Dark matter multiplet stabilized by a non-abelian group

Igor Ivanov

School of Physics and Astronomy, Sun Yat-sen University, Zhuhai

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Based on:
H. Deng, R. Boto, I.P. Ivanov, J.P. Silva, PRD 111 (2025) 5, 055006 = arXiv:2501.05929

中山大學 物理与天文学院
SUN YAT-SEN UNIVERSITY SCHOOL OF PHYSICS AND ASTRONOMY

Dark matter in Extended Higgs sectors

Non-minimal Higgs sectors with symmetries $\rightarrow$ cosmological consequences.

$\Rightarrow$ many talks and posters at this conference.

- Phase transitions in the early Universe $\rightarrow$ GW signals, electroweak baryogenesis, exotic scenarios, see e.g. [Hindmarsch et al, 2008.09136; Athron et al, 2305.02357].
- Scalar DM protected by residual symmetries, e.g. [Ivanov, 1702.03776; Arbey, Mahmoudi, 2104.11488].
- Higgs portal models connecting the visible and the dark sectors [Arcadi, Djouadi, Raidal, 1903.03616].

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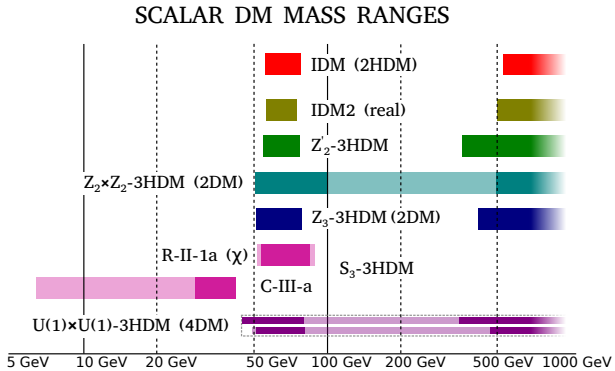
Inert doublet model = 2HDM with an exact $\mathbb{Z}_2$-symmetry [Deshpande, Ma, 1978; Barbieri et al, 2006; Ma, 2006; Lopez Honorez et al, 2006]:

- Colliders, cosmology, DM searches: [Arhrib et al, 1310.0358; Ilnicka, Krawczyk, Robens, 1508.01671; Belyaev et al, 1612.00511] + hundreds more. See also talk by T. Robens on Monday.

Scalar dark matter in NHDM

Numerous multi-Higgs-doublet models with various symmetry options.



[Kuncinas, Osland, Rebelo, 2408.02728].

Scalar DM stabilized by a non-abelian group

- Start with a multi-scalar model with a global symmetry group G. Minimization of the potential leads to a residual symmetry group G_v.
- Usually, one discusses DM stabilized by a single residual symmetry transformation that generates the group $G_v = \mathbb{Z}_2$ or $\mathbb{Z}_{n>2}$ or $U(1)$.

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- Usually, one discusses DM stabilized by a single residual symmetry transformation that generates the group $G_v = \mathbb{Z}_2$ or $\mathbb{Z}_{n>2}$ or $U(1)$.
- However, one can also have a non-abelian residual symmetry group G_v. Moreover, if G is sufficiently large, then a non-abelian G_v can be unavoidable. Such a situation can take place already within the 3HDM.

Our objectives

Is it possible to construct a realistic multi-Higgs-doublet model with a DM multiplet stabilized by a non-abelian residual G_v?

We tried to answer this question in PRD 111 (2025) 5, 055006 = arXiv:2501.05929.

DM in $\Sigma(36)$: attempt 1

[Ivanov, Vdovin, 1210.6553; Darvishi, Pilaftsis, 1912.00887]: the largest Higgs-family group of 3HDM avoiding accidental continuous symmetry: $\Sigma(36) \simeq (\mathbb{Z}_3 \times \mathbb{Z}_3) \rtimes \mathbb{Z}_4$ generated by

$$a = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \omega & 0 \\ 0 & 0 & \omega^2 \end{pmatrix}, \quad b = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix}, \quad d = \frac{i}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega^2 & \omega \\ 1 & \omega & \omega^2 \end{pmatrix}, \quad \omega = e^{2\pi i/3}.$$

The potential:

$$\begin{aligned} V &= -m^2 \left[\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3\right] + \lambda_1 \left[\phi_1^\dagger\phi_1 + \phi_2^\dagger\phi_2 + \phi_3^\dagger\phi_3\right]^2 \\ &\quad -\lambda_2 \left[|\phi_1^\dagger\phi_2|^2 + |\phi_2^\dagger\phi_3|^2 + |\phi_3^\dagger\phi_1|^2 - (\phi_1^\dagger\phi_1)(\phi_2^\dagger\phi_2) - (\phi_2^\dagger\phi_2)(\phi_3^\dagger\phi_3) - (\phi_3^\dagger\phi_3)(\phi_1^\dagger\phi_1)\right] \\ &\quad +\lambda_3 \left(|\phi_1^\dagger\phi_2 - \phi_2^\dagger\phi_3|^2 + |\phi_2^\dagger\phi_3 - \phi_3^\dagger\phi_1|^2 + |\phi_3^\dagger\phi_1 - \phi_1^\dagger\phi_2|^2\right), \end{aligned}$$

- The model is extremely constrained $\rightarrow$ numerous relations among scalar masses and couplings.
- Many features remain intact even if soft breaking terms are added [Varzielas, Ivanov, Levy, 2107.08227].
- Rich dynamics of phase transitions [Yang, Ivanov, 2401.03264].

DM in $\Sigma(36)$: attempt 1

[Ivanov, Nishi, 1410.6139]: up to permutations, the global minimum can only be located at

$$A:\ (\omega,\,1,\,1)\,,\quad A':\ (\omega^2,\,1,\,1)\,,\quad B:\ (1,\,0,\,0)\,,$$
$$C:\ (1,\,1,\,1)\,,\quad (1,\,\omega,\,\omega^2)\,,\quad (1,\,\omega^2,\,\omega)\,.$$

Notation: for example, $(1,1,1)$ denotes the case $v_1 = v_2 = v_3$, that is

$$(\langle\phi_1^0\rangle,\,\langle\phi_2^0\rangle,\,\langle\phi_3^0\rangle) = \frac{v}{\sqrt{6}}\,(1,\,1,\,1)\,.$$

At each minimum, the residual symmetry group is $G_v = S_3$.

DM in $\Sigma(36)$: attempt 1

Parametrizing $\phi_i = \begin{pmatrix} h_i^+ \\ (v_i + h_i + i a_i)/\sqrt{2} \end{pmatrix}$ and choosing $v_i = (v, 0, 0)$, we get the masses:

$$h_{SM} = h_1 : \qquad m_{SM}^2 = 2\lambda_1 v^2 = 2m^2 ,$$

$$H_2^+ = h_2^+ ,\ H_3^+ = h_3^+ : \qquad m_{H^+}^2 = \frac{\lambda_2 v^2}{2} \quad \text{(double degenerate)} ,$$

$$h = \frac{1}{\sqrt{2}}(h_2 + h_3) , \quad a = \frac{1}{\sqrt{2}}(a_2 - a_3) : \qquad m_h^2 = \frac{\lambda_3 v^2}{2} \quad \text{(double degenerate)} ,$$

$$H = \frac{1}{\sqrt{2}}(h_2 - h_3) , \quad A = \frac{1}{\sqrt{2}}(a_2 + a_3) : \qquad m_H^2 = \frac{3\lambda_3 v^2}{2} \quad \text{(double degenerate)} .$$

For $\lambda_2 > \lambda_3$, h and a are the DM candidates.

- Non-abelian S_3 allows for ZHa or ZhA but forbids Zha.
- S_3 allows for semi-annihilation channels such as $hh \to a + \text{SM}$.

DM in $\Sigma(36)$: attempt 1

Extending $\Sigma(36)$ to the quark Yukawa sector (irreps and products in [Grimus, Ludl, 1006.0098]).

label	$\overline{Q}_L$	ϕ	d_R	u_R
case 1-3-3-3	$\mathbf{1}$	$\mathbf{3}$	$\mathbf{3}^*$	$\mathbf{3}$
case 3-3-1-3	$\mathbf{3}^*$	$\mathbf{3}$	$\mathbf{1}$	$\mathbf{3}^*$ or $(\mathbf{3}^{(1)})^*$ or $(\mathbf{3}^{(3)})^*$
case 3-3-3-1	$\mathbf{3}$	$\mathbf{3}$	$\mathbf{3}$ or $\mathbf{3}^{(1)}$ or $\mathbf{3}^{(3)}$	$\mathbf{1}$

- The Yukawa sector is not fully realistic (massless or mass-degenerate quarks); expected from the no-go theorem [González Felipe et al, 1401.5807].
- The real show-stopper is the presence of unavoidable $h\bar{q}_i q_j$, $a\bar{q}_i q_j$ couplings
  $\rightarrow$ h, a are unstable, no DM candidates.

DM in $\Sigma(36)$: attempt 2

Rethinking the strategy:

- Our key goal is to exploit $\Sigma(36) \to S_3$ in the scalar sector.
- Assume that the Yukawa sector is only S_3-invariant.
- Allow for a soft-breaking term in the scalar potential: $\Sigma(36)$ is softly broken but S_3 is preserved:

$$V_{\text{soft}} = \mu^2 \left(\phi_2^\dagger \phi_2 + \phi_3^\dagger \phi_3 \right) , \quad \mu^2 > 0 \text{ to guarantee } (v, 0, 0) \text{ is the global minimum.}$$

- Strictly speaking, the model is only S_3 invariant, but the quartic interactions still possess $\Sigma(36)$.
  $\Rightarrow$ the model inherits many features from the $\Sigma(36)$ 3HDM [Varzielas, Ivanov, Levy, 2107.08227].

We still have mass-degenerate DM candidates h and a.
Can this model survive the LHC and DM constraints?

DM in $\Sigma(36)$: attempt 2

A non-trivial exercise, with several constraints pulling the model in opposite directions!

- Light DM $m_h \leq m_{SM}/2$ is ruled out due to the huge $h_{SM} \to$ invisible.
- For larger DM mass, the case without soft breaking ($\mu^2 = 0$) is also ruled out due $R(h_{SM} \to \gamma\gamma) < 0.8$ for any $m(H^+)$ (no decoupling limit!).

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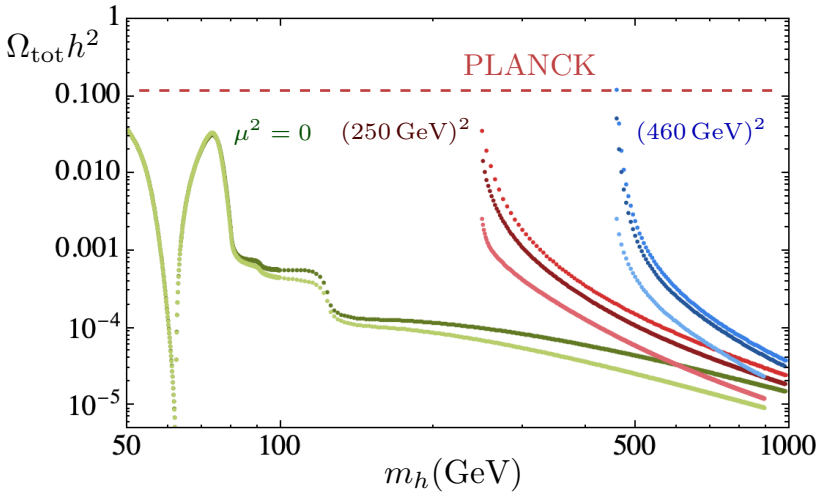
- For larger DM mass, the case without soft breaking ($\mu^2 = 0$) is also ruled out due $R(h_{SM} \to \gamma\gamma) < 0.8$ for any $m(H^+)$ (no decoupling limit!).
- With soft breaking terms but small $\mu^2 \sim (100\,\text{GeV})^2$, the model passes the LHC and relic density constraints (only for subdominant DM!) but badly fails the direct detection constraints.
- The key quantity is the coupling $\bar{\lambda}$ (analogous to the λ_{345} if the IDM):

$$h_{SM}hh,\ h_{SM}aa: \quad \bar{\lambda}v = \frac{m_{SM}^2 + 2m_h^2 - 2\mu^2}{v},$$

  which also controls $hh \to V_L V_L$ annihilation.
- Heavy DM but small $\bar{\lambda}$ requires a large μ^2!

DM in Σ(36): attempt 2



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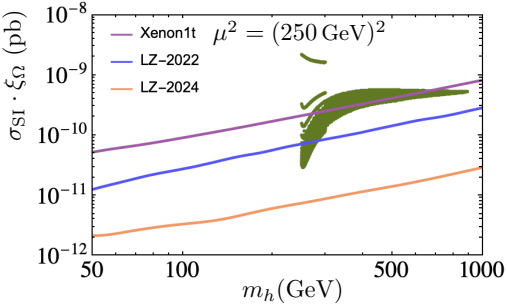

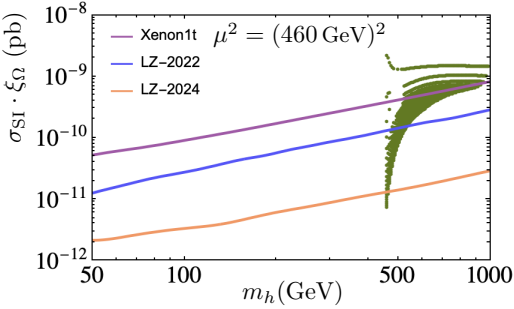

The rescaled DD signal can satisfy the LZ-2024 upper limits 2410.17036 only for $\mu > 450$ GeV.

Conclusions

Our goals:

- DM can be stabilized by a non-abelian residual group G_ν rather than a single symmetry.
- Non-abelian G_ν can appear automatically if the starting symmetry group G is large.
- But a large G implies rigid relationships among observables. So, can it lead to a realistic scalar DM model, e.g. from multiple Higgs doublets?

Our lessons from the $\Sigma(36)$ 3HDM (here, $G_\nu = S_3$ for all minima):

- Due to very little freedom in the model, we repeatedly run into various conflicts with the LHC, Planck, and direct detection constraints.
- The only viable option: softly broken $\Sigma(36)$ with a large parameter $\mu > 450$ GeV.
- Whether it can meet other constraints (LHC, indirect detection, etc.) remains an open question.

A lesson on what can and what cannot be achieved with several Higgs doublets.