Nonperturbative study of singlet-assisted electroweak phase transition

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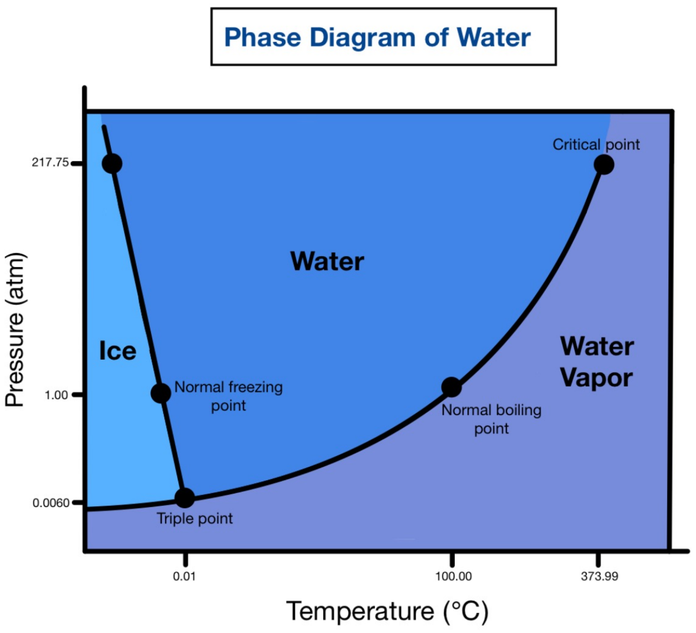
paper access

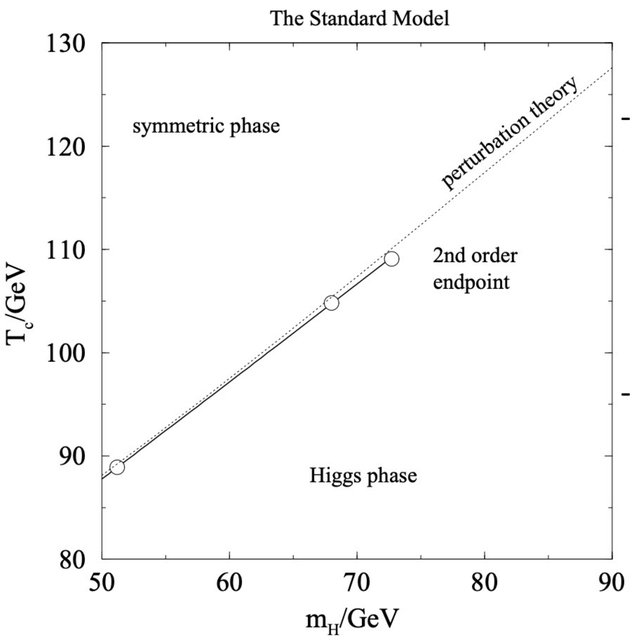
Paper, code, and data are available:
https://github.com/niemilau/su2higgs
https://doi.org/10.5281/zenodo.11099794

Water's Thermal Phase Transition



- Phases are distinguished by Helmholtz free energy when volume is kept constant. Lowest free energy picks the stable phase.
- Solid black curves mark coexistence where two phases share equal free energy.
- Crossing a coexistence curve → first-order transition with latent heat.
- Beyond the critical point, phases blend continuously—only a smooth crossover.

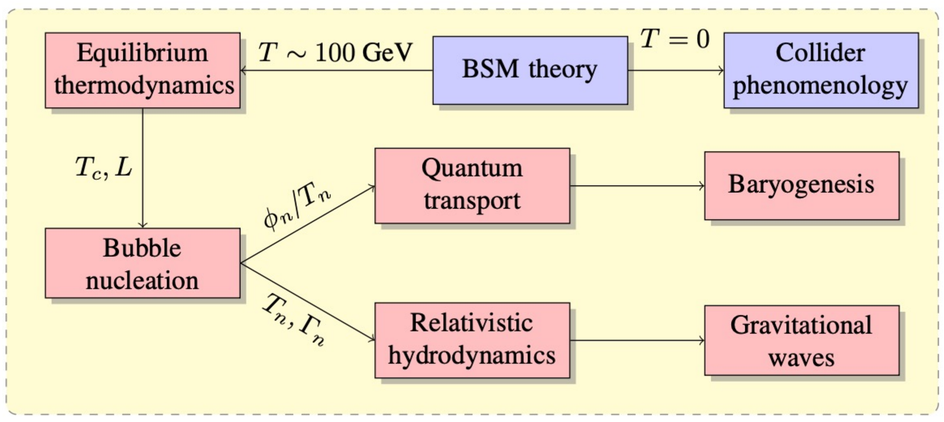
Phase Diagram of SU(2) + Higgs

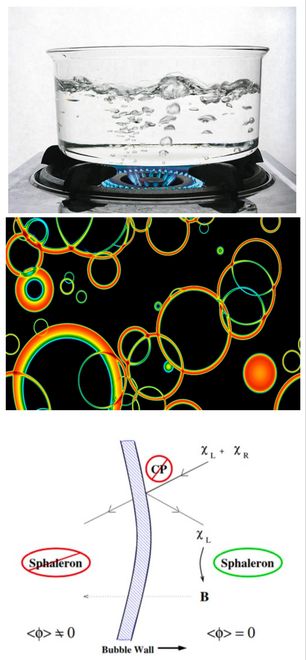


- Perturbation theory always predicts a first-order transition via the effective potential, as IR divergences in thermal QFT spoil the perturbative prediction near the endpoint **(Linde problem)**.
- Distinguishing 1st order vs. 2nd order vs. crossover is a **nonperturbative** problem!

Is There Really A FOEWPT?

In reality, Higgs mass = 125 GeV…

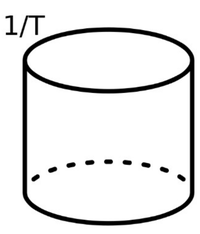




xSM Lattice Simulation Based on 3D EFT

$$S = \int_0^{1/T} d\tau \int d^3x \left\{ \begin{aligned} &\frac{1}{4}\,\mathrm{Tr}\,F_{\mu\nu}^2 + |D_\mu\phi|^2 + m_\phi^2\,\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \\ &+ (\partial_\mu S)^2 + b_1 S + \frac{1}{2}m_S^2 S^2 + \frac{1}{3}b_3 S^3 + \frac{1}{4}b_4 S^4 \\ &+ \frac{1}{2}a_1 S\phi^\dagger\phi + \frac{1}{2}a_2 S^2\phi^\dagger\phi \end{aligned} \right\}$$

$$\langle\phi(-P)\phi(P)\rangle_0 = \frac{1}{\mathbf{p}^2 + \omega_n^2 + m^2}, \quad \omega_n = \begin{cases} 2\pi n T & \text{bosons} \\ 2\pi(n+\frac{1}{2})T & \text{fermions} \end{cases}$$

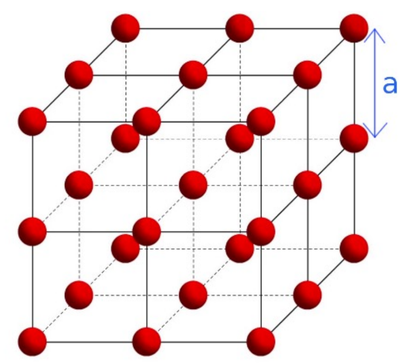


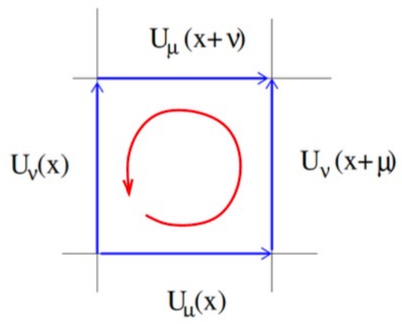
For momenta $\ll \pi T$, can use effective theory framework,
Low-energy description for the bosonic $\omega_n = 0$ modes:

$$S_{3\mathrm{D}}(T) = \frac{1}{T}\int d^3x \left\{ \begin{aligned} &\frac{1}{4}\,\mathrm{Tr}\,F_{ij}^2 + |D_i\phi|^2 + \bar{m}_\phi^2(T)\,\phi^\dagger\phi + \bar{\lambda}(T)(\phi^\dagger\phi)^2 \\ &+ (\partial_i S)^2 + \bar{b}_1(T)S + \frac{1}{2}\bar{m}_S^2(T)S^2 + \frac{1}{3}\bar{b}_3(T)S^3 + \frac{1}{4}\bar{b}_4(T)S^4 \\ &+ \frac{1}{2}\bar{a}_1(T)S\phi^\dagger\phi + \frac{1}{2}\bar{a}_2(T)S^2\phi^\dagger\phi \end{aligned} \right\}$$

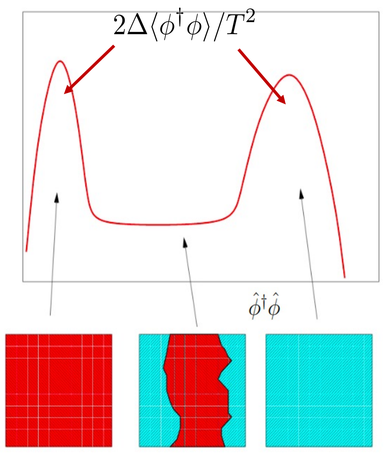
EFT is 3D with T-dependent parameters ("**dimensional reduction**").

Putting fields on lattice





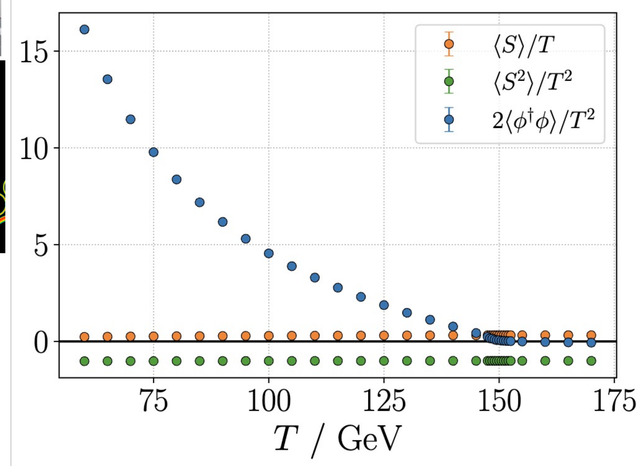
1st order PT: histogram of field configuration at Tc

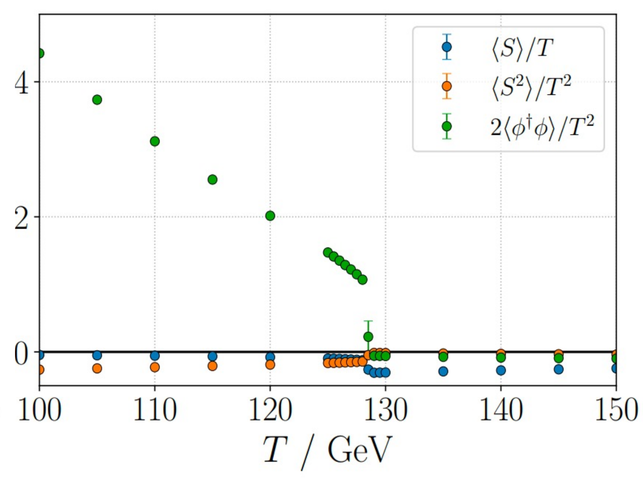


EWPT order parameter $\langle\phi^\dagger\phi\rangle$

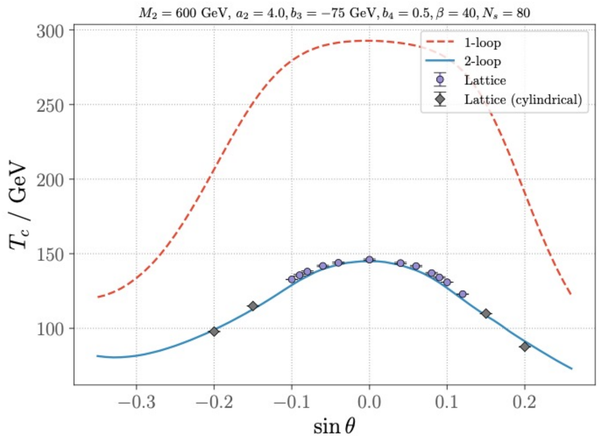
$p(\phi) \propto e^{-S(\phi)}$

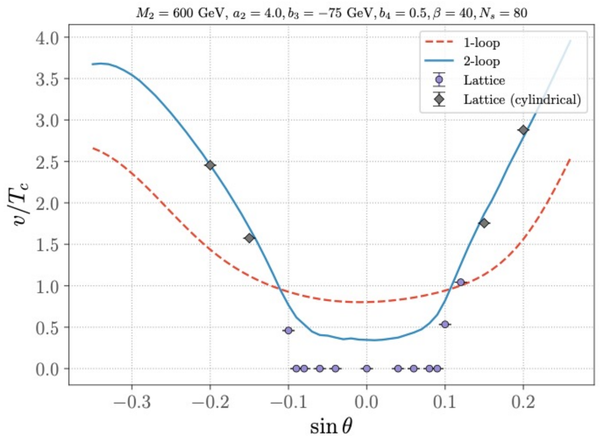
EWPT Thermal History





Conclusions





(a) Critical temperature. (b) Order parameter discontinuity.

- Extending calculation of the thermal effective potential to **2-loop** order greatly improves the predictions for basic thermodynamical quantities, which are **very close to lattice results in strong 1st order phase transition region.**
- **Distinguishing 1st order vs. 2nd order vs. crossover is a nonperturbative problem!**

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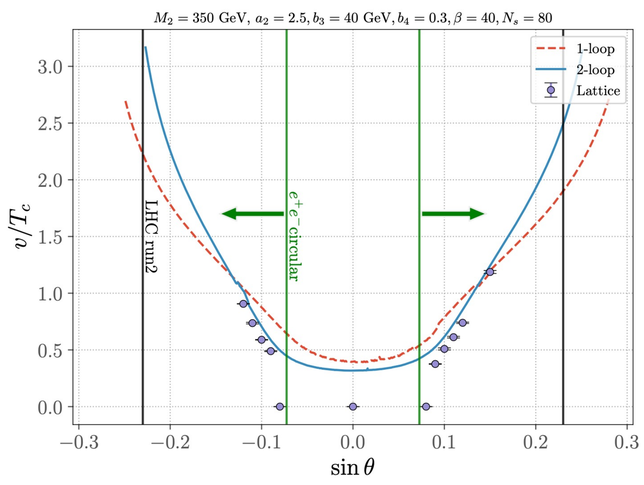
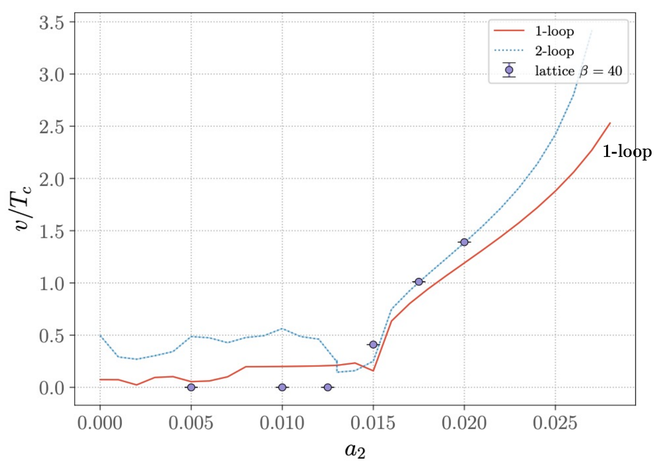
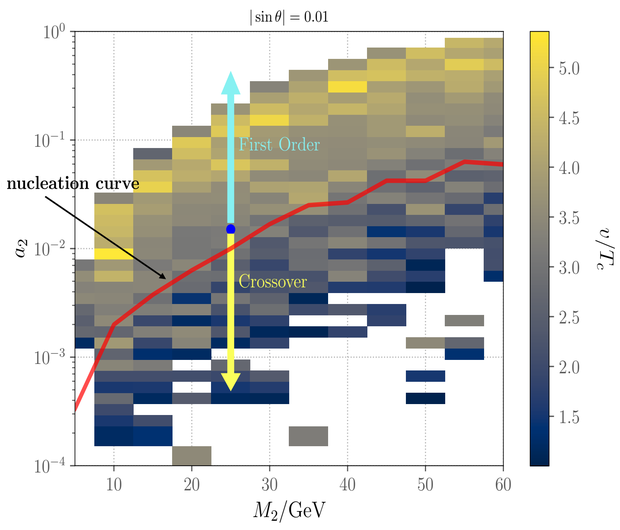
Hints to Phenomenology



Future precision Higgs studies may be sensitive to a greater portion of FOEWPT-viable param space than earlier realized!





Lattice confirms whether parameter points truly yield a first-order electroweak phase transition beyond any perturbative estimates.