Gauge coupling jump and small instantons from a large non-minimal coupling

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Based on [hep-ph 2411.00768] with Wen Yin

QCD vacuum & instanton

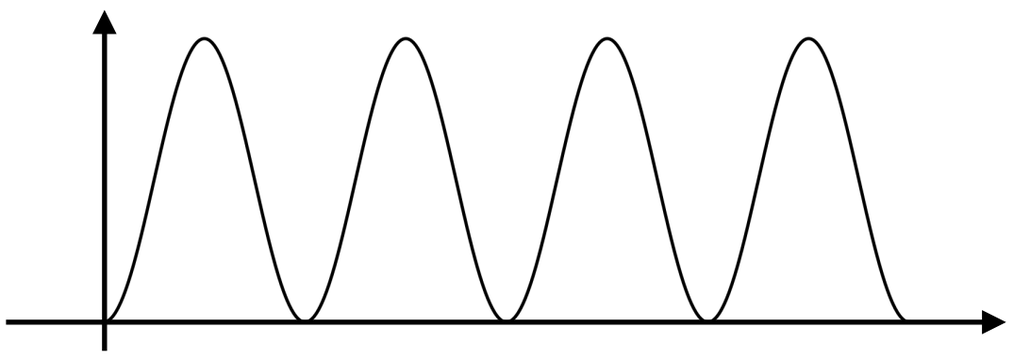
- QCD Instantons

  Instanton solutions connect degenerate vacua in QCD via quantum tunneling, ant it has finite-size.

$$|\theta\rangle = \sum_{\infty}^{\infty} e^{-in\theta} |n\rangle = \cdots + |0\rangle + e^{-i\theta} |1\rangle + \cdots$$

$$Z(\theta) := \langle\theta| e^{-HT} |\theta\rangle |_{T\to\infty} = \sum_{n,m} e^{i(n-m)\theta} \langle m| e^{-HT} |n\rangle$$

By evaluating cross terms between different vacua using instantons, the θ dependent QCD vacuum energy can be computed.



$$E(\theta) \propto -e^{-8\pi/g_s^2} \cos\theta$$

QCD vacuum & instanton

- Instantons size

  Typical size of instantons can be estimated by renormalization group for QCD coupling.

$$g^2(\mu) \simeq \frac{8\pi^2}{\beta_0 \log(\mu/\Lambda_{QCD})}$$

Instantons configuration is suppressed when coupling becomes asymptotically free. Dominant contributions comes from

$$e^{\frac{-8\pi^2}{g_s^2(1/\rho)}} = (\rho\Lambda_{QCD})^{\beta_0} \blacktriangleright \quad \rho \sim 1/\Lambda_{QCD}$$

Large instanton

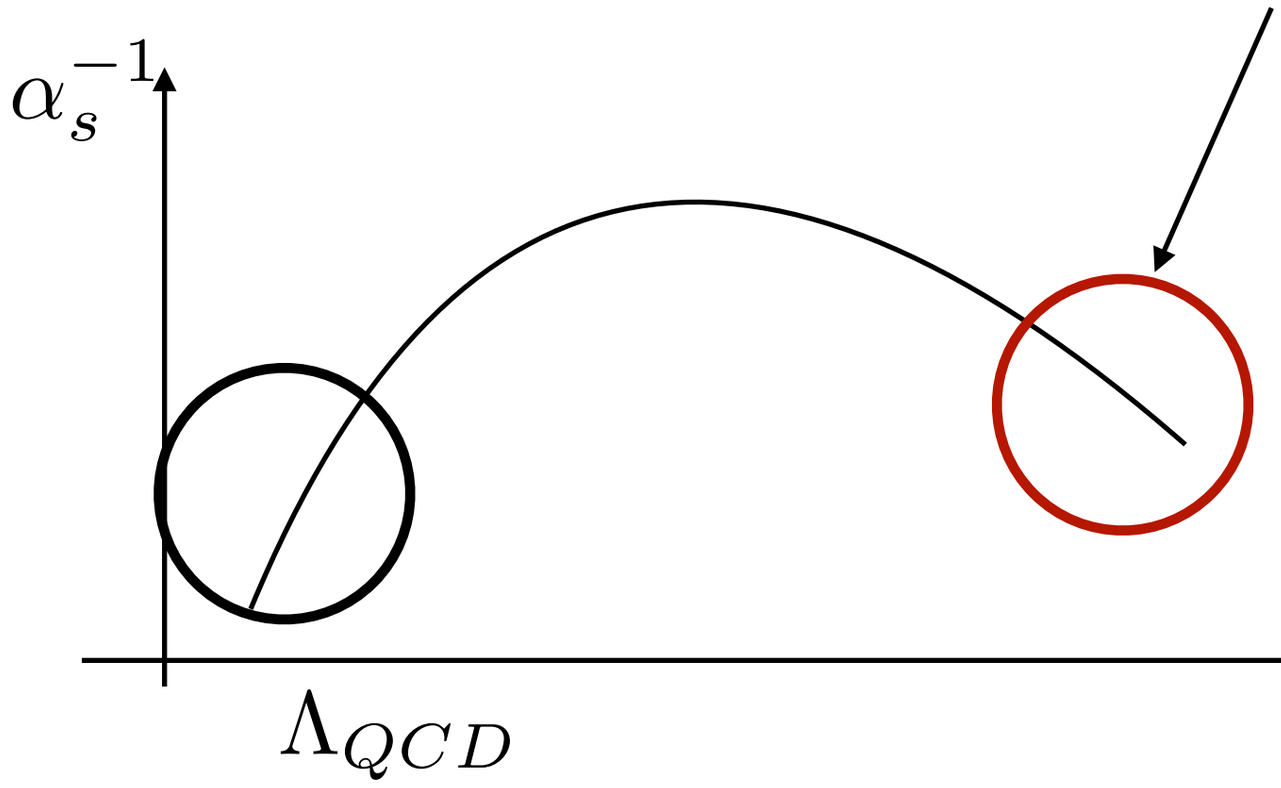
Note: smaller size instantons (small instantons) contribution is small in SM but exists in path integral.

Small instanton

- Large & small Instantons

  If g_s became stronger again at UV scale, instanton contribution is enhanced. Typical size of this instanton is small compared to QCD scale.

$$\rho \sim 1/\Lambda_{UV} << 1/\Lambda_{QCD}$$ Small instanton



It affects axion potential & may revive Strong CP problem.

M. Dine and N. Seiberg, Nucl. Phys. B 273 109 (1986).

R. Kitano and W. Yin, JHEP 07 (2021) 078

Overview of our work

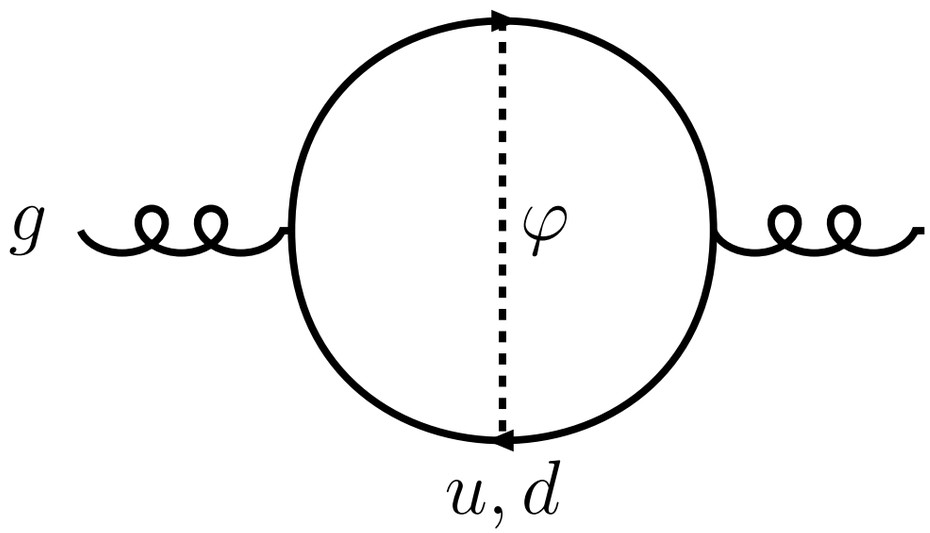
- Small instanton from non-minimal coupling

  Juntaro Wada and Wen Yin [hep-ph 2411.00768]

  In our work, we consider “Higgs” with non-minimal coupling to the gravity, and discuss the impact for scale dependence of QCD gauge coupling.

$$\mathcal{L} \ni \frac{1}{2} M_{\rm pl}^2 \left(1 + \xi \frac{h^2}{M_{\rm pl}^2}\right) g_J^{\mu\nu} \mathcal{R}_{J\mu\nu}$$ @ Jordan frame

This coupling modifies 2 point function for gluon at two loop level.



Non-minimal coupling

- Non-minimal coupling to gravity

  “Higgs” can have non-minimal coupling to gravity:

$$\mathcal{L} \ni \frac{1}{2} M_{\rm pl}^2 \left(1 + \xi \frac{h^2}{M_{\rm pl}^2}\right) g_J^{\mu\nu} \mathcal{R}_{J\mu\nu} \qquad \text{@ Jordan frame}$$

This simple setup has been often discussed in context of inflation.

F. L. Bezrukov and M. Shaposhnikov Phys. Lett. B 659 703 (2008)
F. L. Bezrukov, A. Magnin and M. Shaposhnikov Phys. Lett. B 675 88 (2009)

On the Einstein frame, for canonical normalized higgs φ , Yukawa interaction is given by

$$\mathcal{L}_{\rm quark} \ni \frac{h(\varphi)}{\left(1 + \xi \frac{h(\varphi)^2}{M_{\rm pl}^2}\right)^{1/2}} (\bar{\psi}_u \cdot Y_u \cdot \psi_u + \bar{\psi}_d \cdot Y_d \cdot \psi_d)$$

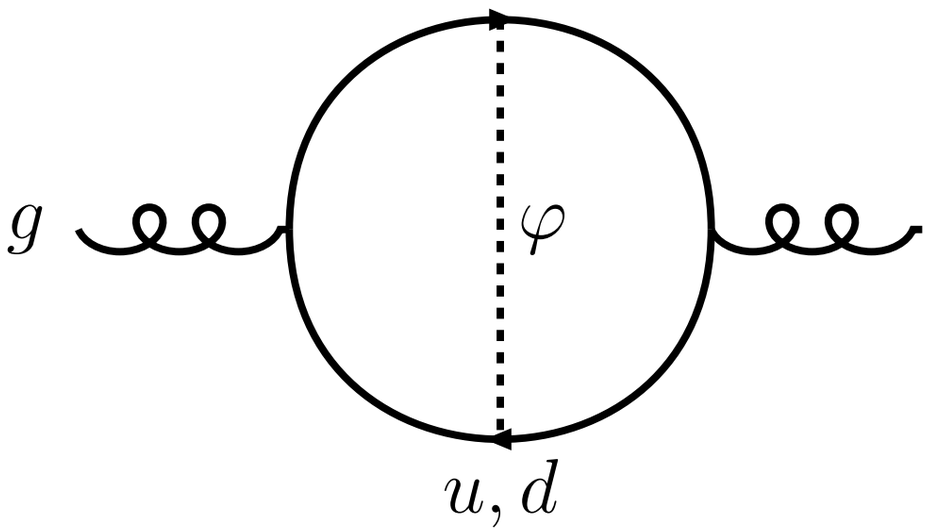
$$=: F(\varphi) \quad \text{Not polynomial function of } \varphi$$

Non-minimal coupling

- Gauge coupling jump from non-minimal coupling

  Juntaro Wada and Wen Yin [hep-ph 2411.00768]

Through Yukawa interaction, gluon 2-point function is modified:



Field dependent counter terms, and additional parameters appear. (Threshold effect)

F. Bezrukov, J. Rubio and M. Shaposhnikov, Phys. Rev. D 92 083512 (2015)

$$\delta\mathcal{L}_{\rm ct} \supset -\left(\partial_\varphi F\right)^2 \mathrm{tr}\, G_{\mu\nu}G^{\mu\nu}\left(\mathcal{O}\left(\frac{1}{(4-d)}\right)+\frac{1}{2}\,\delta g^{-2}\right),$$

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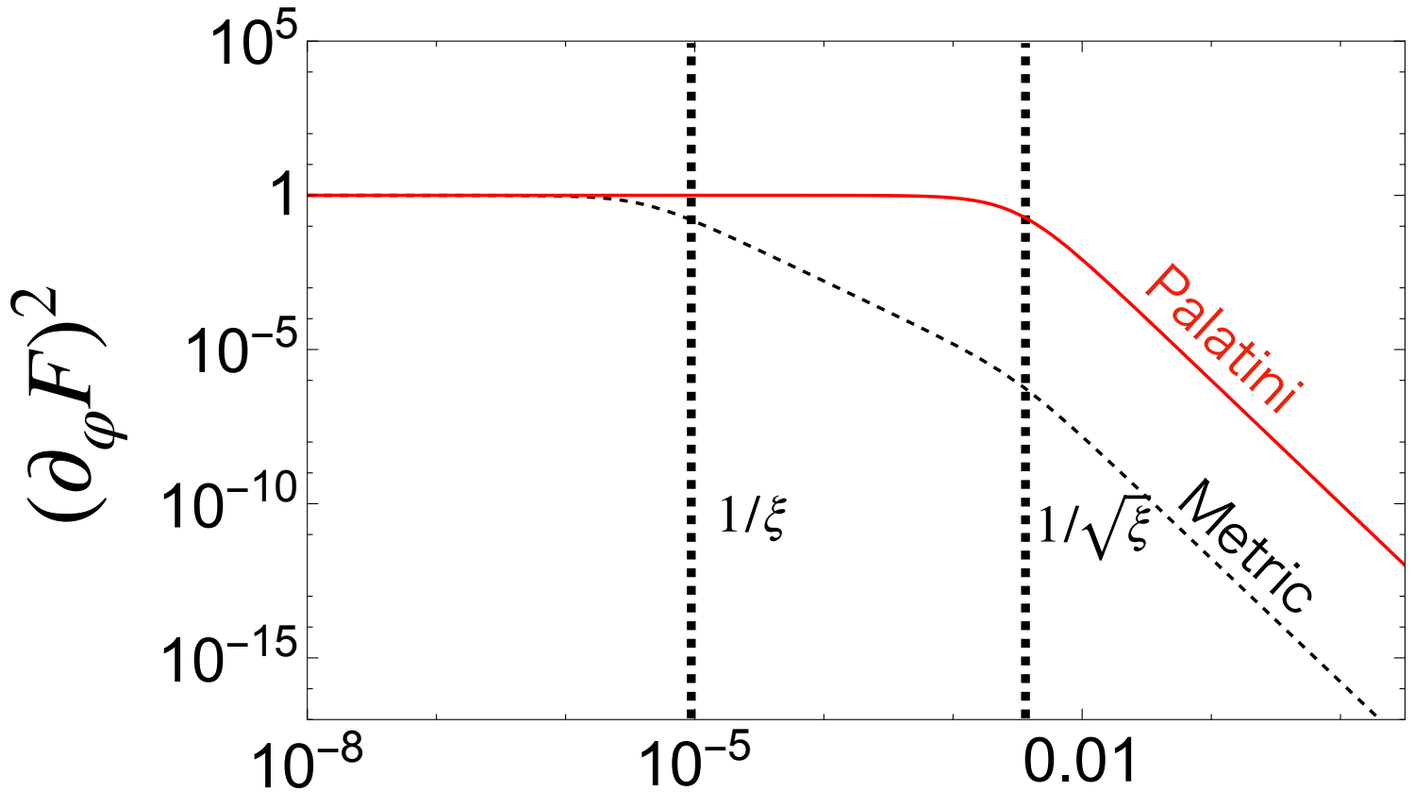
For $\partial_\varphi F = \partial_\varphi F(\bar{\varphi})$ with $\varphi = \bar{\varphi} + \delta\varphi$



083512 (2015)

$$\delta\mathcal{L}_{\rm ct} \supset -(\partial_\varphi F)^2 \,{\rm tr}\, G_{\mu\nu}G^{\mu\nu}\left(\mathcal{O}\left(\frac{1}{(4-d)}\right) + \frac{1}{2}\,\delta g^{-2}\right),$$

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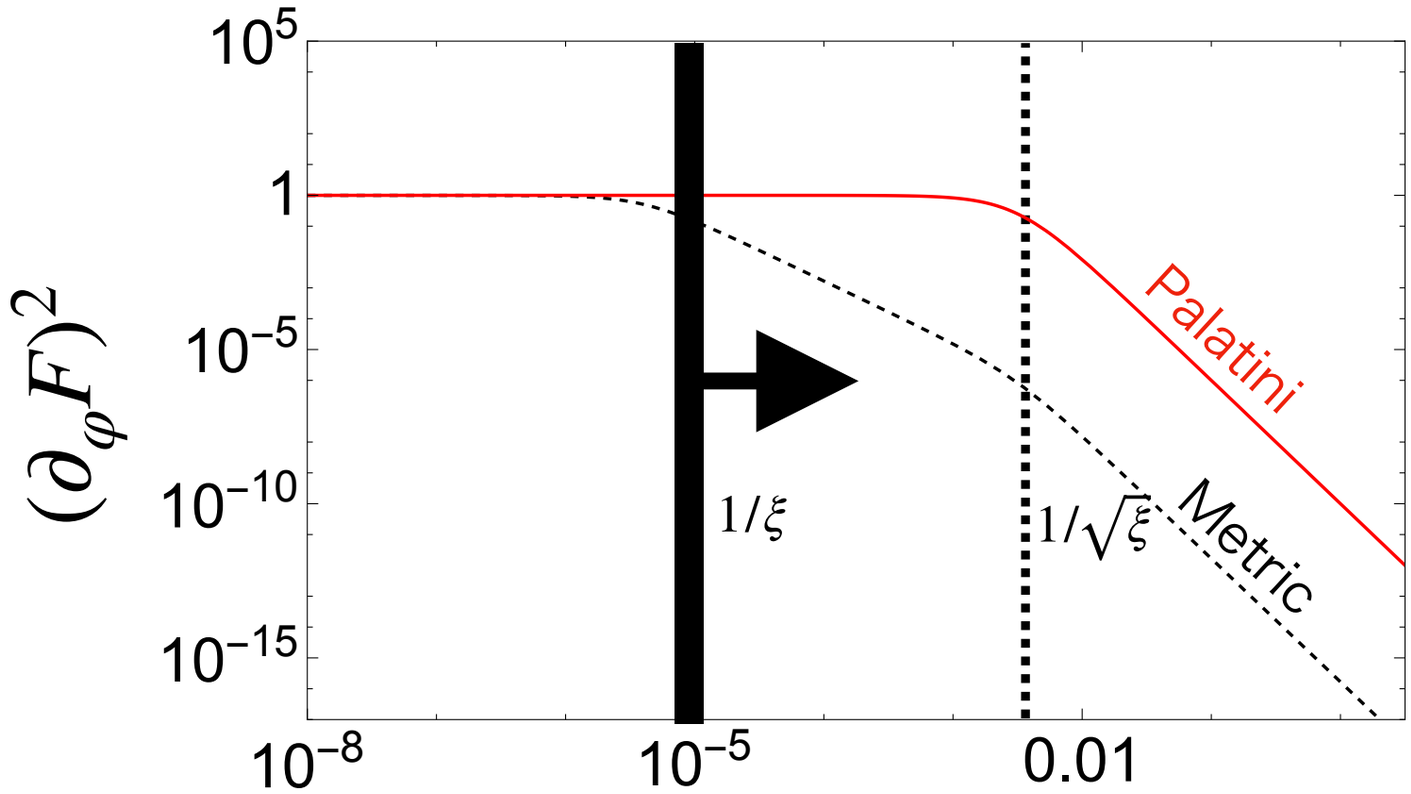
For $\partial_\varphi F = \partial_\varphi F(v_{\rm EW} + \delta\varphi)$



with

We encounter perturbative unitarity violation in

- $M_{\rm pl}/\xi \lesssim \delta\varphi \lesssim M_{\rm pl}/\sqrt{\xi}$

R. N. Lerner and J. McDonald, JCAP 1004 015 (2010)

S. Ferrara, et.al., Phys. Rev. D83 025008 (2011)

F. Bezrukov, et.al, JHEP 01 016 (2011)

We cannot rely on usual RGE for QCD gauge coupling.

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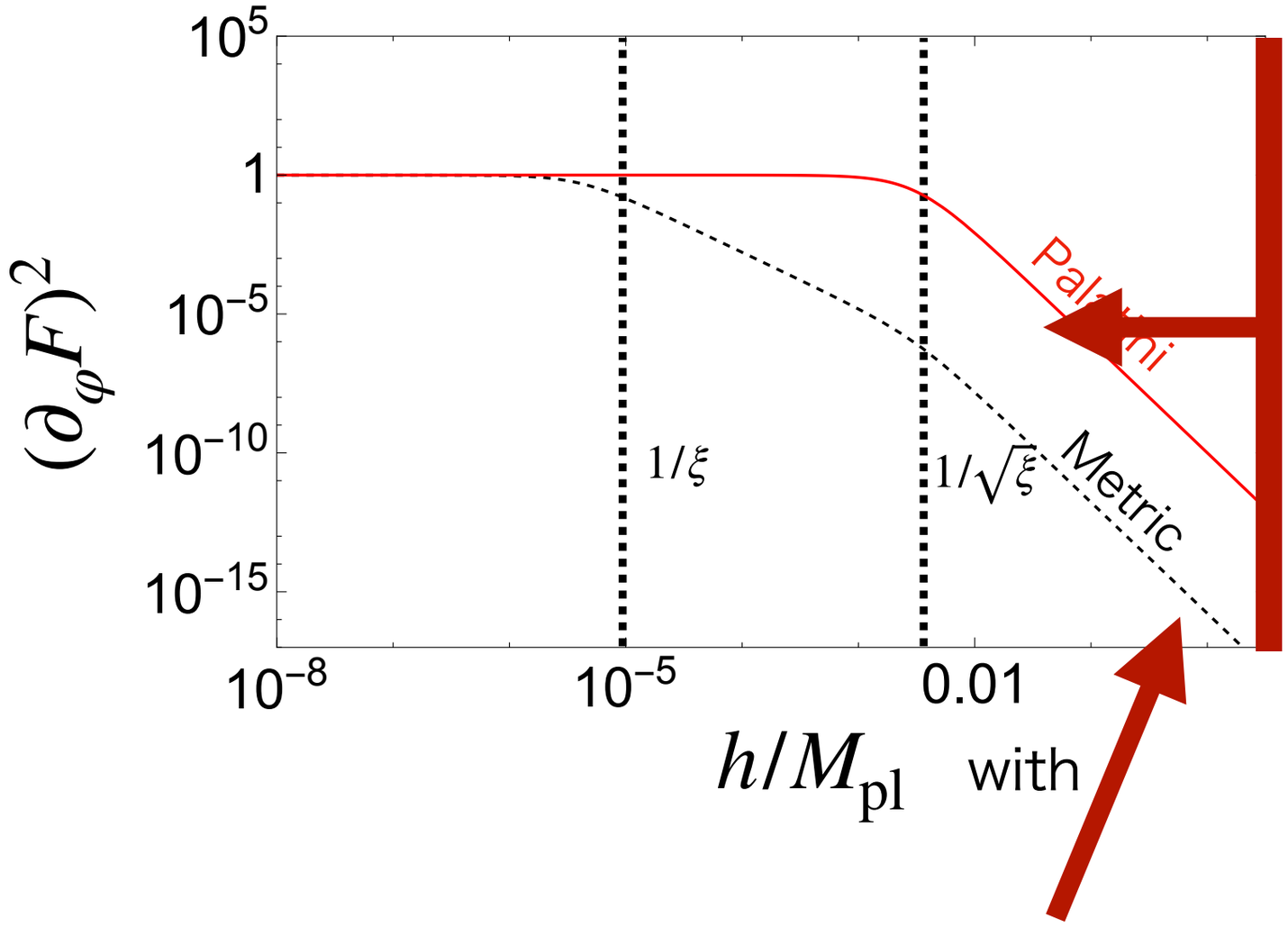
For $\partial_\varphi F = \partial_\varphi F(v_{\rm EW} + \delta\varphi)$



with

Instead RGE for QCD coupling,
We want to evaluate $\partial_\varphi F(v_{\rm EW} + \delta\varphi)$ without perturbative expansion in terms of $\delta\varphi$.

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Cf) Lattice simulation for free scalar field with high momentum mode, $\epsilon\Lambda < k < \Lambda$

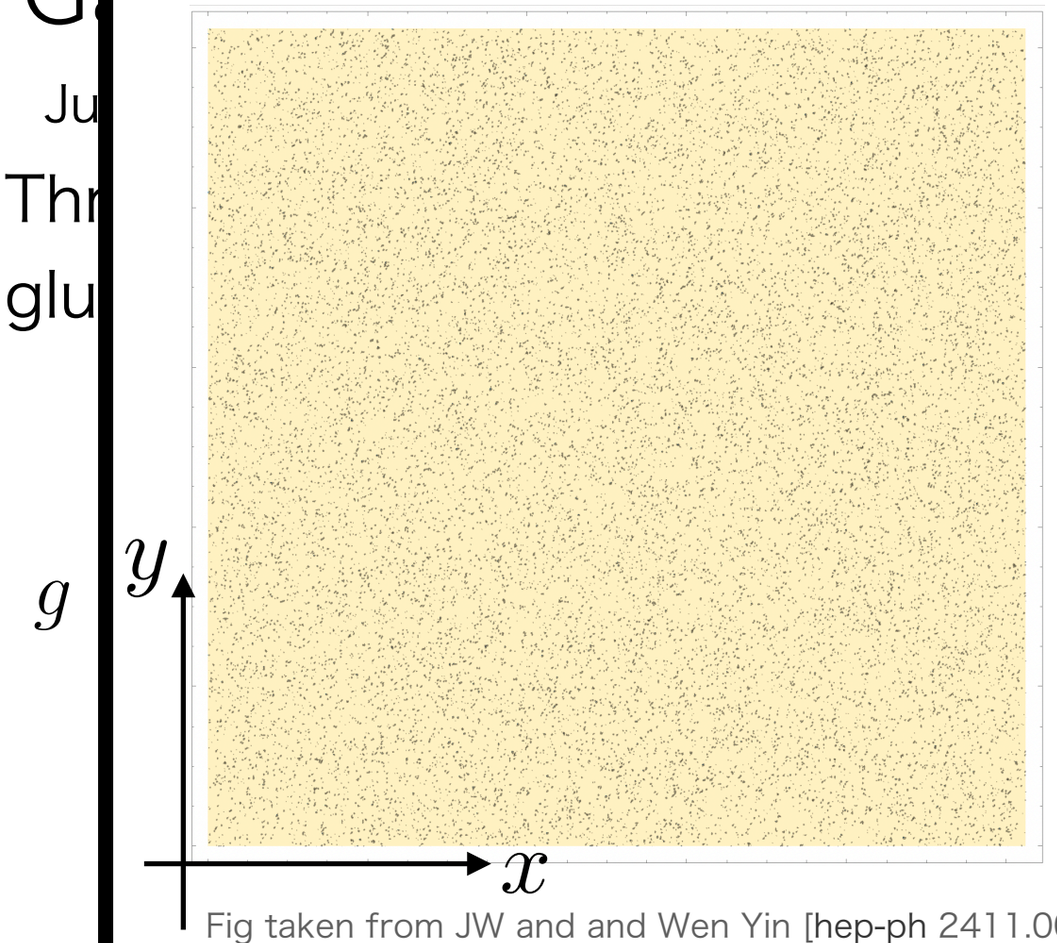
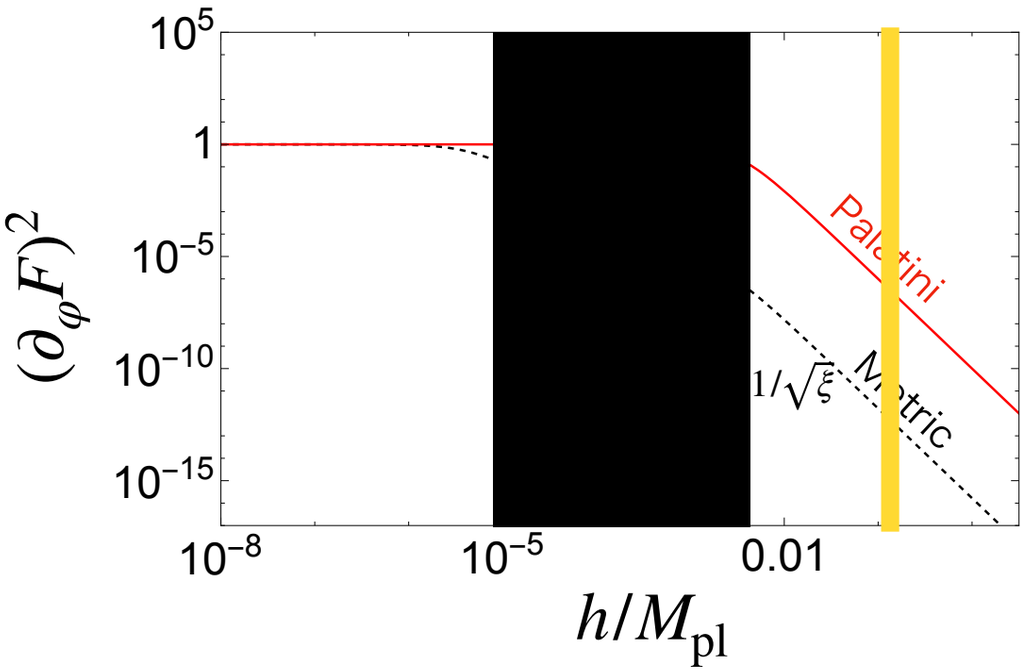


Fig taken from JW and and Wen Yin [hep-ph 2411.00768]



$|\varphi(x)|$ is typically $\Lambda/(4\pi)$, on yellow region, except for black points.

$(\partial_\varphi F(v_{\rm EW}+\delta\varphi))^2 \simeq 0$ for $\Lambda \gtrsim M_{\rm pl}/\sqrt{\xi}$

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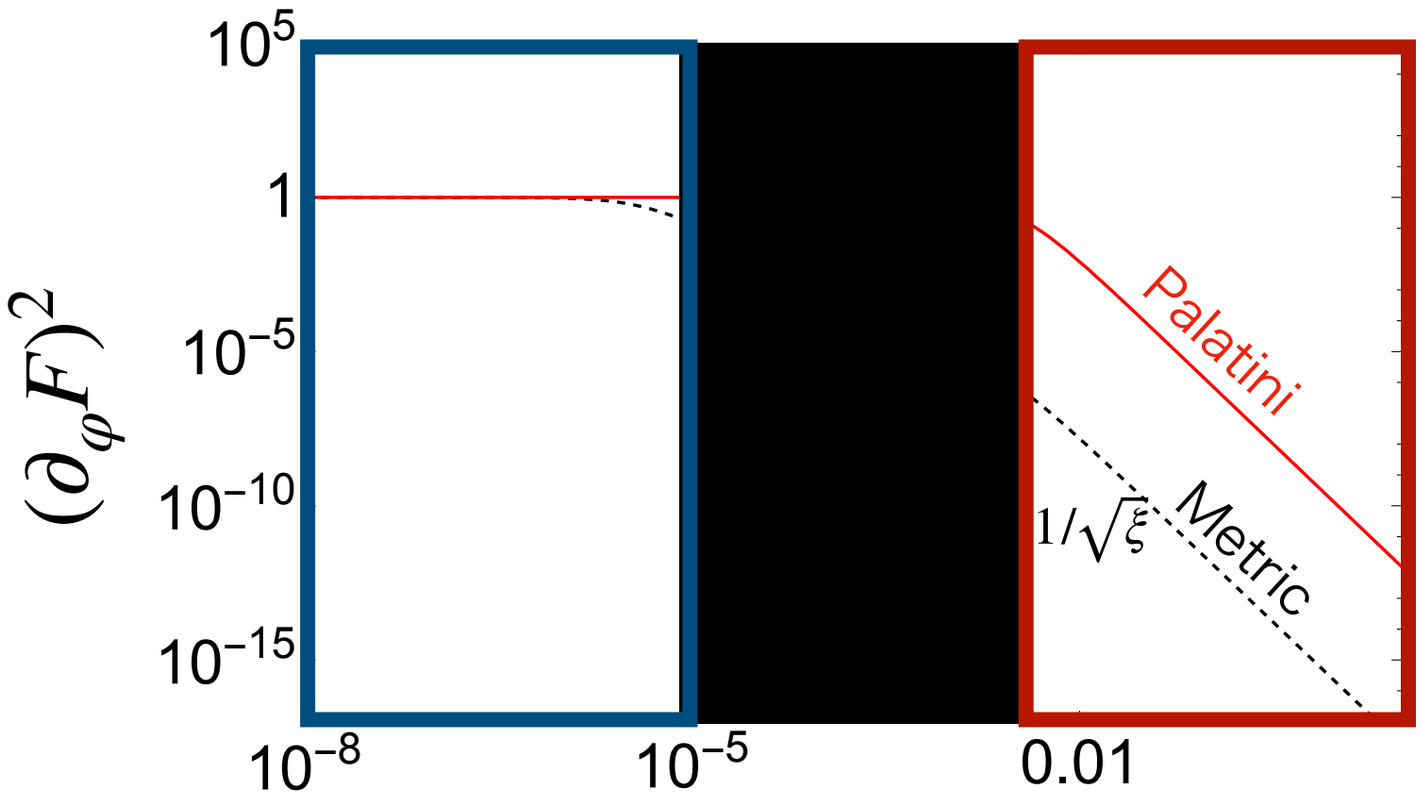
For $\partial_\varphi F = \partial_\varphi F(v_{\rm EW} + \delta\varphi)$ with $h(v_{\rm EW}) := v_{\rm EW}$



$$\frac{1}{g^2_{\rm eff}} \simeq \begin{cases} \dfrac{1}{g^2} + \delta g^{-2}, & \text{for } h \ll \dfrac{M_{\rm pl}}{\xi}, \\ \dfrac{1}{g^2}, & \text{for } h \gg \dfrac{M_{\rm pl}}{\sqrt{\xi}}. \end{cases}$$

Small instanton

If $1/g^2 \lesssim 1,$
$\delta g^{-2} \gtrsim 1$

Summary

- QCD instants has finite size, typically inverse of QCD scale. However, if QCD coupling becomes stronger again at UV scale, small instanton appears.
- If Higgs has non minimal coupling to the gravity, it affects gluon 2 point function, from 2 loop diagram.
- We observe a gauge coupling jump, which can lead to small instantons, between regions separated by regime where perturbative unitarity is violated.

Back up

Non-minimal coupling

- Yukawa interaction @ Einstein frame

F. L. Bezrukov and M. Shaposhnikov Phys. Lett. B 659 703 (2008)
F. L. Bezrukov, A. Magnin and M. Shaposhnikov Phys. Lett. B 675 88 (2009)

$$\mathcal{L}_{\rm quark} \ni \frac{h(\varphi)}{\left(1+\xi\frac{h(\varphi)^2}{M_{\rm pl}^2}\right)^{1/2}}(\bar{\psi}_u\cdot Y_u\cdot\psi_u+\bar{\psi}_d\cdot Y_d\cdot\psi_d) =: F(\varphi)$$

$$F(\varphi)\simeq\begin{cases}\varphi, & \text{for } \varphi\ll\dfrac{M_{\rm pl}}{\xi},\\ \dfrac{M_{\rm pl}}{\sqrt{\xi}}\left(1-e^{-\sqrt{2/3}\varphi/M_{\rm pl}}\right), & \text{for } \varphi\gg\dfrac{M_{\rm pl}}{\sqrt{\xi}}.\end{cases}$$

φ is canonically normalized field @ Einstein frame

Non-minimal coupling

- Perturbative unitarity @ Jordan frame

S. Ferrara, et.al., Phys. Rev. D83 025008 (2011)
F. Bezrukov, et.al, JHEP 01 016 (2011)

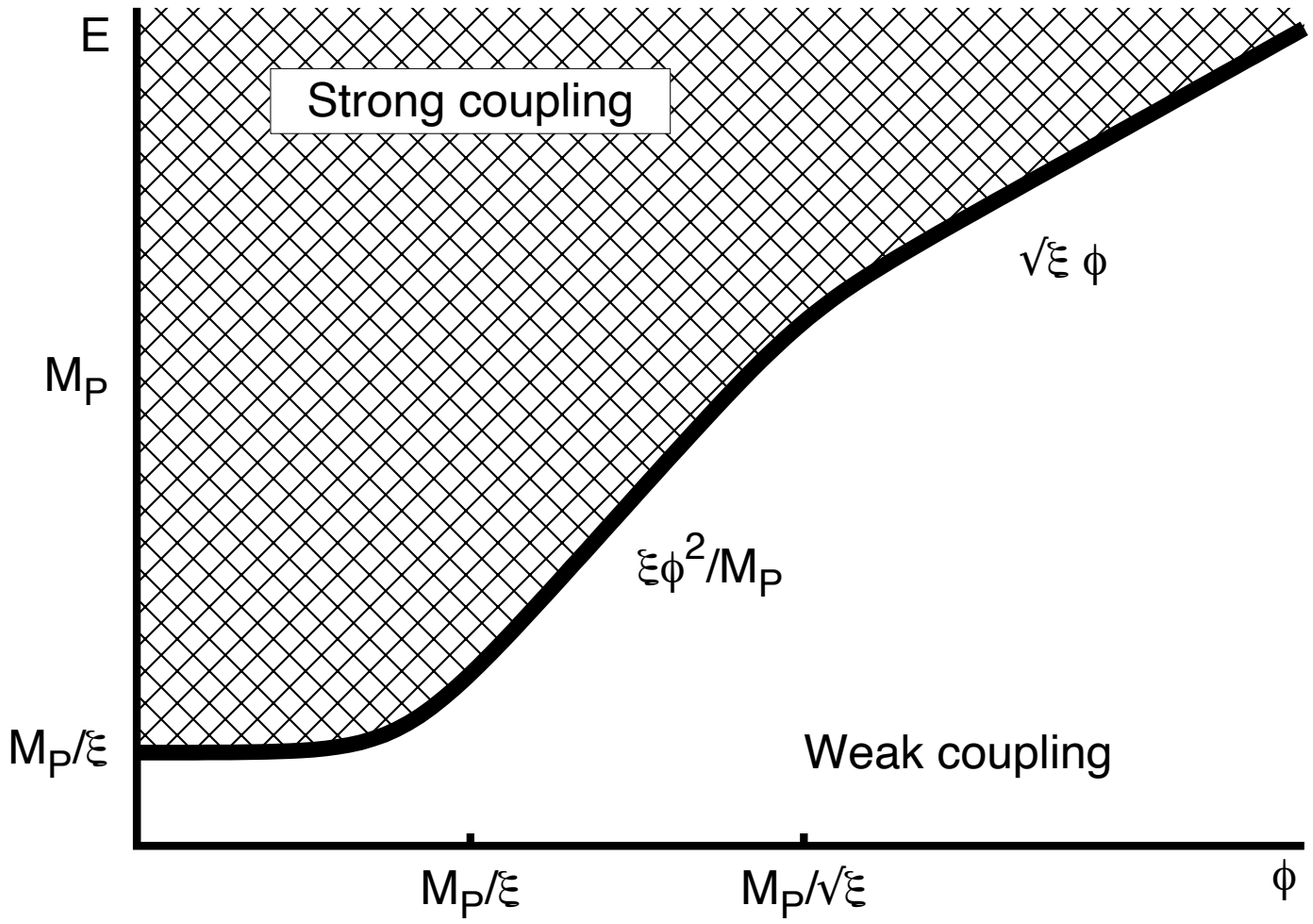


Fig taken from F. Bezrukov, et.al, JHEP 01 016 (2011)

Field dependent cut-off
@ Jordan frame:

$$\mathcal{L}_J \ni \frac{\xi\sqrt{M_{\rm pl}+\xi\bar{\phi}^2}}{M_{\rm pl}+\xi\bar{\phi}^2+6\xi^2\bar{\phi}^2}(\delta\phi)^2\Box h$$

Non-minimal coupling

- Perturbative unitarity @ Einstein frame

C.P. Burgess, H.M. Lee and M. Trott, JHEP 07 007 (2010)

At Einstein frame, before canonical normalization,

$$\mathcal{L}_{\rm Higgs} \supset \frac{Z[h]}{2}\partial_\mu h \partial^\mu h \qquad \Omega := \left(1+\xi\frac{h^2}{M_{\rm pl}}\right)^{1/2}$$

$$Z[h] = \begin{cases} \Omega^{-2} + \frac{3}{2}(M_{\rm pl}\frac{d}{dh}\log(\Omega^2))^2 & \text{(Metric)}, \\ \Omega^{-2} & \text{(Palatini)}, \end{cases}$$

- $\Lambda_{\rm cutoff} = \begin{cases} M_{\rm pl}/\xi & \text{(Metric)}, \\ M_{\rm pl}/\sqrt{\xi} & \text{(Palatini)}, \end{cases}$

Non-minimal coupling

- Perturbative unitarity @ Einstein frame

C.P. Burgess, H.M. Lee and M. Trott, JHEP 07 007 (2010)
F. Bauer and D.A. Demir, Phys. Lett. B 698 425 (2011)

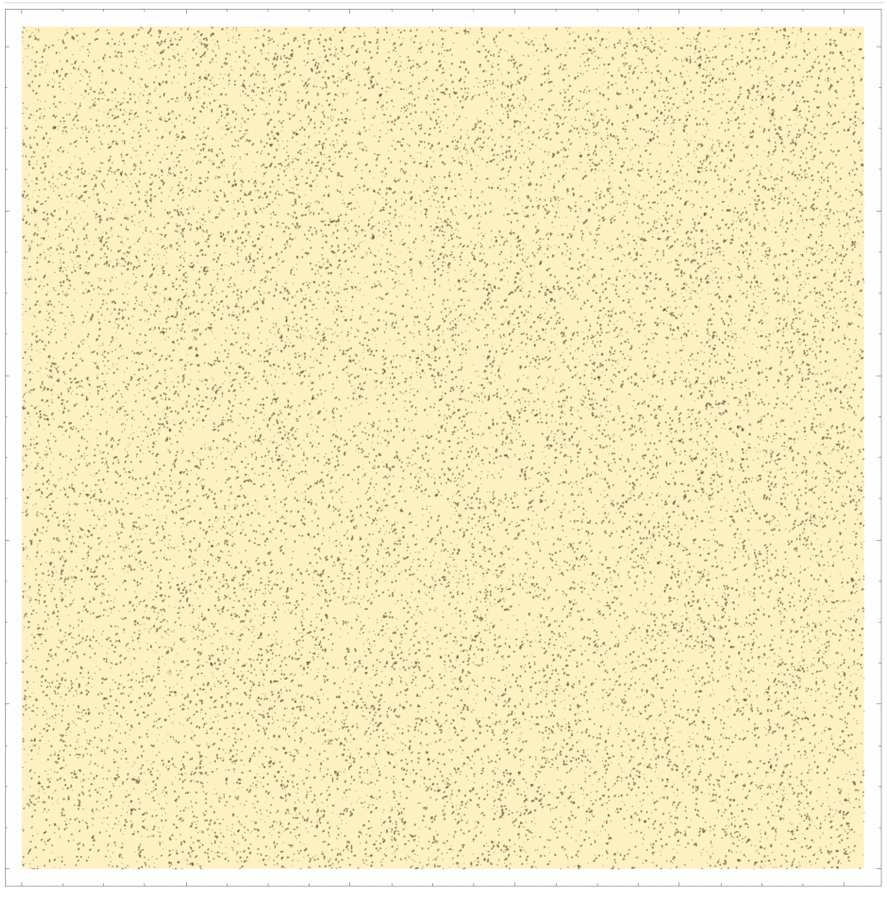
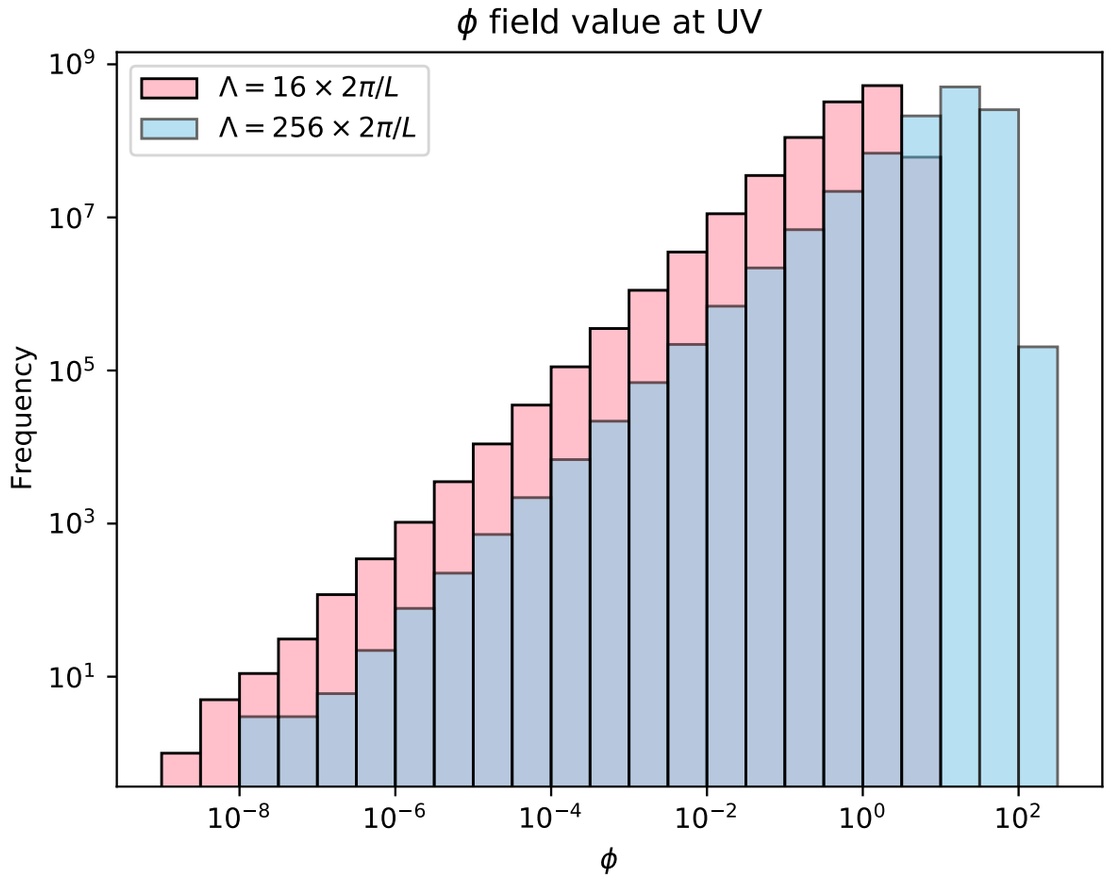
In both the metric and Palatini formalisms, the SM Higgs becomes decoupled from the weak gauge bosons in the large-field regime.

$$m_V^2 \simeq g^2 \frac{h(\varphi)^2}{\Omega(\varphi)^2} \to g^2 \frac{M_{\rm pl}^2}{\xi}$$

This implies existence of cut-off energy for weak boson energy.

- $\Lambda_{\rm cutoff} \simeq \dfrac{M_{\rm pl}}{\sqrt{\xi}}$

Non-minimal coupling

- Histogram for field configuration

  Juntaro Wada and Wen Yin [hep-ph 2411.00768]