Horizontal 3HDMs: A simple framework for flavor hierarchies

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10th June, 2025

♣ ArXiv: 2502.20296

3HDMs are interesting!

- Three Higgs doublet models (3HDMs) offer a unique opportunity to investigate two particular types of Yukawa structures.
- One is a vertical type of Yukawa structure where each Higgs doublet gives mass only to a particular type (up, down, charged-lepton) massive fermion.
- The other has a horizontal Yukawa structure where the Vacuum Expectation Value (VEV) of a particular doublet is responsible for masses of a particular generation of fermions.

Horizontal 3HDM

Objectives:

- We wish to disentangle the sources of masses for different generations of fermion such that the major source of mass for a particular generation of fermions comes from the VEV of a particular doublet.

- In this way, generation wise strong mass hierarchies in the fermion sector can be ascribed to strong hierarchies among the VEVs of the three doublets.

Challenges:

- 3HDMs which are constructed without natural flavor conservation (NFC) are almost invariably accompanied by flavor changing neutral currents (FCNCs) mediated by nonstandard scalars.

- Therefore, one would usually require such nonstandard scalars to be much heavier than the electroweak scale to mitigate the constraints from flavor data.

Interesting features of Horizontal 3HDM

- Textures of the Yukawa matrices are such that the charged fermions of a particular generation primarily receive their masses from a particular Higgs-doublet. Hierarchies among these VEVs are responsible for the mass hierarchies.

- No FCNC couplings in down-quark and charged lepton sector. Although the neutral Higgs couplings in these sectors are flavor diagonal, they are flavor nonuniversal. This aspect of the model may be exploited to selectively enhance or suppress the nonstandard Higgs couplings with fermions of a desired flavor.

- All the FCNCs occur exclusively in the up-quark sector, with their effects being considerably suppressed by the off-diagonal elements of the CKM matrix.

- The model is, at the very least, as economical as the NFC versions of 3HDMs in terms of the number of physical parameters. Therefore, when compared with the NFC avatars of 3HDMs, the predictive abilities are not being compromised.

Implementation of the horizontal 3HDM

- Our construction is based on a 3HDM structure, endowed with a global $U(1)_1 \times U(1)_2$ horizontal symmetry, under which the fields transform as

$$
\begin{aligned}
U(1)_1: &\quad (Q_L)_1 \to e^{-i\theta_1}(Q_L)_1\,, \quad (n_R)_1 \to e^{-2i\theta_1}(n_R)_1\,, \quad \phi_1 \to e^{i\theta_1}\phi_1\,,\\
U(1)_2: &\quad (Q_L)_2 \to e^{-i\theta_2}(Q_L)_2\,, \quad (n_R)_2 \to e^{-2i\theta_2}(n_R)_2\,, \quad \phi_2 \to e^{i\theta_2}\phi_2\,,
\end{aligned}
$$

  where, $Q_L = (p_L\ n_L)^T$ are the usual SM quark $SU(2)_L$ doublets, n_R are the down-type $SU(2)$ quark singlets, and the subscript refers to the different generations in the Lagrangian basis.

- All other quark and scalar fields transform trivially under the flavor symmetry.

Scalar Potential

- Before looking at the resulting textures, we first look at the scalar sector of the model in order to set up the notations that will be used throughout.

- Taking into account the field transformations, the scalar potential can be written as:

$$
\begin{aligned}
V_{U(1)\times U(1)} \quad = \quad & m_{11}^2 \phi_1^\dagger \phi_1 + m_{22}^2 \phi_2^\dagger \phi_2 + m_{33}^2 \phi_3^\dagger \phi_3 \\
& - \left(m_{12}^2 \phi_1^\dagger \phi_2 + m_{13}^2 \phi_1^\dagger \phi_3 + m_{23}^2 \phi_2^\dagger \phi_3 + \text{h.c.} \right) \\
& + \lambda_1 (\phi_1^\dagger \phi_1)^2 + \lambda_2 (\phi_2^\dagger \phi_2)^2 + \lambda_3 (\phi_3^\dagger \phi_3)^2 + \lambda_4 (\phi_1^\dagger \phi_1)(\phi_2^\dagger \phi_2) \\
& + \lambda_5 (\phi_1^\dagger \phi_1)(\phi_3^\dagger \phi_3) + \lambda_6 (\phi_2^\dagger \phi_2)(\phi_3^\dagger \phi_3) + \lambda_7 (\phi_1^\dagger \phi_2)(\phi_2^\dagger \phi_1) \\
& + \lambda_8 (\phi_1^\dagger \phi_3)(\phi_3^\dagger \phi_1) + \lambda_9 (\phi_2^\dagger \phi_3)(\phi_3^\dagger \phi_2) \,,
\end{aligned}
$$

  where m_{12}^2, m_{13}^2, and m_{23}^2 break the global $U(1)_1 \times U(1)_2$ symmetry softly.

- We assume that all the potential parameters (including VEVs) are real.

Notations and Rotations

- The individual VEVs after the EWSB, are further parameterized as follows:

$$v_1 = v\cos\beta_1\cos\beta_2\,, \qquad v_2 = v\sin\beta_1\cos\beta_2\,, \qquad v_3 = v\sin\beta_2\,,$$

with $v = \sqrt{v_1^2 + v_2^2 + v_3^2} = 246$ GeV being the total electroweak VEV.

- For later convenience, we go to an intermediate basis by rotating the fields in the Lagrangian basis as follows:

$$\begin{pmatrix} H_0 \\ R_1 \\ R_2 \end{pmatrix} = O_\beta \begin{pmatrix} h_1 \\ h_2 \\ h_3 \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ A_1' \\ A_2' \end{pmatrix} = O_\beta \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix}, \qquad \begin{pmatrix} G^\pm \\ H_1'^\pm \\ H_2'^\pm \end{pmatrix} = O_\beta \begin{pmatrix} w_1^\pm \\ w_2^\pm \\ w_3^\pm \end{pmatrix},$$

- Here,

$$O_\beta = \begin{pmatrix} \cos\beta_2\cos\beta_1 & \cos\beta_2\sin\beta_1 & \sin\beta_2 \\ -\sin\beta_1 & \cos\beta_1 & 0 \\ -\cos\beta_1\sin\beta_2 & -\sin\beta_1\sin\beta_2 & \cos\beta_2 \end{pmatrix}.$$

Physical scalars

- The fields other than the Goldstones are not the physical mass eigenstates, in general.
- The rotations to the mass eigenstates are as follows,

$$\begin{pmatrix} G^{\pm} \\ H_1^{\pm} \\ H_2^{\pm} \end{pmatrix} = O_{\gamma_2} \begin{pmatrix} G^{\pm} \\ H_1'^{\pm} \\ H_2'^{\pm} \end{pmatrix}, \qquad \begin{pmatrix} G^0 \\ A_1 \\ A_2 \end{pmatrix} = O_{\gamma_1} \begin{pmatrix} G^0 \\ A_1' \\ A_2' \end{pmatrix},$$

  with O_{γ_1} and O_{γ_2} are,

$$\mathcal{O}_{\gamma_1} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_1 & -\sin\gamma_1 \\ 0 & \sin\gamma_1 & \cos\gamma_1 \end{pmatrix}. \quad \mathcal{O}_{\gamma_2} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos\gamma_2 & -\sin\gamma_2 \\ 0 & \sin\gamma_2 & \cos\gamma_2 \end{pmatrix}$$

- For notational convenience, we choose to work in the intermediate basis, defined earlier, under the alignment limit where the scalar H_0 mimics the SM Higgs boson.

Yukawa Sector

- Following the standard conventions, we write the Yukawa Lagrangian as

$$\mathscr{L}_{\rm Y} = -\sum_{k=1}^{3}\left[\bar{Q}_L\Gamma_k\phi_k n_R + \bar{Q}_L\Delta_k\tilde{\phi}_k p_R\right] + \text{h.c.}\,,$$

  where, p_R are the up-type quark $SU(2)_L$ singlets and we suppress the flavor indices (Γ_k and Δ_k are 3×3 matrices in the flavor space). Also, $\tilde{\phi}_i = i\tau_2\phi_i^*$, as usual, with τ_2 being the second Pauli matrix.

- The mass matrices for the down-type and up-type quarks are then

$$M_n = \frac{1}{\sqrt{2}}\sum_{k=1}^{3} v_k\,\Gamma_k\,, \qquad M_p = \frac{1}{\sqrt{2}}\sum_{k=1}^{3} v_k\,\Delta_k\,.$$

Yukawa Textures

The textures of Γ and Δ matrices that follow from the $U(1) \times U(1)$ symmetry are given below:

$$\Gamma_1 = \begin{pmatrix} y_1^d & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \Gamma_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & y_2^d & 0 \\ 0 & 0 & 0 \end{pmatrix}; \quad \Gamma_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & y_3^d \end{pmatrix},$$

$$\Delta_1 = \begin{pmatrix} a_1 & b_1 & c_1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \Delta_2 = \begin{pmatrix} 0 & 0 & 0 \\ a_2 & b_2 & c_2 \\ 0 & 0 & 0 \end{pmatrix}; \Delta_3 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ a_3 & b_3 & c_3 \end{pmatrix}$$

Some Immediate Consequences

- Since the Γ_k's and hence M_n are already diagonal, it becomes obvious that the mass of each generation in the down sector is given by the VEV of a dedicated ϕ_k. Then, if the VEVs of the scalars are hierarchical ($v_1 \ll v_2 \ll v_3$), the hierarchies among the Yukawa coefficients in the down sector will be substantially diluted.

- Although not as obvious as the down sector, it can be shown that the same VEV hierarchy will lead to analogous effects in the up-quark sector, *i.e.*, the VEV of a particular doublet will be the major source of mass for a particular generation of quark in the up-sector as well.

- Since the down-sector is already diagonal, the FCNCs will appear in the up-sector only. We will show the structure of the FCNC couplings in the intermediate basis. The couplings to the physical scalars can later be obtained by suitable rotations originating from the scalar sector alone.

Physical quarks and the CKM matrix

- The physical quark fields are defined as

$$n_L = U_L d_L\,, \quad n_R = U_R d_R\,, \qquad\qquad p_L = V_L u_L\,, \quad p_R = V_R u_R\,,$$

  where, $u \equiv (u\ c\ t)^T$ and $d \equiv (d\ s\ b)^T$ denote the physical quarks.

- After these rotations, the diagonal mass matrices will be obtained as follows:

$$D_d = U_L^\dagger \cdot M_n \cdot U_R = \text{diag}(m_d,\ m_s,\ m_b)\,,$$
$$D_u = V_L^\dagger \cdot M_p \cdot V_R = \text{diag}(m_u,\ m_c,\ m_t)\,,$$

  and the CKM matrix will be given by $V = V_L^\dagger\, U_L$.

- Since M_n is directly diagonal, we can take U_L to be the identity matrix, whereas U_R provides the necessary rephasings such that the elements of D_d are real and positive.

- In practice, since we are already in a basis where the down-sector is diagonal, we may write,

$$\frac{1}{\sqrt{2}}\sum_{k=1}^{3} v_k\,\Gamma_k = \frac{1}{\sqrt{2}}\mathrm{diag}\left(v_1\,y_1^d\,,\;\; v_2\,y_2^d\,,\;\; v_3\,y_3^d\right) = \mathrm{diag}\left(m_d\,,\;\; m_s\,,\;\; m_b\right)\,.$$

- Therefore, the CKM mixing arises exclusively from the up-sector as follows:

$$V = V_L^\dagger\,U_L = V_L^\dagger\,.$$

- The Yukawa couplings of the CP-even scalars are

$$-\mathscr{L}_{\mathrm{Y}}^{\mathrm{CP-even}} = \frac{1}{\sqrt{2}}\sum_{k=1}^{3}\left[\bar{n}_L\,\Gamma_k\,h_k\,n_R + \bar{p}_L\,\Delta_k\,h_k\,p_R\right] + \mathrm{h.c.}$$

$$= \bar{d}_L\left(\frac{1}{\sqrt{2}}\sum_{k=1}^{3}\Gamma_k\,h_k\right)d_R + \bar{u}_L\,V_L^\dagger\left(\frac{1}{\sqrt{2}}\sum_{k=1}^{3}\Delta_k\,h_k\right)V_R\,u_R + \mathrm{h.c.}\qquad.$$

- In the intermediate basis defined earlier, we can decompose h_k as

$$h_k = (O_\beta)_{1k}\,H_0 + (O_\beta)_{2k}\,R_1 + (O_\beta)_{3k}\,R_2\,.$$

- By virtue of being a SM-like Higgs, the couplings of H_0 simplify to

$$-\mathscr{L}_{\rm Y}^{H^0} \;=\; \frac{H_0}{v}\left[\bar{d}_L\, D_d\, d_R + \bar{u}_L\, D_u\, u_R\right] + \text{h.c.} = \frac{H_0}{v}\left[\bar{d}\, D_d\, d + \bar{u}\, D_u\, u\right].$$

- The couplings of R_1 and R_2 that mediate FCNCs can be written as

$$-\mathscr{L}_{\rm Y}^{R} \;=\; R_1\left[\bar{d}_L\, N_1^d\, d_R + \bar{u}_L\, N_1^u\, u_R\right] + R_2\left[\bar{d}_L\, N_2^d\, d_R + \bar{u}_L\, N_2^u\, u_R\right] + \text{h.c.}$$

where,

$$N_1^d = \frac{1}{\sqrt{2}}\sum_{k=1}^{3}(O_\beta)_{2k}\Gamma_k\,, \qquad N_1^u = V_L^\dagger\left(\frac{1}{\sqrt{2}}\sum_{k=1}^{3}(O_\beta)_{2k}\Delta_k\right)V_R\,,$$

$$N_2^d = \frac{1}{\sqrt{2}}\sum_{k=1}^{3}(O_\beta)_{3k}\Gamma_k\,, \qquad N_2^u = V_L^\dagger\left(\frac{1}{\sqrt{2}}\sum_{k=1}^{3}(O_\beta)_{3k}\Delta_k\right)V_R\,.$$

- Since the Yukawa matrices in the down-sector were already diagonal, N_1^d assumes a simple shape:

$$\begin{aligned} N_1^d &= \text{diag}\left(\frac{m_d(O_\beta)_{21}}{v_1}, \frac{m_s(O_\beta)_{22}}{v_2}, \frac{m_b(O_\beta)_{23}}{v_3},\right) \\ &= \text{diag}\left(-\frac{m_d}{v}\frac{t_{\beta_1}t_{\beta_2}}{s_{\beta_2}}, \frac{m_s}{v}\frac{t_{\beta_2}}{t_{\beta_1}s_{\beta_2}}, 0\right) . \end{aligned}$$

- Similarly

$$\begin{aligned} N_2^d &= \text{diag}\left(\frac{m_d(O_\beta)_{31}}{v_1}, \frac{m_s(O_\beta)_{32}}{v_2}, \frac{m_b(O_\beta)_{33}}{v_3},\right) \\ &= \text{diag}\left(-\frac{m_d}{v}t_{\beta_2}, -\frac{m_s}{v}t_{\beta_2}, \frac{m_b}{v}\frac{1}{t_{\beta_2}}\right) . \end{aligned}$$

- We can write N_1^u and N_2^u as

$$\begin{aligned} N_1^u &= -\sin\beta_1 F_1 + \cos\beta_1 F_2 \\ N_2^u &= -\cos\beta_1 \sin\beta_2 F_1 - \sin\beta_1 \sin\beta_2 F_2 + \cos\beta_2 F_3 \end{aligned}$$

where,

$$F_k = \frac{1}{\sqrt{2}} V_L^\dagger . \Delta_k . V_R = \frac{1}{v_k} V_L^\dagger (P_k M_u) V_R$$

- Now, we can write F_k as,

$$F_k = \frac{1}{v_k} V_L^\dagger (P_k M_u) V_R = \frac{1}{v_k} (V_L^\dagger P_k V_L) D_u$$

  where, P_k is the projection matrix which has all elements zero except the (k,k) element.

- Now, we can expand,

$$(V_L^\dagger P_k V_L)_{AB} = V_{Ak}^* V_{Bk}$$

- Therefore

$$(F_k)_{AB} = \frac{1}{v_k} (V_{Ak}^*) V_{Bk} m_B$$

- Using this, we can write N_1^u and N_2^u as

$$(N_1^u)_{AB} = \left(\frac{-t_{\beta_1} t_{\beta_2}}{s_{\beta_2}} (V)_{A1} (V)_{B1}^* + \frac{t_{\beta_2}}{t_{\beta_2} s_{\beta_2}} (V)_{A2} (V)_{B2}^* \right) \frac{(D_u)_{BB}}{v}$$

$$(N_2^u)_{AB} = \left(-t_{\beta_2} (V)_{A1} (V)_{B1}^* - t_{\beta_2} (V)_{A2} (V)_{B2}^* + \frac{1}{t_{\beta_2}} (V)_{A3} (V)_{B3}^* \right) \frac{(D_u)_{BB}}{v}$$

- the pseudoscalar couplings follow the above closely, up to a factor of i and the consequent relative sign from the conjugation

$$\begin{aligned}
-\mathscr{L}_{\mathrm{Y}}^{\mathrm{CP-odd}} &= \frac{1}{\sqrt{2}} \sum_{k=1}^{3} \left[\bar{n}_L \, \Gamma_k \, (i z_k) \, n_R + \bar{p}_L \, \Delta_k \, (i z_k) \, p_R \right] + \text{h.c.} \\
&\supset i A_1' \left[\bar{d} \left(N_1^d \, P_R - N_1^{d\dagger} \, P_L \right) d + \bar{u} \left(N_1^u \, P_R - N_1^{u\dagger} \, P_L \right) u \right] \\
&+ i A_2' \left[\bar{d} \left(N_2^d \, P_R - N_2^{d\dagger} \, P_L \right) d + \bar{u} \left(N_2^u \, P_R - N_2^{u\dagger} \, P_L \right) u \right] .
\end{aligned}$$

- Similarly, for the charged scalars,

$$\begin{aligned}
-\mathscr{L}_{\mathrm{Y}}^{\mathrm{charged}} &= \sum_{i=1}^{3} \left[\bar{p}_L \, \Gamma_i \, \phi_i^+ \, n_R + \bar{p}_R \, \Delta_i^\dagger \, (-\phi_i^+) \, n_L \right] + \text{h.c.} \\
&\supset \sqrt{2} H_1'^+ \left[\bar{u} \left(V N_1^d \, P_R - N_1^{u\dagger} \, V P_L \right) d \right] \\
&+ \sqrt{2} H_2'^+ \left[\bar{u} \left(V N_2^d \, P_R - N_2^{u\dagger} \, V P_L \right) d \right] + \text{h.c.}
\end{aligned}$$

Yukawa Interactions with Physical Quarks

- By virtue of being a SM-like Higgs, the couplings of H_0 simplify to

$$-\mathscr{L}_{\rm Y}^{H_0} = \frac{H_0}{v}\left[\bar{d}\, D_d\, d + \bar{u}\, D_u\, u\right].$$

- The couplings of R_1 and R_2 that mediate FCNCs can be read as

$$-\mathscr{L}_{\rm Y}^{R} = R_1\left[\bar{d}_L\, N_1^d\, d_R + \bar{u}_L\, N_1^u\, u_R\right] + R_2\left[\bar{d}_L\, N_2^d\, d_R + \bar{u}_L\, N_2^u\, u_R\right] + \text{h.c.}$$

- For the pseudoscalar couplings we have,

$$\begin{aligned} -\mathscr{L}_{\rm Y}^{\rm CP-odd} \quad = \quad & i\,A_1'\left[\bar{d}\left(N_1^d\, P_R - N_1^{d\dagger}\, P_L\right) d + \bar{u}\left(N_1^u\, P_R - N_1^{u\dagger}\, P_L\right) u\right] \\ & + i\,A_2'\left[\bar{d}\left(N_2^d\, P_R - N_2^{d\dagger}\, P_L\right) d + \bar{u}\left(N_2^u\, P_R - N_2^{u\dagger}\, P_L\right) u\right]. \end{aligned}$$

FCNC Matrices

The Yukawa interactions are controlled by the matrices $N_{1,2}^{u,d}$.

$$N_1^d = \text{diag}\left(-\frac{m_d}{v}\frac{t_{\beta_1}t_{\beta_2}}{s_{\beta_2}}, \frac{m_s}{v}\frac{t_{\beta_2}}{t_{\beta_1}s_{\beta_2}}, 0\right),$$

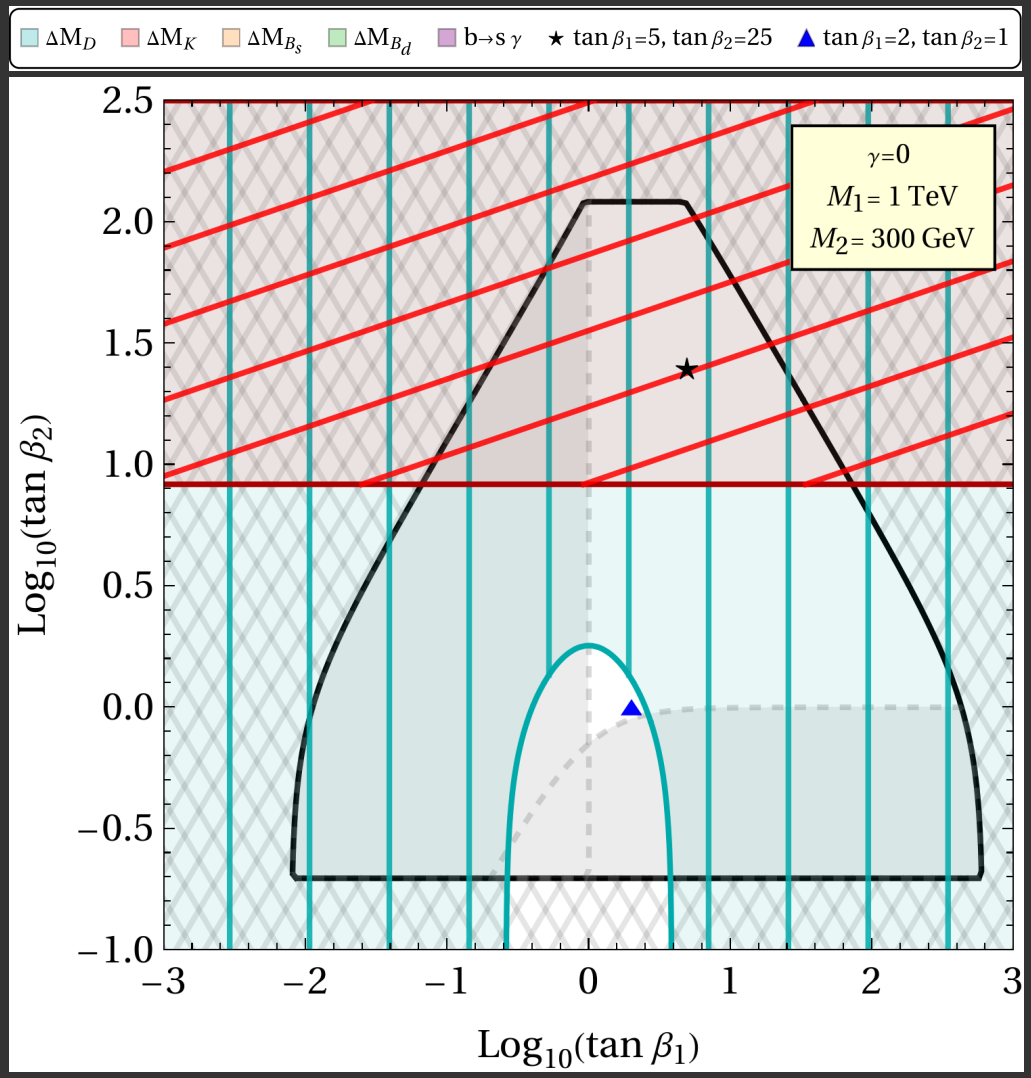
$$N_2^d = \text{diag}\left(-\frac{m_d}{v}t_{\beta_2}, -\frac{m_s}{v}t_{\beta_2}, \frac{m_b}{v}\frac{1}{t_{\beta_2}}\right),$$

$$(N_1^u)_{AB} = \left(\frac{-t_{\beta_1}t_{\beta_2}}{s_{\beta_2}}(V)_{A1}(V)_{B1}^* + \frac{t_{\beta_2}}{t_{\beta_2}s_{\beta_2}}(V)_{A2}(V)_{B2}^*\right)\frac{(D_u)_{BB}}{v}$$

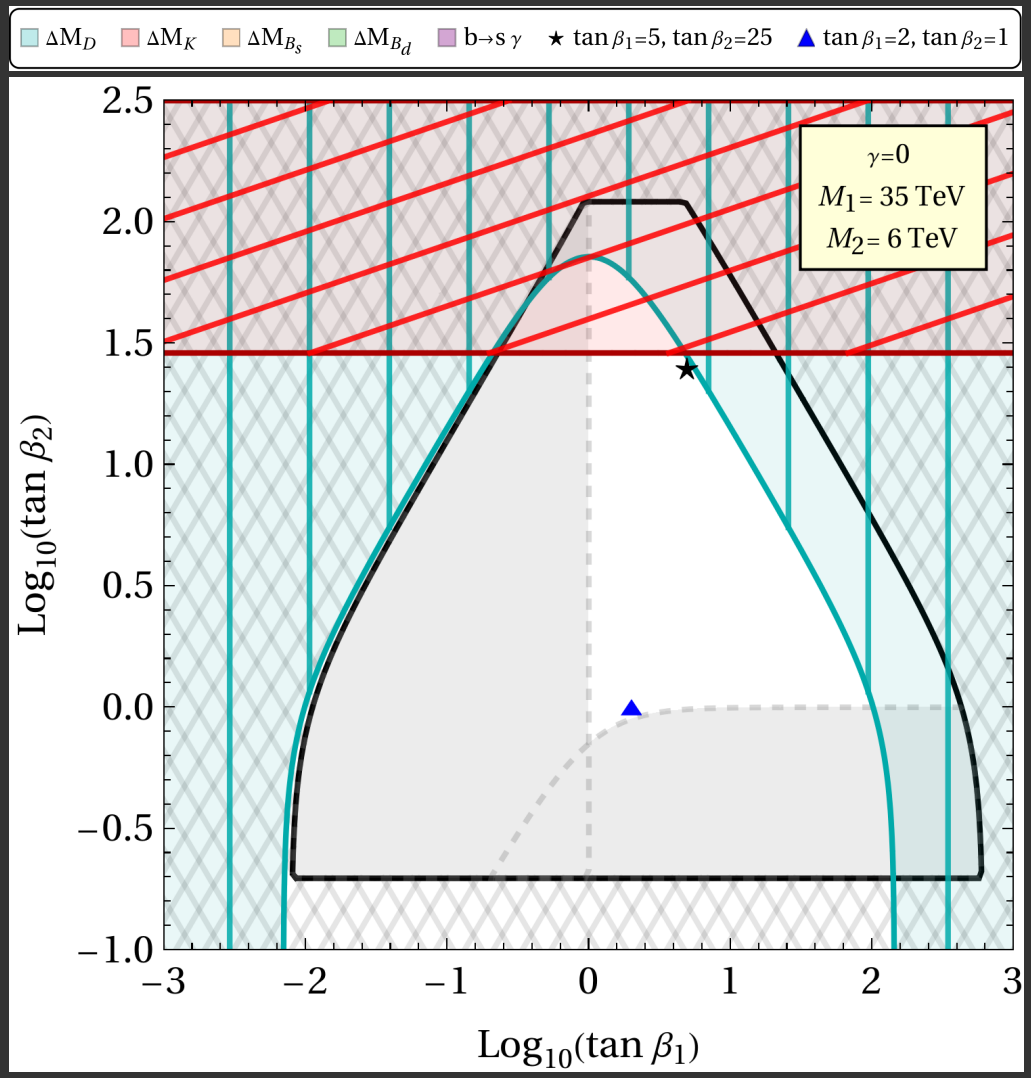
$$(N_2^u)_{AB} = \left(-t_{\beta_2}(V)_{A1}(V)_{B1}^* - t_{\beta_2}(V)_{A2}(V)_{B2}^* + \frac{1}{t_{\beta_2}}(V)_{A3}(V)_{B3}^*\right)\frac{(D_u)_{BB}}{v}$$

$\tan\beta_{1,2}$ are the only new parameters involved. This makes the horizontal-3HDM as economical as the NFC versions of 3HDMs.

Flavor constraints



Flavor constraints



Summary

- The k-th generation fermions receive their masses primarily from the k-th scalar doublet.
- The FCNCs are limited to the up-quark sector only (for the version considered here) and are controlled primarily by the off-diagonal elements of the CKM matrix.
- Flavor non-universal Higgs couplings are an interesting outcome.

THANK YOU !