrt{\alpha_s} \mu$



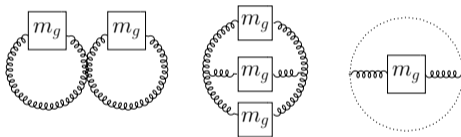
Soft modes



Hard Thermal Loop

Hard Thermal Loop pressure (pure glue) at LO (equivalently NNLO for $m_g = m_E$)

$$\mathcal{P}_{\text{LO}}^{\text{HTL}} = \frac{d_A m_g^4}{(8\pi)^2} \left(\frac{1}{2\varepsilon} + C_{11} - \ln\left(\frac{m_g}{M}\right) + \varepsilon \left(\ln\left(\frac{m_g}{M}\right)^2 + C_{21} \ln\left(\frac{m_g}{M}\right) + C_{22} \right) \right)$$



C_{11} : S. Moggiacci et al. 2013

C_{21}, C_{22} : LF, J-L. Kneur (2021)

$$m_E^2 = 2 \frac{\alpha_s}{\pi} \sum_f \mu_f$$

HTL at NLO (i.e NNNLO)

$$\mathcal{P}_{\text{NLO}}^{\text{HTL}} = \frac{N_c d_A \alpha_s m_g^4}{(8\pi)^2} \left(\frac{p_{-2}}{4\varepsilon^2} + \frac{p_{-1} - 2p_{-2} \ln\left(\frac{m_g}{M}\right)}{2\varepsilon} + 2p_{-2} \ln\left(\frac{m_g}{M}\right)^2 - 2p_{-1} \ln\left(\frac{m_g}{M}\right) + p_0 \right)$$

T. Gorda, A. Kurkela, R. Paatelainen, S. Säppi and A. Vuorinen (GKPSV), PRL 127,162003 (2021)

Hard Thermal Loop: Factorization picture

HTL in factorization picture

$$\mathcal{P}_{\text{LO}}^{\text{HTL}} = \frac{d_A m_g^4}{(8\pi)^2} \left(\frac{1}{2\varepsilon} + C_{11} - \ln \left(\frac{m_g}{M_s} \right) \right)$$

$$\mathcal{P}_{\text{NLO}}^{\text{HTL}} = \frac{N_c d_A \alpha_s m_g^4}{(8\pi)^2} \left(\frac{p_{-2}}{4\varepsilon^2} + \frac{p_{-1} - 2p_{-2} \ln \left(\frac{m_g}{M_s} \right)}{2\varepsilon} + 2p_{-2} \ln \left(\frac{m_g}{M_s} \right)^2 - 2p_{-1}^{\text{fac}} \ln \left(\frac{m_g}{M_s} \right) + p_0^{\text{fac}} \right)$$

UV divergences, they cancel against
IR divergences in hard sector

M_s : Factorization scale

UV vs. IR cancellation argument

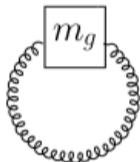
Hard Thermal Loop: Renormalization picture

Organization of a complete EFT calculation (ideally)

$$\mathcal{P}^{\text{Full}} = \mathcal{P}_{\text{HTL}} + (\mathcal{P}^{\text{Full}} - \mathcal{P}_{\text{HTL}}) \Big|_{m_g \text{ expanded}}$$

- \mathcal{P}_{QCD} and \mathcal{P}_{HTL} have different UV behaviors (counterterms, anomalous dimensions). **We renormalize HTL**
- $(\mathcal{P}_{\text{QCD}} - \mathcal{P}_{\text{HTL}}) \Big|_{m_g \text{ expanded}}$ is part of the **EFT matching** contribution. Their IR divergences cancel against IR divergences of $\mathcal{P}^{\text{Hard}}$.
- α_s^2 order is already known exactly.
- We consider here only the *soft* modes.
- m_g is not the Debye mass YET.

HTL at LO



$$\mathcal{P}_{\text{LO}}^{\text{HTL}} = \frac{d_A m_g^4}{(8\pi)^2} \left(\frac{1}{2\epsilon} + C_{11} - \ln \left(\frac{m_g}{M} \right) \right)$$

- 1 There is no parameter at LO that can absorb this divergence.

LO vacuum energy counterterm \rightarrow Anomalous dimension

$$\Delta \mathcal{E}_0^{(2)} = \frac{d_A m_g^4}{(8\pi)^2} \left(\frac{1}{2\epsilon} \right) \equiv z_0^g(g^2) m_g^4$$

At LO, same counterterm as in HTL ($T \neq 0$)
(Andersen, Braaten, Petitgirard, Strickland (2002))

RG definitions

(Massive) Renormalization group operator

$$M \frac{d}{dM} \equiv M \frac{\partial}{\partial M} + \beta(g^2) \frac{\partial}{\partial g^2} - \gamma_m^g(g^2) m_g \frac{\partial}{\partial m_g}$$

γ_m^g : Anomalous mass dimension of the gluon.

$$\gamma_m^g(g^2) = \gamma_0^g g^2 + \gamma_1^g g^4 + \dots$$

$$\gamma_0^g = \frac{11 N_c}{3(4\pi)^2} \equiv b_0^g$$

$$\longrightarrow m_g^B \equiv m_g \mathcal{Z}_{m_g} \simeq m_g \left(1 - g^2 \frac{\gamma_0^g}{2\epsilon} \right) + \mathcal{O}(g^4)$$

Hard Thermal Loop: EFT picture

Renormalization of the EFT

$$\mathcal{P}_{\text{LO}}^{\text{HTL}} = \frac{d_A m_g^4}{(8\pi)^2} \left(\frac{1}{2\varepsilon} + C_{11} - \ln \left(\frac{m_g}{M_s} \right) \right)$$

$$\mathcal{P}_{\text{NLO}}^{\text{HTL}} = \frac{N_c d_A \alpha_s m_g^4}{(8\pi)^2} \left(\frac{p_{-2}}{4\varepsilon^2} + \frac{p_{-1} - 2p_{-2} \ln \left(\frac{m_g}{M_s} \right)}{2\varepsilon} + 2^1 p_{-2} \ln \left(\frac{m_g}{M_s} \right)^2 - 2p_{-1}^{\text{ren}} \ln \left(\frac{m_g}{M_s} \right) + p_0^{\text{ren}} \right)$$

Renormalized by vacuum energy counterterms

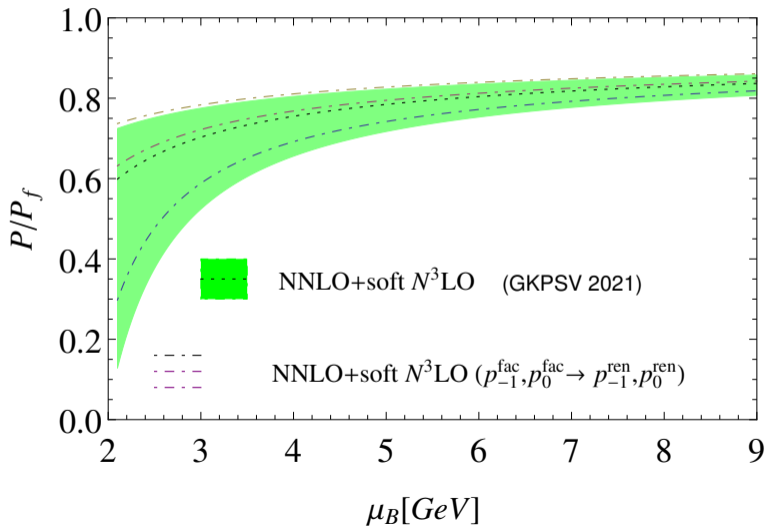
and *nonlocal divergences* by mass counterterms

Weinberg theorem proves renormalizability at massive NLO (i.e NNNLO)

$$p_{-1}^{\text{ren}} = p_{-1}^{\text{fac}} - \frac{8\pi \gamma_0^g}{N_c} \left(C_{11} - \frac{1}{4} \right) \simeq p_{-1}^{\text{fac}} - 0.5381, \quad p_0^{\text{ren}} = p_0^{\text{fac}} - \frac{8\pi \gamma_0^g}{N_c} \left(C_{22} - \frac{C_{11}}{2} \right) \simeq p_0^{\text{fac}} - 0.9229$$

Automatically

HTL pressure in EFT versus Factorization scheme



LL and NLL series

Soft pressure from power counting

$$g^2 = (4\pi)\alpha_s$$

$$\mathcal{P}^{\text{soft}} \sim m_g^4 \sum_{p=1}^{\infty} (g^2)^{p-1} \sum_{l=0}^p a_{p,l} \ln^{p-l} \left(\frac{m_g}{M_s} \right)$$

$$L \equiv \ln \left(\frac{m_g}{M_s} \right)$$

$$\begin{aligned} \frac{\mathcal{P}^{\text{soft}}}{m_g^4} &= a_{1,0} L + g^2 a_{2,0} L^2 + \dots LL \\ &+ a_{1,1} + g^2 a_{2,1} L + \dots NLL \\ &+ g^2 a_{2,2} + \dots \end{aligned}$$

Renormalization group property

The first order coefficient at which a series of (sub)leading logarithm appear is sufficient to determine all order coefficients upon requiring $M \frac{d\mathcal{P}}{dM} = 0$

LL and NLL series

LL series

$$-p a_{p,0} = (4\gamma_0^g + 2b_0^g(p-2)) a_{p-1,0}, \quad p \geq 2$$

NLL series

$$(1-p)a_{p,1} = (4\gamma_0^g + 2b_0^g(p-2)) a_{p-1,1} + (4\gamma_1^g + 2b_1^g(p-3)) a_{p-2,0} + \gamma_0^g(p-1)a_{p-1,0}$$

$$\mathcal{P}^{\text{soft}} = a_{1,0} L + g^2 a_{2,0} L^2 + \dots LL$$

*Post-diction: $a_{2,0} = -2\gamma_0^g a_{1,0} = -2b_0^g a_{1,0}$
Reproduce direct (GKPSV) calculation*

$$+ a_{1,1} + g^2 a_{2,1} L + \dots NLL$$

Input for (Soft !) NLL series, $a_{2,1} \propto p_{-1}^{\text{ren}}$

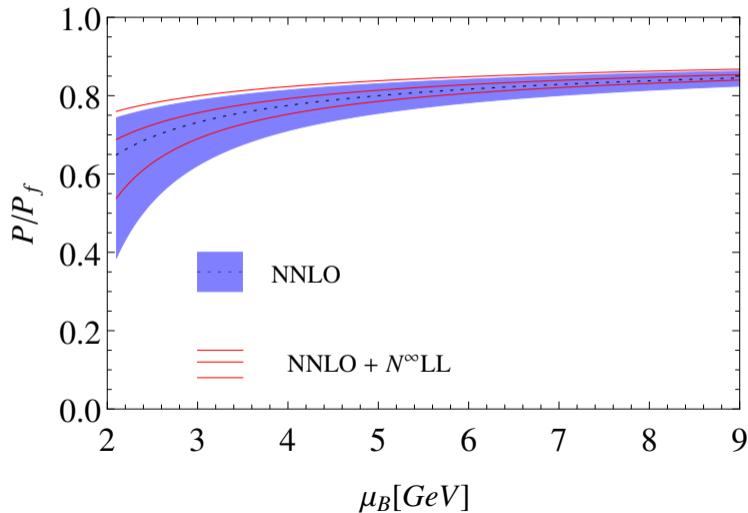
$$+ g^2 a_{2,2} + \dots$$

Resummation formula

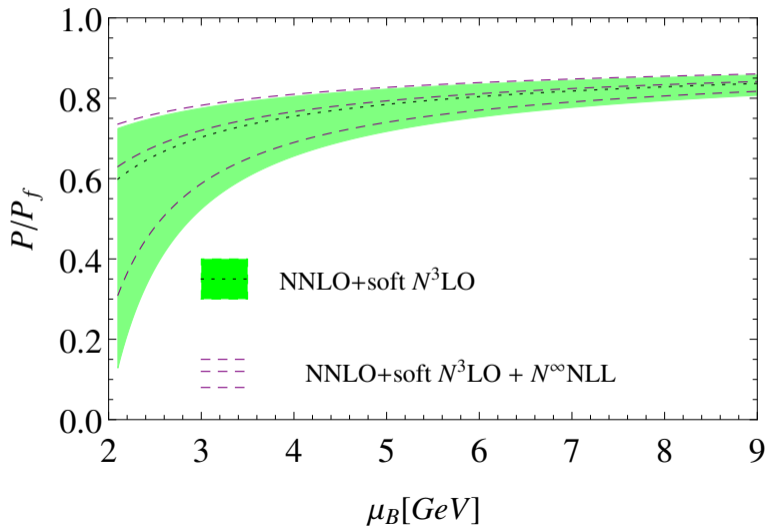
LL series

$$\mathcal{P}_{\text{LL}}^{\text{sum}} = a_{1,0} \ln \left(\frac{m_g}{M} \right) m_g^4 f_1^{1-4\left(\frac{\gamma_0^g}{2b_0^g}\right)} = a_{1,0} \ln \left(\frac{m_g}{M} \right) m_g^4 f_1^{-1},$$
$$f_1 = 1 + 2b_0^g g^2 \ln \frac{m_g}{M}.$$

$N^2\text{LO} + \text{LL}$ series

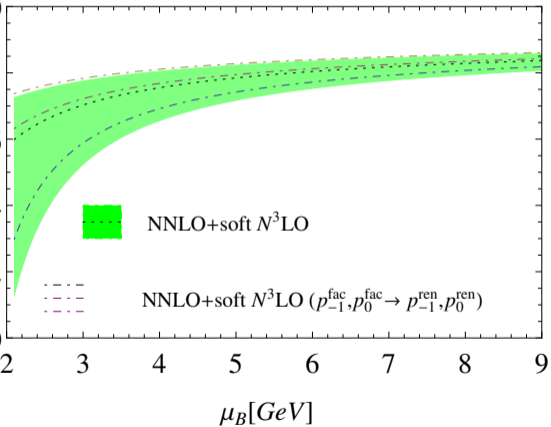


Pressure with only LL and NLL resummation

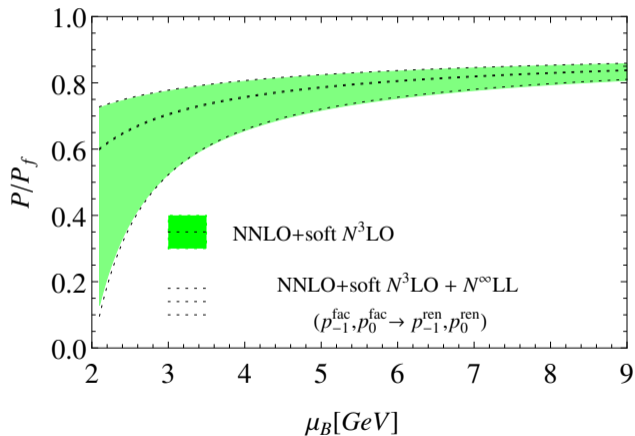
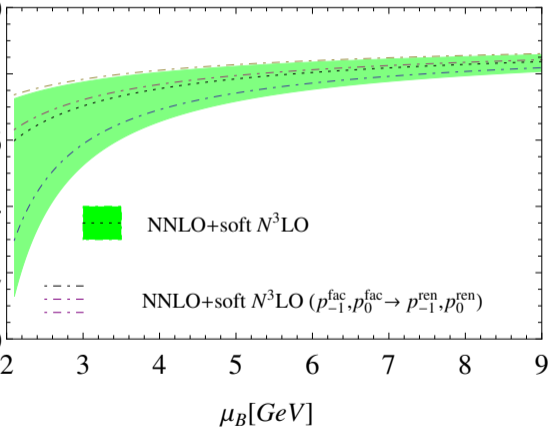


$$M_h \in [\mu, 4\mu]$$
$$M_s = 0.275m_E$$
$$m_g = m_E$$

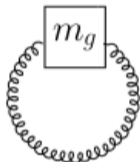
LL resummation



LL resummation



HTL at LO



$$\mathcal{P}_{\text{LO}}^{\text{HTL}} = \frac{d_A m_g^4}{(8\pi)^2} \left(\frac{1}{2\varepsilon} + C_{11} - \ln \left(\frac{m_g}{M} \right) \right)$$

- 1 There is no parameter at LO that can absorb this divergence.
- 2 **Massive** LO pressure is already not RG invariant.
- 3 We need to take into account the unit operator as it mixes: $m_g^4 \mathbb{1}$

Vacuum energy anomalous dimension: $\hat{\Gamma}^g$

NNLO vacuum energy counterterm

$$\Delta\mathcal{E}_0^{(2)} = \frac{d_A m_g^4}{(8\pi)^2} \left\{ \frac{1}{2\varepsilon} + N_c \left(\frac{g^2}{4\pi} \right) \left(-\frac{11}{24\pi\varepsilon^2} + \left(\frac{p_{-1}}{2\varepsilon} - \frac{11}{6\pi\varepsilon} (C_{11} - \frac{1}{4}) \right) \right) \right\} \equiv Z_0^g(g^2) m_g^4$$

At LO, same counterterm as in HTL (Andersen et al.(2002))

Vacuum energy anomalous dimension: $\hat{\Gamma}^g$

NNLO vacuum energy counterterm

$$\Delta \mathcal{E}_0^{(2)} = \frac{d_A m_g^4}{(8\pi)^2} \left\{ \frac{1}{2\varepsilon} + N_c \left(\frac{g^2}{4\pi} \right) \left(-\frac{11}{24\pi\varepsilon^2} + \left(\frac{p_{-1}}{2\varepsilon} - \frac{11}{6\pi\varepsilon} (C_{11} - \frac{1}{4}) \right) \right) \right\} \equiv Z_0^g(g^2) m_g^4$$

At LO, same counterterm as in HTL (Andersen et al. (2002))

$$-\frac{d\mathcal{P}_{\text{HTL}}}{d \ln M} = \frac{d\mathcal{E}_0}{d \ln M} \equiv \hat{\Gamma}_0^g(g^2) m_g^4 = \frac{d}{d \ln M} \left(m_g^4 \sum_{k \geq 0} s_k^g g^{2k-2} \right) \rightarrow \mathcal{E}_0^B \equiv M^{-2\varepsilon} (\mathcal{E}_0(g^2) - m_g^4 Z_0^g(g^2))$$

Renormalization group invariant (RGI) pressure

$$\mathcal{P}_{\text{RGI}}^{\text{HTL}} \equiv \mathcal{P}^{\text{HTL}} - m_g^4 \sum_{k \geq 0} s_k^g g^{2k-2} = \mathcal{P}^{\text{HTL}} - m_g^4 \frac{s_0^g}{g^2} - m_g^4 s_1^g + \dots$$

Matching the perturbative QCD pressure

$$\mathcal{P}^{\text{HTL}}(g^2(M), m(M)) = \mathcal{P}^{\text{HTL}}(M_0) + \int_{g^2(M_0)}^{g^2(M)} dx \left\{ \frac{-\hat{\Gamma}_0^g(x)}{\beta(x)} \exp \left[-4 \int_{g^2(M_0)}^x dy \frac{\gamma_m(y)}{\beta(y)} \right] \right\} m_g^4(M_0)$$

Boundary term

$$\mathcal{P}_{\text{RGI,B.C}}^{\text{HTL}} \equiv \mathcal{P}_{\text{RGI}}^{\text{HTL}} + m_g^4(M_0) \left(\frac{s_0^g}{g^2(M_0)} + s_1^g \right)$$

$$s_0^g = -\frac{a_{1,0}}{2(b_0^g - 2\gamma_0^g)} = \frac{-d_A}{2(8\pi)^2 b_0^g},$$

$$s_1^g = a_{1,1} + \frac{a_{2,1}}{4\gamma_0^g} + \frac{a_{1,0}}{4} + \frac{s_0^g}{2\gamma_0^g} (b_1^g - 2\gamma_1^g).$$

To match the SB limit, M_0 must be of the order of the hard scale $\mathcal{O}(\mu)$.

Natural choice to match the $\text{P}^{\text{N}^2\text{LO}}$: $M_0 = 2\mu$.

Collecting the pieces

RGI pressure: Full LL + soft NLL

$$\mathcal{P}_{\text{RGI}}^{\text{C.Q.M}} = \mathcal{P}_f \left(1 - \frac{2}{\pi} \alpha_s - \frac{N_f}{\pi^2} \alpha_s^2 \ln \alpha_s - 0.874355 \alpha_s^2 - 2d_A \alpha_s^2 \frac{(11N_c - 2N_f)}{3(4\pi)^2} \ln \left(\frac{M_h}{\mu} \right) \right) \leftarrow \text{Full N}^2\text{LO}$$

Collecting the pieces

RGI pressure: Full LL + soft NLL

$$\begin{aligned}
 \mathcal{P}_{\text{RGI}}^{\text{C.Q.M}} = & \mathcal{P}_f \left(1 - \frac{2}{\pi} \alpha_s - \frac{N_f}{\pi^2} \alpha_s^2 \ln \alpha_s - 0.874355 \alpha_s^2 - 2d_A \alpha_s^2 \frac{(11N_c - 2N_f)}{3(4\pi)^2} \ln \left(\frac{M_h}{\mu} \right) \right) \leftarrow \text{Full } N^2\text{LO} \\
 & + \frac{N_c d_A \alpha_s m_E^4}{(8\pi)^2} \left(p_{-2} \ln \left(\frac{m_g}{M_s} \right)^2 - 2p_{-1}^{\text{ren}} \ln \left(\frac{m_g}{M_s} \right) + p_0^{\text{ren}} \right) \leftarrow \text{Soft } N^3\text{LO}
 \end{aligned}$$

Collecting the pieces

RGI pressure: Full LL + soft NLL

$$\begin{aligned}
 \mathcal{P}_{\text{RGI}}^{\text{C.Q.M}} = & \mathcal{P}_f \left(1 - \frac{2}{\pi} \alpha_s - \frac{N_f}{\pi^2} \alpha_s^2 \ln \alpha_s - 0.874355 \alpha_s^2 - 2d_A \alpha_s^2 \frac{(11N_c - 2N_f)}{3(4\pi)^2} \ln \left(\frac{M_h}{\mu} \right) \right) \leftarrow \text{Full N}^2\text{LO} \\
 & + \frac{N_c d_A \alpha_s m_E^4}{(8\pi)^2} \left(p_{-2} \ln \left(\frac{m_g}{M_s} \right)^2 - 2p_{-1}^{\text{ren}} \ln \left(\frac{m_g}{M_s} \right) + p_0^{\text{ren}} \right) \leftarrow \text{Soft N}^3\text{LO} \\
 & + m_E^4 g^4(M_h) a_{3,0} \ln^3 \left(\frac{m_E}{M_s} \right) + m_E^4 g^6(M_h) a_{4,0} \ln^4 \left(\frac{m_E}{M_s} \right) + \dots \leftarrow \text{LL series} \\
 & + m_E^4 g^4(M_h) a_{3,1} \ln^2 \left(\frac{m_E}{M_s} \right) + m_E^4 g^6(M_h) a_{4,1} \ln^3 \left(\frac{m_E}{M_s} \right) + \dots \leftarrow \text{NLL series}
 \end{aligned}$$

Collecting the pieces

RGI pressure: Full LL + soft NLL

$$\begin{aligned}
 \mathcal{P}_{\text{RGI}}^{\text{C.Q.M}} = & \mathcal{P}_f \left(1 - \frac{2}{\pi} \alpha_s - \frac{N_f}{\pi^2} \alpha_s^2 \ln \alpha_s - 0.874355 \alpha_s^2 - 2d_A \alpha_s^2 \frac{(11N_c - 2N_f)}{3(4\pi)^2} \ln \left(\frac{M_h}{\mu} \right) \right) \leftarrow \text{Full N}^2\text{LO} \\
 & + \frac{N_c d_A \alpha_s m_E^4}{(8\pi)^2} \left(p_{-2} \ln \left(\frac{m_g}{M_s} \right)^2 - 2p_{-1}^{\text{ren}} \ln \left(\frac{m_g}{M_s} \right) + p_0^{\text{ren}} \right) \leftarrow \text{Soft N}^3\text{LO} \\
 & + m_E^4 g^4(M_h) a_{3,0} \ln^3 \left(\frac{m_E}{M_s} \right) + m_E^4 g^6(M_h) a_{4,0} \ln^4 \left(\frac{m_E}{M_s} \right) + \dots \leftarrow \text{LL series} \\
 & + m_E^4 g^4(M_h) a_{3,1} \ln^2 \left(\frac{m_E}{M_s} \right) + m_E^4 g^6(M_h) a_{4,1} \ln^3 \left(\frac{m_E}{M_s} \right) + \dots \leftarrow \text{NLL series} \\
 & - s_0^g \left(\frac{m_E^4(M_h)}{g^2(M_h)} - \frac{m_E^4(M_0)}{g^2(M_0)} \right) - s_1^g (m_E^4(M_h) - m_E^4(M_0)) \leftarrow \text{RGI restoring terms}
 \end{aligned}$$

Compact formula

RGI pressure: Full LL + soft NLL

$$\begin{aligned}
 \mathcal{P}_{\text{RGI}}^{\text{C.Q.M}} = & \mathcal{P}_f \left(1 - \frac{2}{\pi} \alpha_s - \frac{N_f}{\pi^2} \alpha_s^2 \ln \alpha_s - 0.874355 \alpha_s^2 - 2d_A \alpha_s^2 \frac{(11N_c - 2N_f)}{3(4\pi)^2} \ln \left(\frac{M_h}{\mu} \right) \right) \\
 & - \frac{s_0^4 m_g^4}{g^2 f_2^{4A_0 - 1}} [R(f_2)]^B \left(1 - \frac{a'_{1,1} g^2}{s_0^g f_2} - \frac{a_{2,2} g^4}{s_0^g f_2^2} \right) - \left(a_{1,1} + a_{1,0} \ln \left(\frac{m_g}{M_s} \right) \right) \\
 & + \frac{m_g^4(M_0)}{g^2(M_0)} s_0^g + m_g^4(M_0) s_1^g
 \end{aligned}$$

$$a'_{1,1} = a_{1,1} - s_1^g$$

To avoid double counting

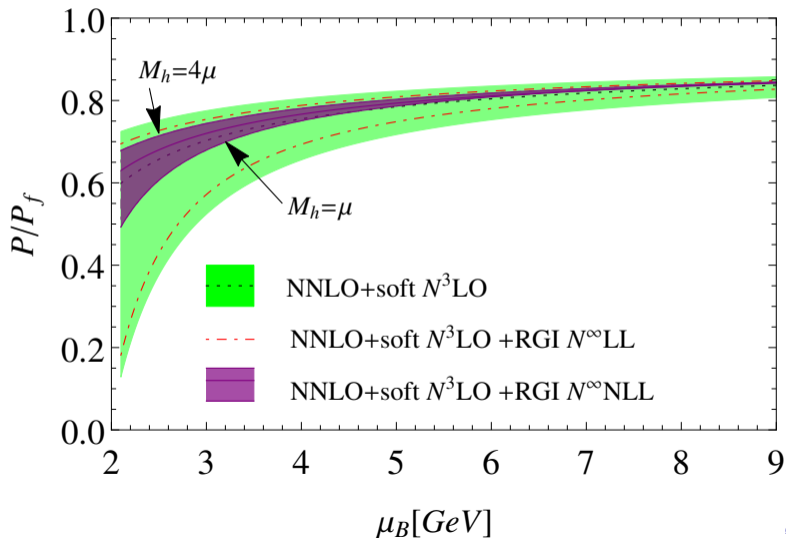
$$R(f_2) = (1 + g^2 b_1^g / (b_0^g f_2)) / (1 + g^2 b_1^g / b_0^g),$$

$$A_0 = \gamma_0^g / (2b_0^g), \quad A_1 = \gamma_1^g / (2b_1^g)$$

$$f_2 \simeq 1 + [2b_0^g g^2 + 2(b_1^g - \gamma_0^g b_0^g) g^4] \ln \frac{m_g}{M} + \mathcal{O}(g^6),$$

$$B = 4(A_1 - A_0), \quad C = b_1^g / (2(b_0^g)^2)$$

Complete RGI pressure results

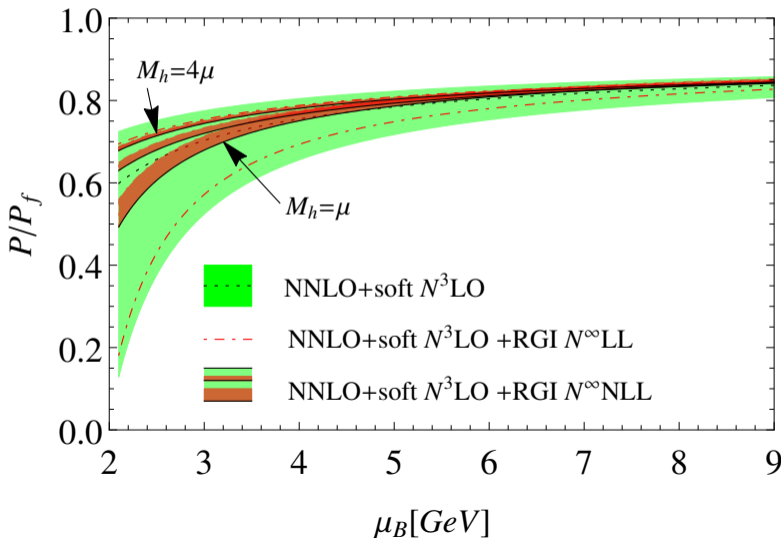


$$M_h \in [\mu, 2\mu, 4\mu]$$

$$M_s = 0.275m_E$$

$$m_g = m_E$$

Complete RGI pressure results



$$M_h \in [\mu, 4\mu]$$

$$M_s \in [1/2, 2] \times 0.275 m_E$$

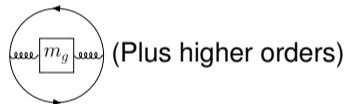
$$m_g = m_E$$

LF, J-L. Kneur
(2021)

arXiv:2109.02410

Conclusion and perspectives

- Our RGI construction exhibits substantially improved remnant scale dependence.
- Evaluation of the HTL fermion loop diagram (beyond pure glue)
- Add mixed NLL to the resummation
- Effects of non-zero quark masses
- Application to neutron stars equation-of-state with expected reduced scale dependence uncertainties.



Thank you for your attention !