Effective field theory for type II seesaw model —symmetric phase v.s. broken phase—

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Currently under reviewed in PRD

Collaborated with

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Which is better EFT, SMEFT or HEFT?

SMEFT v.s. HEFT

Standard Model Effective Field Theory (SMEFT)

$$\mathscr{L}_{\text{SMEFT}} = (D_\mu H)^\dagger D^\mu H + \frac{C_{\varphi\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \cdots$$

Symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$

Field contents SM fields *before* the EW symmetry breaking: $H,\ W_\mu^a,\ B_\mu,\ \cdots$

Power counting $1/\Lambda$ expansion

Higgs Effective Field Theory (HEFT)

$$U = \exp\left(\frac{i}{v}\varphi^a\tau^a\right)$$

φ^a : would-be NG boson

$$\mathscr{L}_{\text{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{v^2}{4}\text{Tr}[(D_\mu U)^\dagger D^\mu U]\left(1 + 2\kappa_W\frac{h}{v} + \kappa_W^{(2)}\left(\frac{h}{v}\right)^2 + \cdots\right) + \cdots$$

Symmetry $SU(3)_C \times SU(2)_L \times U(1)_Y$ * $SU(2)_L \times U(1)_Y$ is non-linearly realized

Field contents SM fields *after* the EW symmetry breaking: $h,\ W_\mu^\pm,\ Z_\mu,\ A_\mu, \cdots$

Power counting Loop expansion

What is each of SMEFT and HEFT particulary good at?

SMEFT v.s. HEFT

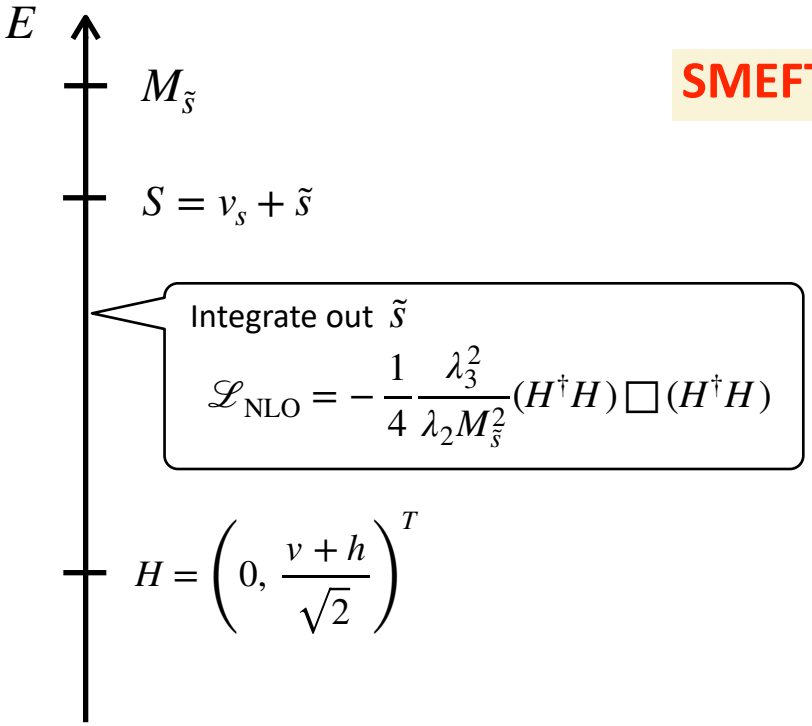
SM + real singlet scalar

$$\mathscr{L} = (D_\mu H)^\dagger D^\mu H + \frac{1}{2}\partial_\mu S \partial^\mu S - V(H,S)$$

$S \in (\mathbf{1},\mathbf{1},0)$, real

Buchalla et al. 1608.03564

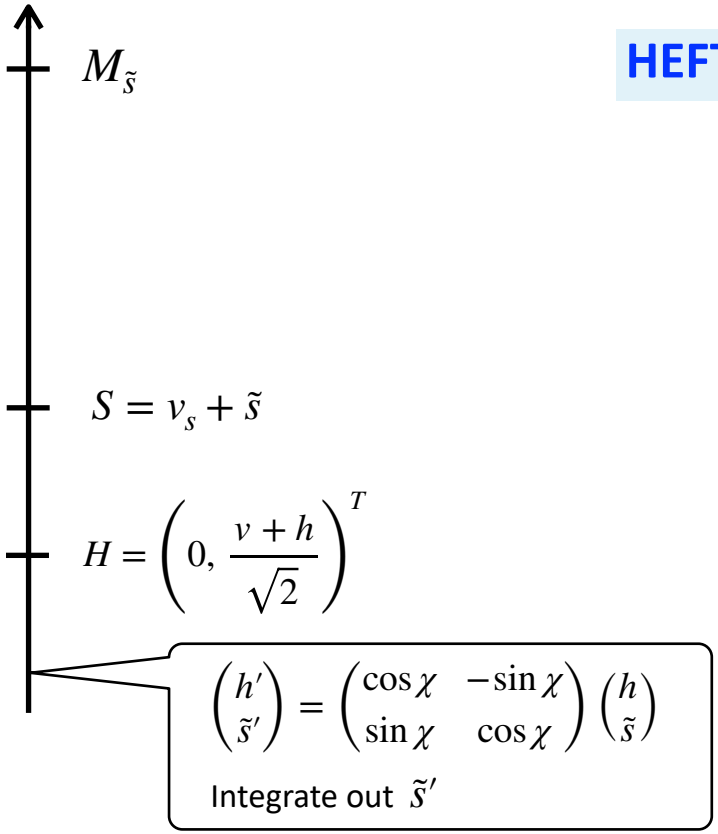
$$V(H,S) = -\frac{\mu_1^2}{2}H^\dagger H - \frac{\mu_2^2}{4}S^2 + \frac{\lambda_1}{4}(H^\dagger H)^2 + \frac{\lambda_2}{16}S^4 + \frac{\lambda_3}{4}H^\dagger H S^2$$



SMEFT

$$(D_\mu H)^\dagger D^\mu H - \frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_{\tilde{s}}^2}(H^\dagger H)\Box(H^\dagger H)$$

$$\mathscr{L}_{\rm SMEFT} = (D_\mu H)^\dagger D^\mu H + \frac{C_{\varphi\Box}}{\Lambda^2}(H^\dagger H)\Box(H^\dagger H) + \cdots$$



HEFT

$$\frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(2c_\chi\frac{h}{v} + \left(c_\chi^4 - s_\chi^3 c_\chi\frac{v}{v_s}\right)\left(\frac{h}{v}\right)^2 + \mathcal{O}(h^3)\right)$$

$$\mathscr{L}_{\rm HEFT} = \frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(1 + 2\kappa_W\frac{h}{v} + \kappa_W^{(2)}\left(\frac{h}{v}\right)^2 + \cdots\right) + \cdots$$

SMEFT v.s. HEFT

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E

SMEFT

$M_{\tilde{s}}$

$S = v_s + \tilde{s}$

Integrate out $\tilde{s}$

$$\mathscr{L}_{\rm NLO} = -\frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_{\tilde{s}}^2}(H^\dagger H)\Box(H^\dagger H)$$

$H = \left(0, \frac{v+h}{\sqrt{2}}\right)^T$

$$(D_\mu H)^\dagger D^\mu H - \frac{1}{4}\frac{\lambda_3^2}{\lambda_2 M_{\tilde{s}}^2}(H^\dagger H)\Box(H^\dagger H)$$

$$\supset m_W^2 W_\mu^- W^{+\mu}\left((2-\chi^2)\frac{h}{v} + (1-2\chi^2)\left(\frac{h}{v}\right)^2 + \mathcal{O}(h^3)\right)$$

$$\chi^2 \doteq \frac{v^2}{2}\frac{\lambda_3^2}{\lambda_2 M_{\tilde{s}}^2}$$

HEFT

$M_{\tilde{s}}$

$S = v_s + \tilde{s}$

$H = \left(0, \frac{v+h}{\sqrt{2}}\right)^T$

$$\begin{pmatrix} h' \\ \tilde{s}' \end{pmatrix} = \begin{pmatrix} \cos\chi & -\sin\chi \\ \sin\chi & \cos\chi \end{pmatrix}\begin{pmatrix} h \\ \tilde{s} \end{pmatrix}$$

Integrate out $\tilde{s}'$

$m_W^2 W_\mu^- W^{+\mu}$

$$\frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(2c_\chi\frac{h}{v} + \left(c_\chi^4 - s_\chi^3 c_\chi \frac{v}{v_s}\right)\left(\frac{h}{v}\right)^2 + \mathcal{O}(h^3)\right)$$

$$c_\chi \simeq 1 - \frac{\chi^2}{2}, \quad s_\chi \simeq 0$$

SMEFT v.s. HEFT

- Specific models

	SMEFT	HEFT
SM + singlet scalar	G. Buchalla, et. al., 1608.03564	
SM + triplet scalar	T. Cohen, et. al., 2008.08597	H. Song, et. al., 2412.00355
type-II seesaw model	X. Li, et. al., 2201.05082	Today's talk
Two Higgs doublet model	S. Dawson, et. al., 2305.07689 Ian Banta, et. al., 2304.09884	G. Buchalla, et. al., 2312.13885

- General discussion

Models that cannot be matched to SMEFT but only to HEFT

R. Alonso et. al., 1605.03602	Target space of the scalar kinetic term
A. Falkowski et. al., 1902.05936	Singularity of scalar potential
T. Cohen, et. al., 2008.08597	Both scalar kinetic term and potential

- Bottom-up approach

R. Gómez-Ambrosio et. al., 2204.01763
R. Gómez-Ambrosio et. al., 2207.09848
A. Salas-Bernardez, et. al., 2211.09605
R. L. Delgado, et. al., 2311.04280
S. Mahmud, et. al., 2406.03522

Why type-II seesaw model?

New scalar's VEV must be much smaller than the EW scale

$$\rho = \frac{v_H^2 + 2v_\Delta^2}{v_H^2 + 4v_\Delta^2} = 1.00038 \pm 0.00020 \quad \Rightarrow \quad v_\Delta \lesssim 10^{-2}\, v_{\rm EW} \qquad v_{\rm EW} = 246\,{\rm GeV}$$

Matching type-II seesaw onto two EFTs

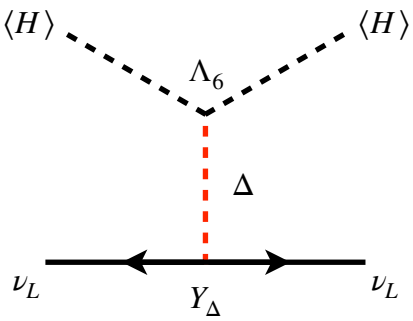
Type-II seesaw

- In order to realize tiny neutrino mass, triplet scalar $\Delta \in (\mathbf{3},1)$ is introduced.

$$\Delta = \begin{pmatrix} \delta^+/\sqrt{2} & \delta^{++} \\ \delta^0 & -\delta^+/\sqrt{2} \end{pmatrix}$$

- Neutrino mass is generated by the $U(1)_L$ violating interactions

$$\mathscr{L}_{\not L} = \Lambda_6 H^T \Delta^\dagger H - Y_\Delta^{ij} \bar{L}_i^C \Delta L_j + \mathrm{H.c.}$$

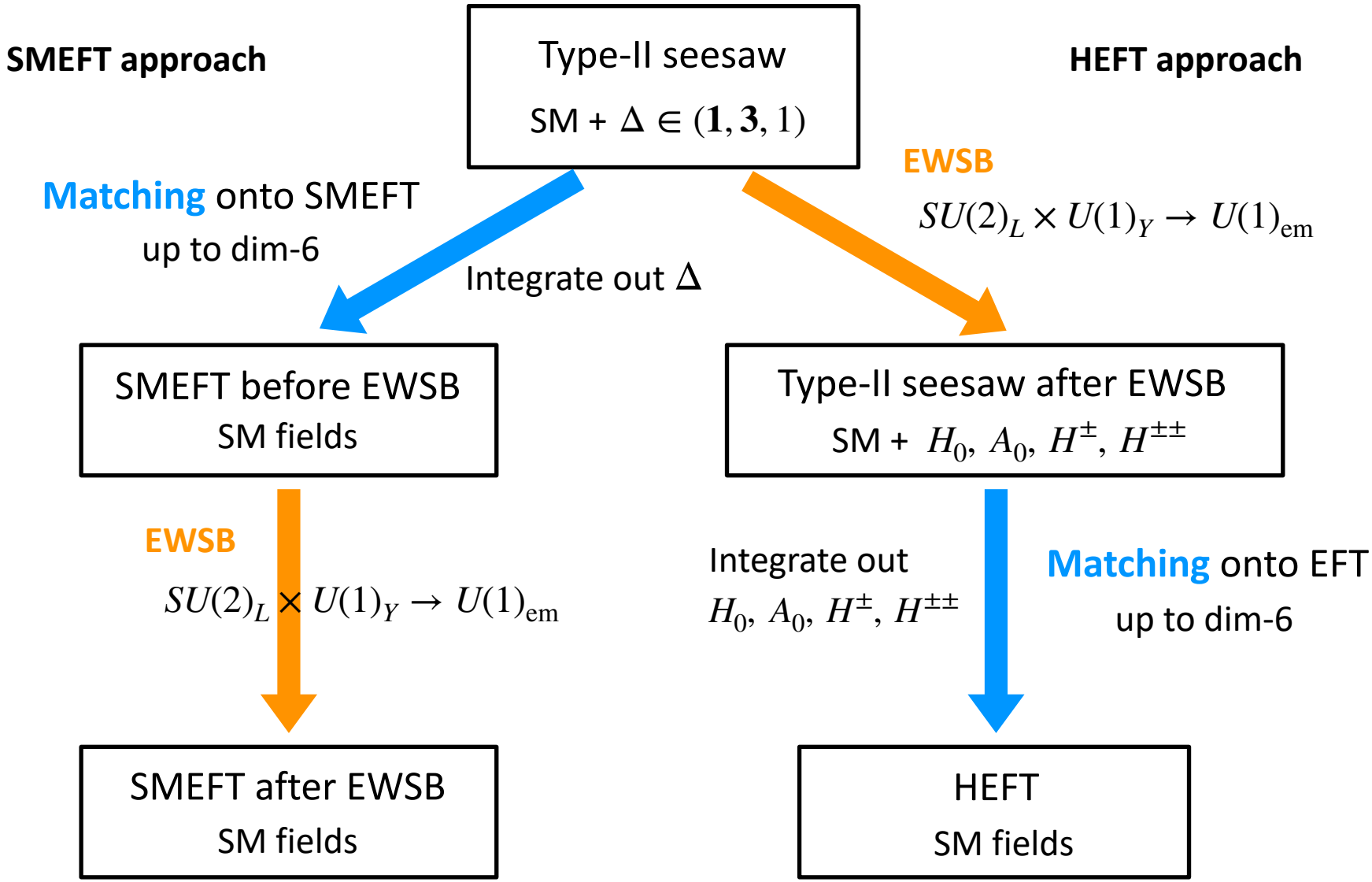


- Triplet VEV induces large deviations in ρ parameter from unity

$$\rho = \frac{v_H^2 + 2v_\Delta^2}{v_H^2 + 4v_\Delta^2} = 1.00038 \pm 0.00020 \quad \Rightarrow \quad v_\Delta \lesssim 10^{-2}\, v_{\mathrm{EW}} \qquad v_{\mathrm{EW}} = 246\,\mathrm{GeV}$$

We perform double expansion w.r.t. ϵ and $1/M_\Delta$ $\qquad \epsilon := \dfrac{v_\Delta}{v_{\mathrm{EW}}}$

Strategy


SMEFT approach
Type-II seesaw
SM + $\Delta \in (\mathbf{1}, \mathbf{3}, 1)$
HEFT approach
Matching onto SMEFT
up to dim-6
Integrate out Δ
EWSB
$SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$
SMEFT before EWSB
SM fields
Type-II seesaw after EWSB
SM + H_0, A_0, $H^{\pm}$, $H^{\pm\pm}$
EWSB
$SU(2)_L \times U(1)_Y \to U(1)_{\rm em}$
Integrate out
H_0, A_0, $H^{\pm}$, $H^{\pm\pm}$
Matching onto EFT
up to dim-6
SMEFT after EWSB
SM fields
HEFT
SM fields

Matching onto SMEFT

This is already done by IHEP group

X. Li, D. Zhang, S. Zhou, JHEP04(2022)038, 2201.05082

Symbol	Operator	dimensionless WC
dim 5		
$O_{pr}^{(5)}$	$\bar{L}^p \tilde{H} \tilde{H}^T L^{rC}$	$-\frac{\Lambda_6}{2M_\Delta} Y_\Delta^{*pr}$
dim 6		
O_H	$(H^\dagger H)^3$	$\frac{1}{2}(8\lambda - \lambda_4 + \lambda_5)\frac{\Lambda_6^2}{M_\Delta^2} - \frac{\Lambda_6^4}{M_\Delta^4}$
$O_{H\Box}$	$(H^\dagger H)\Box(H^\dagger H)$	$\frac{\Lambda_6^2}{2M_\Delta^2}$
O_{HD}	$(H^\dagger D_\mu H)^\dagger (H^\dagger D^\mu H)$	$\frac{\Lambda_6^2}{M_\Delta^2}$
O_{eH}^{pr}	$(H^\dagger H)(\bar{L}^p H e_R^r)$	$\frac{\Lambda_6^2}{2M_\Delta^2} Y_l^{pr}$
O_{uH}^{pr}	$(H^\dagger H)(\bar{Q}^p \tilde{H} u_R^r)$	$\frac{\Lambda_6^2}{2M_\Delta^2} Y_{\rm u}^{pr}$
O_{dH}^{pr}	$(H^\dagger H)(\bar{Q}^p H d_R^r)$	$\frac{\Lambda_6^2}{2M_\Delta^2} Y_{\rm d}^{pr}$
$O_{\ell\ell}^{prst}$	$(\bar{L}^p \gamma_\mu L^r)(\bar{L}^s \gamma^\mu L^t)$	$\frac{1}{4} Y_\Delta^{*ps} Y_\Delta^{rt}$

We rewrite the SMEFT in the broken phase.

VEV shift:

$$\mathscr{L}^{\rm smeft}_{\rm dim6} \supset m_H^2 H^\dagger H - \frac{\lambda}{2}(H^\dagger H)^2 - C_H (H^\dagger H)^3$$

new VEV: $\langle H^\dagger H \rangle = \frac{v^2}{2}\left(1 + \frac{3C_H}{2\lambda}\right) \equiv \frac{v_{\rm ew}^2}{2}$

Normalization:

$$\mathscr{L}^{\rm smeft}_{\rm dim6} \supset (D_\mu H)^\dagger D^\mu H + \frac{C_{H\Box}}{M_\Delta^2}(H^\dagger H)\Box(H^\dagger H) + \frac{C_{HD}}{M_\Delta^2}(H^\dagger D_\mu H)^\dagger (H^\dagger D^\mu H)$$

$$\xrightarrow{H = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v+h \end{pmatrix}} \frac{1}{2}\left[1 - 2c_{H,\rm kin}\left(1 + \frac{h}{\tilde{v}_T}\right)^2\right]\partial_\mu h \partial^\mu h$$

To make h canonically normalized,

$$\tilde{h} = \frac{1}{\sqrt{1 - 2c_{H,\rm kin}}} h$$

Matching onto HEFT

Difficulty

In the broken phase, there are so many fields to be integrated out: $\{H_0,\ A_0,\ H^{\pm},\ H^{\pm\pm}\}$

Solution

There is "Z_2 symmetry" in the $\mathscr{L}_{\rm UV}$

$U(1)_L$ violating terms

$$V(H,\Delta) = V_{\rm SM}(H) + \lambda HH\Delta\Delta + (\Delta\Delta)^2 + \Lambda_6 H\Delta H + Y_\Delta L\Delta L$$

$V(H,\Delta)$ is invariant under

$$\Lambda_6 \to -\Lambda_6 \qquad Y_\Delta \to -Y_\Delta \qquad \Delta \to -\Delta$$

After EWSB, this "Z_2 symmetry" still holds

$$\epsilon \to -\epsilon \qquad Y_\Delta \to -Y_\Delta \qquad \Phi_{\rm heavy} \to -\Phi_{\rm heavy}$$

$\Phi_{\rm heavy} \in \{H_0, A_0, H^{\pm}, H^{\pm\pm}\}$

$$\epsilon := \frac{v_\Delta}{\sqrt{v_H^2 + 2v_\Delta^2}}$$

This suggests the solution of E.o.M. for $\Phi_{\rm heavy}$ must take the form of

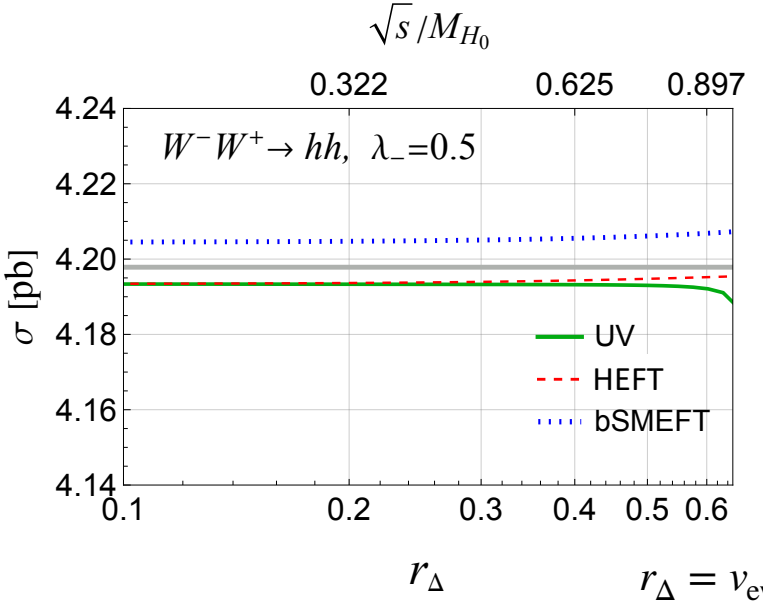
$$\Phi_{\rm heavy}[\phi_{\rm SM}] \ \propto\ \epsilon^m \left(Y_\Delta \bar{L} L\right)^n \qquad m+n = \text{odd}$$

We keep only ϵ^2 order

$$H_0^4 \ \propto\ \epsilon^2 \left(\frac{Y_\Delta}{M_\Delta^2}\right)^2,\ \epsilon \left(\frac{Y_\Delta}{M_\Delta^2}\right)^3,\ \left(\frac{Y_\Delta}{M_\Delta^2}\right)^4$$ can be neglected

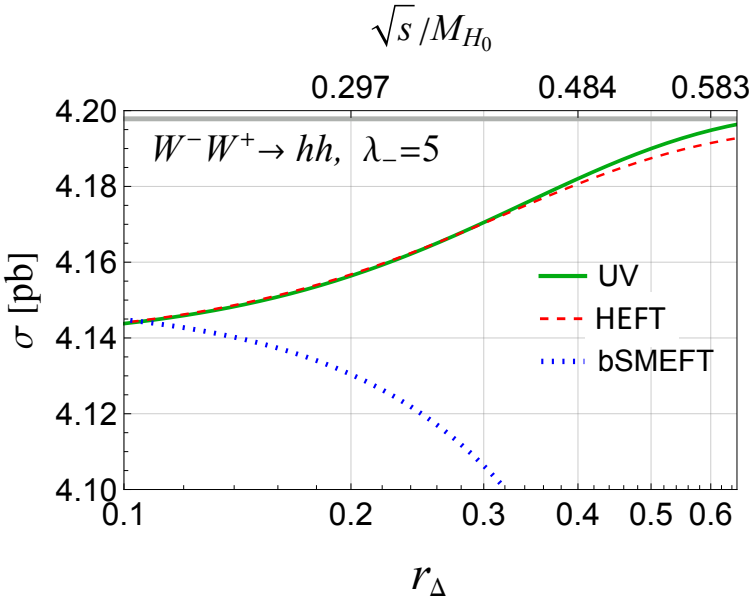
Numerical evaluations

$W^-W^+ \to hh$



$r_\Delta = v_{\rm ew}/M_\Delta$

$$
\begin{aligned}
&V(H,\Delta) \\
&= -m_H^2(H^\dagger H) + \frac{\lambda}{2}(H^\dagger H)^2 \\
&\quad + M_\Delta^2 \mathrm{Tr}(\Delta^\dagger\Delta) + \frac{\lambda_1}{2}\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 \\
&\quad + \frac{\lambda_2}{2}\left(\left[\mathrm{Tr}(\Delta^\dagger\Delta)\right]^2 - \mathrm{Tr}\left[(\Delta^\dagger\Delta)^2\right]\right) \\
&\quad + \lambda_4(H^\dagger H)\mathrm{Tr}(\Delta^\dagger\Delta) + \lambda_5 H^\dagger[\Delta^\dagger,\Delta]H \\
&\quad + \left(\frac{\Lambda_6}{\sqrt{2}} H^T i\sigma_2 \Delta^\dagger H + \mathrm{h.c.}\right)
\end{aligned}
$$



$\sqrt{s} = 400\,\mathrm{GeV},$

$\epsilon = v_\Delta/v_{\rm ew} = 10^{-2},$

$\lambda_- := (\lambda_4 - \lambda_5)/2$

$\lambda_1 = \lambda_2 = \lambda_5 = 1$

HEFT reproduces UV theory more accurately than SMEFT when $v_{\rm ew}/M_\Delta$ is large.

Why HEFT is better than SMEFT ?

✔ WCs of dim-3 operators are same in the UV and the HEFT

$$\mathscr{L}_{\rm dim3} \supset C_{hW^2}\, hW^-_\mu W^{+\mu} + C_{hZ^2}\, hZ_\mu Z^\mu + C_{\overline{\nu}\nu}\, \overline{\nu^c_L}\nu_L$$

$$C^{\rm UV}_{hW^2} = C^{\rm HEFT}_{hW^2} \neq C^{\rm SMEFT}_{hW^2}$$
$$C^{\rm UV}_{hZ^2} = C^{\rm HEFT}_{hZ^2} \neq C^{\rm SMEFT}_{hZ^2}$$
$$C^{\rm UV}_{\overline{\nu}\nu} = C^{\rm HEFT}_{\overline{\nu}\nu} \neq C^{\rm SMEFT}_{\overline{\nu}\nu}$$

✔ Operators absent in SMEFT but present in HEFT

dim 4 $Z_\mu Z^\mu Z_\nu Z^\nu$

dim 5 $\Delta L = 0$ $hW^-_\mu W^{+\mu} W^-_\nu W^{+\nu}$ $hW^-_\mu W^+_\nu W^{-\mu} W^{+\nu}$ $hZ_\mu Z^\mu Z_\nu Z^\nu$ $hW^-_\mu W^{+\mu} Z_\nu Z^\nu$
$hW^-_\mu W^+_\nu Z^\mu Z^\nu$ $ihW^-_\mu W^+_\nu Z^{\mu\nu}$ $i(h\overleftrightarrow{D}_\mu W^-_\nu) W^{+\mu} Z^\nu$ $W^-_\mu W^{+\mu}(\bar{\psi}_L \psi_R)$ $Z_\mu Z^\mu (\bar{\psi}_L \psi_R)$

$\Delta L = 2$ $W^-_\mu W^{+\mu}(\nu^T_L C \nu_L)$ $W^+_\mu W^{+\mu}(e^T_L C e_L)$ $W^+_\mu Z^\mu (\nu^T_L C e_L)$ $Z_\mu Z^\mu (\nu^T_L C \nu_L)$

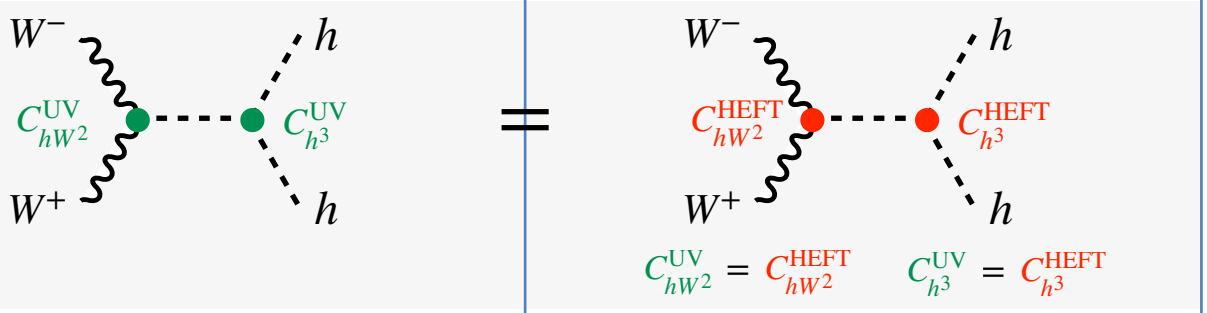
dim 6 $\Delta L = 0$ $(\partial_\mu h)(\partial_\nu h) W^{-\mu} W^{+\nu}$ $hh(D_\mu W^{-\mu})(D_\nu W^{+\nu})$ $hh(D_\mu D_\nu W^{-\mu}) W^{+\nu}$ …

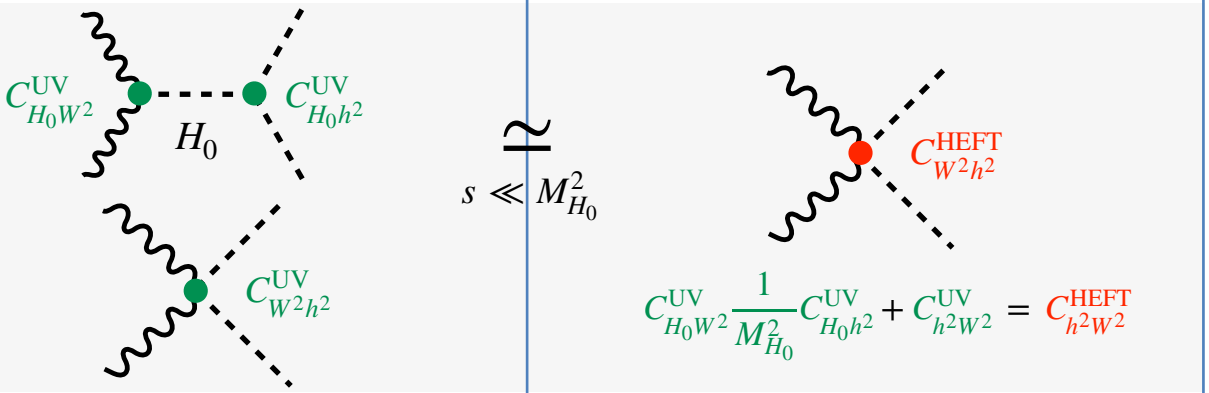
In the SMEFT, non-derivative quartic gauge interactions arise from dim-8

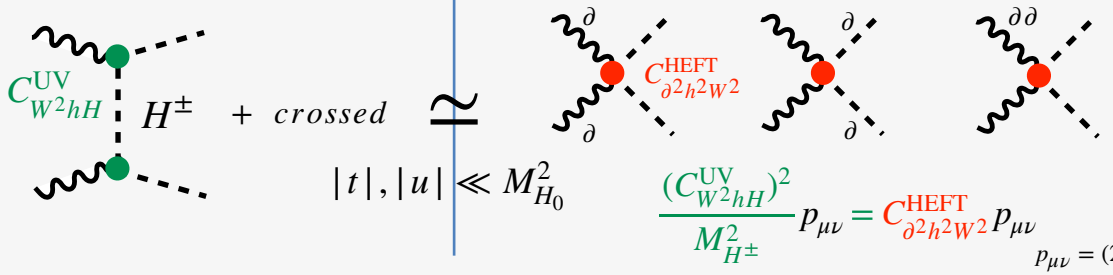
$$\mathscr{L}^{\rm smeft}_{\rm dim8} = \left[(D_\mu H)^\dagger D^\mu H\right]^2 \supset W^-_\mu W^{+\mu} W^-_\nu W^{+\nu}$$

Why HEFT is better than SMEFT ?

type-II seesaw | HEFT | Broken phase SMEFT







- ✔ UV and HEFT have one-to-one correspondence
- ✔ SMEFT treat heavy scalars as one multiplet, so cannot take into account scalar mixing at the lower order of perturbation

Summary

- We match type-II seesaw model onto SMEFT and HEFT and examine which EFT more accurately reproduces UV theory.

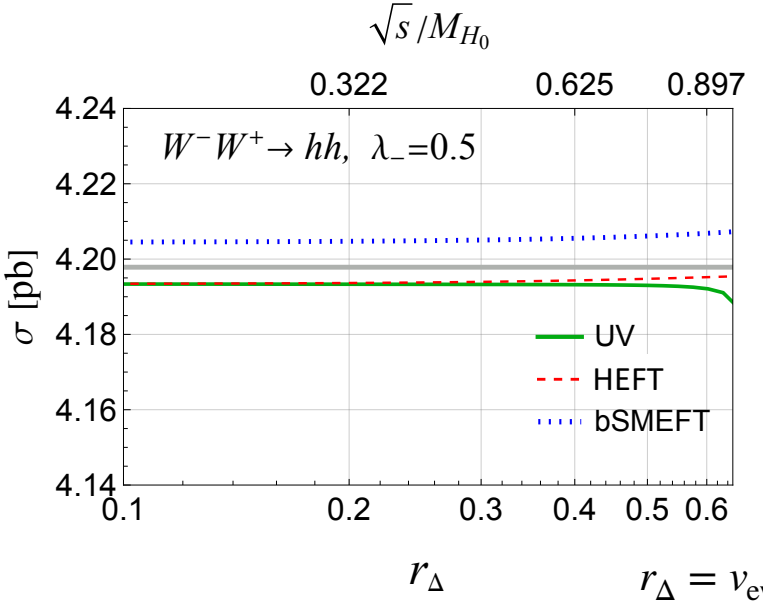
- We rewrite previous studies matching Type-II seesaw onto SMEFT in a basis after the EW symmetry breaking.

- We perform matching the Type-II seesaw onto the HEFT.

- We performed numerical calculations and found that HEFT reproduces UV theory more accurately than SMEFT in the regime that v_T/M_Δ is close to unity.

Back Up

$W^-W^+ \to hh$



$r_\Delta = v_{\rm ew}/M_\Delta$

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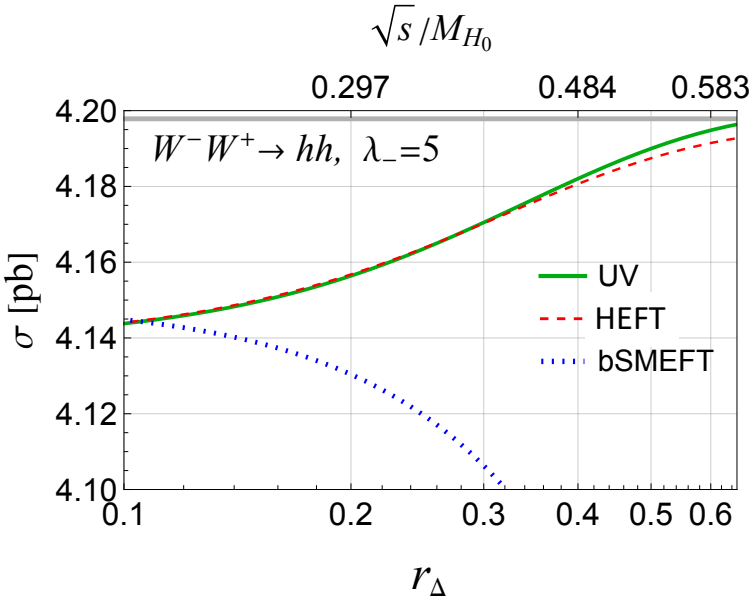

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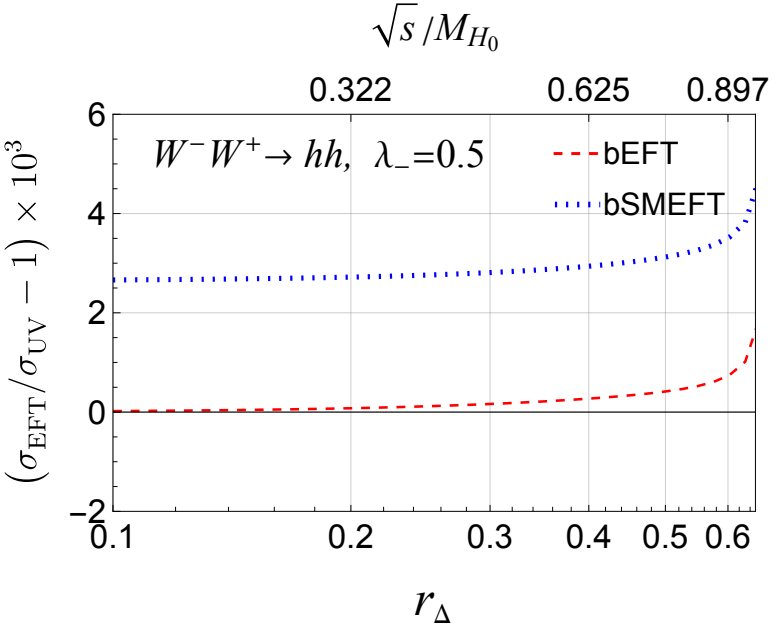

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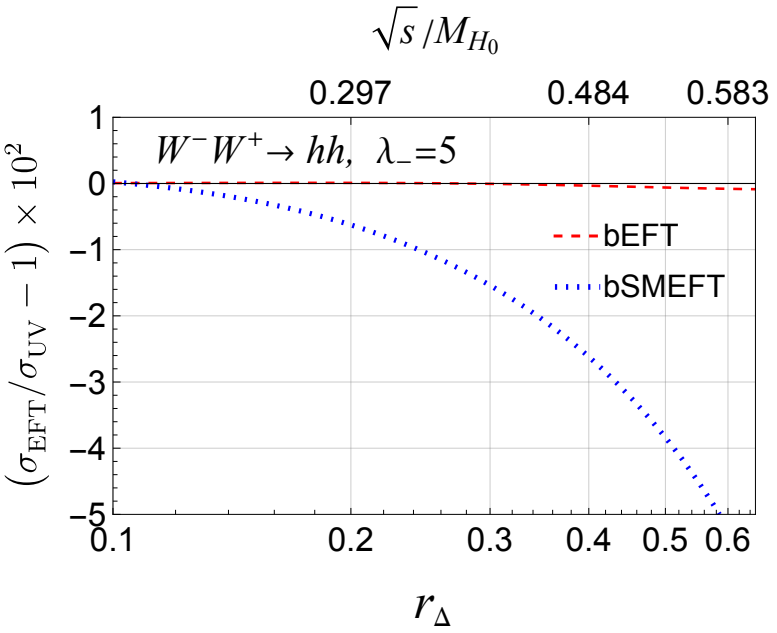

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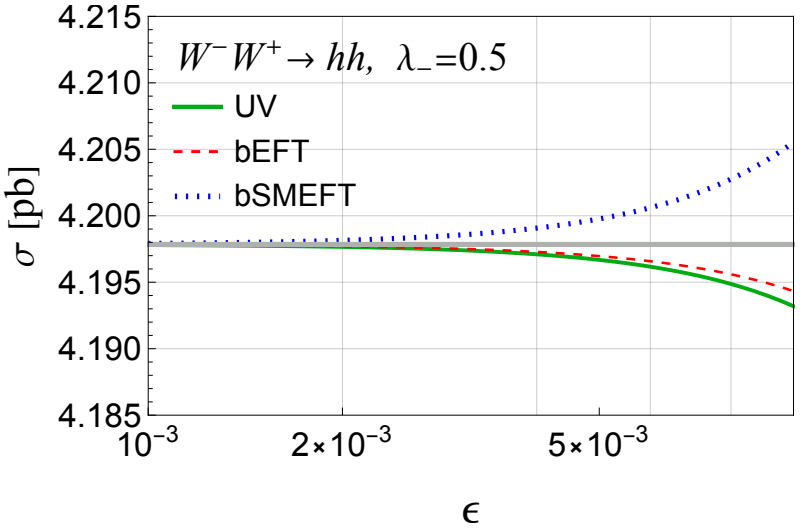

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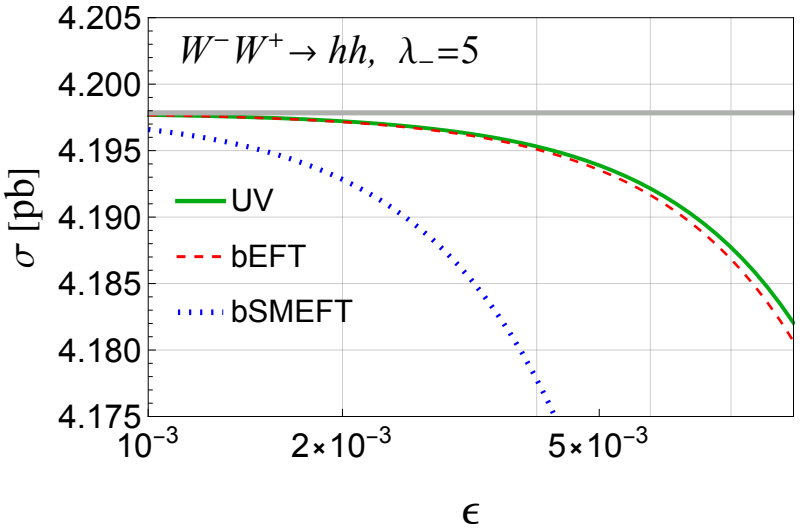
\end{aligned}
$$



$$\sqrt{s} = 400\,\mathrm{GeV},$$

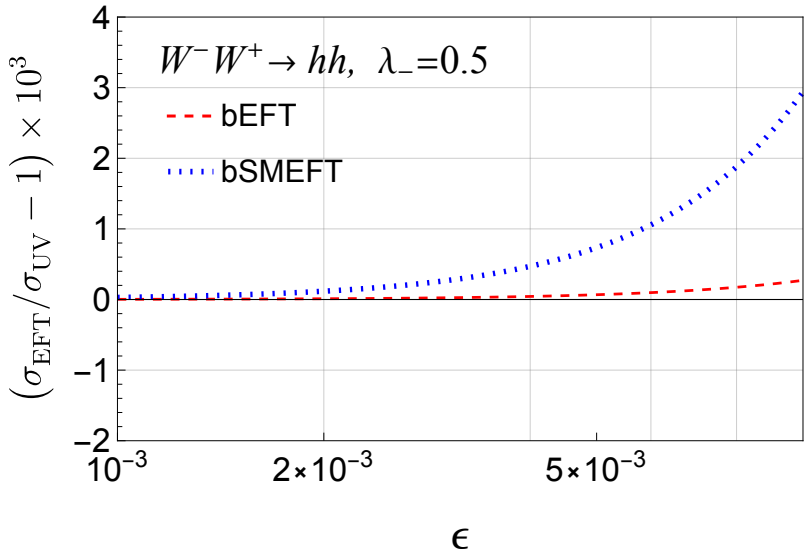
$$r_\Delta = v_\Delta / M_\Delta = 0.4$$

$$\lambda_- := (\lambda_4 - \lambda_5)/2$$

$$\lambda_1 = \lambda_2 = \lambda_5 = 1$$

HEFT reproduces UV theory more accurately than SMEFT when v_{ew}/M_Δ is large.

$W^-W^+ \to hh$



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\begin{aligned}
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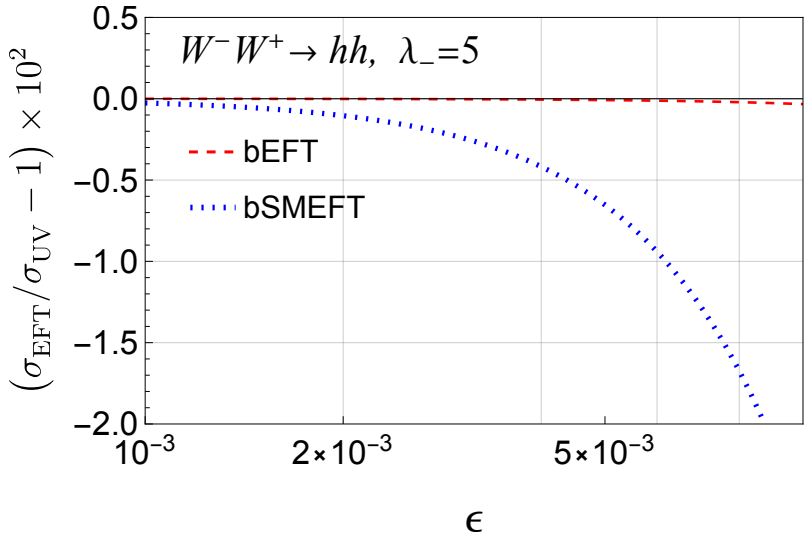

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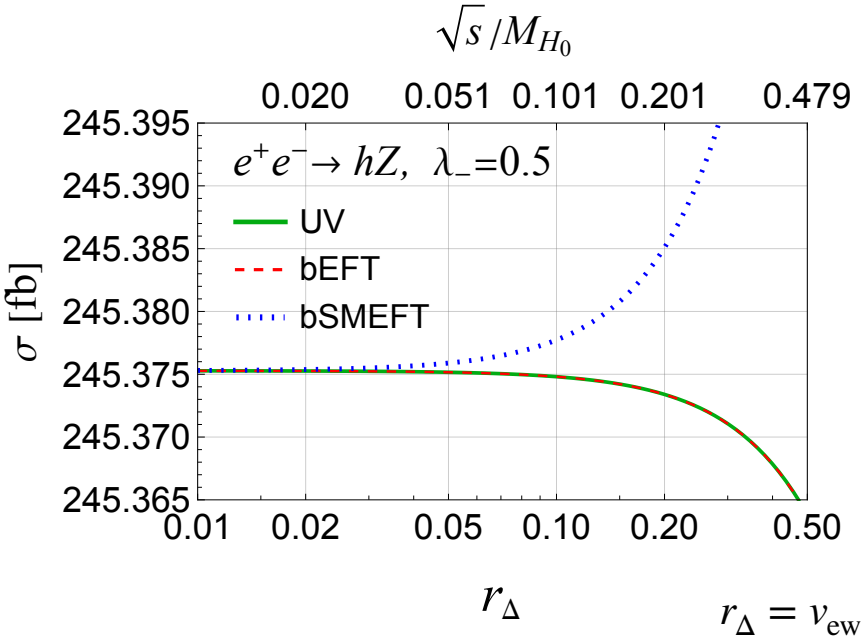
$e^+e^- \to Zh$



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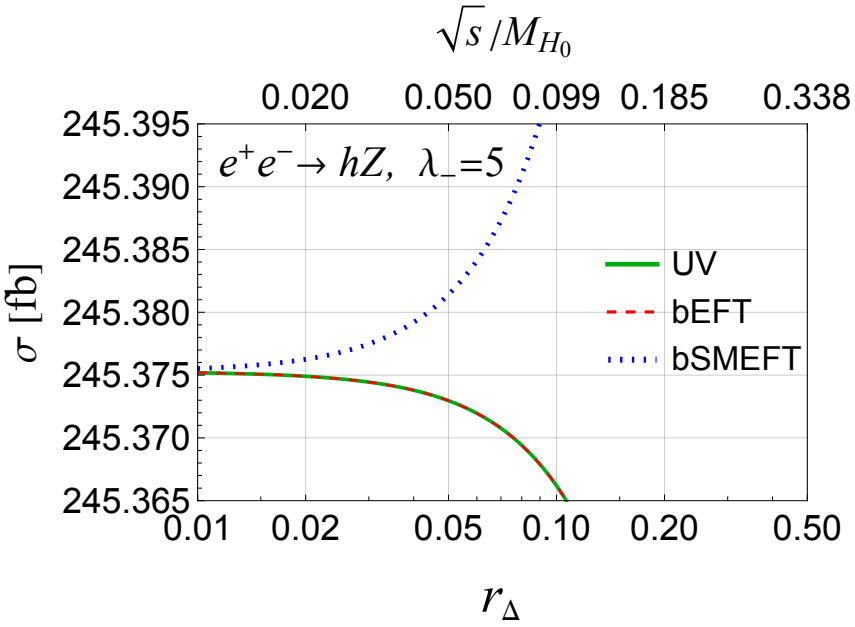

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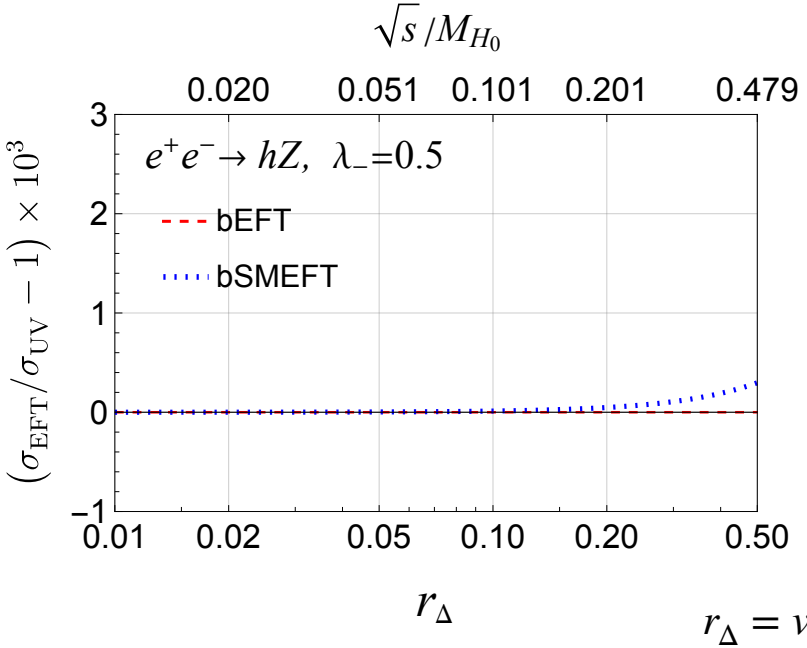
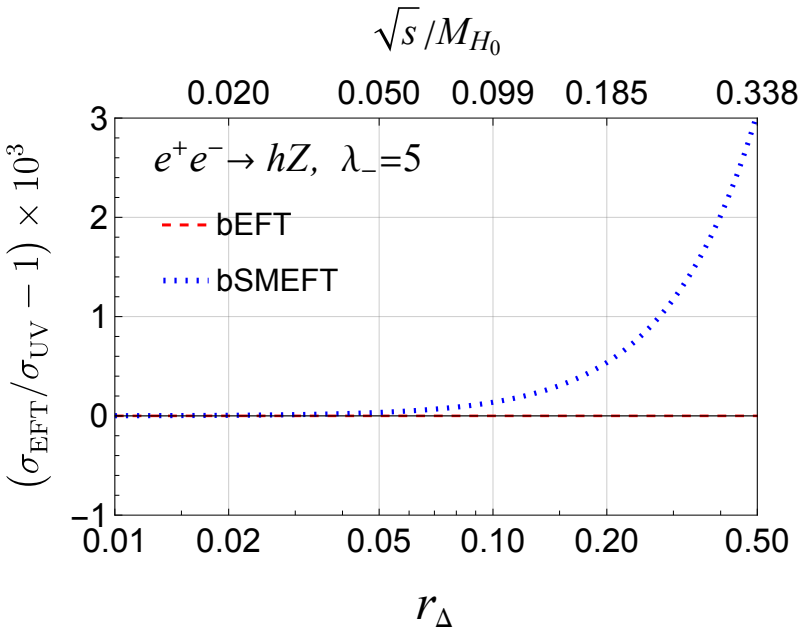


$$r_\Delta = v_{\rm ew}/M_\Delta$$



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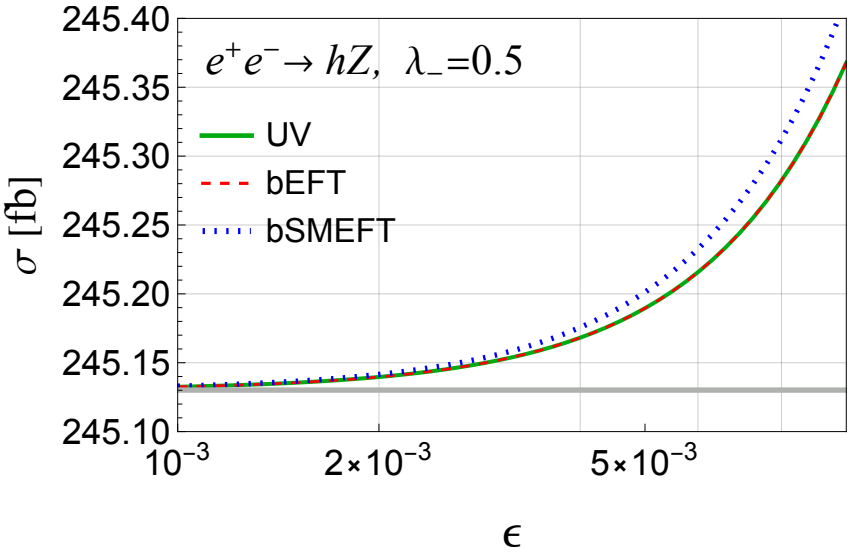

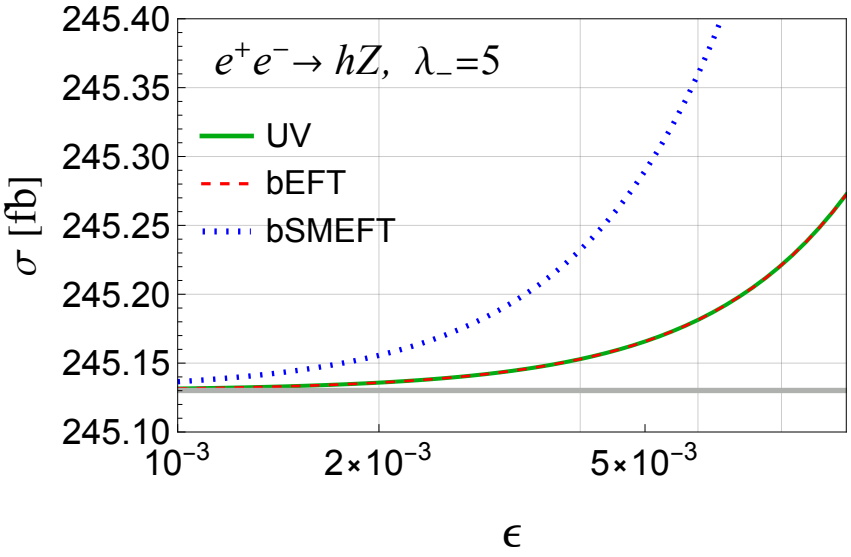

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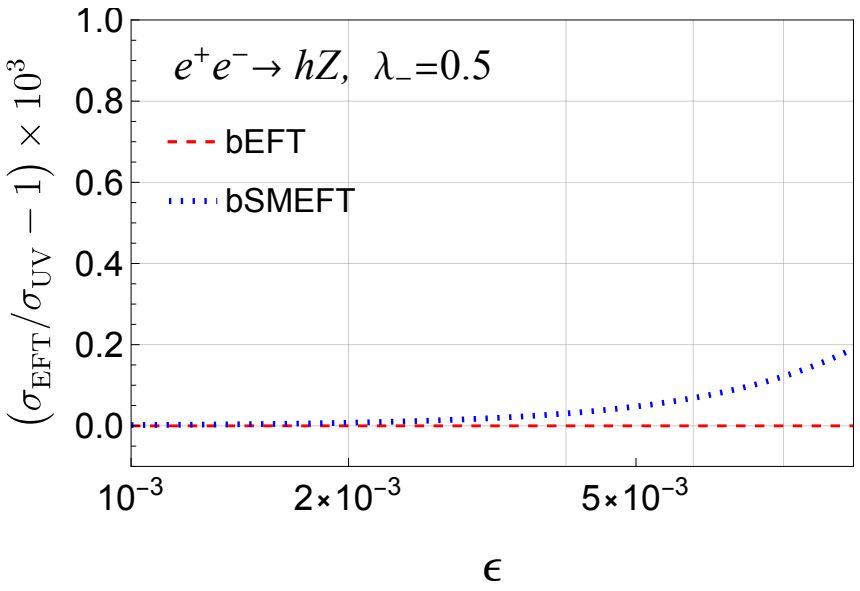

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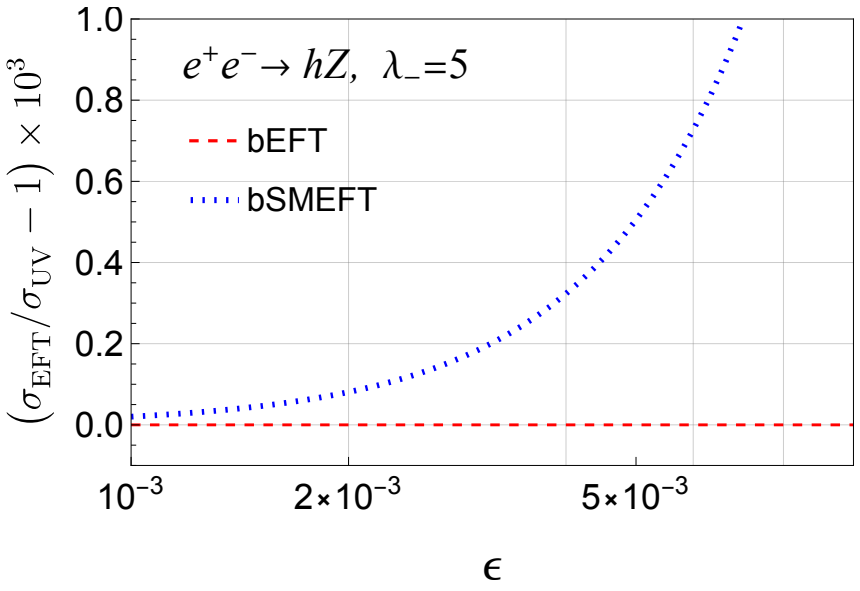

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Matching onto HEFT

Difficulty #1

In the broken phase, $SU(2)_L \times U(1)_Y$ is not really broken, but is non-linearly realized

Polar decomposition, $\widetilde{\Delta} = U\left(\Delta^a \frac{\sigma^a}{2}\right) U^\dagger \qquad \widetilde{\Sigma} = \frac{1}{\sqrt{2}}\left((v+h)\mathbf{1} + i\rho^a\sigma^a\right)U$

Real triplet case
H. Song, et. al., 2412.00355

Integrating out heavy fields from $(\star)$, we get the HEFT for type-II seesaw model.

Solution #1

We just take a unitary guage : $U = \mathbf{1}_{2\times 2}$

Difficulties in HEFT

Difficulty #1

In the broken phase, $SU(2)_L \times U(1)_Y$ is not really broken, but is non-linearly realized

SM + singlet scalar $\quad \mathscr{L} = (D_\mu H)^\dagger D^\mu H + \frac{1}{2}\partial_\mu S \partial^\mu S - V(H,S)$

Substituting polar decomposition, $\quad H = \frac{v+h_1}{\sqrt{2}} U \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad S = \frac{v_s + h_2}{\sqrt{2}} \qquad \begin{pmatrix} h \\ \tilde{s} \end{pmatrix} = \begin{pmatrix} c_\chi & -s_\chi \\ s_\chi & c_\chi \end{pmatrix} \begin{pmatrix} h_1 \\ h_2 \end{pmatrix}$

we get $\quad \mathscr{L} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{1}{2}\partial_\mu \tilde{s} \partial^\mu \tilde{s} - V(h,\tilde{s})$

$$+\frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(1 + \frac{2c_\chi}{v}h + \frac{2s_\chi}{v}\tilde{s} + \frac{c_\chi^2}{v^2}h^2 + \frac{s_\chi^2}{v^2}\tilde{s}^2 + \frac{2s_\chi c_\chi}{v^2}h\tilde{s}\right) \qquad \ldots (\star)$$

$(\star)$ is invariant under non-linerly realized $SU(2)_L \times U(1)_Y$

$$U \to \mathfrak{g}_L U \mathfrak{g}_Y^\dagger \qquad \mathfrak{g}_L = \exp\left(i\theta_L^a \frac{\sigma^a}{2}\right) \qquad \mathfrak{g}_Y = \exp\left(i\theta_Y \frac{\sigma^3}{2}\right)$$

Integrating out $\tilde{s}$ from $(\star)$, we get

$$\mathscr{L}_{\mathrm{HEFT}} = \frac{1}{2}\partial_\mu h \partial^\mu h + \frac{v^2}{4}\mathrm{Tr}[(D_\mu U)^\dagger D^\mu U]\left(1 + 2\kappa_W \frac{h}{v} + \kappa_W^{(2)}\left(\frac{h}{v}\right)^2 + \cdots\right) + \cdots$$

Difficulty #2

In the broken phase, there are so many fields to be integrated out

Symmetric phase: $\Delta \in (\mathbf{1}, \mathbf{3}, 1)$ Broken phase: $\{H_0,\ A_0,\ H^\pm,\ H^{\pm\pm}\}$

HEFT

Higgs sector & Yukawa sector in HEFT

Higgs sector in SM

$$\mathcal{L}^{\rm SM}_{\rm Higgs} = (D_\mu H)^\dagger D^\mu H + \mu^2 (H^\dagger H) - \lambda (H^\dagger H)^2$$

$$= \frac{1}{2}{\rm Tr}\left[(D_\mu \hat{\Phi})^\dagger D^\mu \hat{\Phi}\right] + \frac{\mu^2}{2}{\rm Tr}\left[\hat{\Phi}^\dagger \hat{\Phi}\right] - \frac{\lambda}{4}{\rm Tr}\left[\hat{\Phi}^\dagger \hat{\Phi}\right]^2$$

$\hat{\Phi} = (\tilde{H}, H)$: bi-doublet　　但し $\tilde{H} := i\sigma_2 H^*$

$$D_\mu \hat{\Phi} = \partial_\mu \hat{\Phi} - igW^a_\mu \frac{\tau^a}{2}\hat{\Phi} + ig'\hat{\Phi} B_\mu \frac{\tau^3}{2}$$

Higgs sector in HEFT

h : (125GeV Higgs) singlet under non-linearly realized SU(2)xU(1)

$w^\pm, z$: (would-be Goldstone) transform nontrivially under non-linearly realized SU(2)xU(1)

➡ We must separate h from $w^\pm, z$ because they transform differently

Polar decomposition : $\hat{\Phi} = (v+h)U$　　$U = \exp\left(\frac{w^\pm + z^0}{v}\right)$

$$\mathcal{L}^{\rm SM}_{\rm Higgs} = \frac{v^2}{4}\left(1+\frac{h}{v}\right)^2 {\rm Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V_{\rm SM}(h)$$

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Higgs sector & Yukawa sector in HEFT

SM Yukawa

$$\mathcal{L}^{\rm SM}_{\rm Yukawa} = -y_u\,\bar{q}_L \tilde{H} u_R - y_d\,\bar{q}_L H d_R - y_e\,\bar{l}_L \tilde{H} e_R + \text{h.c.}$$

$$= -y_u\,\bar{q}_L \hat{\Phi} P_+ q_R - y_d\,\bar{q}_L \hat{\Phi} P_- q_R - y_e\,\bar{l}_L \hat{\Phi} P_- l_R + \text{h.c.}$$

$\hat{\Phi} = (\tilde{H}, H)$: bi-doublet　　但し $\tilde{H} := i\sigma_2 H^*$

$q_R = \begin{pmatrix} u_R \\ d_R \end{pmatrix}$, $l_R = \begin{pmatrix} 0 \\ e_R \end{pmatrix}$: right-handed fermions　　$P_+ = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $P_- = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$: projection operator

HEFT Yukawa

Substituting $\hat{\Phi} = (v+h)U$ into $\mathcal{L}^{\rm SM}_{\rm Yukawa}$, we get

$$\mathcal{L}^{\rm SM}_{\rm Yukawa} = -v\,y_u\left(1+\frac{h}{v}\right)\bar{q}_L U P_+ q_R - v\,y_d\left(1+\frac{h}{v}\right)\bar{q}_L U P_- q_R - v\,y_e\left(1+\frac{h}{v}\right) U P_- l_R + \text{h.c.}$$

Replacement

$$\mathcal{L}^{\rm HEFT}_{\rm Yukawa} = -v\quad Y_u(h)\quad \bar{q}_L U P_+ q_R - v\quad Y_d(h)\quad \bar{q}_L U P_- q_R - v\quad Y_e(h)\quad U P_- l_R + \text{h.c.}$$

Buchalla et al., 1307.5017

HEFT (LO)

$$\mathcal{L}_{\text{HEFT, LO}} = \mathcal{L}_0 + \mathcal{L}_{Uh}$$

$$\mathcal{L}_{Uh} = \frac{v^2}{4} F(h) \text{Tr}\left[(D_\mu U)^\dagger D^\mu U\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h)$$

Higgs sector

$$- v\left[Y_u(h)\, \bar{q}_L U P_+ q_R + Y_d(h)\, \bar{q}_L U P_- q_R + Y_e(h)\, \bar{l}_L U P_- l_R + \text{h.c.}\right]$$

Yukawa項

$$U = \exp\left(\frac{w^\pm + z^0}{v}\right)$$

$$SU(2)_L \times U(1)_Y \ : \ \hat{\Phi} \to \mathfrak{g}_L \hat{\Phi} \mathfrak{g}_Y^\dagger \quad \longleftrightarrow \quad h \to h\ , \quad U \to \mathfrak{g}_L\, U\, \mathfrak{g}_Y^\dagger$$

$$\mathcal{L}_0 = -\frac{1}{2}\text{Tr}\left[G_{\mu\nu} G^{\mu\nu}\right] - \frac{1}{2}\text{Tr}\left[W_{\mu\nu} W^{\mu\nu}\right] - \frac{1}{4} B_{\mu\nu} B^{\mu\nu}$$
$$+ \bar{q}_L i \not{D} q_L + \bar{l}_L i \not{D} l_L + \bar{u}_R i \not{D} u_R + \bar{d}_R i \not{D} d_R + \bar{e}_R i \not{D} e_R$$

$$q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix} \in \mathbf{2}_{1/6} \qquad l_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \in \mathbf{2}_{-1/2}$$

Identical to those in SM

$$u_R \in \mathbf{1}_{2/3} \qquad d_R \in \mathbf{1}_{-1/3} \qquad e_R \in \mathbf{1}_{-1}$$

CCWZ

Concept of Higgs Effective Field Theory

Symmetry： $O(4)$

Matter： $\vec{\phi} = (\phi^1, \phi^2, \phi^3, \phi^4)^T \in \mathbf{4}$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{1}{2}\mu^2(\vec{\phi})^2 - \frac{\lambda}{4}(\vec{\phi})^4$$

$$\langle \phi \rangle \neq 0$$

$$O(4) \to O(3)$$

Symmetry： $O(3)$

Matter： $\underbrace{(\pi^1, \pi^2, \pi^3)}_{\text{Goldstone}} \in \mathbf{3}$, $\sigma \in \mathbf{1}$

$$\vec{\phi} = (v + \sigma) e^{i \frac{\pi^a}{v} X^a} \vec{F}$$

$$\mathcal{L} = \frac{1}{2}\left(1 + \frac{\sigma}{v}\right)^2 \left[\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{(\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 - |\vec{\pi}|^2 (\partial_\mu \vec{\pi})^2}{3v^2} + \cdots\right] + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - V(\sigma)$$

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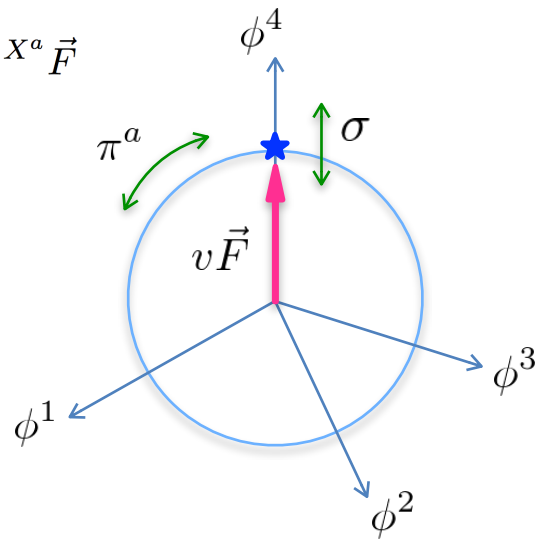

X^a：broken generator

$\vec{F} = (0, 0, 0, 1)^T$

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$O(4) \to O(3)$

All we know

Symmetry : $O(3)$

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$$\vec{\phi} = (v + \sigma) e^{i \frac{\pi^a}{v} X^a} \vec{F}$$

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Let's assume we don't know UV theory

⇨ **EFT approach**

Write down all the $O(3)$ invariant operators

Q1. Should we write $(\vec{\pi} \cdot \vec{\pi})\,,\ (\vec{\pi} \cdot \vec{\pi})^2\,,\ (\vec{\pi} \cdot \vec{\pi})^3$? ✕ No

Q2. Should we write $|\vec{\pi}|^2 (\partial_\mu \vec{\pi})^2$? ○ Yes

What are the criteria for inclusion or exclusion of Goldstone operator?

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Matter： $\underbrace{(\pi^1, \pi^2, \pi^3)}_{\text{Goldstone}} \in \mathbf{3}$, $\sigma \in \mathbf{1}$

$$\vec{\phi} = (v + \sigma) e^{i \frac{\pi^a}{v} X^a} \vec{F}$$

$$\mathcal{L} = \frac{1}{2}\left(1 + \frac{\sigma}{v}\right)^2 \left[\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{(\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 - |\vec{\pi}|^2 (\partial_\mu \vec{\pi})^2}{3v^2} + \cdots\right] + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - V(\sigma)$$

What did we miss ?

If the symmetry $\mathcal{H}$ is due to a SSB of the larger symmetry $\mathcal{G}$,
we should write invariance under the non-linearly realized $\mathcal{G}$, rather than $\mathcal{H}$.

We should write non-linear $O(4)$ invariance,

rather than $O(3)$ invariance

Concept of Higgs Effective Field Theory

Symmetry： $O(4)$

Matter： $\vec{\phi} = (\phi^1, \phi^2, \phi^3, \phi^4)^T \in \mathbf{4}$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{1}{2}\mu^2(\vec{\phi})^2 - \frac{\lambda}{4}(\vec{\phi})^4$$

$$\langle \phi \rangle \neq 0$$

$$O(4) \to O(3)$$

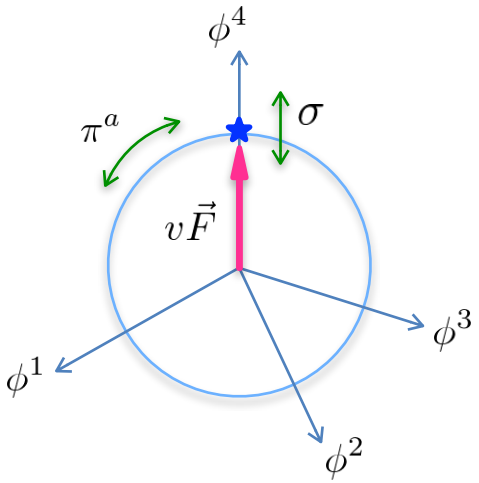
Symmetry： $O(3)$

Matter： $\underbrace{(\pi^1, \pi^2, \pi^3)}_{\text{Goldstone}} \in \mathbf{3}\,,\ \sigma \in \mathbf{1}$

$$\vec{\phi} = (v+\sigma) e^{i\frac{\pi^a}{v}X^a} \vec{F}$$

$$\mathcal{L} = \frac{1}{2}\left(1+\frac{\sigma}{v}\right)^2 \left[\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 - |\vec{\pi}|^2(\partial_\mu\vec{\pi})^2}{3v^2} + \cdots\right] + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma)$$

What is non-linearly realized symmetry ?



$$O(4): \quad \vec{\phi} \to \mathfrak{g} \cdot \vec{\phi} \qquad \mathfrak{g} \in O(4)$$

Substituting $\vec{\phi} = (v+\sigma) e^{i\frac{\pi^a}{v}X^a} \vec{F}$

$$(v+\sigma) e^{i\frac{\pi^a}{v}X^a} \vec{F} \to (v+\sigma)\, \mathfrak{g} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot \mathfrak{h} \cdot \vec{F} \qquad \mathfrak{h} \in O(3)$$

$$O(4): \quad \sigma \to \sigma \qquad e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g} \cdot e^{i\frac{\pi^a}{v}X^a} \cdot \mathfrak{h} = e^{i\frac{\pi'^a}{v}X^a} e^{i\varphi^a S^a} \cdot \mathfrak{h}$$

Concept of Higgs Effective Field Theory

Symmetry： $O(4)$

Matter： $\vec{\phi} = (\phi^1, \phi^2, \phi^3, \phi^4)^T \in \mathbf{4}$

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$$\vec{\phi} = (v+\sigma) e^{i\frac{\pi^a}{v}X^a} \vec{F}$$

$$\mathcal{L} = \frac{1}{2}\left(1+\frac{\sigma}{v}\right)^2 \left[\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{(\vec{\pi}\cdot\partial_\mu\vec{\pi})^2 - |\vec{\pi}|^2(\partial_\mu\vec{\pi})^2}{3v^2} + \cdots\right] + \frac{1}{2}\partial_\mu\sigma\partial^\mu\sigma - V(\sigma)$$

What is non-linearly realized symmetry ?

$$O(4): \quad \vec{\phi} \ \to\ \mathfrak{g}\cdot\vec{\phi} \qquad \mathfrak{g} \in O(4)$$

non-linear function of π

$$\pi'^a = \overbrace{v + \mathcal{M}^a{}_b \pi^b + \mathcal{N}^a{}_{bc}\pi^b\pi^c + \mathcal{O}(\pi^3)}$$

Substituting $\vec{\phi} = (v+\sigma) e^{i\frac{\pi^a}{v}X^a}\vec{F}$

$$(v+\sigma) e^{i\frac{\pi^a}{v}X^a}\vec{F} \ \to\ (v+\sigma)\,\mathfrak{g}\cdot e^{i\frac{\pi^a}{v}X^a}\cdot\mathfrak{h}\cdot\vec{F} \qquad \mathfrak{h}\in O(3)$$

Non-linearly realized $O(4)$: $\quad \sigma \to \sigma \qquad e^{i\frac{\pi^a}{v}X^a} \to \mathfrak{g}\cdot e^{i\frac{\pi^a}{v}X^a}\cdot\mathfrak{h} = e^{i\frac{\pi'^a}{v}X^a} e^{i\varphi^a S^a}\cdot\mathfrak{h}$

Concept of Higgs Effective Field Theory

Symmetry： $O(4)$

Matter： $\vec{\phi} = (\phi^1, \phi^2, \phi^3, \phi^4)^T \in \mathbf{4}$

$$\mathcal{L} = \frac{1}{2}(\partial_\mu \vec{\phi})^2 + \frac{1}{2}\mu^2(\vec{\phi})^2 - \frac{\lambda}{4}(\vec{\phi})^4$$

$\langle \phi \rangle \neq 0$

$O(4) \to O(3)$

Symmetry： $O(3)$

Matter： $(\pi^1, \pi^2, \pi^3) \in \mathbf{3}$, $\sigma \in \mathbf{1}$

Goldstone

$$\vec{\phi} = (v + \sigma) e^{i \frac{\pi^a}{v} X^a} \vec{F}$$

$$\mathcal{L} = \frac{1}{2}\left(1 + \frac{\sigma}{v}\right)^2 \left[\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{(\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 - |\vec{\pi}|^2 (\partial_\mu \vec{\pi})^2}{3v^2} + \cdots\right] + \frac{1}{2}\partial_\mu \sigma \partial^\mu \sigma - V(\sigma)$$

Let’s assume we don’t know UV theory

Concept of Higgs Effective Field Theory

Symmetry： $O(4)$

Matter： $\vec{\phi} = (\phi^1, \phi^2, \phi^3, \phi^4)^T \in \mathbf{4}$

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Symmetry： Non-linear $O(4)$

Matter： $\sigma \to \sigma$

$e^{i\frac{\pi^a}{v} X^a} \to \mathfrak{g} \cdot e^{i\frac{\pi^a}{v} X^a} \cdot \mathfrak{h}$

Let’s assume we don’t know UV theory

correct EFT is …

$$\mathcal{L}_{\text{HEFT}} = \frac{1}{2} F(h) \left[\partial_\mu \vec{\pi} \cdot \partial^\mu \vec{\pi} + \frac{(\vec{\pi} \cdot \partial_\mu \vec{\pi})^2 - |\vec{\pi}|^2 (\partial_\mu \vec{\pi})^2}{3v^2} + \cdots\right] + \frac{1}{2}\partial_\mu h \partial^\mu h - V(h)$$

$F(h)$, $V(h)$ ：arbitrary function of h