Higgs mass naturalness and Gravity

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Introduction - Quantum Gravity and CC problem

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Einstein-Hilbert Action : $S_{\rm grav} = \frac{1}{16\pi G}\int \mathrm{d}^4x\,\sqrt{g}\,(-R+2\Lambda)$

Spherical background $\bar{g}_{\mu\nu} = g^{(a)}_{\mu\nu}$; $S^{(a)}_{\rm grav} = \frac{\pi\Lambda}{3G}a^4 - \frac{2\pi}{G}a^2$

One-loop correction $\delta S^{1l}_{\rm grav}$

One-loop corrections to $\frac{\Lambda}{G}$ and $\frac{1}{G}$ through coefficients of a^4 and a^2 in $\delta S^{1l}_{\rm grav}$

$S_{\rm gf} = \frac{1}{32\pi G\xi}\int \mathrm{d}^4x\sqrt{g^{(a)}}\left[\nabla_\mu\left(h^\mu_\nu - \frac{1}{2}\delta^\mu_\nu\, h^\sigma_\sigma\right)\right]^2$

$S_{\rm ghost} = \frac{1}{32\pi G}\int \mathrm{d}^4x\sqrt{g^{(a)}}\,g^{(a)\,\mu\nu}\,v^*_\mu\left(-\nabla_\rho\nabla^\rho - \frac{3}{a^2}\right)v_\nu$

Path integral measure - Fradkin-Vilkovisky

$$\left[\mathcal{D}\mu\right] \equiv \prod_x \left[g^{(a)\,00}(x) \left(g^{(a)}(x)\right)^{-1} \left(\prod_{\alpha\leq\beta} \mathrm{d}h_{\alpha\beta}(x)\right)\left(\prod_\rho \mathrm{d}v^*_\rho(x)\right)\left(\prod_\sigma \mathrm{d}v_\sigma(x)\right)\right]$$

Now : $g^{(a)}_{\mu\nu} = a^2\,\widetilde{g}_{\mu\nu}$ $\widetilde{g}_{\mu\nu}$ metric of a sphere of unitary radius, $a = 1$

Redefinition : $\widehat{h}_{\mu\nu} = (32\pi G)^{-1/2}\, a^{-1} h_{\mu\nu}$, $v_\mu \to (32\pi G)^{\frac{1}{2}}\, v_\mu$

$$\left[\mathcal{D}\mu\right] \sim \prod_x \left[\left(\prod_{\alpha\leq\beta} \mathrm{d}\widehat{h}_{\alpha\beta}(x)\right)\left(\prod_\rho \mathrm{d}v^*_\rho(x)\right)\left(\prod_\sigma \mathrm{d}v_\sigma(x)\right)\right] \quad \text{No } a!$$

In terms of $\widehat{h}_{\mu\nu}$ and v_μ $\delta S^{(2)} = S_2 + S_{\mathrm{gf}} + S_{\mathrm{ghost}}$ contains

only dimensionless operators

... All this boils down to ...

Dimensionless Laplace-Beltrami operator $-\widetilde{\Box}^{(s)} \equiv -a^2\,\Box_a^{(s)}$

$-\Box_a^{(s)}$ Laplace-Beltrami for sphere of radius a; s spins: $s = 0, 1, 2$

Dimensionless eigenvalues $\lambda_n^{(s)}$ and corresponding degeneracies $D_n^{(s)}$

$$\lambda_n^{(s)} = n^2 + 3n - s \quad ; \quad D_n^{(s)} = \tfrac{2s+1}{3}\left(n+\tfrac{3}{2}\right)^3 - \tfrac{(2s+1)^3}{12}\left(n+\tfrac{3}{2}\right) \quad ; \quad n = s, s+1, \ldots$$

Expanding $\widehat{h}_{\mu\nu}$, v_ρ^* and v_σ for $\delta S^{1l}_{\rm grav}$

$$\delta S^{1l}_{\rm grav} = -\frac{1}{2}\log\frac{\det_1[-\widetilde{\Box}^{(1)} - 3]\,\det_2[-\widetilde{\Box}^{(0)} - 6]}{\det_0[-\widetilde{\Box}^{(2)} - 2a^2\Lambda + 8]\,\det_2[-\widetilde{\Box}^{(0)} - 2a^2\Lambda]} + \ldots$$

Laplacians of a **unit sphere** although we have a sphere of **radius** a

Now we have to calculate the $\det[-\widetilde{\Box} + \alpha]$

Calculation of det with two different strategies

Eigenvalues ; Proper-time

Fluctuation determinants - Eigenvalues

$$\delta S^{1l}_{\text{grav}} = \frac{1}{2}\sum_{n=2}^{N-2}\left[D_n^{(2)}\log\left(\lambda_n^{(2)} - 2a^2\Lambda + 8\right) + D_n^{(0)}\log\left(\lambda_n^{(0)} - 2a^2\Lambda\right) - D_n^{(1)}\log\left(\lambda_n^{(1)} - 3\right) - D_n^{(0)}\log\left(\lambda_n^{(0)} - 6\right)\right] + \dots$$

N: numerical UV cutoff on the number of eigenvalues

De Sitter solution for the classical action $a_{\text{dS}} = \sqrt{\frac{3}{\Lambda_{\text{cc}}}}$

a_{dS} size of the universe $\Longrightarrow$ $\Lambda \sim M_P = \frac{N}{a_{\text{dS}}} = N\sqrt{\frac{\Lambda_{\text{cc}}}{3}}$

Expanding for $N \gg 1$

$$(\delta S^{1l}_{\text{grav}} = -\left(\Lambda_{\text{cc}}^2\log N^2\right)a^4 + \Lambda_{\text{cc}}\left(-N^2 + 8\log N^2\right)a^2 + \frac{N^4}{24}\left(-1 + 2\log N^2\right) + \frac{N^2}{36}\left(203 - 75\log N^2\right) - \frac{779}{90}\log N^2)$$

$$\delta S^{1l}_{\text{grav}} = -\left(\Lambda_{\text{cc}}^2\log\frac{3\Lambda^2}{\Lambda_{\text{cc}}}\right)a^4 + \left(-3\Lambda^2 + 8\Lambda_{\text{cc}}\log\frac{3\Lambda^2}{\Lambda_{\text{cc}}}\right)a^2 + \dots$$

Fluctuation determinants - Proper time

$(-\widetilde{\Box}^{(s)} - \alpha)$ dimensionless $\Longrightarrow$ $\det_i$ regularized with a dimensionless proper-time τ (lower cut: $N_{\rm pt} \gg 1$) with kernel kernel $\mathrm{K}_i^{(s)}(\tau)$

$$\det{}_i(-\Box_{a=1}^{(s)} - \alpha) = e^{-\int_{1/N_{\rm pt}^2}^{+\infty} \frac{\mathrm{d}\tau}{\tau}\, \mathrm{K}_i^{(s)}(\tau)} \quad ; \quad \mathrm{K}_i^{(s)}(\tau) = \sum_{n=s+i}^{+\infty} D_n^{(s)}\, e^{-\tau\left(\lambda_n^{(s)} - \alpha\right)}$$

After integration over τ, sum over n with EML sum formula. Expanding for $N \gg 1$

$$\delta S_{\rm grav}^{1l} = -\left(\Lambda_{\rm cc}^2 \log N^2\right) a^4 + \Lambda_{\rm cc}\left(-N^2 + 8\log N^2\right) a^2 - \frac{N^4}{12} + \frac{17}{3}N^2 - \frac{1859}{90}\log N^2 + \dots$$

$$\text{With} \quad \Lambda \equiv \frac{N}{a_{\rm dS}} = \sqrt{\frac{\Lambda_{\rm cc}}{3}}\, N \qquad \Longrightarrow$$

$$\delta S_{\rm grav}^{1l} = -\left(\Lambda_{\rm cc}^2 \log \frac{3\Lambda^2}{\Lambda_{\rm cc}}\right) a^4 + \left(-3\Lambda^2 + 8\Lambda_{\rm cc}\log\frac{3\Lambda^2}{\Lambda_{\rm cc}}\right) a^2 + \dots$$

The two methods give the same result

Coefficients of a^4 and a^2 : give one-loop corrections to $\frac{\Lambda_{cc}}{G}$ and $\frac{1}{G}$

$$\frac{\Lambda_{cc}^{1l}}{G^{1l}} = \frac{\Lambda_{cc}}{G}\left(1 - \frac{3G\Lambda_{cc}}{\pi}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right) \quad ; \quad \frac{1}{G^{1l}} = \frac{1}{G}\left[1 + \frac{G}{2\pi}\left(3\Lambda^2 - 8\Lambda_{cc}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

Unexpected result: *only* logarithmic corrections to $\rho_{vac} = \frac{\Lambda_{cc}}{8\pi G}$

Taking for G the natural value $G \sim M_P^{-2}$ we see that quantum corrections do not spoil the naturalness of this relation

No naturalness problem with the renormalization of the Newton constant

$$G \sim G^{1l} \sim M_P^{-2}$$

Usual result: $\rho_{vac} \sim M_P^4 \quad \Longrightarrow \quad$ bare value of $\rho_{vac} \sim M_P^4$ with coefficient to be enormously fine-tuned to cancel the one-loop M_P^4 correction

Our result: loop corrections $\rightarrow$ only mild (log) correction to ρ_{vac}

No Naturalness Problem in pure gravity

No need for Bare $\Lambda_{cc} \sim M_P^2$

We may naturally have $\Lambda_{cc} \ll M_P^2$

$$\Lambda_{cc}^{1l} \sim \Lambda_{cc}$$

... Unexpected Result ...

Why usually powerlike divergences?

Connect for a moment N and Λ through $\Lambda = \frac{N}{a}$ (rather than through $\Lambda = \frac{N}{a_{\rm dS}}$)

Corresponds to identify Λ with the maximal eigenvalue $\widetilde{\lambda}^{(s)}_{\rm max}$ of $-\Box^{(s)}_a$:

$$\left[\delta S^{1/}_{\rm grav} = -\left(\Lambda^2_{\rm cc}\log N^2\right)a^4 + \Lambda_{\rm cc}\left(-N^2 + 8\log N^2\right)a^2 - \frac{N^4}{12} + \frac{17}{3}N^2 - \frac{1859}{90}\log N^2 + \ldots\right]$$

$$\delta S^{1/}_{\rm grav} = -\left[\frac{\Lambda^4}{12} + \Lambda_{\rm cc}\Lambda^2 + \Lambda^2_{\rm cc}\log\left(\Lambda^2 a^2\right)\right]a^4 + \left[\frac{17}{3}\Lambda^2 + 8\Lambda_{\rm cc}\log\left(\Lambda^2 a^2\right)\right]a^2 - \frac{1859}{90}\log\left(\Lambda^2 a^2\right)$$

spurious dependence on $g^{(a)}_{\mu\nu}$ in $\delta S^{1/}_{\rm grav}$

Scalar theory non minimally coupled to gravity

$$S = \frac{1}{16\pi G}\int \mathrm{d}^4x\,\sqrt{g}\,(-R+2\Lambda_{\rm cc}) + \int \mathrm{d}^4x\,\sqrt{g}\left[\frac{1}{2}\,g^{\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + \frac{\xi}{2}R\phi^2 + V(\phi)\right]$$

Taking the metric $g^{(a)}_{\mu\nu}$ of a sphere of radius a

$$S^{(a)}[\phi] = \frac{\pi\Lambda_{\rm cc}}{3G}\,a^4 - \frac{2\pi}{G}\,a^2 + \int \mathrm{d}^4x\,\sqrt{g^{(a)}}\left[\frac{1}{2}\,g^{(a)\,\mu\nu}\,\partial_\mu\phi\,\partial_\nu\phi + \frac{\xi}{2}\,\frac{12}{a^2}\,\phi^2 + V(\phi)\right]$$

One-loop correction δS^{1l} $\qquad$ ($\phi = \Phi + \eta$; expand S up to η^2)

$$e^{-\delta S^{1l}} = \int \Big[\mathcal{D}\mu(\eta)\Big]\; e^{-S_2}$$

where $\Big[\mathcal{D}\mu\Big]$ Fradkin-Vilkovisky measure (arXiv:2506.05100)

$$S_2 \equiv \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{g^{(a)}}\;\eta\Big[-\Box_a + \frac{12\,\xi}{a^2} + V''(\Phi)\Big]\eta \quad ; \quad \Big[\mathcal{D}\mu(\eta)\Big] = \prod_x \left[\left(g^{(a)\,00}(x)\right)^{\frac{1}{2}}\left(g^{(a)}(x)\right)^{\frac{1}{4}}\,\mathrm{d}\eta(x)\right]$$

Dimensionless operators

$-\Box_a$ spin-0 Laplace-Beltrami operator for a sphere of radius a

$\left(g^{(a)\,00}(x)\right)^{1/2}\left(g^{(a)}(x)\right)^{1/4}$ comes from the integration over conjugate momenta (FV)

$g^{(a)}_{\mu\nu} = a^2\,\widetilde{g}_{\mu\nu}$ where the elements of $\widetilde{g}_{\mu\nu}$ are dimensionless and a-independent

$$\left(g^{(a)\,00}(x)\right)^{1/2}\left(g^{(a)}(x)\right)^{1/4} = a\left(\widetilde{g}^{\,00}(x)\right)^{1/2}\left(\widetilde{g}(x)\right)^{1/4}$$

Convenient redefinition: $\widehat{\eta} \equiv a\eta$

$$S_2 = \frac{1}{2}\int \mathrm{d}^4x\,\sqrt{\widetilde{g}}\,\widehat{\eta}\Big[-\widetilde{\Box} + 12\,\xi + a^2\,V''(\Phi)\Big]\widehat{\eta}$$

$-\widetilde{\Box}$ dimensionless spin-0 Laplace-Beltrami operator defined as $-\widetilde{\Box} \equiv -a^2\,\Box_a$

Since $\qquad \mathrm{d}\eta(x) = a^{-1}\mathrm{d}\widehat{\eta}\,(x)$

$\Big[\mathcal{D}\mu(\eta)\Big]$ can be written as $\qquad \Big[\mathcal{D}\mu(\eta)\Big] = \mathcal{A}\prod_x\Big[\mathrm{d}\widehat{\eta}\,(x)\Big]$

where a-independent terms such as $\prod_x\left(\widetilde{g}^{\,00}(x)\right)^{1/2}\left(\widetilde{g}(x)\right)^{1/4}$ are included in the factor $\mathcal{A}$

Dimensionless operators

$-\widetilde{\Box}$ dimensionless Laplacian $\Longrightarrow$ expand $\widehat{\eta}(x)$ with eigenfunctions $\phi_n^{(i)}(x)$ of $-\widetilde{\Box}$

$$\widehat{\eta} = \sum_{n,i} c_n^{(i)} \phi_n^{(i)}$$

Eigenvalues λ_n of $-\widetilde{\Box}$ and degeneracies D_n

$$\lambda_n = n^2 + 3n \qquad ; \qquad D_n = \frac{1}{3}\left(n+\frac{3}{2}\right)^3 - \frac{1}{12}\left(n+\frac{3}{2}\right)$$

Quantum correction to the action

$$e^{-\delta S} = \int \left[\mathcal{D}\mu(\eta)\right] e^{-S_2} = \widetilde{\mathcal{A}} \int \prod_{n,i} \mathrm{d}c_n^{(i)} e^{-\frac{1}{2}\sum_{n,i}\left[c_n^{(i)}\right]^2\left(\lambda_n + 12\xi + a^2 V''(\Phi)\right)}$$

$$\Longrightarrow \qquad \Gamma = S^{(a)}[\Phi] + \frac{1}{2}\log\left[\det\left(-\widetilde{\Box} + 12\xi + a^2 V''(\Phi)\right)\right] + \mathcal{C}$$

Had we missed $\left(g^{(a)\,00}(x)\right)^{1/2}\left(g^{(a)}(x)\right)^{1/4}$ $\Longrightarrow$ a-dependence of the determinant altered $\Longrightarrow$ determinant dimensionful $\Longrightarrow$ arbitrary scale μ needed to make argument of Log dimensionless

Calculation of $\log \det \left(-\widetilde{\Box} + 12\xi + a^2 V''(\Phi)\right)$

1 - Product of eigenvalues

Finite number N ($\gg 1$) of eigenvalues λ_n. Largest eigenvalue: $\lambda_N \sim N^2$ ($N-2$ for convenience)

$$\delta S^{1l} = \frac{1}{2}\sum_{n=0}^{N-2}\left[D_n \log\left(\lambda_n + 12\xi + a^2 V''(\Phi)\right)\right] + \mathcal{C}$$

N = numerical UV cutoff

Expanding for $N \gg 1$

$$\delta S^{1l} = \frac{8\pi^2}{3}a^4\left[-\frac{\left(V''(\Phi)\right)^2}{64\pi^2}\log N^2 + \frac{12}{a^2}\frac{V''(\Phi)}{384\pi^2}\left(N^2 + 2\left(1-6\xi\right)\log N^2\right)\right]$$
$$+ \frac{N^4}{48}\left(-1 + 2\log N^2\right) - \frac{N^2}{72}\left(13 - 72\xi + 3\log N^2\right) + \left(2\xi\left(1-3\xi\right) - \frac{29}{180}\right)\log N^2 + \mathcal{C}$$

Consider Φ^4-theory : $V(\Phi) = \frac{m^2}{2}\Phi^2 + \frac{\lambda}{4!}\Phi^4$

One-loop effective action $\Gamma^{1l} = S^{(a)}[\Phi] + \delta S^{1l}$

$$\Gamma^{1l} = \frac{8\pi^2}{3}a^4\Big\{ -\frac{1}{16\pi G}\left[1 - \frac{G\,m^2}{24\pi}\left(N^2 + 2(1-6\xi)\log N^2\right)\right]\frac{12}{a^2}$$
$$+ \frac{\Lambda_{cc}}{8\pi G}\left[1 - \frac{G\,m^4}{8\pi\Lambda_{cc}}\log N^2\right] + \frac{\xi}{2}\left[1 + \frac{\lambda}{384\pi^2\,\xi}\left(N^2 + 2\,(1-6\xi)\log N^2\right)\right]\frac{12}{a^2}\Phi^2$$
$$+ \frac{m^2}{2}\left[1 - \frac{\lambda}{32\pi^2}\log N^2\right]\Phi^2 + \frac{\lambda}{4!}\left[1 - \frac{3\lambda}{32\pi^2}\log N^2\right]\Phi^4\Big\}$$
$$+ \frac{N^4}{48}\left(-1 + 2\log N^2\right) - \frac{N^2}{72}\left(13 - 72\xi + 3\log N^2\right) + \left(2\xi\,(1-3\xi) - \frac{29}{180}\right)\log N^2$$

2 - Proper time

Since $(-\widetilde{\Box} + 12\xi + a^2 V''(\Phi))$ dimensionless $\Longrightarrow$ regularize the determinant with dimensionless proper-time τ (lower cut $N \gg 1$). (λ_n and D_n eigenvalues and degeneracies)

$$\det(-\widetilde{\Box} + 12\xi + a^2 V''(\Phi)) = e^{-\int_{1/N^2}^{+\infty} \frac{d\tau}{\tau} \mathrm{K}(\tau)} \quad ; \quad \mathrm{K}(\tau) = \sum_{n=0}^{+\infty} D_n \, e^{-\tau \left(\lambda_n + 12\xi + a^2 V''(\Phi)\right)}$$

Perform first integration over τ and then sum over n with EML

$$\Big[\sum_{n=n_i}^{n_f} f(n) = \int_{n_i}^{n_f} \mathrm{d}x \, f(x) + \frac{f(n_f)+f(n_i)}{2} + \sum_{k=1}^{p} \frac{B_{2k}}{(2k)!} \left(f^{(2k-1)}(n_f) - f^{(2k-1)}(n_i) \right) + R_{2p}$$

p is an integer ; B_m are Bernoulli numbers ; R_{2p} is the rest

$$R_{2p} = \sum_{k=p+1}^{\infty} \frac{B_{2k}}{(2k)!} \left(f^{(2k-1)}(n_f) - f^{(2k-1)}(n_i) \right) = \frac{(-1)^{2p+1}}{(2p)!} \int_{n_i}^{n_f} \mathrm{d}x \, f^{(2p)}(x) B_{2p}(x - [x])$$

$B_n(x)$ Bernoulli polynomials ; $[x]$ integer part of x ; $f^{(i)}$ i-th derivative of f $\Big]$

Expanding the resulting expression of δS^{1l} for $N \gg 1$

$$\delta S^{1/} = \frac{8\pi^2}{3} a^4 \Big[- \frac{\big(V''(\Phi)\big)^2}{64\pi^2} \log N^2 + \frac{12}{a^2} \frac{V''(\Phi)}{384\pi^2} \Big(N^2 + 2\,(1-6\xi) \log N^2 \Big) \Big]$$
$$- \frac{N^4}{24} - \frac{1-6\xi}{6} N^2 + \left(2\xi\,(1-3\xi) - \frac{29}{180} \right) \log N^2 + \mathcal{C}$$

Taking $V(\Phi) = \frac{m^2}{2}\Phi^2 + \frac{\lambda}{4!}\Phi^4 \quad \Longrightarrow \quad \Gamma^{1/}\ (= S^{(a)} + \delta S^{1/})$ is

$$\Gamma^{1/} = \frac{8\pi^2}{3} a^4 \Big\{ - \frac{1}{16\pi G} \Big[1 - \frac{G\,m^2}{24\pi} \Big(N^2 + 2(1-6\xi) \log N^2 \Big) \Big] \frac{12}{a^2} + \frac{\Lambda_{\rm cc}}{8\pi G} \Big[1 - \frac{G\,m^4}{8\pi\Lambda_{\rm cc}} \log N^2 \Big]$$
$$+ \frac{\xi}{2} \left[1 + \frac{\lambda}{384\pi^2\,\xi} \Big(N^2 + 2\,(1-6\xi) \log N^2 \Big) \right] \frac{12}{a^2} \Phi^2$$
$$+ \frac{m^2}{2} \Big[1 - \frac{\lambda}{32\pi^2} \log N^2 \Big] \Phi^2 + \frac{\lambda}{4!} \left[1 - \frac{3\lambda}{32\pi^2} \log N^2 \right] \Phi^4 \Big\}$$
$$- \frac{N^4}{24} - \frac{1-6\xi}{6} N^2 + \left(2\xi\,(1-3\xi) - \frac{29}{180} \right) \log N^2$$

Apart from irrelevant a- and Φ-independent terms, the two results for $\Gamma^{1/}$ coincide

One-loop corrections to $\frac{\Lambda_{cc}}{G}$, $\frac{1}{G}$, ξ, m^2 and λ

Comparing Γ^{1l} with $S^{(a)}[\Phi]$ we immediately read the corrections to $\frac{\Lambda_{cc}}{G}$, $\frac{1}{G}$, ξ, m^2 and λ

$$\frac{1}{G^{1l}} = \frac{1}{G}\left[1 - \frac{G\,m^2}{24\pi}\left(N^2 + 2(1-6\xi)\log N^2\right)\right]$$

$$\frac{\Lambda_{cc}^{1l}}{G^{1l}} = \frac{\Lambda_{cc}}{G}\left[1 - \frac{G\,m^4}{8\pi\Lambda_{cc}}\log N^2\right]$$

$$m_{1l}^2 = m^2\left[1 - \frac{\lambda}{32\pi^2}\log N^2\right]$$

$$\lambda^{1l} = \lambda\left[1 - \frac{3\lambda}{32\pi^2}\log N^2\right]$$

$$\xi^{1l} = \xi\left[1 + \frac{\lambda}{384\pi^2\,\xi}\left(N^2 + 2\,(1-6\xi)\log N^2\right)\right]$$

Since Λ_{cc}^{1l} and G^{1l} renormalized cosmological and Newton constant $\Longrightarrow$ must be positive

$$\Lambda_{cc}^{1l} > 0 \text{ and } G^{1l} > 0$$

For this to be the case, it must be $\quad \Lambda_{cc} > 0$ and $G > 0$

From $N \Longrightarrow \Lambda$; Effective Action

Connect N with Λ ($\sim M_P$ or string scale M_s) $\qquad \Lambda = \frac{N}{a_{\rm dS}} = N\sqrt{\frac{\Lambda_{\rm cc}}{3}} \qquad \Longrightarrow$

$$
\begin{aligned}
\Gamma^{1/} = \frac{8\pi^2}{3}a^4\Big\{ & -\frac{1}{16\pi G}\Big[1-\frac{G\,m^2}{24\pi}\left(\frac{3\Lambda^2}{\Lambda_{\rm cc}}+2(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{\rm cc}}\right)\Big]\frac{12}{a^2} \\
& +\frac{\Lambda_{\rm cc}}{8\pi G}\Big[1-\frac{G\,m^4}{8\pi\Lambda_{\rm cc}}\log\frac{3\Lambda^2}{\Lambda_{\rm cc}}\Big] \\
& +\frac{\xi}{2}\left[1+\frac{\lambda}{384\pi^2\,\xi}\left(\frac{3\Lambda^2}{\Lambda_{\rm cc}}+2\,(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{\rm cc}}\right)\right]\frac{12}{a^2}\Phi^2 \\
& +\frac{m^2}{2}\left[1-\frac{\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{\rm cc}}\right]\Phi^2+\frac{\lambda}{4!}\left[1-\frac{3\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{\rm cc}}\right]\Phi^4\Big\} \\
& -\frac{3\Lambda^4}{8\Lambda_{\rm cc}^2}-\frac{1-6\xi}{2}\,\frac{\Lambda^2}{\Lambda_{\rm cc}}+\left(2\xi\,(1-3\xi)-\frac{29}{180}\right)\log\frac{3\Lambda^2}{\Lambda_{\rm cc}}
\end{aligned}
$$

... Then read how the couplings get modified ...

$$\frac{\Lambda_{cc}^{1/}}{8\pi G^{1/}} = \frac{\Lambda_{cc}}{8\pi G}\left[1 - \frac{G\,m^4}{8\pi\Lambda_{cc}}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \frac{1}{G^{1/}} = \frac{1}{G}\left[1 - \frac{G\,m^2}{24\pi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

$$m_{1/}^2 = m^2\left[1 - \frac{\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \lambda^{1/} = \lambda\left[1 - \frac{3\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right]$$

$$\xi^{1/} = \xi\left[1 + \frac{\lambda}{384\pi^2\,\xi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2\,(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

Quartic self-coupling - only mild logarithmic correction (coincides with flat space-time result)

Scalar mass $\delta m^2 \sim \log\Lambda$ rather than $\sim \Lambda^2$: *no quadratic divergence*

Usual result $\delta m^2 \sim \Lambda^2$: enormus fine-tuning

This result $m_{1/}^2 \sim m^2 \Rightarrow m^2(\Lambda)$ may well be $m^2(\Lambda) \ll \Lambda^2$

$\Longrightarrow$ no naturalness problem for scalar mass

If SM embedded in SUSY, GUT, ..., fields of heavy mass M coupled to Higgs $\Rightarrow \delta m^2 \propto M^2$.
Physical mechanism that disposes of these contributions and makes $m_H^2 \sim (125\,\text{GeV})^2$ to be found

Still important result. Here absence of divergences not due to "technical tricks" (dimensional, zeta function regularization, ...) nor to physical cancellations (SUSY, ...)

Physical result within Wilsonian framework, where **physical cutoff built-in**

... still reading the couplings ...

$$\frac{\Lambda_{cc}^{1l}}{G^{1l}} = \frac{\Lambda_{cc}}{G}\left[1 - \frac{G\,m^4}{8\pi\Lambda_{cc}}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \frac{1}{G^{1l}} = \frac{1}{G}\left[1 - \frac{G\,m^2}{24\pi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

$$m_{1l}^2 = m^2\left[1 - \frac{\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \lambda^{1l} = \lambda\left[1 - \frac{3\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right]$$

$$\xi^{1l} = \xi\left[1 + \frac{\lambda}{384\pi^2\,\xi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2\,(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

Non-minimal coupling ξ : besides a mild logarithmic correction (present in previous literature)

ξ receives a quadratically divergent contribution

UV sensitivity of m^2 and ξ inverted : $m^2 \sim \log\Lambda \quad ; \quad \xi \sim \Lambda^2$.

Phenomenological remarks

Higgs boson mass confronted with measured $m_H^2 \sim (125\,\text{GeV})^2 \Longrightarrow$ quadratic sensitivity to Λ gives rise to severe naturalness problem. Much less is known on the experimental value of ξ. Correction $\delta\xi \sim \Lambda^2$ can be easily handled, and does not seem to be worrisome

... still reading the couplings ...

$$\frac{\Lambda_{cc}^{1/}}{G^{1/}} = \frac{\Lambda_{cc}}{G}\left[1 - \frac{G\,m^4}{8\pi\Lambda_{cc}}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \frac{1}{G^{1/}} = \frac{1}{G}\left[1 - \frac{G\,m^2}{24\pi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

$$m_{1/}^2 = m^2\left[1 - \frac{\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right] \quad ; \quad \lambda^{1/} = \lambda\left[1 - \frac{3\lambda}{32\pi^2}\log\frac{3\Lambda^2}{\Lambda_{cc}}\right]$$

$$\xi^{1/} = \xi\left[1 + \frac{\lambda}{384\pi^2\,\xi}\left(\frac{3\Lambda^2}{\Lambda_{cc}} + 2\,(1-6\xi)\log\frac{3\Lambda^2}{\Lambda_{cc}}\right)\right]$$

Vacuum energy $\rho_{\rm vac} = \frac{\Lambda_{cc}}{8\pi G}$. Radiative correction $\sim \log\Lambda$: *no quartic/quadratic divergence*

Note : $\log\Lambda$ correction multiplied by $m^4 \Longrightarrow$ for SM masses $m \sim \mu_{\rm F}$ still left with (at least) 50 orders of magnitude discrepancy with measured vacuum energy. Absence of Λ^4 and Λ^2 corrections sheds some light on the CC problem, and makes it less severe

Inverse Newton constant - Quadratically UV-sensitive contribution as usual

Comparison with the literature

Calculation usually performed within the heat kernel expansion. Quadratically divergent contribution to the mass typically found (obviously dimensional regularization excluded from these considerations: trick to cancel powerlike divergences)

Let us go back to $\Gamma^{1/}$: temporarily connect N and Λ through

$$\Lambda = \frac{N}{a} \quad \text{rather than} \quad \Lambda = \frac{N}{a_{\rm dS}}$$

$$\begin{aligned}
\Gamma^{1/} = \frac{8\pi^2}{3} a^4 \Big\{ & - \frac{1}{16\pi G} \Big[1 + \frac{1-6\xi}{12\pi} G\Lambda^2 - \frac{G\,m^2}{24\pi} \Big(2(1-6\xi) \log\left(a^2\Lambda^2\right) \Big) \Big] \frac{12}{a^2} \\
& + \frac{\Lambda_{\rm cc}}{8\pi G} \Big[1 - \frac{G}{8\pi\Lambda_{\rm cc}} \Lambda^4 + \frac{m^2 G}{4\pi\Lambda_{\rm cc}} \Lambda^2 - \frac{G\,m^4}{8\pi\Lambda_{\rm cc}} \log\left(a^2\Lambda^2\right) \Big] \\
& + \frac{\xi}{2} \left[1 + \frac{\lambda}{384\pi^2\,\xi} \Big(2\,(1-6\xi) \log\left(a^2\Lambda^2\right) \Big) \right] \frac{12}{a^2} \Phi^2 \\
& + \frac{m^2}{2} \Big[1 + \frac{\lambda\,\Lambda^2}{32\pi^2\,m^2} - \frac{\lambda}{32\pi^2} \log\left(a^2\Lambda^2\right) \Big] \Phi^2 + \frac{\lambda}{4!} \left[1 - \frac{3\lambda}{32\pi^2} \log\left(a^2\Lambda^2\right) \right] \Phi^4 \Big\} \\
& + \left(2\xi\,(1-3\xi) - \frac{29}{180} \right) \log\left(a^2\Lambda^2\right)
\end{aligned}$$

From $\frac{N}{a_{\rm dS}}$ to $\frac{N}{a}$

$$\Gamma^{1l} = \frac{8\pi^2}{3} a^4 \Big\{ -\Big[1 - \frac{G\,m^2}{24\pi}\Big(\mathbf{N^2} + 2(1-6\xi)\log N^2\Big)\Big]\frac{1}{16\pi G}\frac{12}{a^2} + \Big[1 - \frac{G\,m^4}{8\pi\Lambda_{\rm cc}}\log N^2\Big]\frac{\Lambda_{\rm cc}}{8\pi G}$$
$$+ \Big[1 + \frac{\lambda}{384\pi^2\,\xi}\Big(\mathbf{N^2} + 2\,(1-6\xi)\log N^2\Big)\Big]\frac{\xi}{2}\frac{12}{a^2}\Phi^2 + \Big[1 - \frac{\lambda}{32\pi^2}\log N^2\Big]\frac{m^2}{2}\Phi^2 + \Big[1 - \frac{3\lambda}{32\pi^2}\log N^2\Big]\frac{\lambda}{4!}\Phi^4 \Big\}$$
$$- \frac{\mathbf{N^4}}{24} - \frac{1-6\xi}{6}\mathbf{N^2} + \Big(2\xi\,(1-3\xi) - \frac{29}{180}\Big)\log N^2$$

Legenda: **Green** $\rightarrow \frac{\Lambda_{\rm cc}}{8\pi G}$; **Red** $\rightarrow m^2$; **Blue** $\rightarrow \frac{1}{16\pi G}$

$$\Gamma^{1l} = \frac{8\pi^2}{3} a^4 \Big\{ -\Big[1 + \frac{1-6\xi}{12\pi}G\mathbf{\Lambda^2} - \frac{G\,m^2}{24\pi}\Big(2(1-6\xi)\log\Big(a^2\Lambda^2\Big)\Big)\Big]\frac{1}{16\pi G}\frac{12}{a^2}$$
$$+ \Big[1 - \frac{G}{8\pi\Lambda_{\rm cc}}\mathbf{\Lambda^4} + \frac{m^2 G}{4\pi\Lambda_{\rm cc}}\mathbf{\Lambda^2} - \frac{G\,m^4}{8\pi\Lambda_{\rm cc}}\log\Big(a^2\Lambda^2\Big)\Big]\frac{\Lambda_{\rm cc}}{8\pi G}$$
$$+ \Big[1 + \frac{\lambda}{384\pi^2\,\xi}\Big(2\,(1-6\xi)\log\Big(a^2\Lambda^2\Big)\Big)\Big]\frac{\xi}{2}\frac{12}{a^2}\Phi^2$$
$$+ \Big[1 + \frac{\lambda\,\mathbf{\Lambda^2}}{32\pi^2\,m^2} - \frac{\lambda}{32\pi^2}\log\Big(a^2\Lambda^2\Big)\Big]\frac{m^2}{2}\Phi^2 + \Big[1 - \frac{3\lambda}{32\pi^2}\log\Big(a^2\Lambda^2\Big)\Big]\frac{\lambda}{4!}\Phi^4 \Big\}$$
$$+ \Big(2\xi\,(1-3\xi) - \frac{29}{180}\Big)\log\Big(a^2\Lambda^2\Big)$$

Speculations (Conclusions ?)

Curved vs. flat spacetime

Quantum fluctuations usually computed directly in flat spacetime ... However ...

Evidence of positive vacuum energy $\frac{\Lambda_{cc}}{8\pi G} > 0$: Flat spacetime not suitable cosmological description

Quantum corrections should be computed on curved background

Flat spacetime as limit ... different from flat ab initio ...

Our calculations indicate that usual methods may fail ...

Quadratic divergence in ξ instead of m^2 cannot be detected in flat spacetime computations, since in this case $R\phi^2 = 0$