Complex Langevin boundary terms in lattice models

Dénes Sexty

KFU Graz

Collaborators: Nina Maria Lampl, Michael Westh Hansen, Manuel Scherzer, Erhard Seiler, Nucu Stamatescu

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1. Introduction to Complex Langevin

2. Boundary terms; test in full QCD, 3d XY model; correction

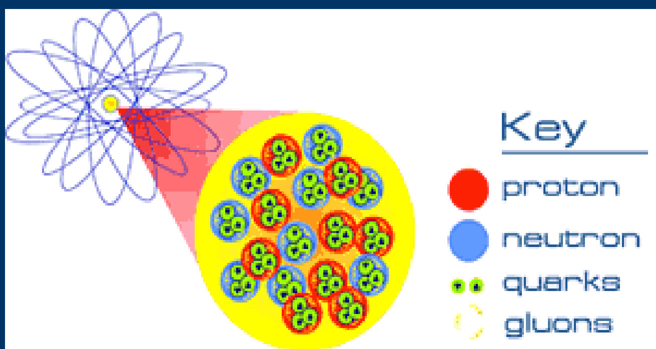
3. Results for full QCD: EoS and phase diagram

From the action to the phenomenology of QCD

Action of QCD, the theory of strong interactions

$$S = -\frac{1}{4}\mathrm{Tr}\, F_{\mu\nu} F^{\mu\nu} + \sum_1^6 \bar{\psi}_f \left(i \gamma^\mu D_\mu + m_f \right) \psi_f$$

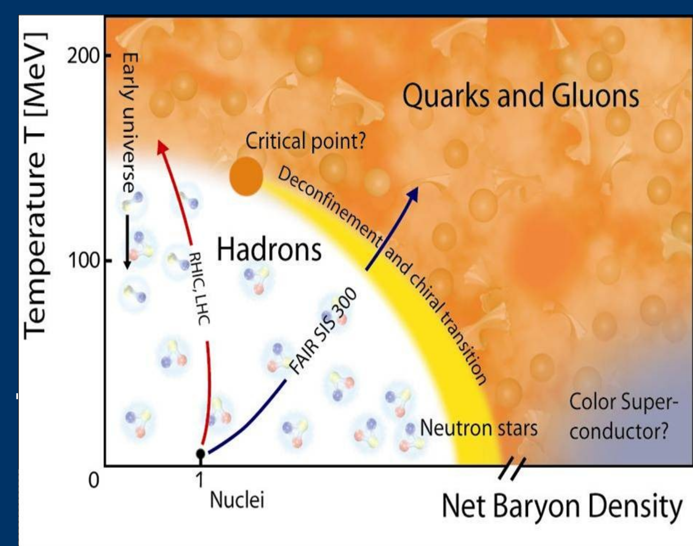
1 gauge coupling
6 quark masses



Confinement mechanism?
Mass of hadrons?
Scattering cross sections?
Phases transition to Quark-gluon plasma?
  Critical point at nonzero density?
  Equation of state?
Compressibility of quark matter? (in neutron stars)
Exotic phases:
  Color superconducting phases?
  Quarkyonic phase?
QCD in magnetic fields?
.... and so on

How?

Perturbation theory – asymptotic freedom
Kinetic theory
Effective models (NJL, Polyakov-NJL, SU(3) spin model, .
Functional methods (FRG, 2PI, Dyson-Schwinger eq.)
Lattice

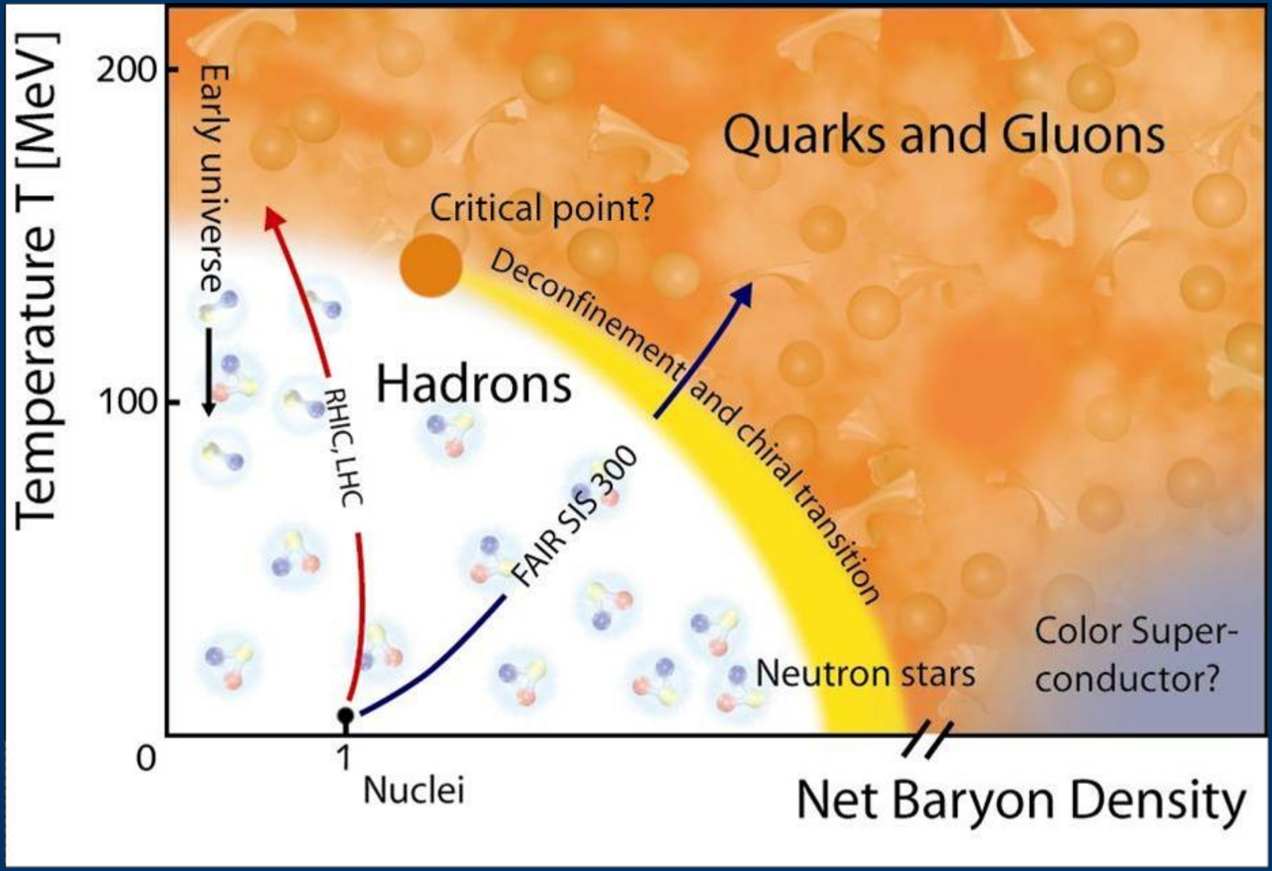


Phase diagram of QCD

Zero density axis well known

transition temperature

zero temperature:
hadron masses
scattering amplitudes, etc.



At nonzero density much less solid knowledge

What phases are present?
Is there a critical point?
compressibility of nuclear matter?

Why is non-zero density so hard?

Importance Sampling and Sign Problem

We are interested in a system
Described with the partition sum: $Z = \int D\phi \, e^{-S} = \mathrm{Tr}\, e^{-\beta(H - \mu N)} = \sum_C W[C]$

Usually: Markov Chain Monte Carlo $\ldots \rightarrow C_{i-1} \rightarrow C_i \rightarrow C_{i+1} \rightarrow \ldots$

Probability of visiting C $p(C) \sim W[C]$ $\langle X \rangle = \frac{1}{N} \sum_i X[C_i]$

This works if we have $W[C] \geq 0$ Otherwise we have a Sign problem

Workaround: Dual variables, Density of States,
Reweighting, Taylor expansion, Imaginary potentials, etc.

Using analyticity: Lefschetz Thimbles, Complex Langevin

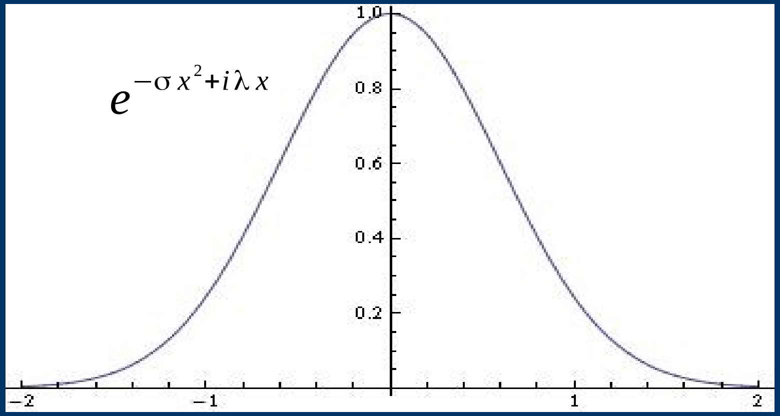
Toy model with sign problem

$$Z=\int_{-\infty}^{\infty} e^{-(\sigma x^2+i\lambda x)}dx$$

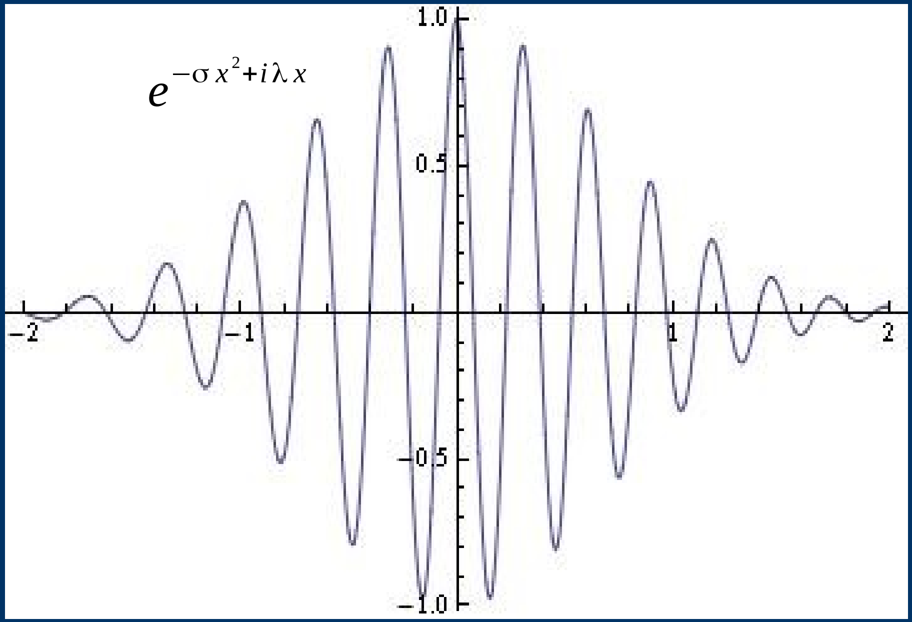
$$\langle x^2\rangle=\frac{1}{Z}\int x^2 e^{-(\sigma x^2+i\lambda x)}dx=?$$

Sampling method: draw uniform random $-a\leqslant x_i\leqslant a$

$$\int_{-a}^{a} f(x)d(x)\approx\frac{1}{N}\sum_i f(x_i)$$



$$\sigma=\sqrt{2},\ \lambda=0$$



$$\sigma=1+i,\ \lambda=20$$

100 samples to have 10% relative error

$$\Delta\sim\frac{1}{\sqrt{N}}$$

$$Z\approx 10^{-22}$$

$\sim 10^{46}$ samples to have 10% relative error

How to solve the sign problem (of QCD)?

Extrapolation from a positive ensemble

Reweighting $$\langle X\rangle_W=\frac{\sum_c W_c X_c}{\sum_c W_c}=\frac{\sum_c W'_c(W_c/W'_c)X_c}{\sum_c W'_c(W_c/W'_c)}=\frac{\langle (W/W')X\rangle_{W'}}{\langle W/W'\rangle_{W'}}$$

Taylor expansion $$Z(\mu)=Z(\mu=0)+\frac{1}{2}\mu^2\partial_\mu^2 Z(\mu=0)+\ldots$$

Analytic continuation from imaginary sources
(chemical potentials, theta angle,..)

Using analyticity (for complexified variables)

Complex Langevin

Complexified variables – enlarged manifolds

Lefschetz thimble (not yet for QCD)

Integration path shifted onto complex plane

In QCD direct simulation only possible at $\mu=0$

Taylor extrapolation, Reweighting, continuation from imaginary μ, canonical ens.
all break down around

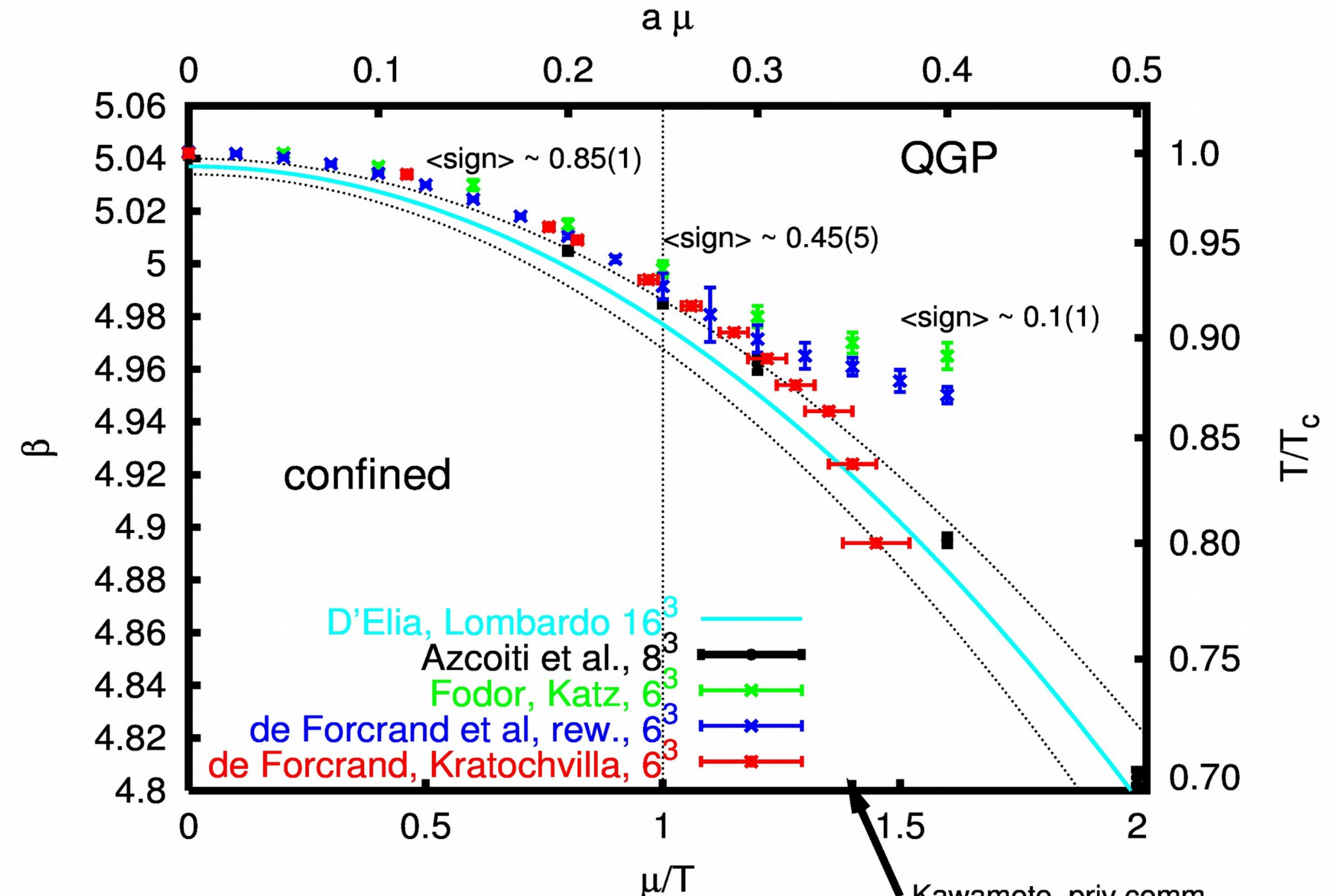
$$\frac{\mu_q}{T}\approx 1-1.5 \qquad \frac{\mu_B}{T}\approx 3-4.5$$

Around the transition temperature
Breakdown at $\mu_q\approx 150-200\,\mathrm{MeV}$ $\mu_B\approx 450-600\,\mathrm{MeV}$

Results on

$$N_T=4, N_F=4, ma=0.05$$

using
Imaginary mu,
Reweighting,
Canonical ensemble



Agreement only at $\mu/T<1$

Langevin Equation (aka. stochatic quantisation)

Given an action $S(x)$

Stochastic process for x: $$\frac{dx}{d\tau} = -\frac{\partial S}{\partial x} + \eta(\tau)$$

Gaussian noise

$$\langle \eta(\tau) \rangle = 0$$
$$\langle \eta(\tau)\eta(\tau') \rangle = \delta(\tau - \tau')$$

Random walk in configuration space

Averages are calculated along the trajectories:

$$\langle O \rangle = \lim_{T \to \infty} \frac{1}{T} \int_0^T O(x(\tau))\, d\tau = \frac{\int e^{-S(x)} O(x)\, dx}{\int e^{-S(x)}\, dx}$$

Numerically,
results are extrapolated to $\Delta\tau \to 0$

Complex Langevin Equation

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Gaussian noise

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$$\frac{dx}{d\tau} = -\frac{\partial S}{\partial x} + \eta(\tau)$$

The field is complexified

real scalar $\longrightarrow$ complex scalar

link variables: SU(N) $\longrightarrow$ SL(N,C)

compact non-compact

$$det(U) = 1, \quad U^{+} \neq U^{-1}$$

Analytically continued observables

$$\frac{1}{Z} \int P_{comp}(x) O(x)\, dx = \frac{1}{Z} \int P_{real}(x, y) O(x+iy)\, dx\, dy$$

$$\langle x^2 \rangle_{real} \rightarrow \langle x^2 - y^2 \rangle_{complexified}$$

Gaussian Example

$$S[x]=\sigma x^2+i\lambda x$$

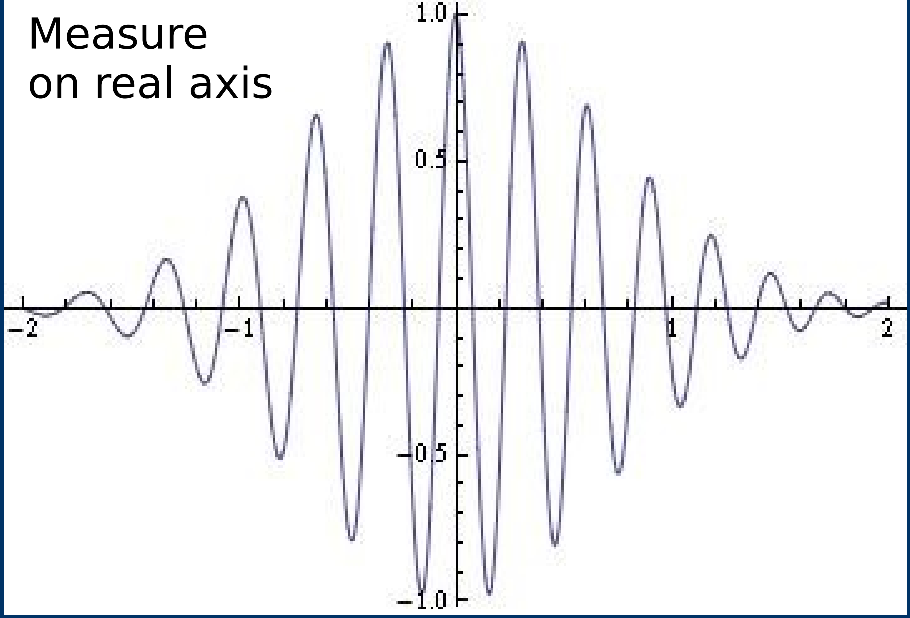
CLE

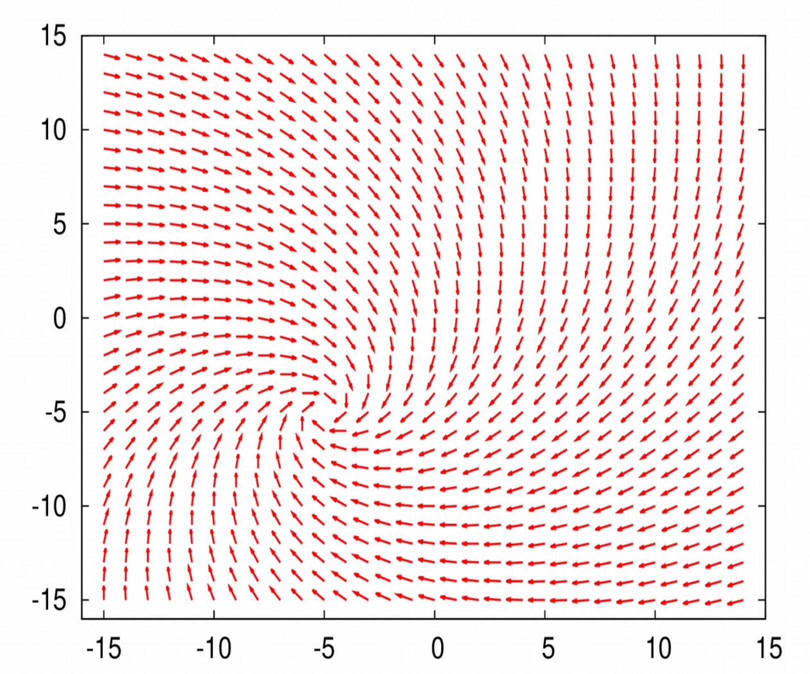
$$\frac{d}{d\tau}(x+iy)=-2\sigma(x+iy)-i\lambda+\eta$$

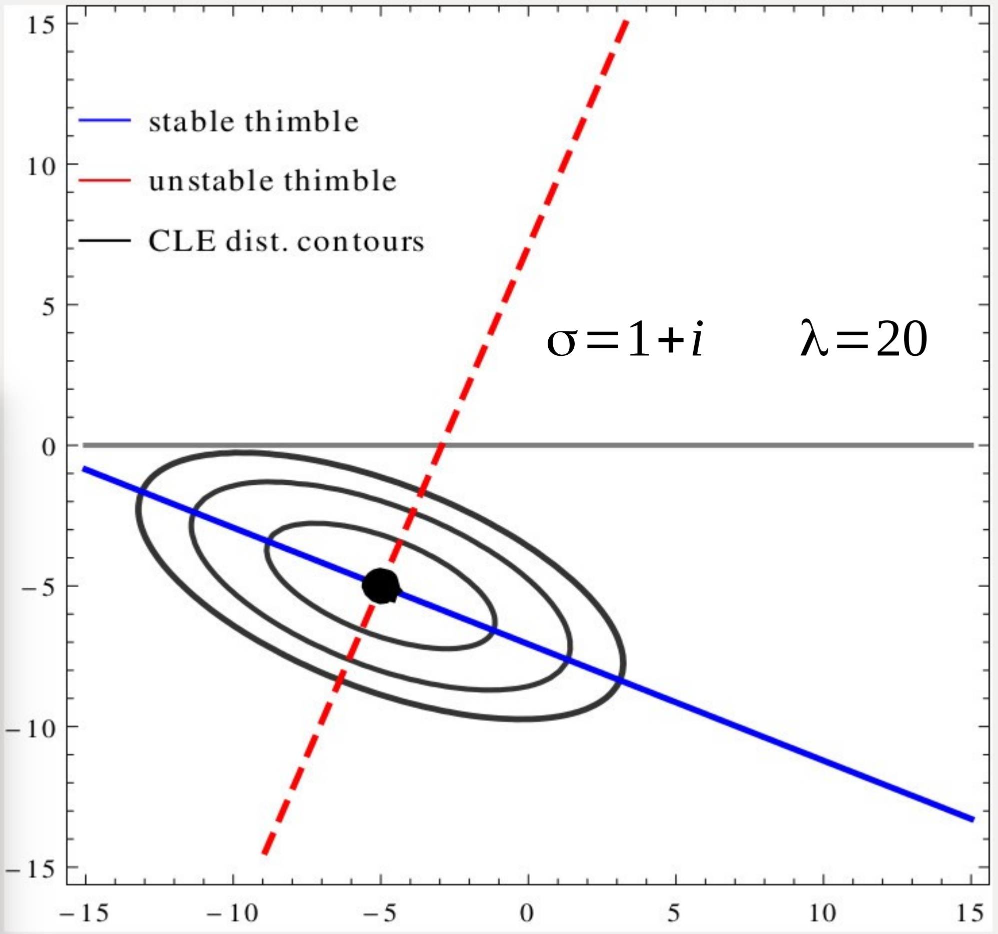
$$P(x,y)=e^{-a(x-x_0)^2-b(y-y_0)^2-c(x-x_0)(y-y_0)}$$

Gaussian distribution
around critical point

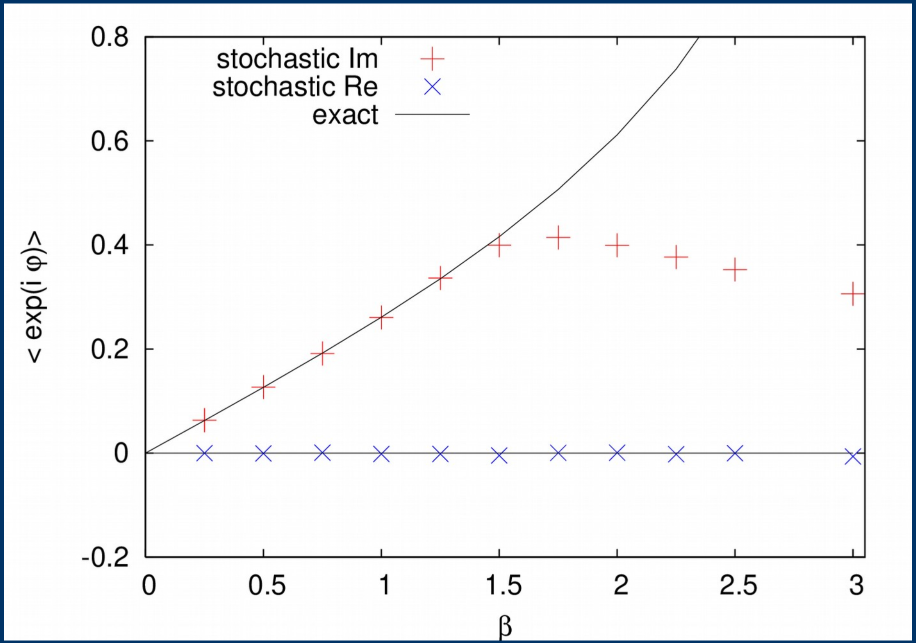
$$\left.\frac{\partial S(z)}{\partial z}\right|_{z_0}=0$$







“troubled past”: Convergence to wrong results
Lack of theoretical understanding
Runaway trajectories



$$S(\varphi) = i\beta\cos\varphi + i\varphi$$

Correct in one parameter region
Incorrect in an other

Convergent in both

Klauder '83, Parisi '83, Hueffel, Rumpf '83, Karsch. Wyld '84, Gausterer, Klauder '86. Matsui, Nakamura '86, ...
Interest went down as difficulties appeared
Renewed interest in connection of otherwise unsolvable problems
applied to nonequilibrium: Berges, Stamatescu '05, ...
aimed at nonzero density QCD: Aarts, Stamatescu '08 ... many important results since revival

Argument for correctness of CLE results

If there is fast decay $P(x,y) \rightarrow 0$ as $x, y \rightarrow \infty$

and a holomorphic action $S(x)$

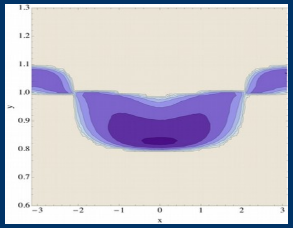
then CLE converges to the correct result

[Aarts, Seiler, Stamatescu (2009)
Aarts, James, Seiler, Stamatescu (2011)]

Loophole 1: Non-holomorphic action for nonzero density

$$S = S_W[U_\mu] + \ln \mathrm{Det}\, M(\mu)$$

measure has zeros $(Det\, M = 0)$
complex logarithm has a branch cut
→ meromorphic drift



No problems if poles are not ‘touched’ by distribution

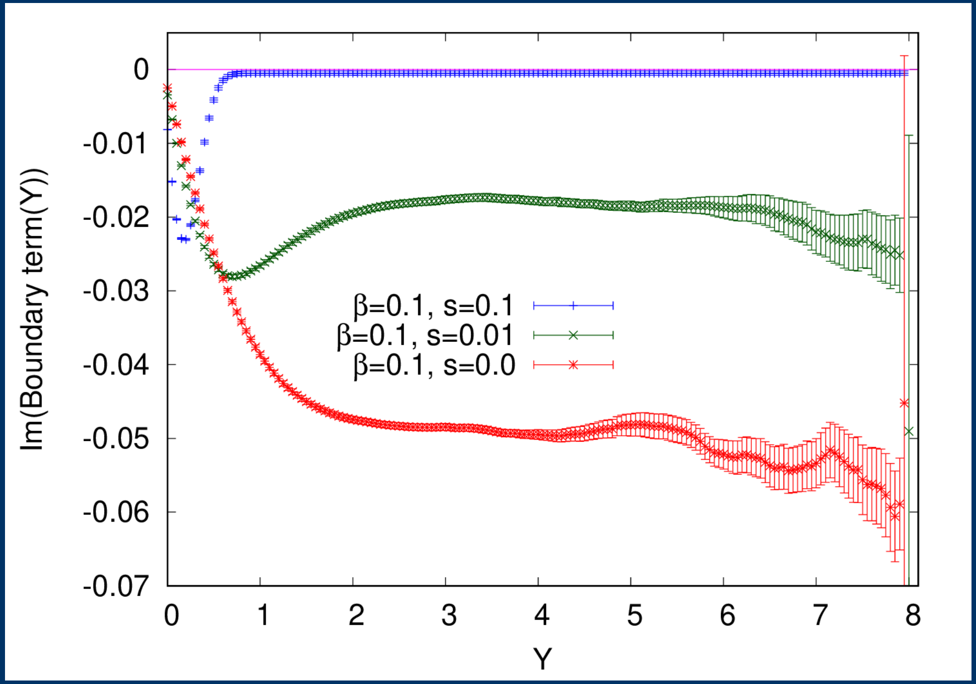
satisfied for: HDQCD, full QCD at high temperatures

[Aarts, Seiler, Sexty, Stamatescu ‘17]

Loophole 2: decay not fast enough

boundary terms can be nonzero
explicit calculation of boundary terms possible

[Scherzer, Seiler, Sexty, Stamatescu (2018)+(2019)]



Unambigous detection of boundary terms

given by plateau as 'cutoff' $Y \rightarrow \infty$

Observable cheap also for lattice systems

Measuring “corrected observable”

in case boundary term nonzero

Details below...

Sketch of the proof

$P(x,y,t)$: probability density on the complex plane at Langevin time t

Real Fokker-Planck equation

$$\frac{\partial P}{\partial \tau}=\frac{\partial}{\partial x}\left(\frac{\partial P}{\partial x}-K_x P\right)-\frac{\partial}{\partial y}\left(K_y P\right) \quad \text{with} \quad K_i=-\partial_i S$$

Real action $\rightarrow$ $K_y=0$, positive eigenvalues of H_{FP}

$$P(x,y,\infty)=\delta(y)\exp(-S(x))$$

$\rho(x,t)$: complex measure evolving with the complex Fokker-Planck equation
(not associated to a stochastic process)

$$\partial_t \rho(x,t)=\partial_x(\partial_x-K_x)\rho(x,t)=L_c^T \rho(x,t)$$

Stationary solution: $\rho(x,\infty)=\exp(-S(x))$

Assuming spectrum of L_c is fine

CLE works, if

What we want		What we get with CLE
$\int dx\, \rho(x) O(x)$	$=$	$\int dx\, dy\, P(x,y) O(x+iy)$

$$\langle O(x)\rangle_{\rho(t)} = \langle O(x+iy)\rangle_{P(t)}$$

$$O(z,t) = e^{L_c t} O(z,0)$$

with $L_c = (\partial_z + K(z))\partial_z$

Interpolating function:

$$F(t,\tau) = \int P(x,y,t-\tau) O(x+iy,\tau)\, dx\, dy$$

$$F(t,0) = \langle O(x+iy)\rangle_{P(t)}$$

$$F(t,t) = \int P(x,y,0) O(x+iy,t)\, dx\, dy$$

$$= \int \rho(x,0)\delta(y) O(x+iy,t)\, dx\, dy$$

$$= \int \rho(x,0) O(x,t)\, dx = \int \rho(x,t) O(x,0)\, dx = \langle O(x)\rangle_{\rho(t)}$$

Choose to be
on real axis initially

$\partial_\tau F(t,\tau) = 0$ can be seen with partial integrations
using Cauchy-Riemann eqs. for $\partial_x O(x+iy,\tau)$ QED

Boundary term defined on a surface

$$\partial_\tau F_O(t,\tau) = B_O(Y,t,\tau) = \int K_y(x,Y) P(x,Y,t-\tau) O(x+iY,\tau)\, dx$$
$$- \int K_y(x,-Y) P(x,-Y,t-\tau) O(x-iY,\tau)\, dx$$

Loophole 2: Spectrum on the wrong side

[Seiler, Sexty et. al. in prep.]

Fokker Planck operator

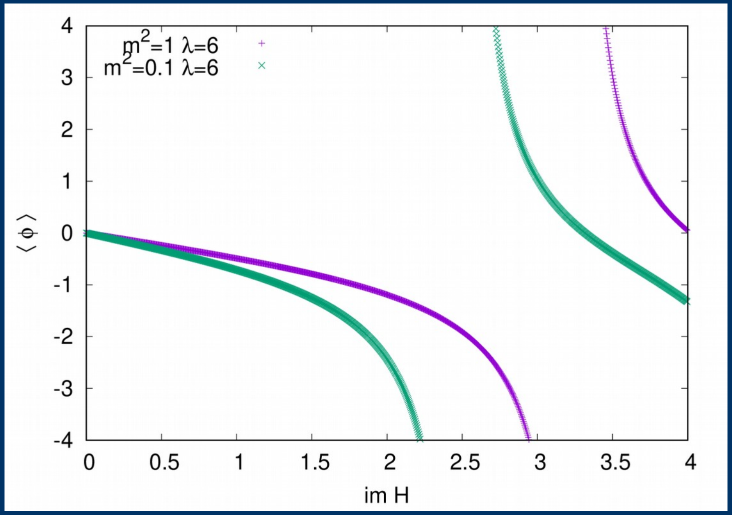
$$L_c = \sum_i \partial_i^2 + K_i \partial_i$$

Determines $\rho(x,t) = e^{t L_c^T} \rho(x,0)$

Toy model:

$$S = \frac{1}{2} m^2 \phi^2 + \frac{\lambda}{24} \phi^4 + H\phi \quad \rightarrow \quad L_c = \partial_z^2 + \left(-m^2 z - \frac{\lambda}{6} z^3 - H \right) \partial_z$$

At imaginary magnetic field
Lee-Yang zeroes appear



At each Lee-Yang zero
an eigenvalue appears with $\mathrm{Re}(\lambda) > 0$

Slow decay is also present: → Boundary terms signal also this problem

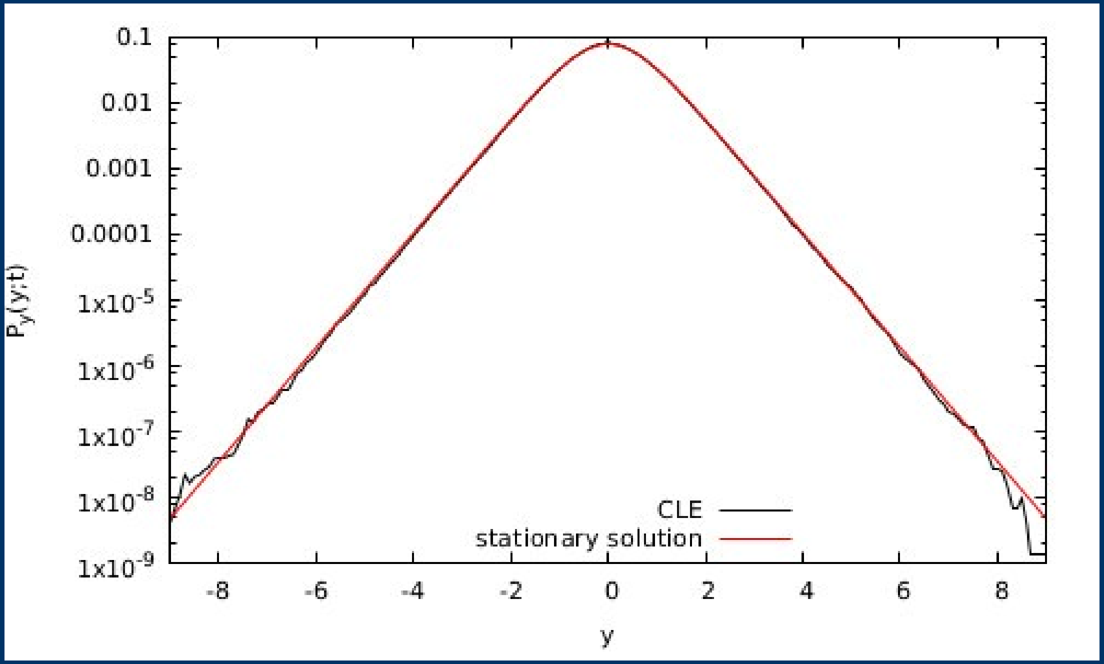
One plaquette model

$$S(\varphi) = i\beta\cos(\varphi) \qquad \langle e^{ikx}\rangle = (-i)^k \frac{J_k(\beta)}{J_0(\beta)}$$

Exact stationary solution of Fokker-Planck eq. [Salcedo, 2017]

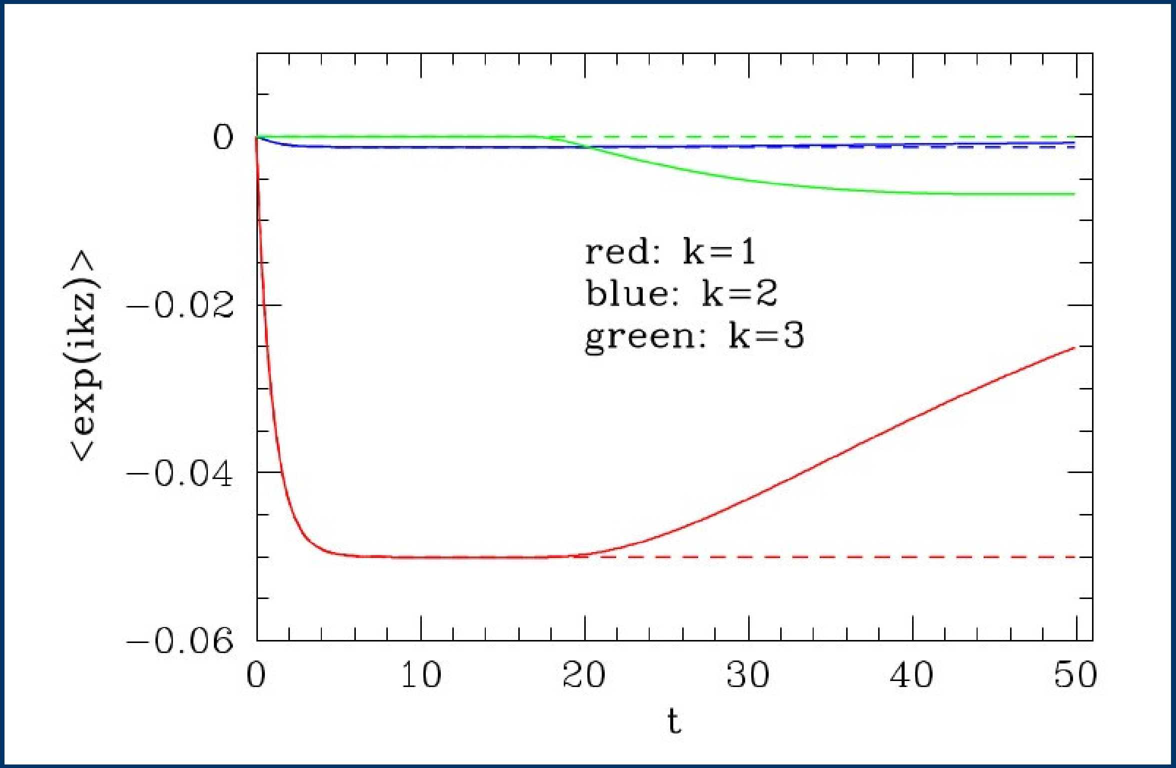
$$P_a(x,y) = \frac{1}{4\pi\cosh^2 y} \qquad \text{independent of } x \text{ and } \beta$$

$$\langle e^{ix}\rangle_{P_a} = 0, \quad \langle e^{ikx}\rangle_{P_a} \text{ for } k \geq 2 \text{ is undefined or divergent}$$



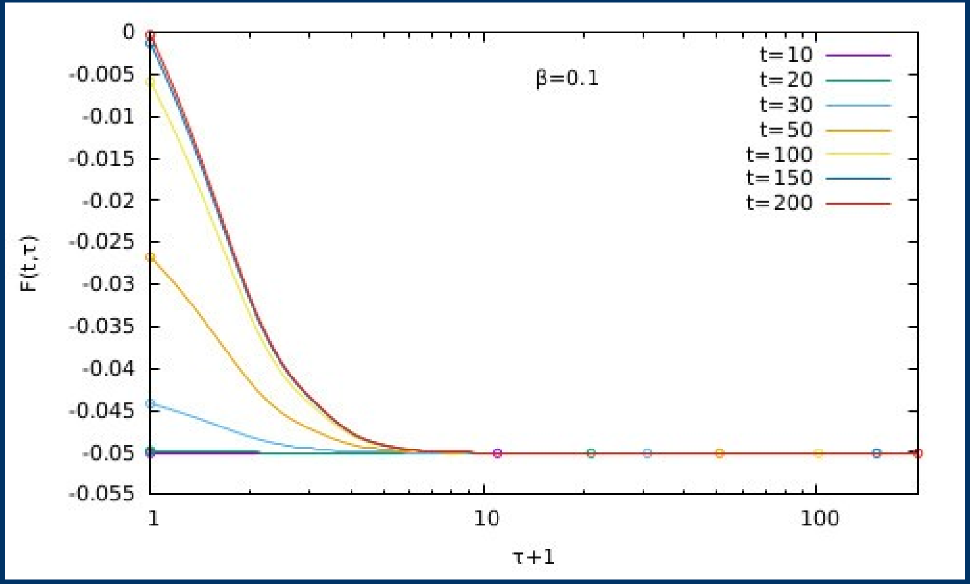
CLE reproduces this (incorrect) solution

Langevin time evolution



For short times
plateau at the correct value

asymptotic result incorrect



$F(t,0)-F(t,t)$ gets >0 above $t=20$

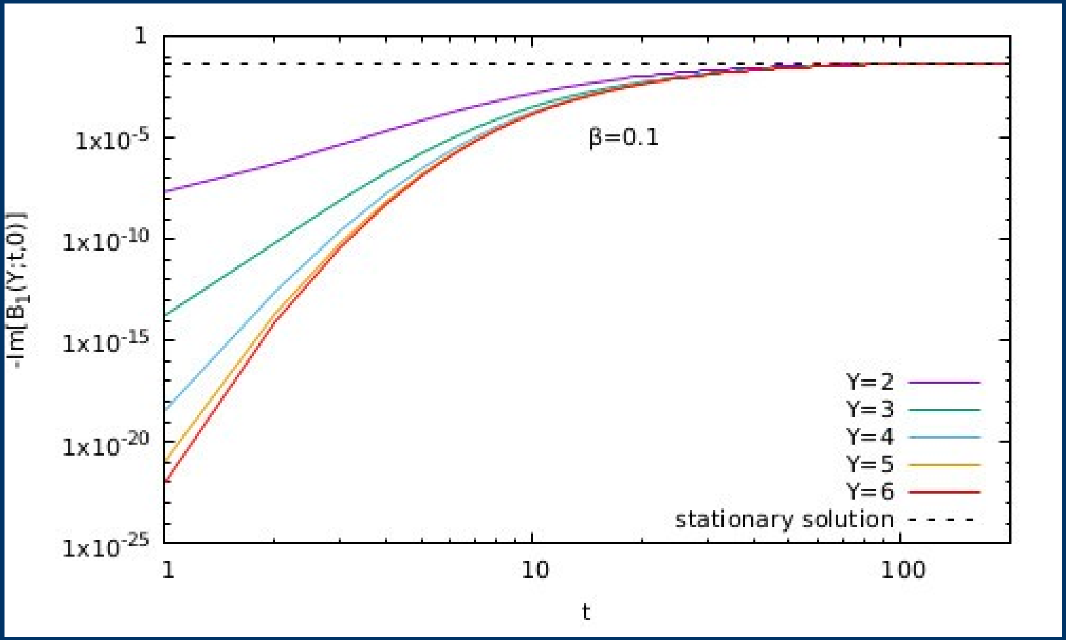
Largest slope at $\tau=0$

$$\partial_\tau F_{(t,\tau=0)}=B(t,0)$$

seems like a good proxy for

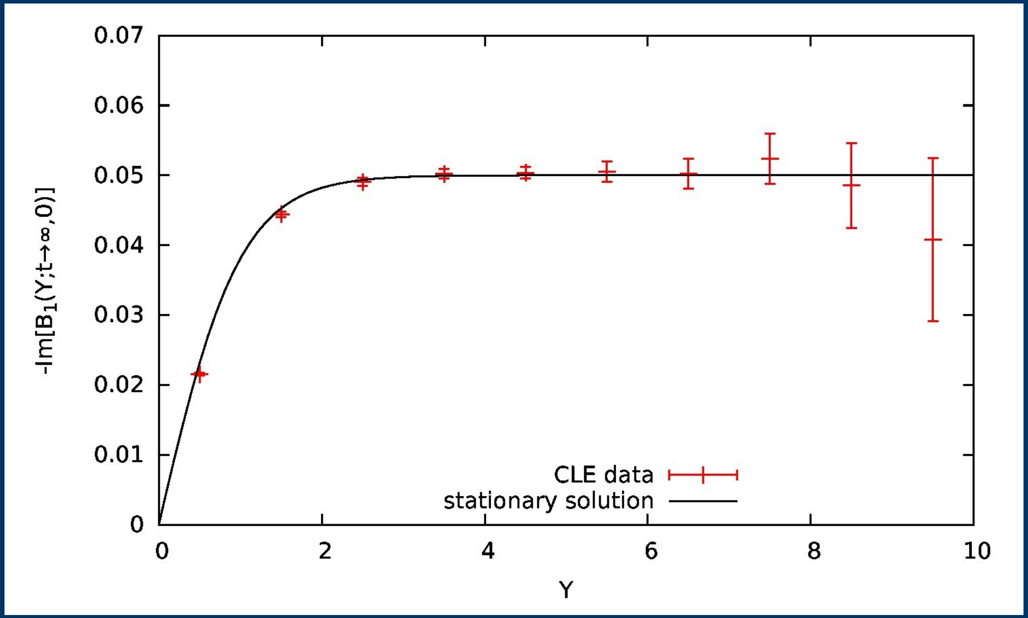
$$F(t,0)-F(t,t)$$

Boundary term



Boundary term

Calculated using Fokker-Planck discretised on a 2d grid



Using Complex Langevin only

Plateau clearly visible
At high cutoff statistics is worse

Need to measure on some surface inconvenient in many dimensions

Boundary terms as a volume integral

[Scherzer, Seiler, Sexty, Stamatescu (2018+2019)]

Calculating an observable defined on a compact boundary in many dimensions can be inconvenient

$$\partial_\tau F_O(Y,t,\tau=0) = B_O(Y,t,\tau=0) =$$
$$\int_{-Y}^{Y} P(x,y,t) L_c O(x+iy) - \int_{-Y}^{Y} (L^T P) O(x+iy,0)$$

Observable with a cutoff
easy to do in many dimensions

Vanishes as process equilibrates

$L_c O(x+iy)$ consistency conditions $\approx$ Schwinger-Dyson eqs.

Order of limits crucial

$$\lim_{t\to\infty} \lim_{Y\to\infty} \int_{-Y}^{Y} P(x,y,t) L_c O(x+iy) \text{ can be undefined}$$

Measuring boundary terms

$$\int_{-Y}^{Y} P(x,y,t) L_c O(x+iy) = \int P(x,y,t) L_c O(x+iy) \theta(Y-y)$$

$$L_c = \sum \partial_i^2 + K_i \partial_i$$

Many variables: define cutoff to extend SU(N) manifold
to compact submanifold of SL(N,C)

$$\text{e.g.} \quad \mathrm{Im}\, z\,; \quad \max_i Tr\left(U_i^{+} U_i - 1\right)^2$$

Measure "unitarity norm" and observable

Analyze for any cutoff

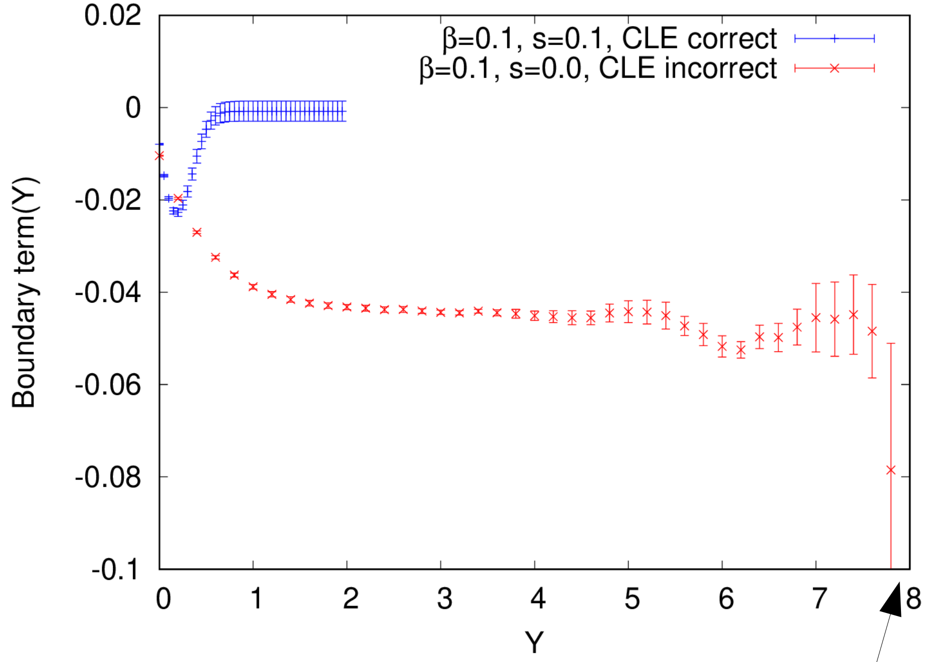
Trick for second term:

$$\sum K_i \partial_i O = \frac{1}{\epsilon} O\left(z(\tau+\epsilon, \eta=0) - z(\tau)\right)$$

Measure observable after doing a noiseless update step with stepsize ϵ

One plaquette model

$$S(x) = i\beta\cos(x) + \frac{s}{2}x^2$$



last point with no cutoff
large fluctuations
consistent with zero?

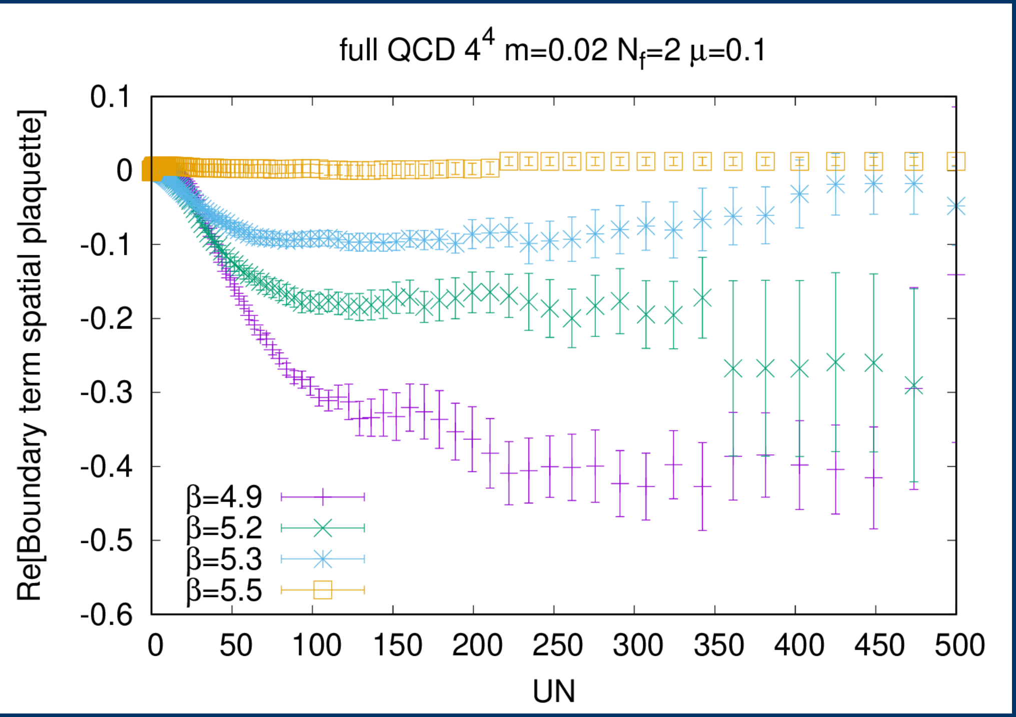
Unambigous detection of boundary terms

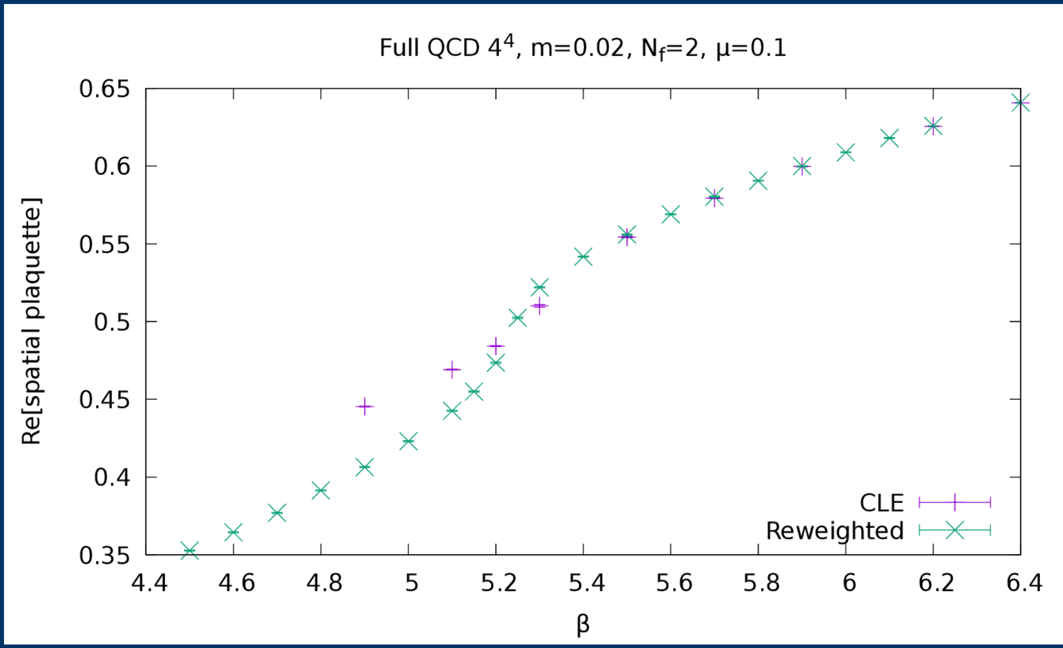
Observable cheap also for lattice systems

In full QCD this confirms already known signals
Quantifies error

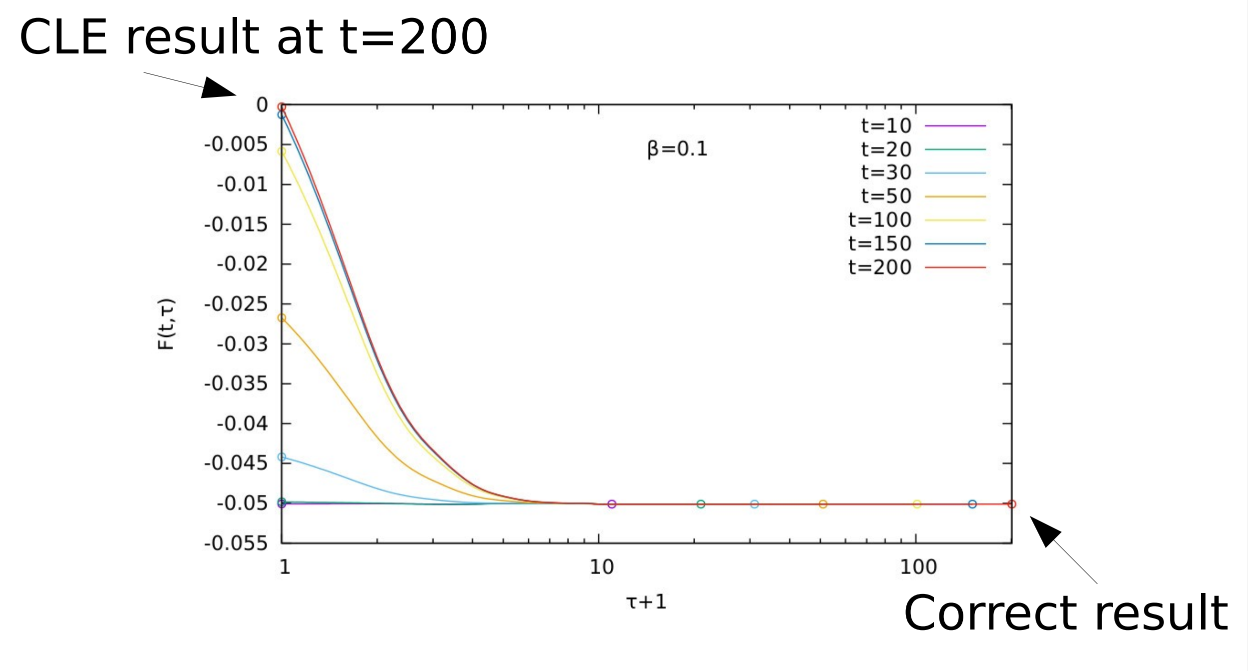
Faster than exponential decay of histograms of observables
Drift criterion = same for drift term observable

Boundary terms appear at small β = large lattice spacing





Correcting CLE using boundary terms



Interpolation function

$$F(t,\tau)=\sum A_n \exp(-\omega_n \tau)$$

Higher order boundary terms

$$\frac{\partial^n F(t,\tau)}{\partial \tau^n}=B_n=\langle L_c^n O\rangle$$

Ansatz

$$F(t,\tau)=A_0+A_1 \exp(-\omega_1 \tau)$$

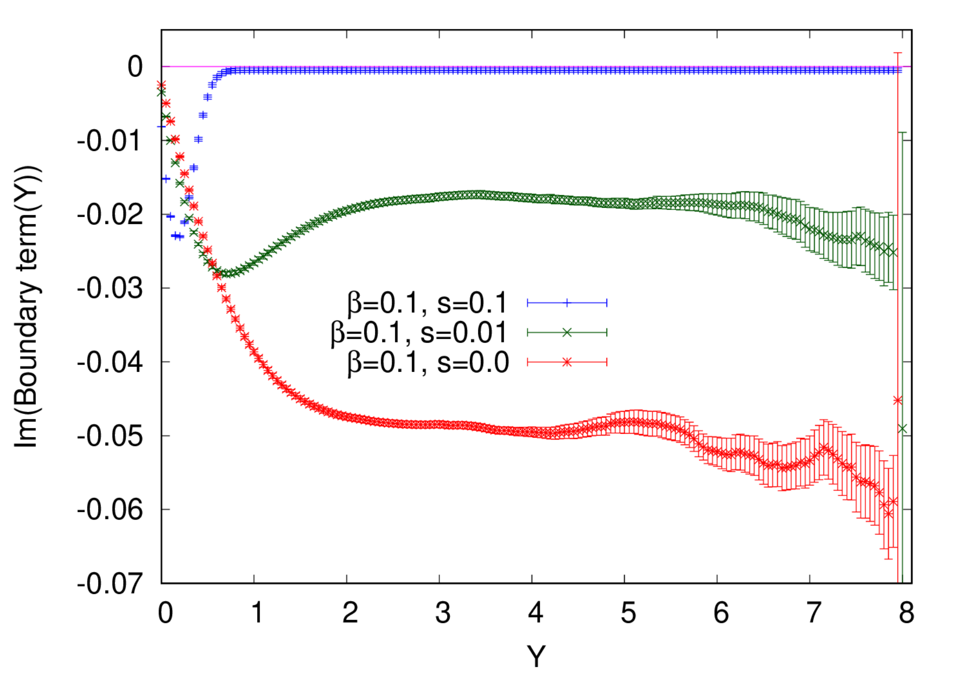
Systematic error of CLE

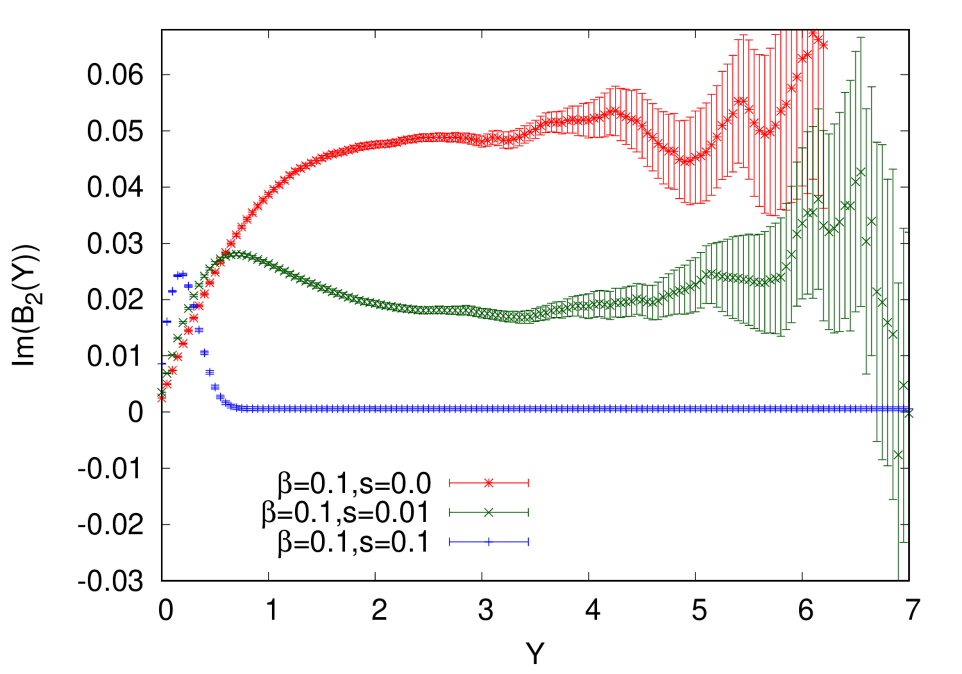
$$F(t,0)-F(t,t)=B_1^2/B_2$$

Correction using Boundary terms in U(1) toy model

$$S(x) = i\beta\cos(x) + \frac{s}{2}x^2$$

Measuring B_1, B_2 allows correction of results when CLE fails





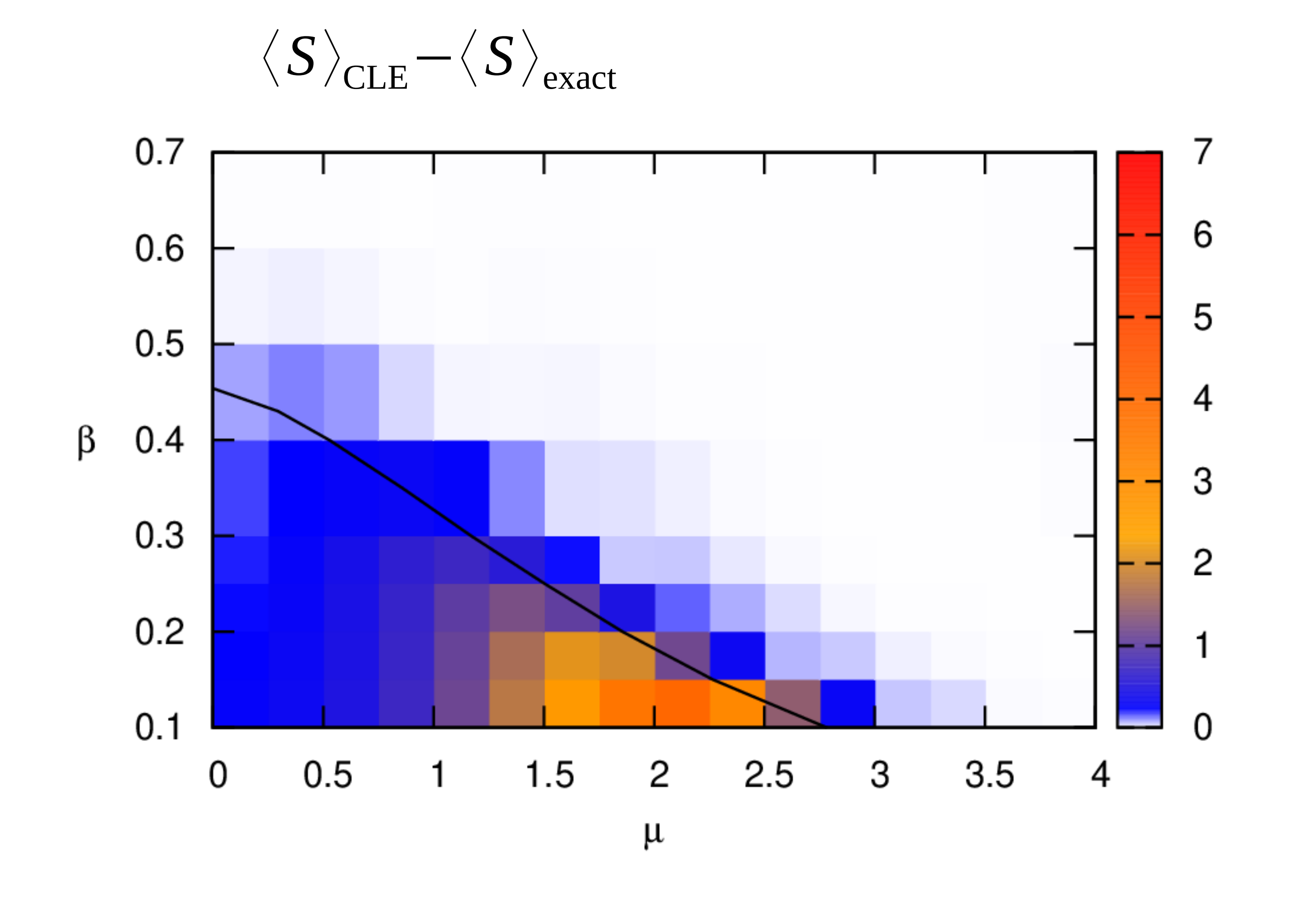
β, s	B_1	B_2	B_1^2/B_2	CL error	CL	correct	corrected CL
0.1, 0	-0.04859(45)	0.0493(11)	0.04786(79)	0.04891(45)	-0.00115(45)	-0.05006	-0.04901(62)
0.1, 0.01	-0.01795(49)	0.01801(80)	0.01789(60)	0.01689(50)	-0.03318(50)	-0.05006	-0.05106(40)
0.1, 0.1	-0.00048(30)	0.00057(35)	0.00039(28)	0.00049(31)	-0.04957(31)	-0.05006	-0.04997(6)
0.5, 0	-0.2474(11)	0.237(11)	0.258(11)	0.25818(23)	0.00003(23)	−0.25815	-0.258(11)
0.5, 0.3	−0.05309(86)	0.0552(51)	0.0507(41)	0.04183(70)	-0.19658(70)	−0.23841	−0.2473(37)

XY model in d=3

$$S = -\beta \sum_x \sum_{\nu=1}^{3} \cos\left(\phi_x - \phi_{x+\nu} - i\mu\delta_{\nu 0}\right)$$

Can be solved exactly using dual variables (worldlines)

CLE fails in one of the phases

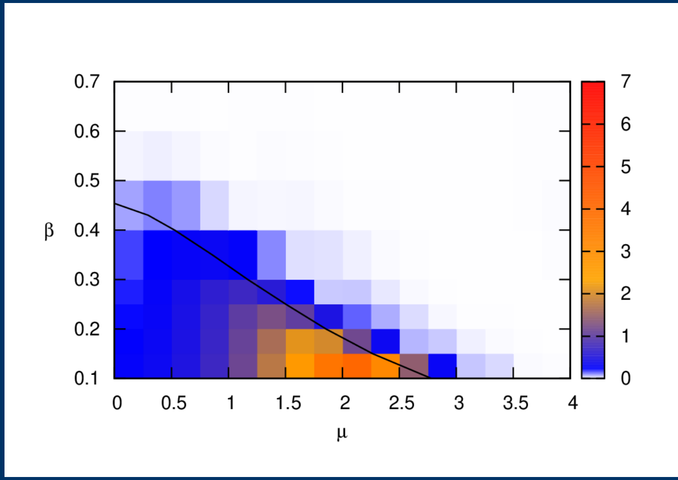


[plot from: Aarts and James (2010)]

Boundary terms in 3d XY model

CLE is actually wrong in the whole phase diag.

Boundary term is very small in one of the phases



Correction

B_2 is very noisy, hard to measure

Step in the right direction

$\mathcal{O}$	β, μ^2	B_1	B_2	B_1^2/B_2	CL error	CL	worldline	corrected CL
S	$0.2, 10^{-6}$	$0.02567(21)$	$-0.0730(47)$	$-0.00902(46)$	$-0.013029(65)$	$-0.075316(65)$	$-0.062288(17)$	$-0.06630(53)$
	$0.2, 0.1$	$0.03309(25)$	$-0.0903(79)$	$-0.01213(89)$	$-0.0169974(91)$	$-0.0792922(91)$	$-0.062295(18)$	$-0.06716(90)$
	$0.2, 0.2$	$0.03941(28)$	$-0.109(13)$	$-0.0142(17)$	$-0.0205408(80)$	$-0.0828399(80)$	$-0.062299(11)$	$-0.0686(17)$
	$0.7, 10^{-6}$	$1.440(15)10^{-4}$	$-7.33(17)10^{-4}$	$-2.834(46)10^{-5}$	$-1.23(33)10^{-4}$	$-1.482311(33)$	$-1.48219(35)$	$-1.482283(34)$
	$0.7, 0.1$	$0.004783(50)$	$-0.0082(23)$	$-0.00278(69)$	$-0.002791(31)$	$-1.526766(31)$	$-1.52398(35)$	$-1.52399(72)$
	$0.7, 0.2$	$0.006013(38)$	$-0.00873(96)$	$-0.00414(45)$	$-0.002488(29)$	$-1.568899(29)$	$-1.56641(20)$	$-1.56476(48)$
n	$0.2, 10^{-6}$	$4.8(1.6)10^{-5}$	$-0.00021(124)$	$1.3(3.7)10^{-5}$	$1.36(31)10^{-5}$	$1.36(31)10^{-5}$	$-1.2(1.1)10^{-8}$	$0.89(7.65)10^{-6}$
	$0.2, 0.1$	$-0.01147(15)$	$0.0286(32)$	$0.00460(24)$	$0.0058177(41)$	$0.0058182(41)$	$4.9(2.1)10^{-7}$	$0.00122(69)$

Pressure of the QCD Plasma at non-zero density

$$\frac{p}{T^4} = \frac{\ln Z}{V\,T^3}$$ → Derivatives of the pressure are directly measureable
→ Integrate from T=0

[Engels et. al. (1990)]

Other strategies:

Measure the Stress-momentum tensor using gradient flow

[Suzuki, Makino (2013-)]

Shifted boundary conditions
[Giusti, Pepe, Meyer (2011-)]

Non-equilibrium quench
[Caselle, Nada, Panero (2018)]

First integrate along the temperature axis, then explore $\mu > 0$

Taylor expansion [Bielefeld-Swansea (2002-)]

Simulating at imaginary μ to calculate susceptibilities
[de Forcrand, Philipsen (2002-)]

Pressure of the QCD Plasma at non-zero density

$$\Delta\left(\frac{p}{T^4}\right)=\frac{p}{T^4}(\mu=\mu_q)-\frac{p}{T^4}(\mu=0)$$

If we want to stay at $\mu=0$

$$\Delta\left(\frac{p}{T^4}\right)=\sum_{n>0,even} c_n(T)\left(\frac{\mu}{T}\right)^n$$

$$c_2=\frac{1}{2}\frac{N_T}{N_s^3}\frac{\partial^2 \ln Z}{\partial\mu^2}$$

$$c_4=\frac{1}{24}\frac{1}{N_s^3 N_T}\frac{\partial^4 \ln Z}{\partial\mu^4}$$

Measuring the coefficients of the Taylor expansion

$$\frac{\partial^2 \ln Z}{\partial\mu^2}=N_F^2\langle T_1^2\rangle+N_F\langle T_2\rangle$$

$$\frac{\partial^4 \ln Z}{\partial\mu^4}=-3\left(\langle T_2\rangle+\langle T_1^2\rangle\right)^2+3\langle T_2^2\rangle+\langle T_4\rangle$$

$$+\langle T_1^4\rangle+4\langle T_3 T_1\rangle+6\langle T_1^2 T_2\rangle$$

$$T_1/N_F=\mathrm{Tr}\left(M^{-1}\partial_\mu M\right)$$

$$T_{i+1}=\partial_\mu T_i$$

$$T_2/N_F=\mathrm{Tr}\left(M^{-1}\partial_\mu^2 M\right)-\mathrm{Tr}\left(\left(M^{-1}\partial_\mu M\right)^2\right)$$

$$T_3/N_F=\mathrm{Tr}\left(M^{-1}\partial_\mu^3 M\right)-3\,\mathrm{Tr}\left(M^{-1}\partial_\mu M\, M^{-1}\partial_\mu^2 M\right)$$
$$+2\,\mathrm{Tr}\left(\left(M^{-1}\partial_\mu M\right)^3\right)$$

$$T_4/N_F=\mathrm{Tr}\left(M^{-1}\partial_\mu^2 M\right)-4\,\mathrm{Tr}\left(M^{-1}\partial_\mu M\, M^{-1}\partial_\mu^3 M\right)$$
$$-3\,\mathrm{Tr}\left(M^{-1}\partial_\mu^2 M\, M^{-1}\partial_\mu^2 M\right)-6\,\mathrm{Tr}\left(\left(M^{-1}\partial\mu\, M\right)^4\right)$$
$$+12\,\mathrm{Tr}\left(\left(M^{-1}\partial_\mu M\right)^2 M^{-1}\partial_\mu^2 M\right)$$

Pressure of the QCD Plasma using CLE

[Sexty (2019)]

If we can simulate at $\mu>0$

$$\Delta\left(\frac{p}{T^4}\right)=\frac{p}{T^4}(\mu=\mu_q)-\frac{p}{T^4}(\mu=0)=\frac{1}{V\,T^3}\left(\ln Z(\mu)-\ln Z(0)\right)$$

$$\ln Z(\mu)-\ln Z(0)=\int_0^\mu d\mu\,\frac{\partial \ln Z(\mu)}{\partial\mu}=\int_0^\mu d\mu\,n(\mu)$$

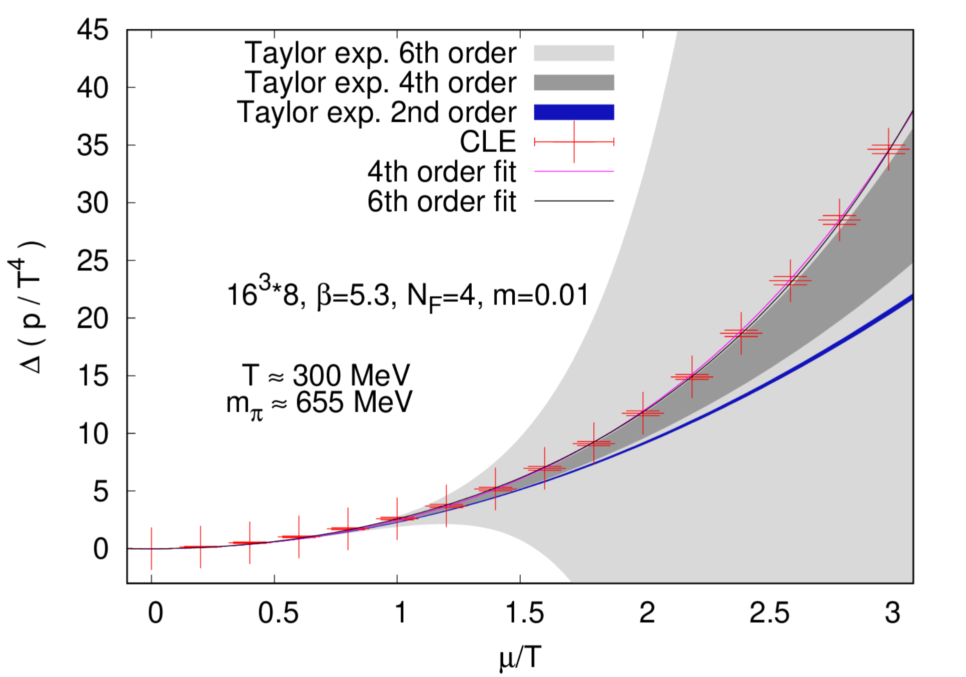
$$n(\mu)=\langle \mathrm{Tr}\left(M^{-1}(\mu)\,\partial_\mu M(\mu)\right)\rangle$$

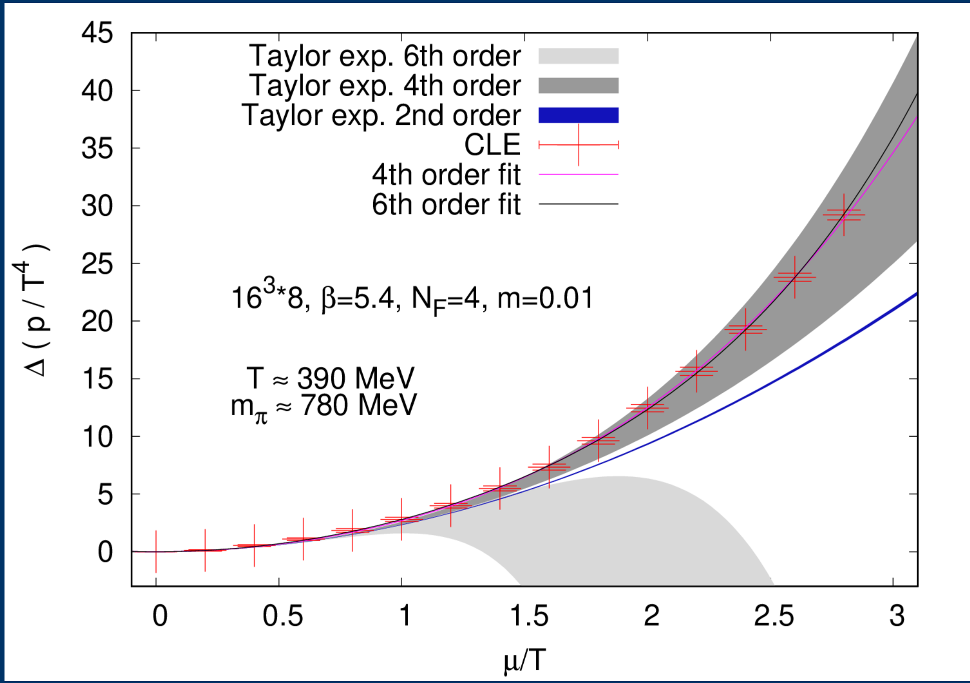
Using CLE it’s enough to measure the density – much cheaper

Pressure calculated with CLE

$\mu_B = 3\mu$

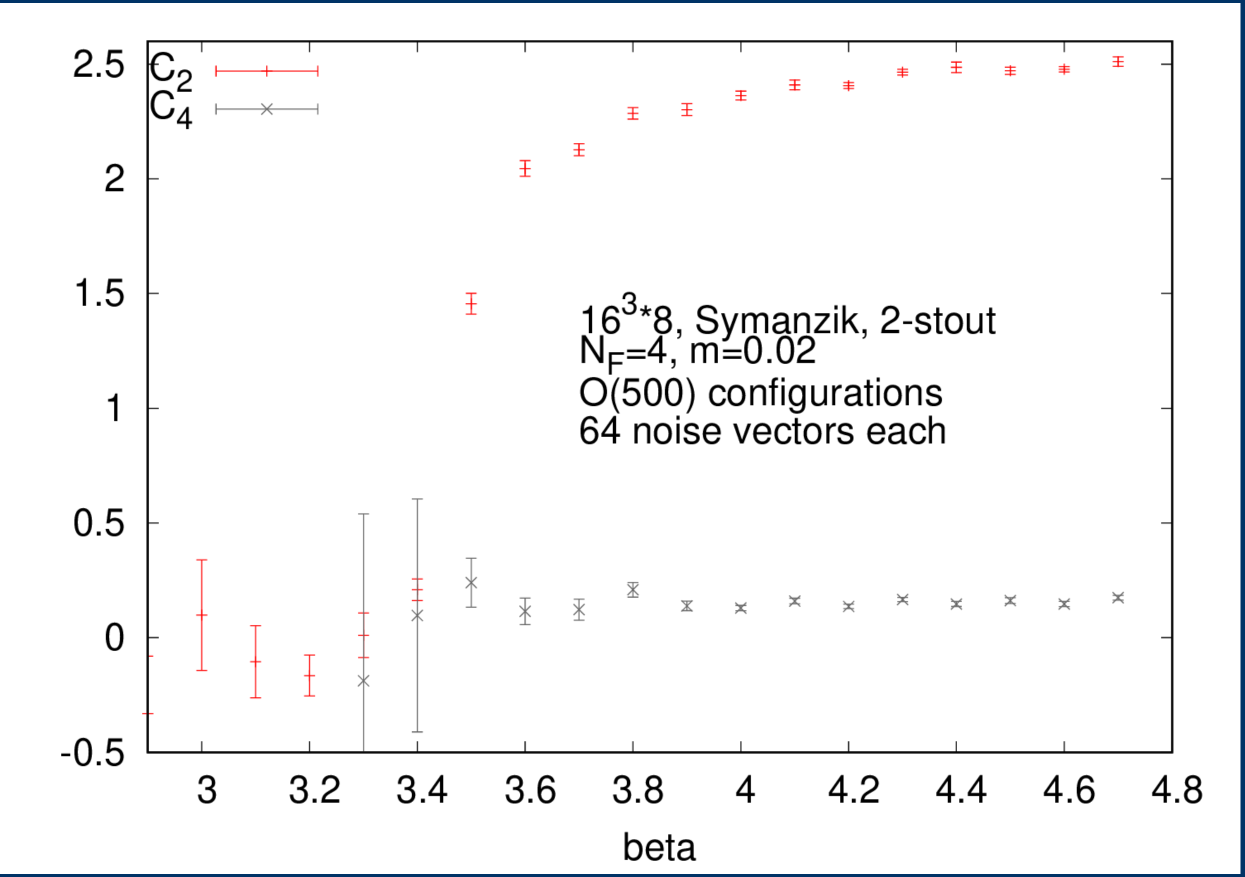
Integration performed numerically
Jackknife error estimates



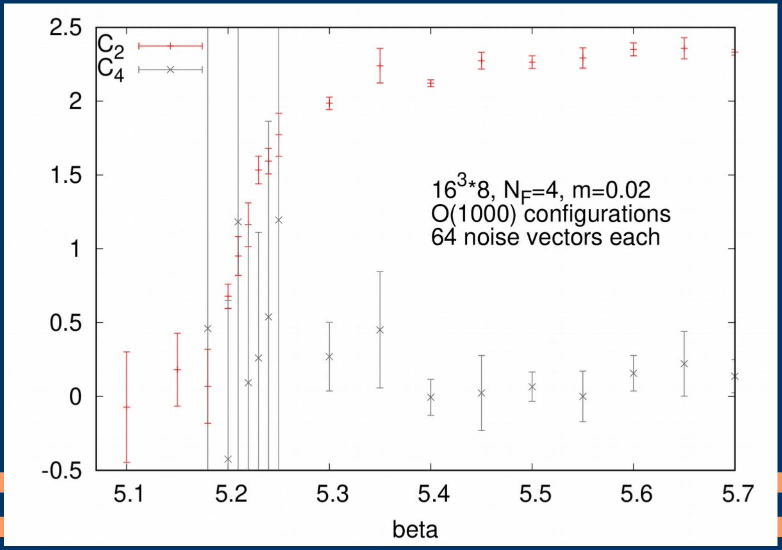


β	a (fm)	c_2 HMC	c_4 HMC	c_2 CLE	c_4 CLE
5.3	0.099 ± 0.001	1.986 ± 0.042	0.27 ± 0.23	2.117 ± 0.1	0.152 ± 0.05
5.6	0.052 ± 0.0013	2.351 ± 0.044	0.16 ± 0.12	2.168 ± 0.1	0.200 ± 0.05

Pressure with improved action



naiv action

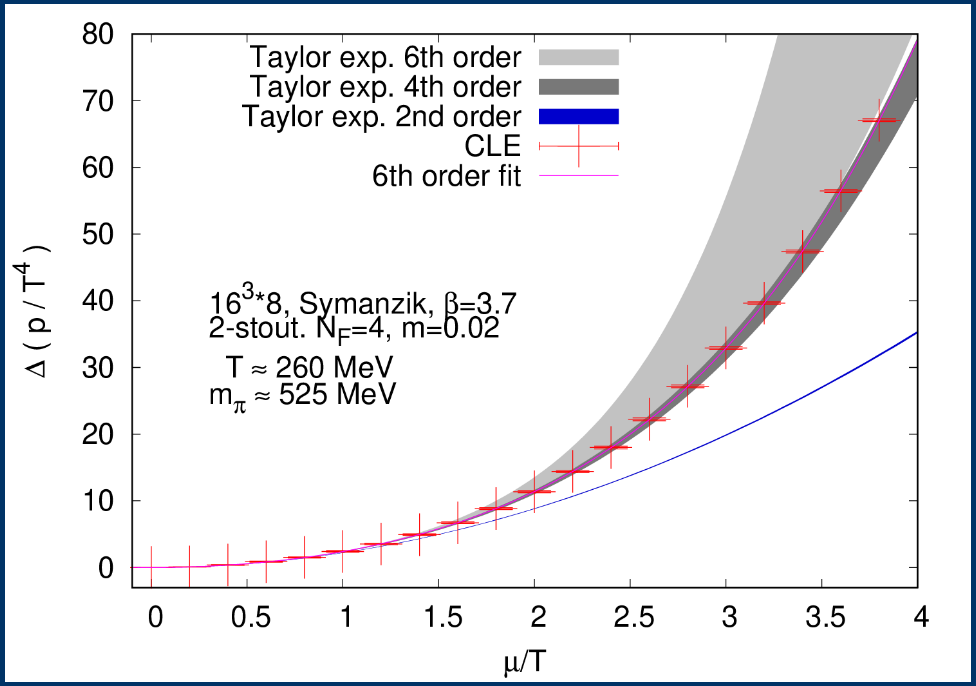


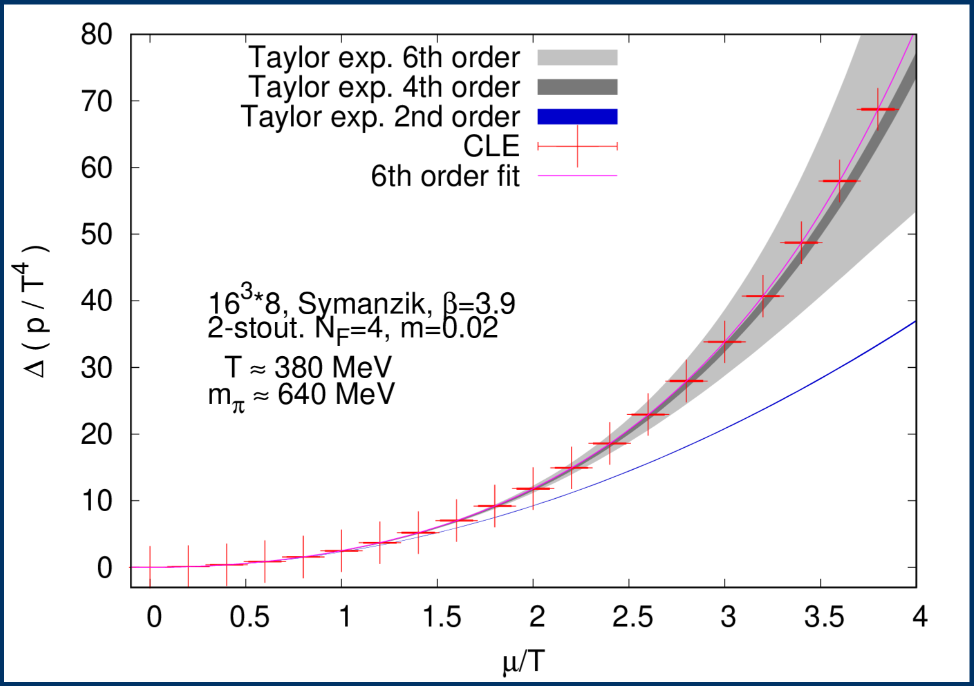
C_4 is measurable with this action
at high T (with O(500) configs.)

Pressure with improved action

[Sexty (2019)]

In deconfined phase
Symanzik gauge action
stout smeared staggered fermions





β	c_2 Taylor exp.	c_4 Taylor exp.	c_6 Taylor exp.	c_2 CLE	c_4 CLE	c_6 CLE
3.7	2.206 ± 0.009	0.156 ± 0.016	0.016 ± 0.013	2.33 ± 0.1	0.13 ± 0.02	0.002 ± 0.001
3.9	2.312 ± 0.007	0.150 ± 0.007	0.001 ± 0.005	2.36 ± 0.04	0.14 ± 0.01	0.002 ± 0.001

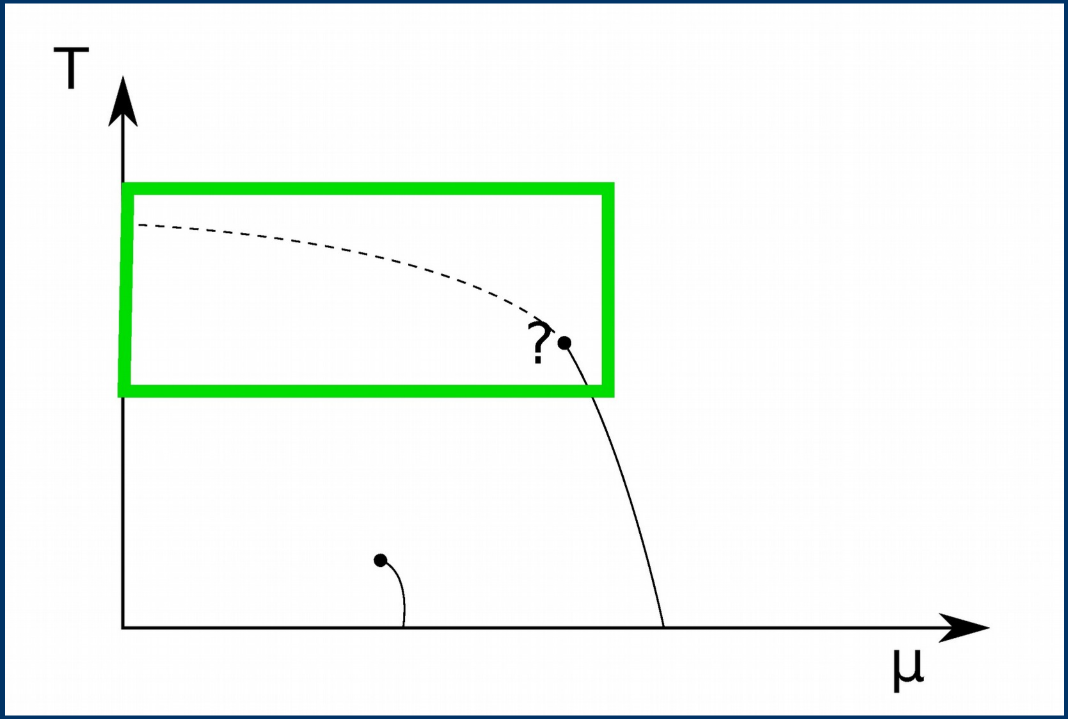
Good agreement at small μ
CLE calculation is much cheaper

further interesting quantities: Energy density, quark number susceptibility, ...

See also Felix Ziegler's talk using CLE and Dyn. Stab. (11:25 today)

Mapping out the phase transition line

[Scherzer, Sexty, Stamatescu (2020)]



Follow the phase transition line starting from $\mu = 0$

Using Wilson fermions

Fixed lattice spacing and spatial vol.
N_t scan

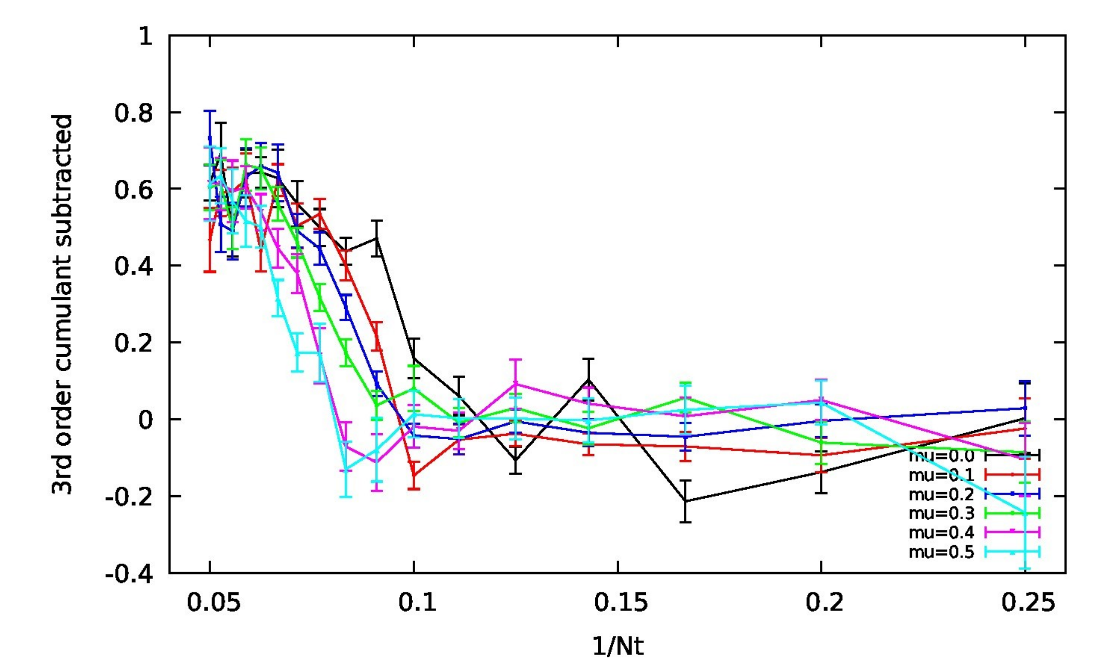
Detection of the phase transition line

Binder cumulant

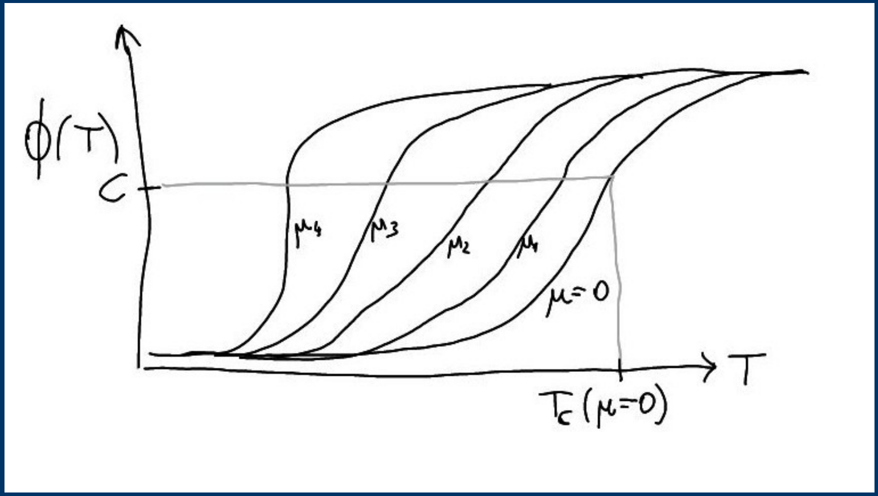
$$B_3 = \frac{\langle O^3 \rangle}{\langle O^2 \rangle^{3/2}}$$

$$O = P - \langle P \rangle \quad \text{with} \quad P = \sqrt{P_{bare} P_{bare}^{-1}}$$

no renormalization
zero crossing defines transition



Shift method



Define $T_c(\mu)$ as $\phi(T_c(\mu),\mu) = C$

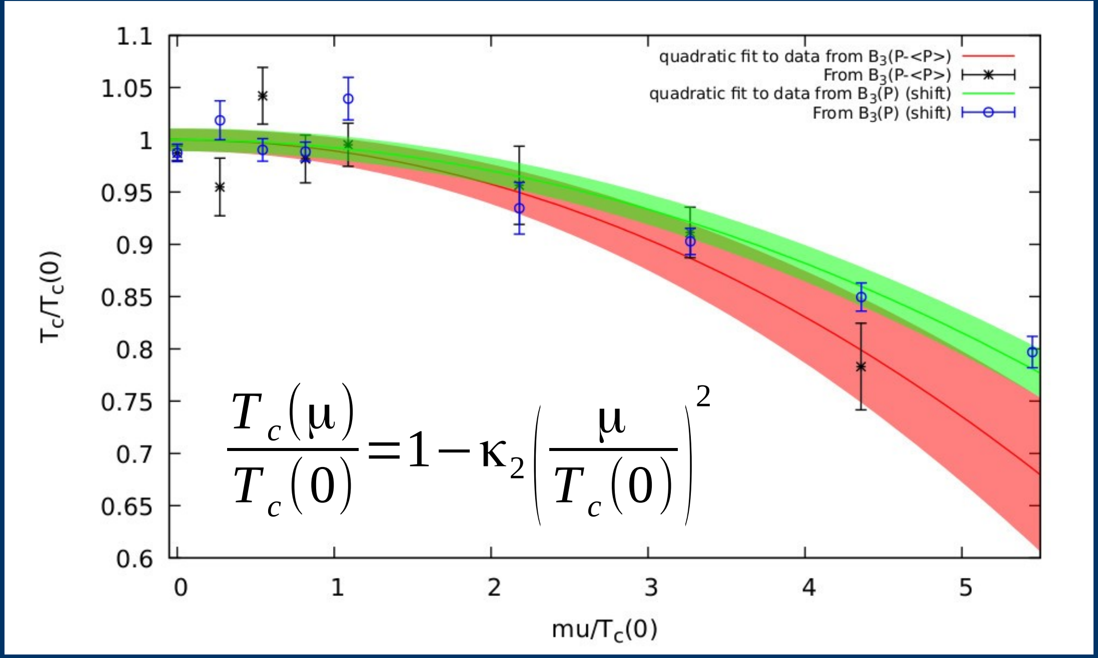
e.g. $B_{3,}$ chiral condensate,
baryon number susceptibility

Works well for small μ
Critial point at μ_4

Lattice spacing: $a = 0.065$ fm

Pion mass: $m_\pi = 1.3$ GeV

Volumes: $8^3, 12^3, 16^3$



Finite size effects large

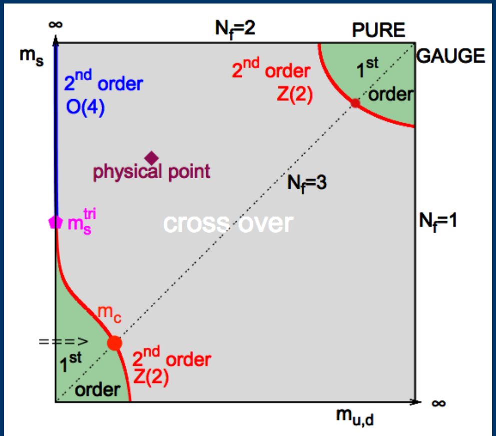
$\kappa_2 \approx 0.0012$

Consistent results

In literature
For physical pion mass
$\kappa_2 = 0.015$

Can follow the line to
quite high μ/T

Open questions
Possible for lighter quarks?
Finite size scaling?
Where is the upper right corner of Columbia plot?
Critical point nearby?



Summary

CLE has potential problems with boundary terms and poles

Monitoring of the process is required:
measuring Boundary terms

lattice models with cheap observable
Correction with higher order boundary terms

Results for the EoS Phase diag. of QCD

Extra slides

HDQCD boundary terms