New renormalization scheme of two Higgs doublet models for future precision measurements

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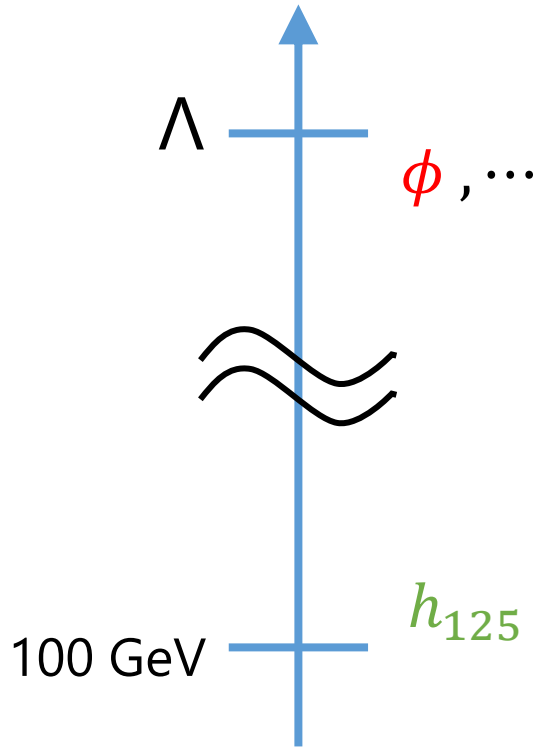
Phys. Lett. B 858 (2024) 139050

HPNP2025 (Osaka U.)　　2025/06/09-13

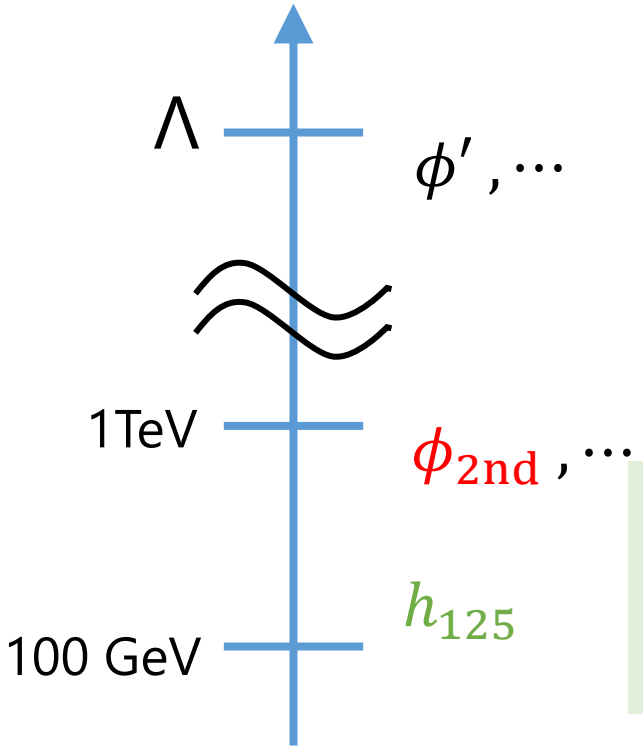
Current status of Higgs physics

Current data indicate nearly alignment $g_{hXX} \simeq g_{hVV}^{SM}$.

Alignment with decoupling



Alignment w/o decoupling



TeV scale NP
- Radiative seesaw
- WIMP DM
- EW baryogenesis

Alignment in Higgs Physics

- In extended Higgs models, alignment is an important feature.
- What is the "alignment"? → Modifications of the h_{125} properties from the SM predictions.
- Numerical definition → Deviations in h_{125}-couplings = mixing angle parameter (Tree level)

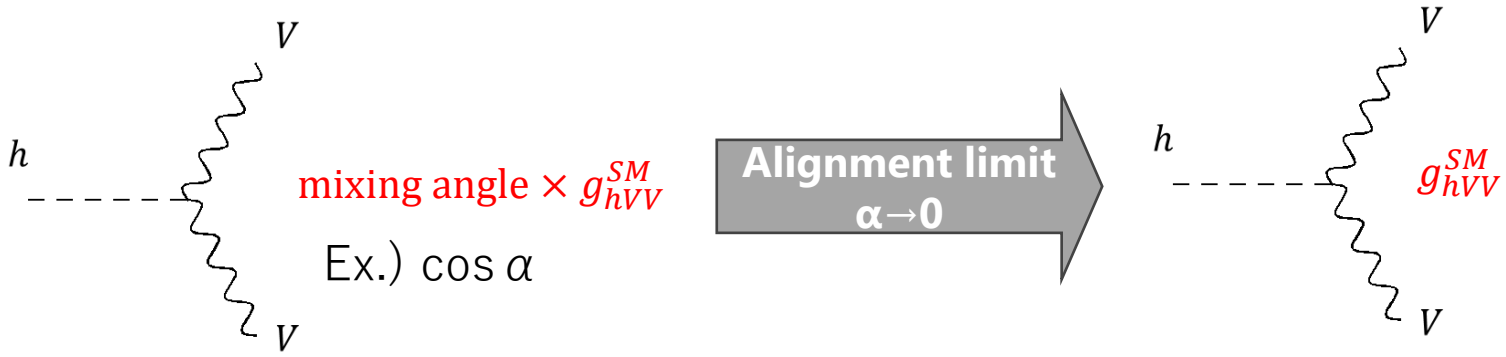
➢ Field mixing

$$\begin{pmatrix} h_1 \\ h_2 \end{pmatrix} = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} \begin{pmatrix} h \\ H \end{pmatrix}$$

➢ VEV mixing

$v^2 = v_1^2 + a_2 v_2^2 + \cdots$

(v≃246 GeV)



Mixing angles = "Alignmentness" [degree of alignment]

- The measurements of h_{125}-couplings provide information about alignmentness.

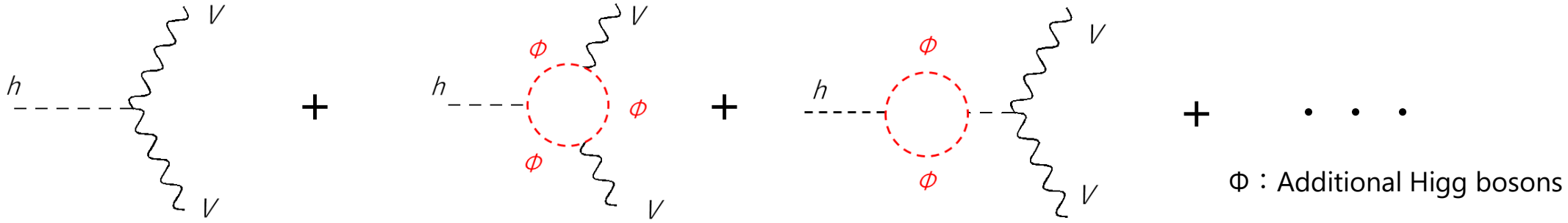
Alignment at loop level

- Precise calculations are necessary for comparison with future precision measurements.
- Loop corrections to various Higgs observables have been studied extensively.
- At loop level, mixing angles are no longer parameters that represents alignmentness,

$$g_{hXX}^{\mathrm{Reno}}\Big|_{\mathrm{no-mixing}} \neq g_{hXX}^{\mathrm{Reno}}\Big|_{\mathrm{SM}}$$

Mixing angle ≠ Alignmentness

because loop corrections receive contributions from various model parameters.



- It is complicated when comparing precision measurements with theoretical predictions.

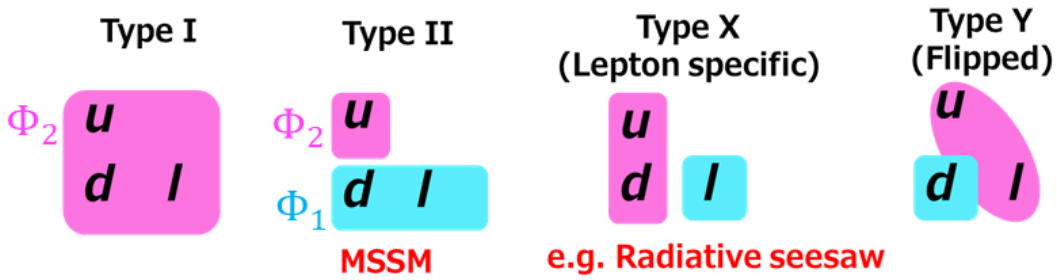
We propose a new renormalization scheme; alignmentness is expressed by mixing angles even at loop level.

Two Higgs doublet models (2HDMs)

We focus on 2HDMs .

- 2HDMs with the softly broken Z_2 symmetry as an example.

$\Phi_1 \rightarrow +\Phi_1$
$\Phi_2 \rightarrow -\Phi_2$
Can avoid FCNC.
4 types of Yukawa interactions ⇒



- CP-conserving case
- Mass eigenstates

Higgs basis

$$\Phi = \begin{pmatrix} G^+ \\ \frac{1}{\sqrt{2}}(h'_1 + v + iG^0) \end{pmatrix} \qquad \Phi' = \begin{pmatrix} H^+ \\ \frac{1}{\sqrt{2}}(h'_2 + iA) \end{pmatrix} \qquad \begin{pmatrix} h'_1 \\ h'_2 \end{pmatrix} = \begin{pmatrix} \cos(\beta-\alpha) & \sin(\beta-\alpha) \\ -\sin(\beta-\alpha) & \cos(\beta-\alpha) \end{pmatrix} \begin{pmatrix} H \\ h_{125} \end{pmatrix}$$

h_{125}, H, A, $H^\pm$

◆ 8 parameters in Higgs potential

$$m_h^2 \quad m_H^2 \quad m_A^2 \quad m_{H^\pm}^2 \quad M^2\left(\equiv \frac{m_3^2}{s_\beta c_\beta}\right) \quad v \quad \alpha \quad \beta$$

Tree level couplings & Decay rates

◆h_{125}-couplings

$$\kappa_X^h = \frac{g_{hXX}}{g_{hXX}^{\mathrm{SM}}}$$

Alignment limit

hWW, hZZ	$\kappa_V^h = \sin(\beta-\alpha)$	$\to 1$
hff	$\kappa_f^h = \sin(\beta-\alpha) + \zeta_f \cos(\beta-\alpha)$	$\to 1$

The form of κ_f^h depends on the Higgs field that couples to the fermion.

	ζ_u	ζ_d	ζ_e
Type-I	$\cot\beta$	$\cot\beta$	$\cot\beta$
Type-II	$\cot\beta$	$-\tan\beta$	$-\tan\beta$
Type-X (lepton specific)	$\cot\beta$	$\cot\beta$	$-\tan\beta$
Type-Y (flipped)	$\cot\beta$	$-\tan\beta$	$\cot\beta$

◆Partial decay rates of h_{125}

Deviations in decay rates are directly parameterized by mixing angles.

$$\Gamma_{\mathrm{2HDM}}[h \to XX] = \left(\kappa_X^h\right)^2 \Gamma_{\mathrm{SM}}[h \to XX]$$

Renormalization

Here, focus on on-shell renormalization in Higgs potential.

Counterterms

δm_{φ}^2 $(\varphi = h, H, A, H^{\pm})$ $\quad \delta m_{12}^2 \quad \delta v \quad \delta\alpha \quad \delta\beta$ 8 counterterms via parameter shifts

Field shift

$$\begin{pmatrix} H \\ h \end{pmatrix} \to \begin{pmatrix} 1+\delta Z_H & \delta Z_{Hh} \\ \delta Z_{hH} & 1+\delta Z_h \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix} \qquad \delta Z_{Hh} \equiv \delta C_{Hh} + \delta\alpha \qquad \delta Z_{hH} \equiv \delta C_{hH} - \delta\alpha$$

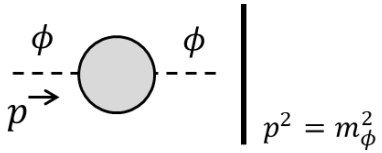
$$\begin{pmatrix} G^0 \\ A \end{pmatrix} \to \begin{pmatrix} 1+\delta Z_G & \delta Z_{GA} \\ \delta Z_{AG} & 1+\delta Z_A \end{pmatrix} \begin{pmatrix} G^0 \\ A \end{pmatrix} \qquad \delta Z_{GA} \equiv \delta C_{GA} + \delta\beta \qquad \delta Z_{AG} \equiv \delta C_{AG} - \delta\beta$$

$$\begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix} \to \begin{pmatrix} 1+\delta Z_{G^{\pm}} & \delta Z_{G^{\pm}H^{\mp}} \\ \delta Z_{H^{\pm}G^{\mp}} & 1+\delta Z_{H^{\pm}} \end{pmatrix} \begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix} \qquad \delta Z_{G^{\pm}H^{\mp}} \equiv \delta C_{G^{\pm}H^{\mp}} + \delta\beta \qquad \delta Z_{H^{\pm}G^{\mp}} \equiv \delta C_{H^{\pm}G^{\mp}} - \delta\beta$$

12 wave function renormalitions via field shifts

Renormalization conditions

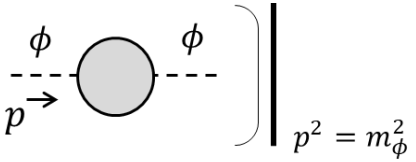
δm_ϕ^2



$= 0 \quad \Rightarrow \quad \delta m_\phi^2 = \tilde{\Pi}_{\phi\phi}^{1\mathrm{PI}}[m_\phi^2] \qquad \times 4 \qquad (\varphi = h, H, A, H^\pm)$

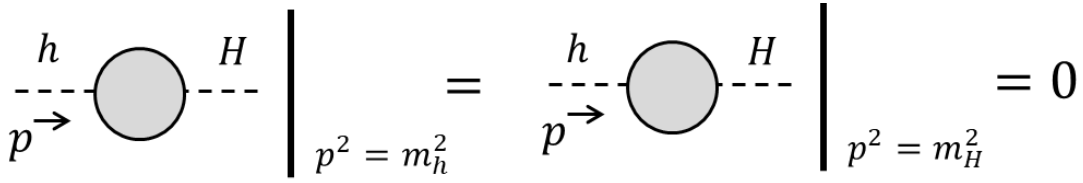
δZ_ϕ

$$\frac{\mathrm{d}}{\mathrm{d}p^2}$$



$= 0 \quad \Rightarrow \quad \delta Z_\phi = -\frac{\mathrm{d}}{\mathrm{d}p^2}\Pi_{\phi\phi}^{1\mathrm{PI}}[m_\phi^2] \qquad \times 6 \qquad (\phi = h, H, A, G^0, H^\pm, G^\pm)$

$\delta Z_{\phi_1\phi_2}, \delta Z_{\phi_2\phi_1}$ 2 × 3 on-shell conditions



$= 0 \quad \Rightarrow$

$$\delta Z_{Hh} = \frac{1}{(m_H^2 - m_h^2)}\Pi_{Hh}^{1\mathrm{PI}}[m_h^2]$$

$$\delta Z_{hH} = \frac{1}{(m_H^2 - m_h^2)}\Pi_{Hh}^{1\mathrm{PI}}[m_H^2] \qquad \times 3$$

Denner,Sack, Nuclear Physics B347 (1990) 203—216
Sirlin, Nuclear Physics B 71 (1974) 29-51
Pilaftsis, Nuclear Physics B 504 (1997) 61-107

δZ_{AG}, δZ_{GA}, $\delta Z_{H^\pm G^\mp}, \delta Z_{G^\mp H^\pm}$ are also determined by similar conditions.

S. Kanemura, Y. Okada, E. Senaha and C.P. Yuan, Phys. Rev. D 70 (2004) 115002
Marcel Krause, Robin Lorenz, Margarete Muhlleitner, Rui Santosb, Hanna ZiescheaJHEP09(2016)143

δM^2 by the MS-bar scheme, which absorbs only the divergent part in the *hhh*.

δv by EW renormalization. M. Bohm, H. Spiesberger, W. Hollik, Fortschr. Phys. 34 (1986) 687.

We determine $\delta\alpha, \delta\beta$ by other conditions.

How to determine counterterm of $\delta\beta - \delta\alpha$

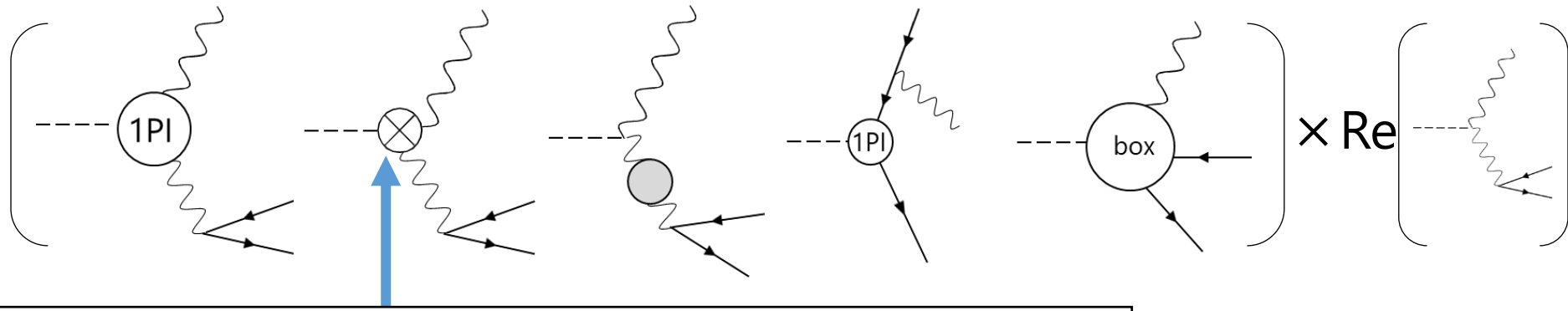
$$\delta\beta\alpha \equiv \delta\beta - \delta\alpha$$

Although the renormalized hZZ vertex includes $\delta\beta\alpha$, the on-shell momentum is not allowed..

➡ We determine $\delta\beta\alpha$ by $\Gamma[h \to ZZ^* \to Z\tau\tau]$.

$$\Gamma[h(\to ZZ^*) \to Z\tau\tau] = \Gamma_0[h \to Z\tau\tau](1 + \Delta_{EW}^{Z\tau\tau})$$

$$\Delta_{EW}^{Z\tau\tau} = \frac{2}{\left|\quad\right|^2} \text{Re}$$



$$\delta\Gamma^1_{hVV} = \frac{2m_V^2}{v} s_{\beta-\alpha}\left[\frac{\delta m_V^2}{m_V^2} - \frac{\delta v}{v} + \delta Z_V + \frac{1}{2}\delta Z_h + \cot(\beta-\alpha)\,(\delta\beta - \delta\alpha) + \cot(\beta-\alpha)\delta Z_{Hh}\right]$$

Renormalization conditions:

$$\Delta_{\text{EW}}^{Z\tau\tau} = \Delta_{\text{EW}}^{Z\tau\tau}\Big|_{\text{SM}}$$

➡

$$\delta\beta\alpha = -\tan(\beta-\alpha)\left(\frac{\delta m_Z^2}{m_Z^2} - \frac{\delta v}{v} + \delta Z_Z + \frac{1}{2}\delta Z_h\right) - \delta Z_{Hh}$$ + (1PI box-vertex-self energy diagrams)

NP loop corrections are absorbed into δβα via renormalization.

Decay rates ($h \to VV^*$)

Renormalization conditions: $\Delta_{\rm EW}^{Z\tau\tau} = \Delta_{\rm EW}^{Z\tau\tau}\Big|_{\rm SM}$

- Deviations in decay rate can be expressed by only mixing angles!!

$$\Gamma[h \to Z\tau\tau] = \Gamma_0[h \to Z\tau\tau]\left(1 + \Delta_{\rm EW}^{Z}\right) = \sin^2(\beta-\alpha)\Gamma[h \to Z\tau\tau]\Big|_{\rm SM}$$

$\sin^2(\beta-\alpha)\Gamma_0[h \to Z\tau\tau]\big|_{\rm SM}$ $\qquad$ $\Delta_{\rm EW}^{Z\tau\tau}\big|_{\rm SM}$

- Input measured $\Gamma[h \to Z\tau\tau]$ values → We obtain information of alignmentness (sin(β−α))

- $\Gamma[h \to Wff']$ is a predicted quantity.

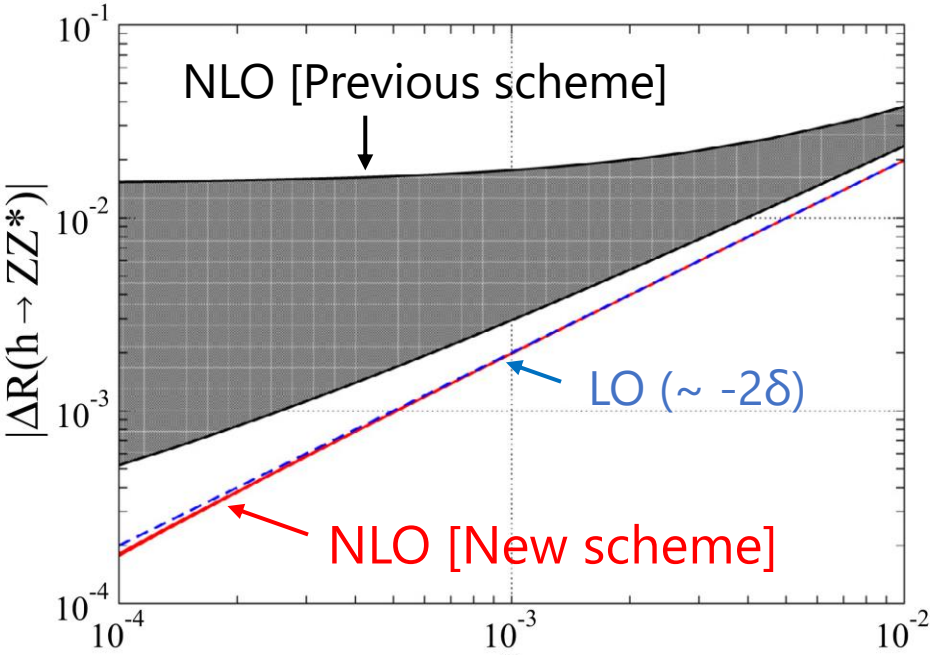
$$\Delta_{\rm EW}^{Wff} = 2\left(\frac{\delta m_W^2}{m_W^2} - \frac{\delta m_Z^2}{m_Z^2} + \delta Z_W - \delta Z_Z\right)_{\rm fin} + \Delta_{\rm rem}^{Wff} - \Delta_{\rm rem}^{Z\ell\ell} + \Delta_{\rm EW}^{Z\ell\ell}\Big|_{\rm SM},$$

The counterterms common with the hZZ vertex ($\delta Z_h, \delta v$) are absorbed into $\delta\beta\alpha$ ⇒ cancel out

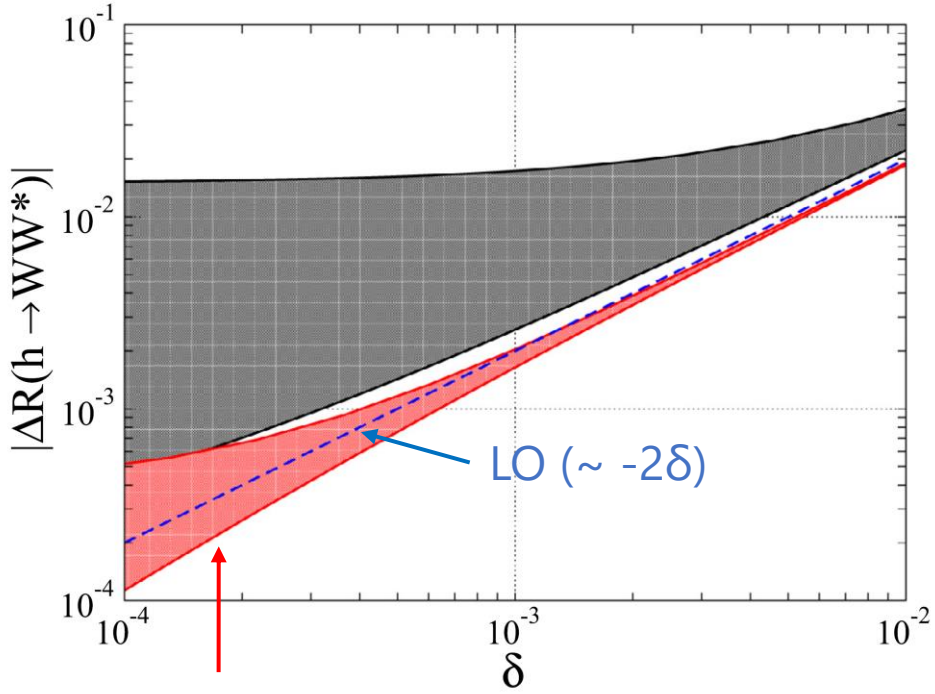
Numerical results

$$\Delta R \equiv \frac{\Gamma[h \to XY] - \Gamma[h \to XY]|_{\rm SM}}{\Gamma[h \to XY]|_{\rm SM}}$$

Type-I $m_{H^\pm} = m_A = m_H = 300\ \text{GeV}, \tan\beta = 2, \cos(\beta-\alpha) > 0,$ Scan M^2 under theoretical constraints.



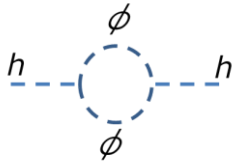
$$\delta \equiv 1 - \sin(\beta - \alpha)$$



$$\Delta R[h \to Wff'] \simeq -2\delta + (\Delta^W_{EW} - \Delta^W_{EW}\big|_{\rm SM})$$

The new scheme result (h→ZZ*) shows quite good agreement with the LO result.

δZ_h-contribution vanishes in New scheme prediction.
The contribution is renormalized by δβα.



$$\sim \frac{1}{16\pi^2}\frac{m_\Phi^2}{v^2}\left(1 - \frac{M^2}{m_\Phi^2}\right)^2$$

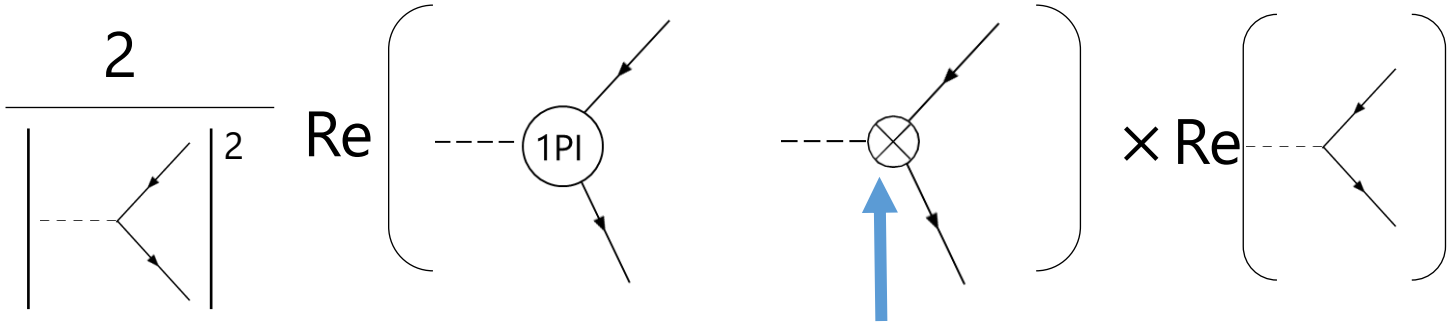
How to determine δβ

$\delta\beta$

We determine $\delta\beta$ by $\Gamma[h \to \tau\tau]$.

$$\Gamma[h(\to \tau\tau] = \Gamma_0[h \to \tau\tau](1 + \Delta^{\tau}_{EW})$$

$$\Delta^{\tau}_{EW} = \frac{2}{\left|\quad\right|^2}\ \mathrm{Re}\left[\ \text{1PI}\ \right] \times \mathrm{Re}\left[\quad\right]$$

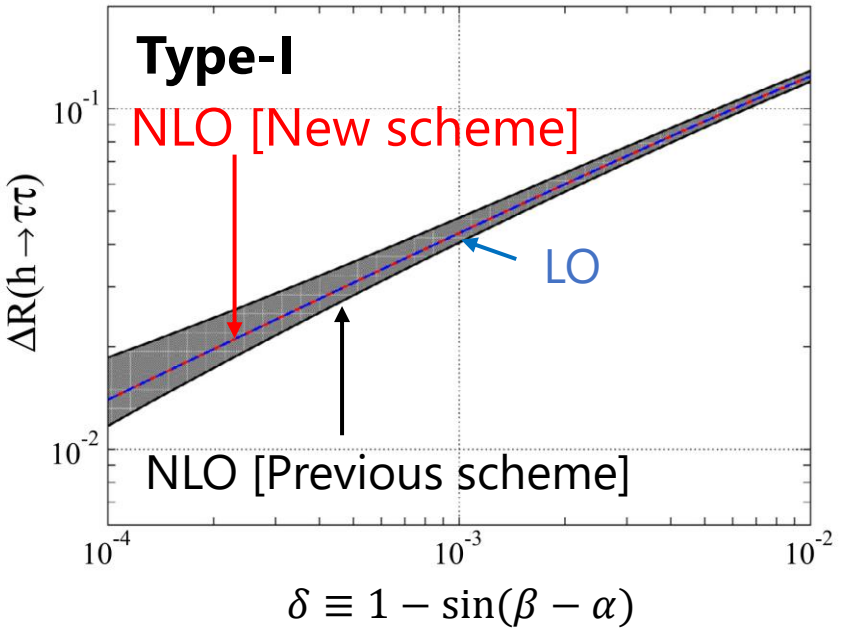


$$\delta\Gamma^{S}_{hff} = -\frac{m_f}{v}\xi^f_h\left(\frac{\delta m_f}{m_f} - \frac{\delta v}{v} + \delta Z^f_V + \frac{1}{2}\delta Z_h\right) - \frac{m_f}{v}\xi^f_H\left(\delta Z_{Hh} + (\delta\beta - \delta\alpha)\right) + \frac{m_f}{v}c_{\beta-\alpha}\xi_{f\beta}\delta\beta,$$

Renormalization conditions: $\Delta^{\tau}_{\mathrm{EW}} = \Delta^{\tau}_{\mathrm{EW}}\Big|_{\mathrm{SM}}$

Deviations in decay rate can be expressed by only mixing angles!!

$$\Gamma[h \to \tau\tau] = (\kappa^h_\tau)^2\,\Gamma[h \to \tau\tau]\Big|_{\mathrm{SM}}$$



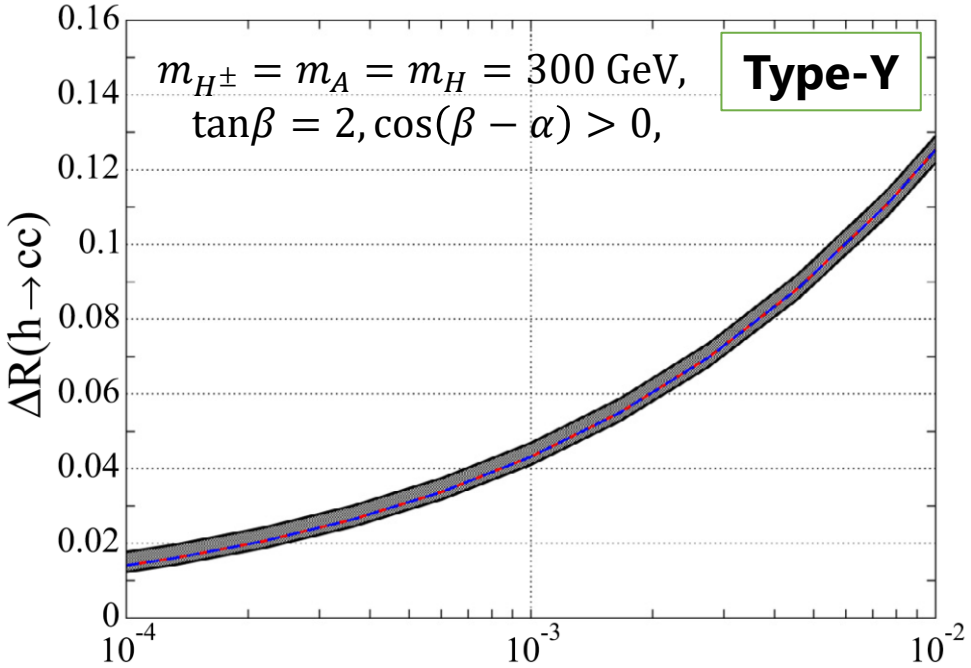
Deviations in $h \to ff$

Loop corrections depend on the zeta-factor.

$$\kappa_f^h = \sin(\beta-\alpha) + \zeta_f \cos(\beta-\alpha)$$

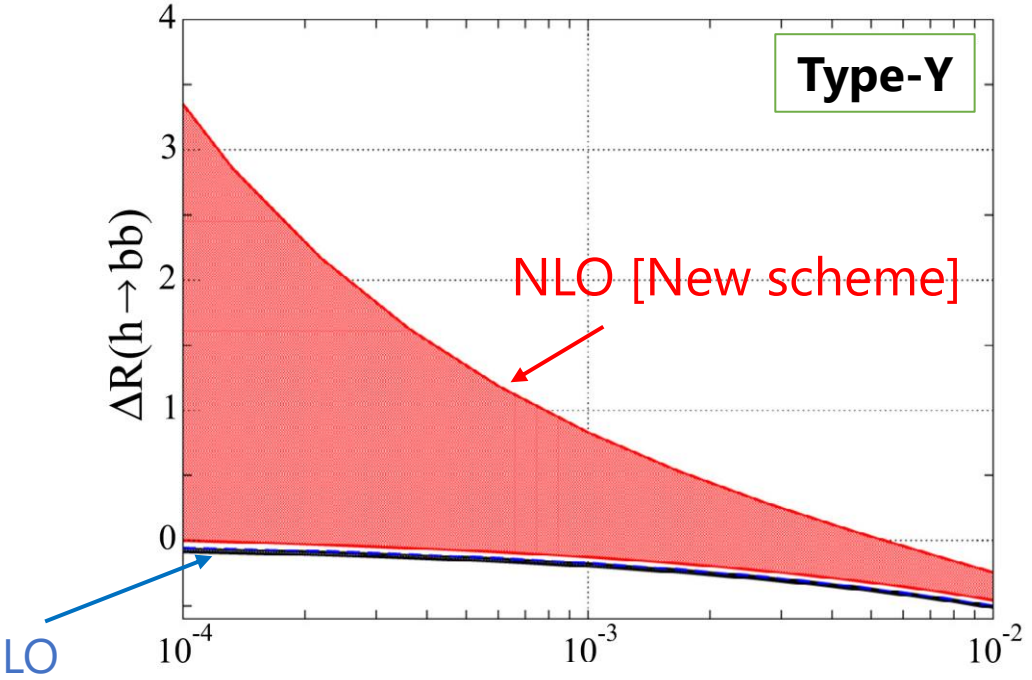
	ζ_c	ζ_b	ζ_τ
Type-Y	$\cot\beta$	$-\tan\beta$	$\cot\beta$

In case with $\zeta_f = \zeta_\tau$



$$\delta \equiv 1 - \sin(\beta-\alpha)$$

In case with $\zeta_f \neq \zeta_\tau$



$$\delta \equiv 1 - \sin(\beta-\alpha)$$

$$\Delta_{\mathrm{EW}}^{\mathrm{f}} = 2\left(\frac{\delta m_f}{\mathrm{m_f}} - \frac{\delta m_\tau}{m_\tau} + \delta Z_V^f - \delta Z_V^\tau\right) + \Delta_{rem}^f - \Delta_{rem}^\tau + \Delta_{\mathrm{EW}}^\tau\Big|_{\mathrm{SM}}$$

$$\Delta_{EW}^f \supset \frac{2}{\cos(\beta-\alpha)} \zeta_f \delta Z_h$$

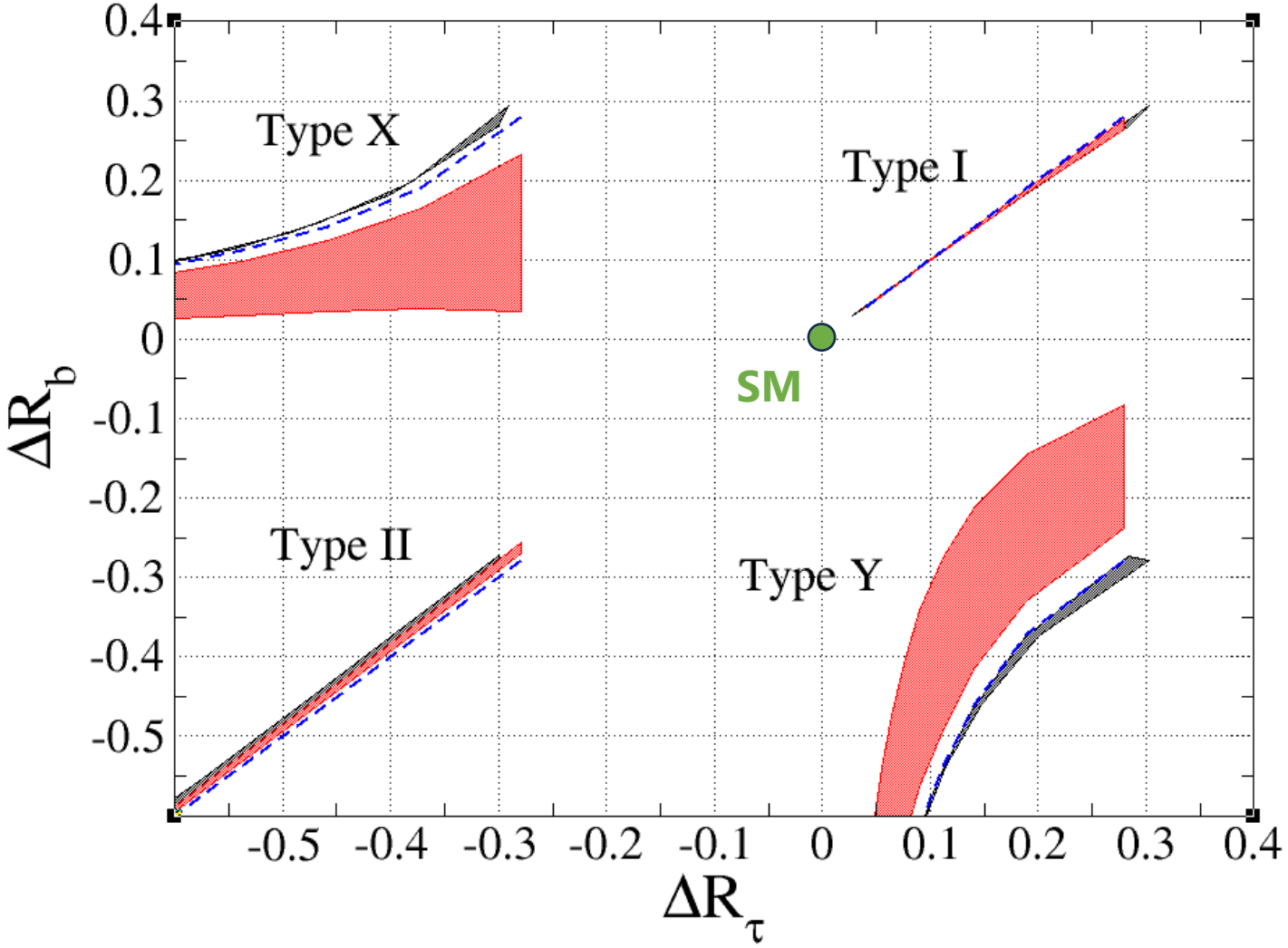
δZ_h-contribution vanishes in New scheme prediction same as h→ZZ*.

Δ_{EW}^f includes contributions with $\frac{1}{\cos(\beta-\alpha)}$ factor.

⇒ The loop corrections enhance decay rate as δ→0.

h→bb vs h → ττ

sin(β−α)=0.99 (ΔR(h→Zττ)=0.98)



$m_H = m_A = m_{H+} = 300$ GeV
$1 < \tan\beta < 5$, $0 < M < 300$ GeV
$\cos(\beta-\alpha)>0$

LHC Run-II (2σ, CMS):
$\kappa_b = 0.99\ (+0.34, -0.32)$
$\kappa_\tau = 0.92\ (\pm 0.16)$ Nature 607 (2022) 60

HL-LHC :
$\kappa_b = (\pm 3.6\%)$
$\kappa_\tau = (\pm 1.9\%)$ 1902.00134

ILC500:
$\kappa_b = (\pm 0.59\%)$
$\kappa_\tau = (\pm 1.9\%)$ Snowmass 2021 report [arXiv:2203.07622]

Summary

- We develop a renormalization scheme to make "alignmentness" clear by comparing observables.

- In the new scheme, we determine δα and δβ by decay processes "h→Zττ" and "h→ττ".

$$\Gamma[h \to Z\tau\tau] = (\kappa_Z^h)^2 \Gamma[h \to Z\tau\tau]\Big|_{\mathrm{SM}}$$

$$\Gamma[h \to \tau\tau] = (\kappa_\tau^h)^2 \, \Gamma[h \to \tau\tau]\Big|_{\mathrm{SM}}$$

Input experimental values

We obtain information of alignmentness (mixing angles)

- The new scheme results of $h \to Zff$, $h \to \tau\tau$ show quite good agreement with the LO result.

- $\Delta R_{NLO}(h \to Wff')$ follows the behavior of the LO prediction, and loop corrections is small.

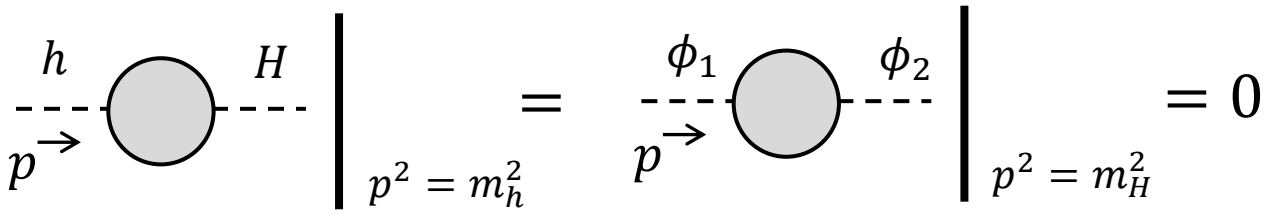
- $h \to ff$ depends on the type of Yukawa interactions and the type of fermions.
  $\Delta R_{NLO}(h \to ff)$ can be quite large. ⇒ Parameter regions allowing large deviations may be testable.

Renormalization conditions in field mixings

Previous scheme (KOSY scheme)

$\delta C_{Hh}, \delta C_{hH}, \delta\alpha$

2 on-shell conditions



1 assumed condition; $\delta C_{Hh} = \delta C_{hH}$

3 counterterms can be determined

$$\delta\alpha = \frac{1}{2(m_H^2 - m_h^2)}\left(\tilde{\Pi}^{1\mathrm{PI}}_{Hh}[m_h^2] + \tilde{\Pi}^{1\mathrm{PI}}_{Hh}[m_H^2]\right)$$

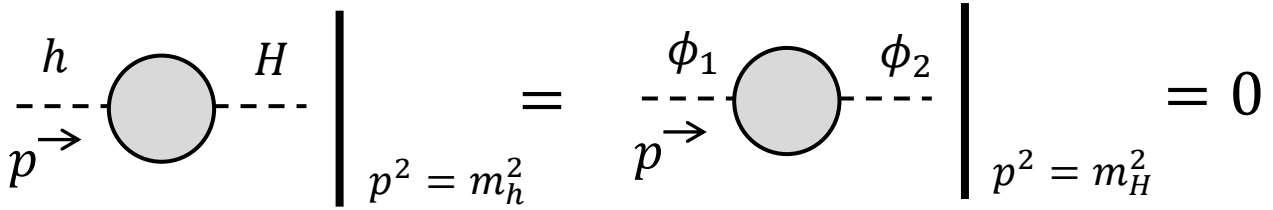
$$\delta C_{hH} = \delta C_{Hh} = \frac{1}{m_H^2 - m_h^2}\left(\Pi^{1\mathrm{PI}}_{hH}[m_h^2] - \Pi^{1\mathrm{PI}}_{hH}[m_H^2]\right)$$

δC_{AG}, δC_{GA}, $\delta C_{H^\pm G^\mp}, \delta C_{G^\mp H^\pm}, \delta\beta$ are also determined by similar conditions.

New scheme

$\delta Z_{Hh}, \delta Z_{hH}$

2 on-shell conditions



$$\delta Z_{Hh} = \frac{1}{(m_H^2 - m_h^2)}\Pi^{1\mathrm{PI}}_{Hh}[m_h^2]$$

$$\delta Z_{hH} = \frac{1}{(m_H^2 - m_h^2)}\Pi^{1\mathrm{PI}}_{Hh}[m_H^2]$$

δZ_{AG}, δZ_{GA}, $\delta Z_{H^\pm G^\mp}, \delta Z_{G^\mp H^\pm}$ are also determined by similar conditions.

We determine $\delta\alpha, \delta\beta$ by other conditions.

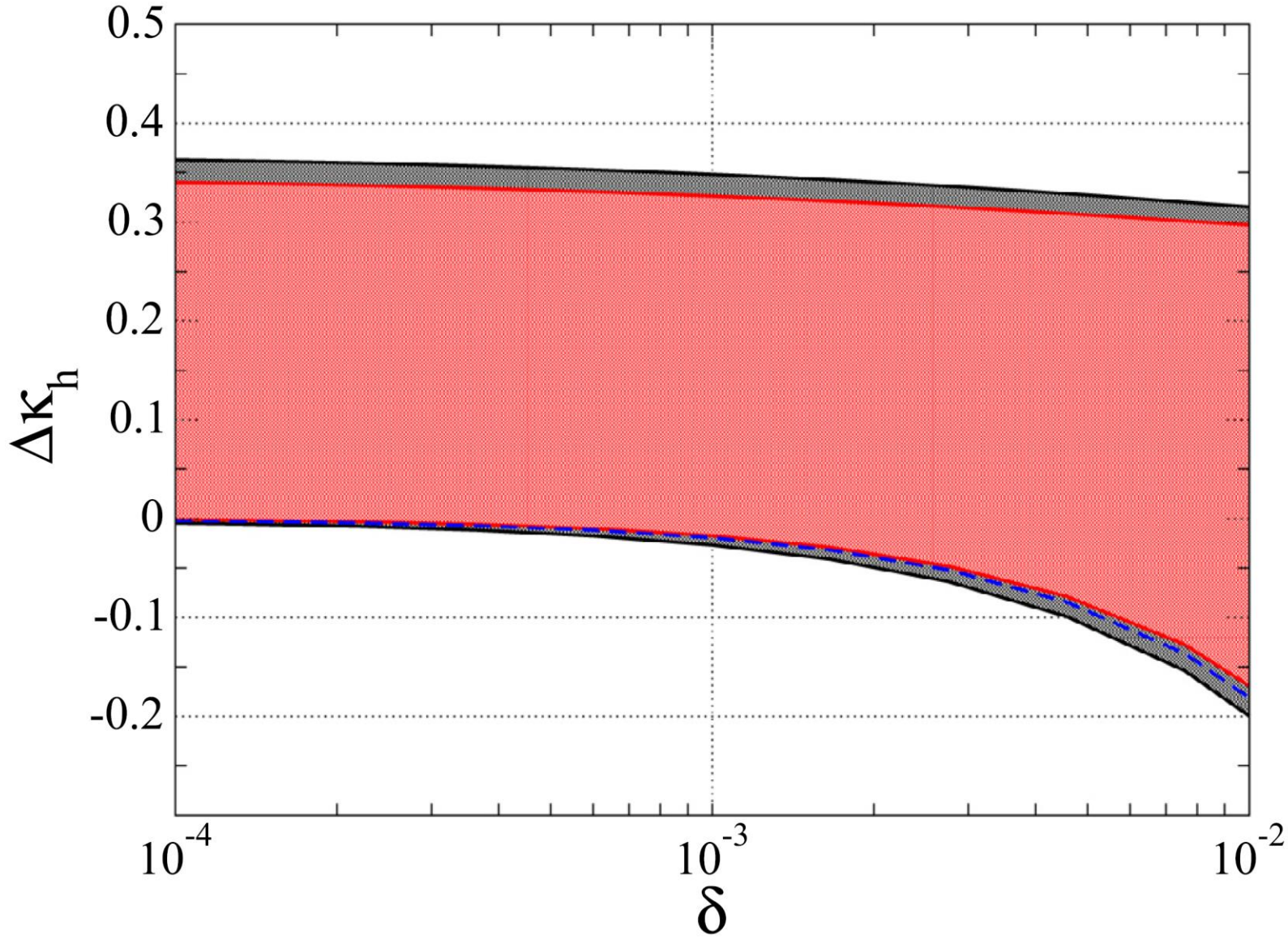
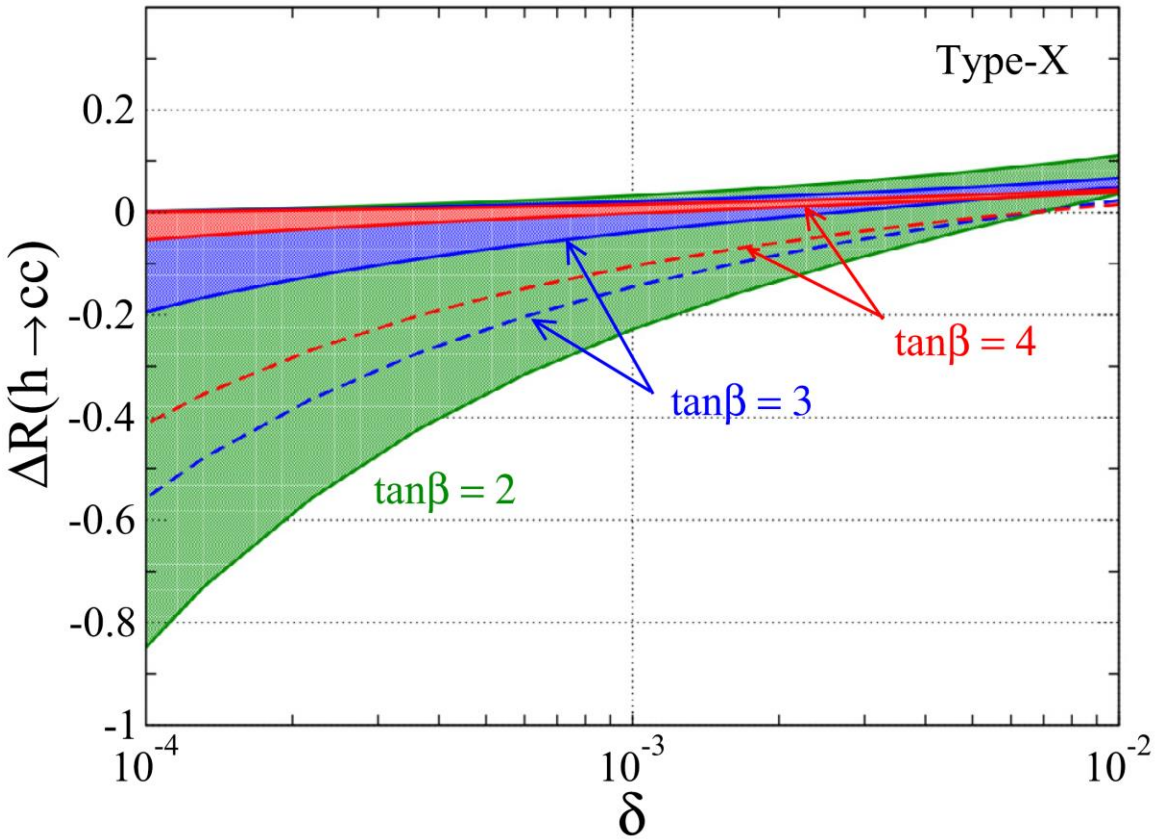


Fig. 3. Deviation in the hhh coupling $\Delta\kappa_h$ defined in Eq. (69) as a function of δ in the Type-I 2HDM with $m_{H^\pm} = m_A = m_H = 300$ GeV, $\cos(\beta-\alpha) > 0$, $\tan\beta = 2$ and scan M^2. We take the renormalization scale $\mu = 300$ GeV.

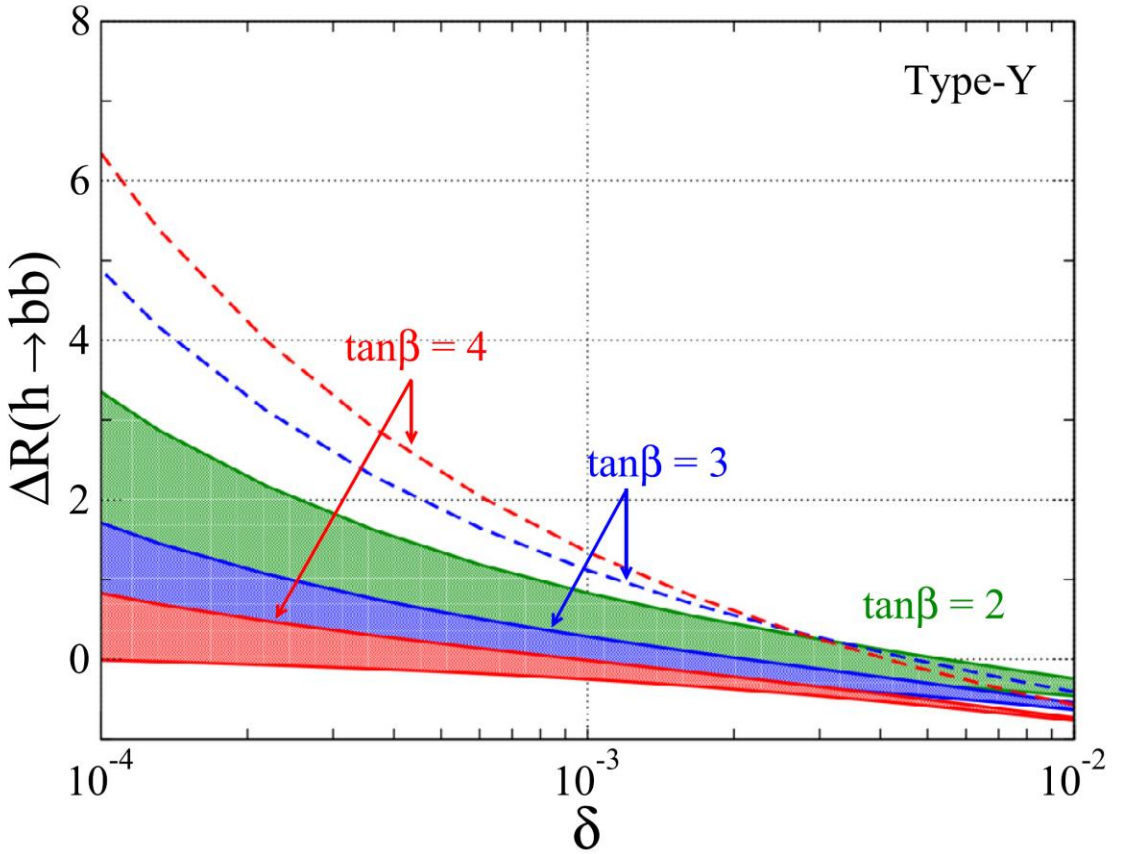
Tanβ dependence

	ζ_c	ζ_b	ζ_τ
Type-X	$\cot\beta$	$\cot\beta$	$-\tan\beta$

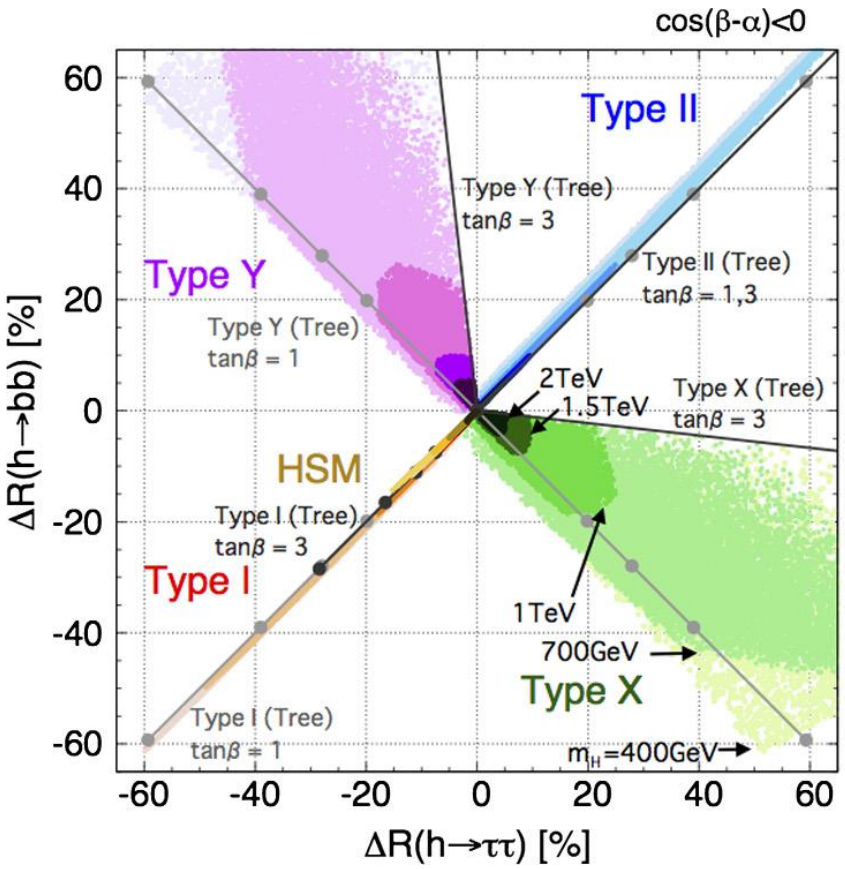


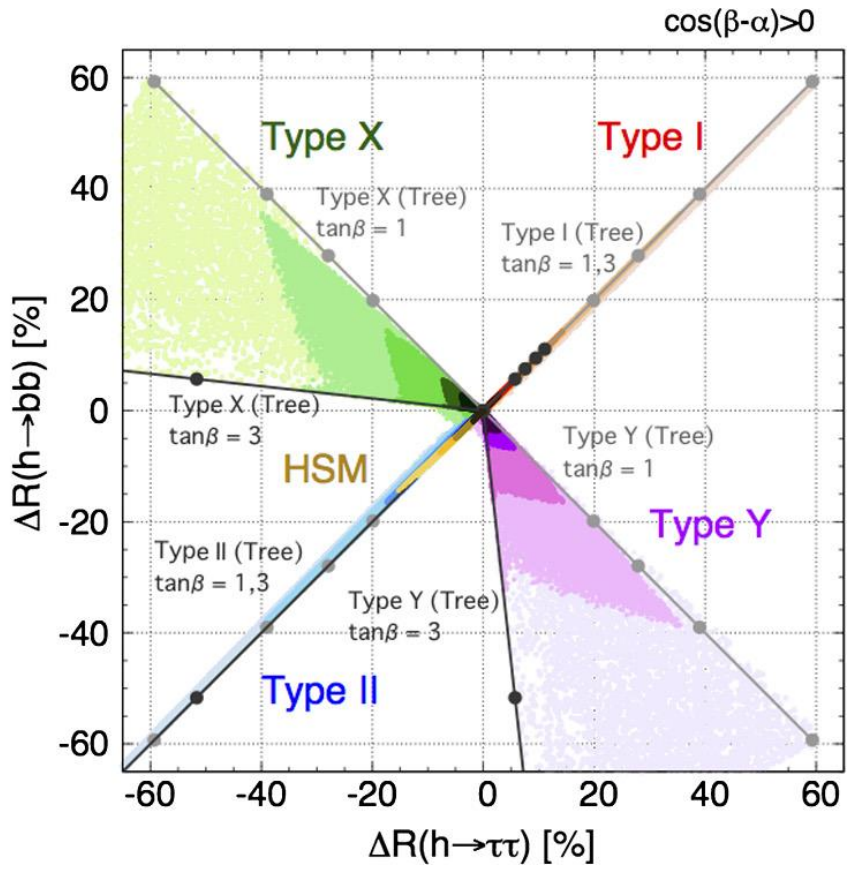
$$\Delta_{EW}^{f} \supset \frac{2}{\cos(\beta-\alpha)}\frac{1}{\tan\beta}\delta Z_h$$

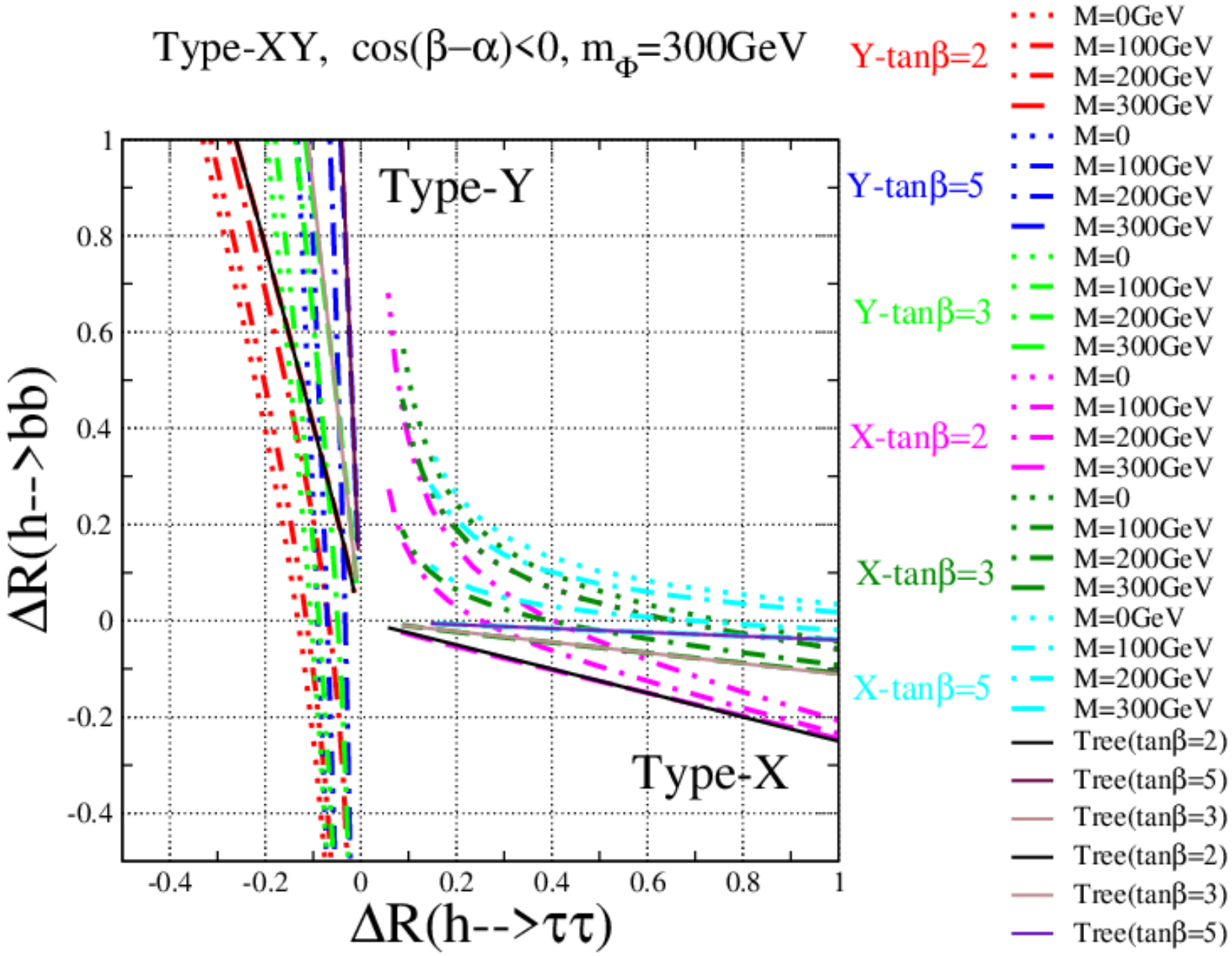
	ζ_c	ζ_b	ζ_τ
Type-Y	$\cot\beta$	$-\tan\beta$	$\cot\beta$



$$\Delta_{EW}^{f} \supset \frac{2}{\cos(\beta-\alpha)}\tan\beta\delta Z_h$$


cos(β-α)<0
Type II
Type Y (Tree)
tanβ = 3
Type Y
Type II (Tree)
tanβ = 1,3
Type Y (Tree)
tanβ = 1
2TeV
1.5TeV
Type X (Tree)
tanβ = 3
HSM
Type I (Tree)
tanβ = 3
Type I
1TeV
700GeV
Type X
Type I (Tree)
tanβ = 1
m_H=400GeV
ΔR(h→bb) [%]
ΔR(h→ττ) [%]
60
40
20
0
-20
-40
-60


cos(β-α)>0
Type X
Type I
Type X (Tree)
tanβ = 1
Type I (Tree)
tanβ = 1,3
Type X (Tree)
tanβ = 3
HSM
Type Y (Tree)
tanβ = 1
Type Y
Type II (Tree)
tanβ = 1,3
Type Y (Tree)
tanβ = 3
Type II
ΔR(h→bb) [%]
ΔR(h→ττ) [%]
60
40
20
0
-20
-40
-60

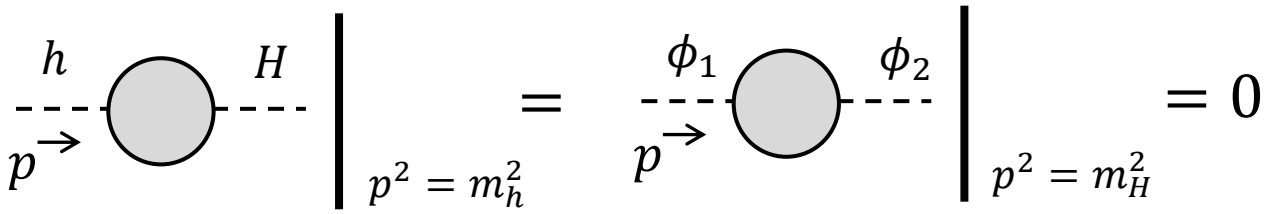

Type-XY, cos(β–α)<0, m_Φ=300GeV
Type-Y
Type-X
ΔR(h-->bb)
ΔR(h-->ττ)
-0.4
-0.2
0
0.2
0.4
0.6
0.8
1
Y-tanβ=2
Y-tanβ=5
Y-tanβ=3
X-tanβ=2
X-tanβ=3
X-tanβ=5
M=0GeV
M=100GeV
M=200GeV
M=300GeV
M=0
M=100GeV
M=200GeV
M=300GeV
M=0
M=100GeV
M=200GeV
M=300GeV
M=0
M=100GeV
M=200GeV
M=300GeV
M=0
M=100GeV
M=200GeV
M=300GeV
M=0GeV
M=100GeV
M=200GeV
M=300GeV
Tree(tanβ=2)
Tree(tanβ=5)
Tree(tanβ=3)
Tree(tanβ=2)
Tree(tanβ=3)
Tree(tanβ=5)

Renormalization conditions in field mixings

Previous scheme (KOSY scheme)

$\delta C_{Hh}, \delta C_{hH}, \delta\alpha$

2 on-shell conditions



3 counterterms can be determined

$$\delta\alpha = \frac{1}{2(m_H^2 - m_h^2)}\left(\tilde{\Pi}^{1\mathrm{PI}}_{Hh}[m_h^2] + \tilde{\Pi}^{1\mathrm{PI}}_{Hh}[m_H^2]\right)$$

$$\delta C_{hH} = \delta C_{Hh} = \frac{1}{m_H^2 - m_h^2}\left(\Pi^{1\mathrm{PI}}_{hH}[m_h^2] - \Pi^{1\mathrm{PI}}_{hH}[m_H^2]\right)$$

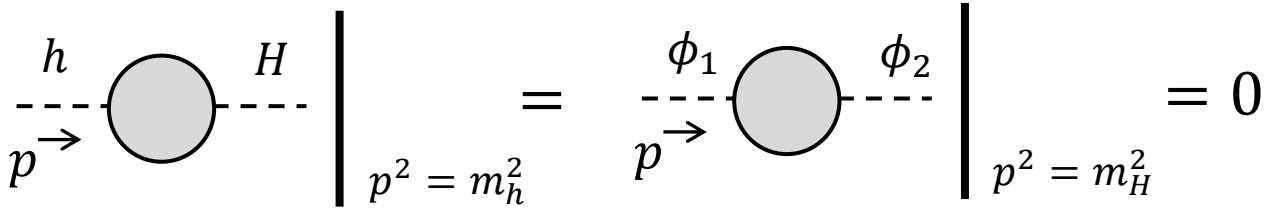
1 assumed condition; $\delta C_{Hh} = \delta C_{hH}$

δC_{AG}, δC_{GA}, $\delta C_{H^\pm G^\mp}, \delta C_{G^\mp H^\pm}, \delta\beta$ are also determined by similar conditions.

New scheme

$\delta Z_{Hh}, \delta Z_{hH}$

2 on-shell conditions



$$\delta Z_{Hh} = \frac{1}{(m_H^2 - m_h^2)}\Pi^{1\mathrm{PI}}_{Hh}[m_h^2]$$

$$\delta Z_{hH} = \frac{1}{(m_H^2 - m_h^2)}\Pi^{1\mathrm{PI}}_{Hh}[m_H^2]$$

δZ_{AG}, δZ_{GA}, $\delta Z_{H^\pm G^\mp}, \delta Z_{G^\mp H^\pm}$ are also determined by similar conditions.

We determine $\delta\alpha, \delta\beta$ by other conditions.

Renormalization

Here, focus on on-shell renormalization in Higgs potential.

◆ Counterterms

δm_{φ}^2 $(\varphi = h, H, A, H^{\pm})$ δm_{12}^2 δv $\delta\alpha$ $\delta\beta$ 8 counterterms via parameter shifts

Field shift

$$\begin{pmatrix} H \\ h \end{pmatrix} \to \begin{pmatrix} 1+\delta Z_H & \delta Z_{Hh} \\ \delta Z_{hH} & 1+\delta Z_h \end{pmatrix} \begin{pmatrix} H \\ h \end{pmatrix} \qquad \delta Z_{Hh} \equiv \delta C_{Hh} + \delta\alpha \qquad \delta Z_{hH} \equiv \delta C_{hH} - \delta\alpha$$

$$\begin{pmatrix} G^0 \\ A \end{pmatrix} \to \begin{pmatrix} 1+\delta Z_G & \delta Z_{GA} \\ \delta Z_{AG} & 1+\delta Z_A \end{pmatrix} \begin{pmatrix} G^0 \\ A \end{pmatrix} \qquad \delta Z_{GA} \equiv \delta C_{GA} + \delta\beta \qquad \delta Z_{AG} \equiv \delta C_{AG} - \delta\beta$$

$$\begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix} \to \begin{pmatrix} 1+\delta Z_{G^{\pm}} & \delta Z_{G^{\pm}H^{\mp}} \\ \delta Z_{H^{\pm}G^{\mp}} & 1+\delta Z_{H^{\pm}} \end{pmatrix} \begin{pmatrix} G^{\pm} \\ H^{\pm} \end{pmatrix} \qquad \delta Z_{G^{\pm}H^{\mp}} \equiv \delta C_{G^{\pm}H^{\mp}} + \delta\beta \qquad \delta Z_{H^{\pm}G^{\mp}} \equiv \delta C_{H^{\pm}G^{\mp}} - \delta\beta$$

12 wave function renormalitions via field shifts

文献リスト（特に非対角wave functions, mixing angles に関する文献）

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KOSY

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033003