NANOGrav Signal and PBH from the Modified Higgs Inflation

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Collaborators:
Kingman Cheung (NTHU),
C.J. Ouseph(Seoul Tech)

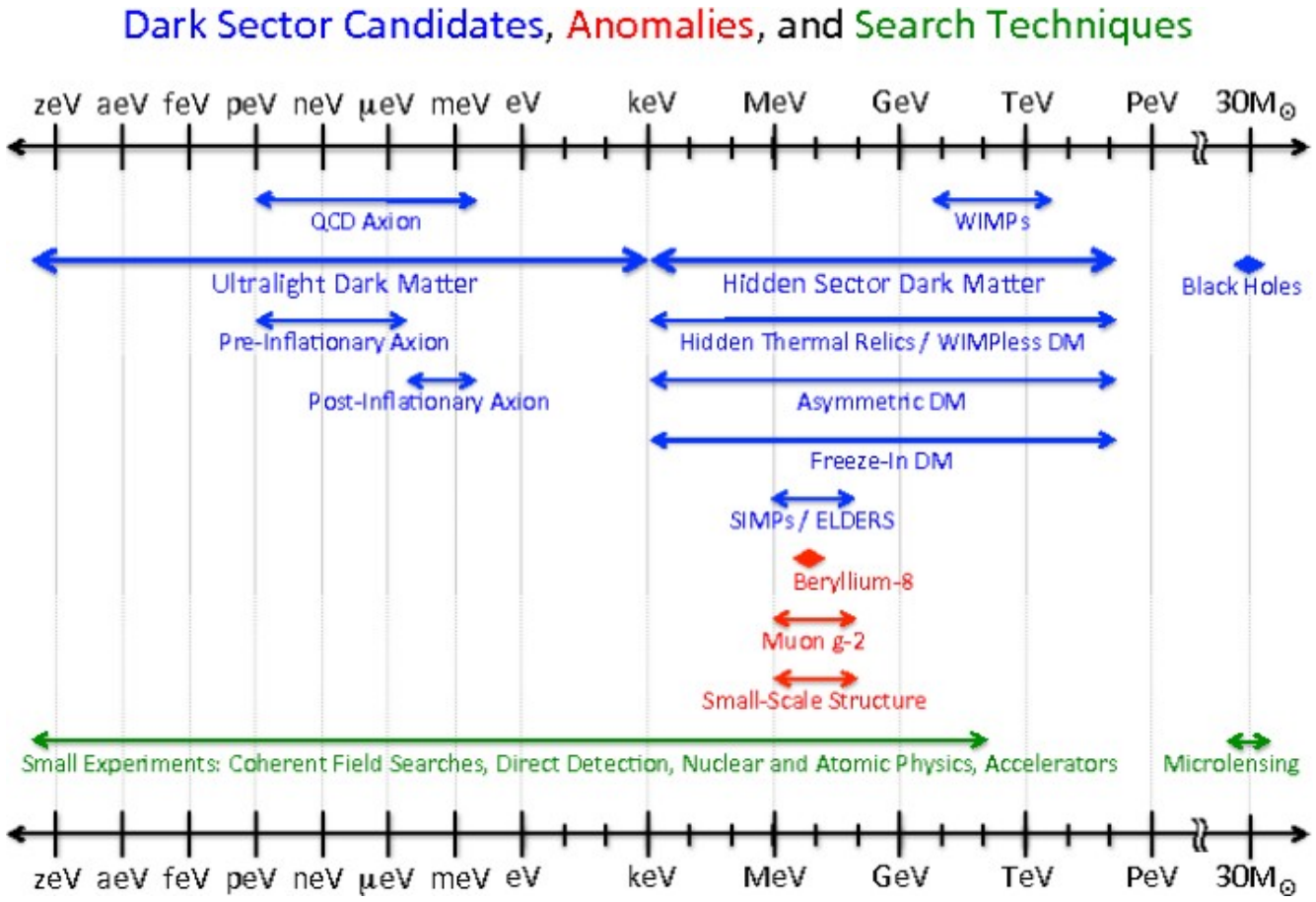
Reference: Eur.Phys.J.C 84(2024)9, 906

HPNP2025 The 7th International Workshop on "Higgs as a Probe of New Physics 2025"

9.-13. June 2025, University of Osaka, Japan

DM candidate

- Range of DM mass:



CERN document Server: US:Cosmic Visions: New ideas in dark matter 2017

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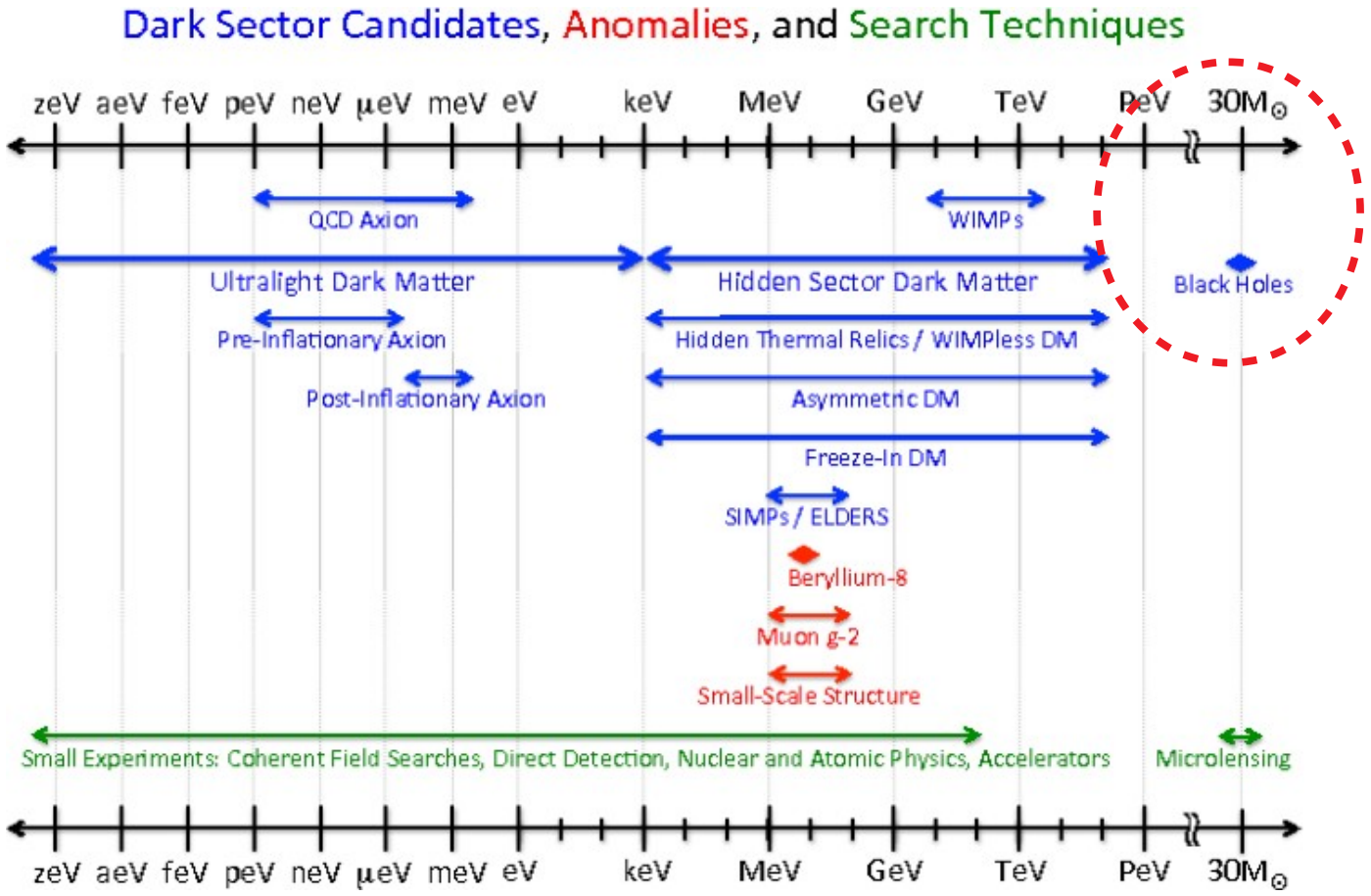
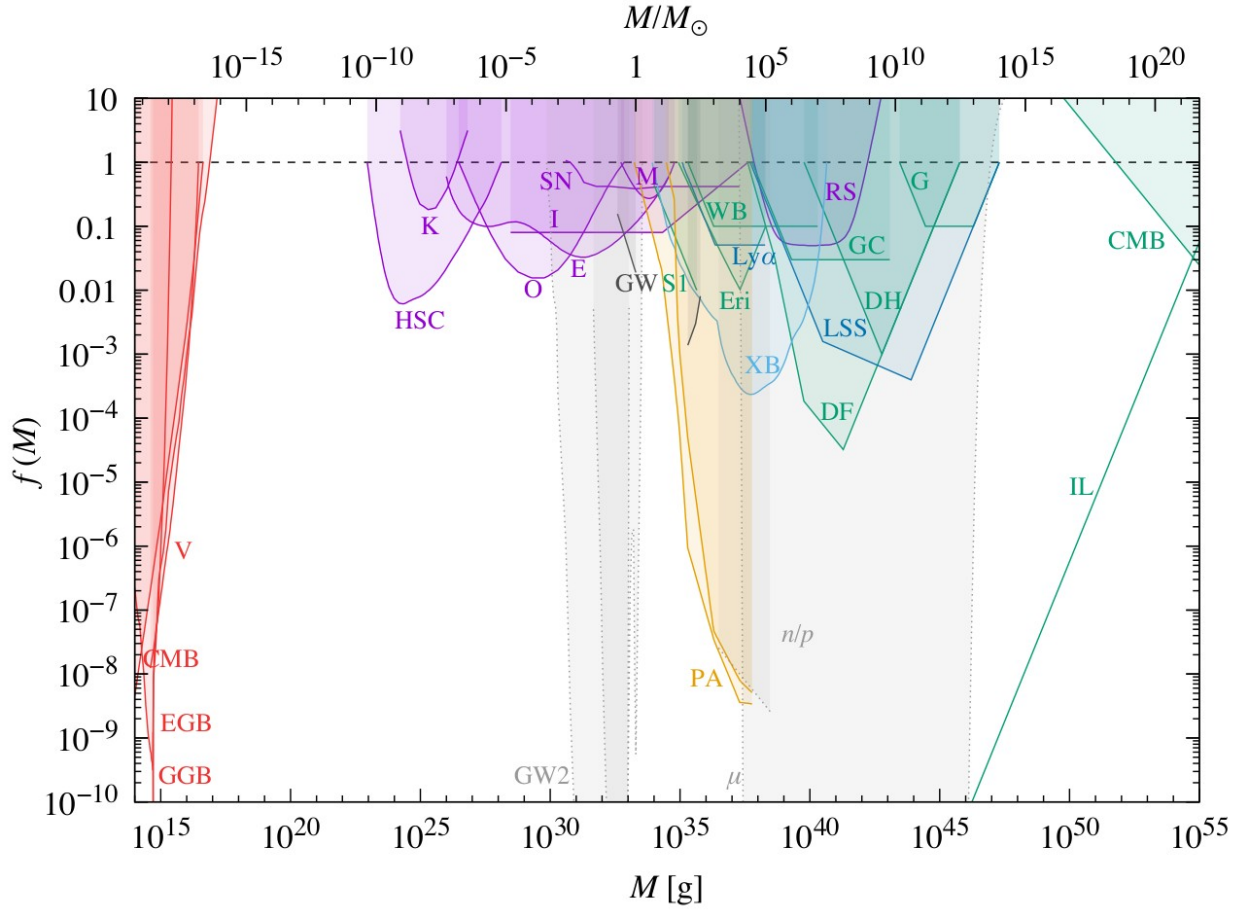


CERN document Server: US:Cosmic Visions: New ideas in dark matter 2017

PBH constraints

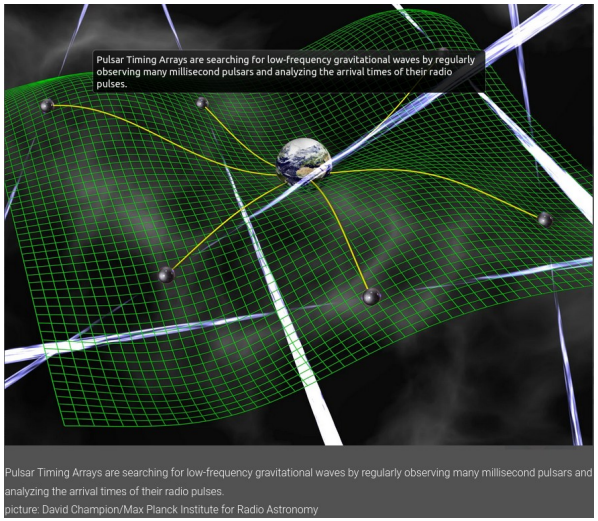
- Mass range of PBH (Primordial Black Holes):



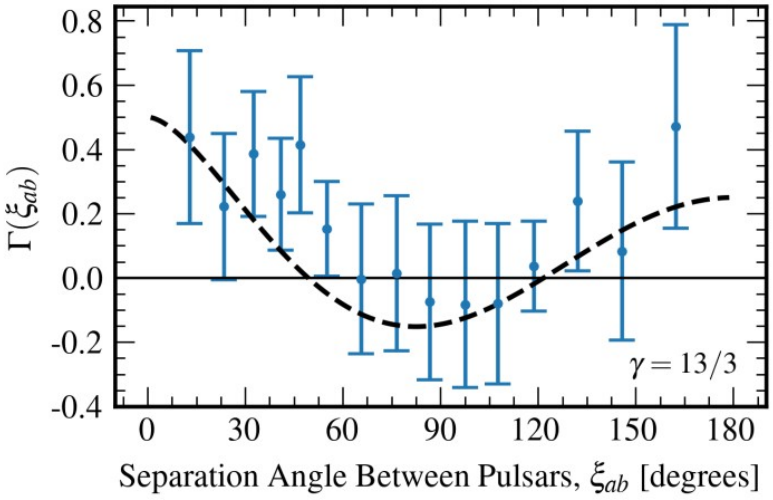
B.Carr, K.Kohri, Y.Sendouda, and J.Yokoyama, arXiv:2002.12778

GWs: Pulsar Timing Array

- The millisecond pulsars provide accurate timing signals, and are exploited to measure the gravitational waves (GW).
- The NANOGrav collaboration observed 67 pulsars for 15 yr and reported the correlations of Helling-Downs pattern, pointing to the stochastic GW.



Pulsar Timing Arrays | Max Planck Institute for Gravitational Physics (Albert Einstein Institute)



Helling-Downs pattern from NANOGrav 15 yr, arXiv:2306.16213

Higgs inflation model

- The Higgs inflation model discussed the possibility of the SM Higgs as the inflaton. F.L.Bezrukov, M.Shaposhnikov: Phys.Lett.B659(2008),703-706.
- Simplest model introduces a non-minimal coupling, where the SM Higgs is coupled to the Ricci scalar with a non-minimal coupling strength:

$$\mathcal{S}_{\mathcal{J}} = \int d^4x \sqrt{-g}\left[-\frac{M_{\rm PL}^2 + \xi h^2}{2}\mathcal{R} + \frac{\partial_\mu h \partial^\mu h}{2} - \frac{\lambda}{4}(h^2 - v^2)^2\right]$$

D.S.Salopek, J.R.Bond, J.M.Bardeen: PRD40(1989)1753.
D.I.Kaiser: PRD52(1995) 4295-4306.
E.Komatsu, T.Furamase: PRD59(1999) 064029.

- Several attempts to address inflation and PBH production in the Higgs-inflation framework D.Y.Cheong, S.M.Lee, S.C.Park: JCAP 01(2021)032.
R.Kawaguchi, S.Tsujikawa: PRD107(2023) no.6,063508.
J.M.Ezquiaga, et.al: PLB776(2018), 345-349.
A.Gundhi, C.F.Steinwachs: EPJC81(2021) no.5,460.

Higgs inflation model

- The action in the Einstein frame is

$$\mathcal{S}_{\mathcal{E}} = \int d^4x\sqrt{-\hat{g}}\Big[-\frac{M_{\rm PL}^2}{2}\hat{\mathcal{R}} + \frac{\partial_\mu\phi\partial^\mu\phi}{2} - U(\phi)\Big]$$

in which also provides the exponentially flat effective potential for the Higgs field:

$$U(\phi) = \frac{\lambda M_{PL}^2}{4\xi^2}\left(1+\exp\left(-\frac{2\phi}{\sqrt{6}M_{PL}}\right)\right)^{-2}$$

- The constraint on the Higgs self and non-minimal couplings can be obtained from COBE normalization $\frac{U}{\epsilon} = (0.027 M_{\rm PL})^4$

$$\frac{\lambda}{\xi^2} = 4.41026\times10^{-10}$$

Higgs inflation model

- The scalar power spectrum of curvature perturbation in slow-roll condition:

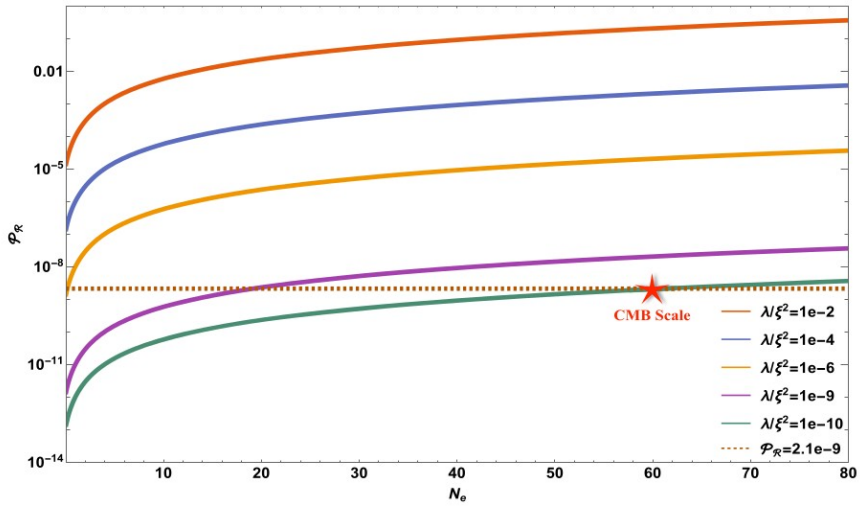
D.Baumann: arXiv: hep-th/0907.5424.

$$\mathcal{P}_{\mathcal{R}} = \frac{1}{12\pi^2}\frac{U^3(\phi)}{M_{PL}^6 U'^2(\phi)}$$

- More precise determination of $\mathcal{P}_{\mathcal{R}}$ can be obtain by solving the Mukhanov-Sasaki equation.

M.Sasaki: Prog.Theor.Phys.76,1036(1986).
E.D.Stewart, D.H.Lyth: PLB 302,171-175 (1993).

- The CMB observations suggested the value of $\mathcal{P}_{\mathcal{R}} = 2.1 \times 10^{-9}$ at CMB pivot scale



Modified Higgs Potential

- To produce PBHs in the early Universe, the magnitude of the curvature power spectrum needs to be 10^{-3} to 10^{-2}.

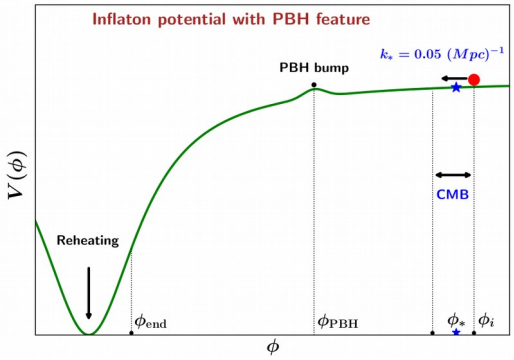
- We add a Gaussian dip/bump to the Higgs potential

S.S.Mishra, V.Sahni: JCAP 04 (2020)007.

$$\pm \left[A e^{-\frac{(h(\phi)-h_0(\phi))^2}{2\sigma^2}} \right]$$

- After the conformal transformation, the potential becomes

$$U_{eff}(\phi) = \frac{\lambda h^4(\phi)}{4} \frac{\left(1 \pm A e^{-\frac{(h(\phi)-h_0(\phi))^2}{2\sigma^2}}\right)}{\left(\frac{h^2(\phi)\xi}{M_{PL}^2} + 1\right)^2}$$



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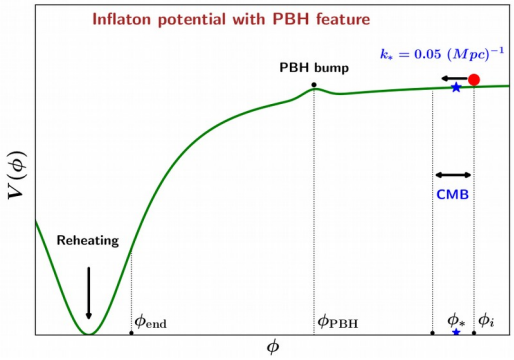
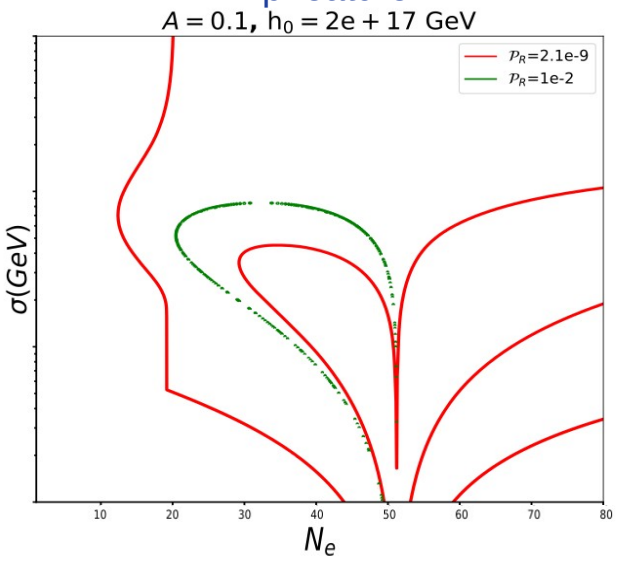


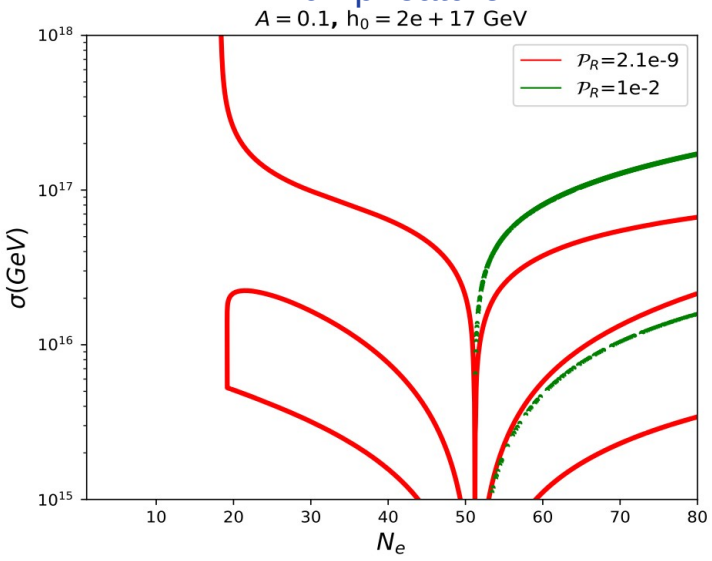
Parameter space search: dip or bump

- They yields a value of $\mathcal{P}_{\mathcal{R}} = 10^{-2} - 10^{-3}$ for PBH formation and $\mathcal{P}_{\mathcal{R}} = 2.1 \times 10^{-9}$ for a successful inflation model at CMB scale.
- A dip feature produces a desired PBH formation and inflation:

Dip feature:



Bump feature:



K.Cheung, C.J.Ouseph, P.Y.Tseng: EPJC 84 (2024) 9,906.

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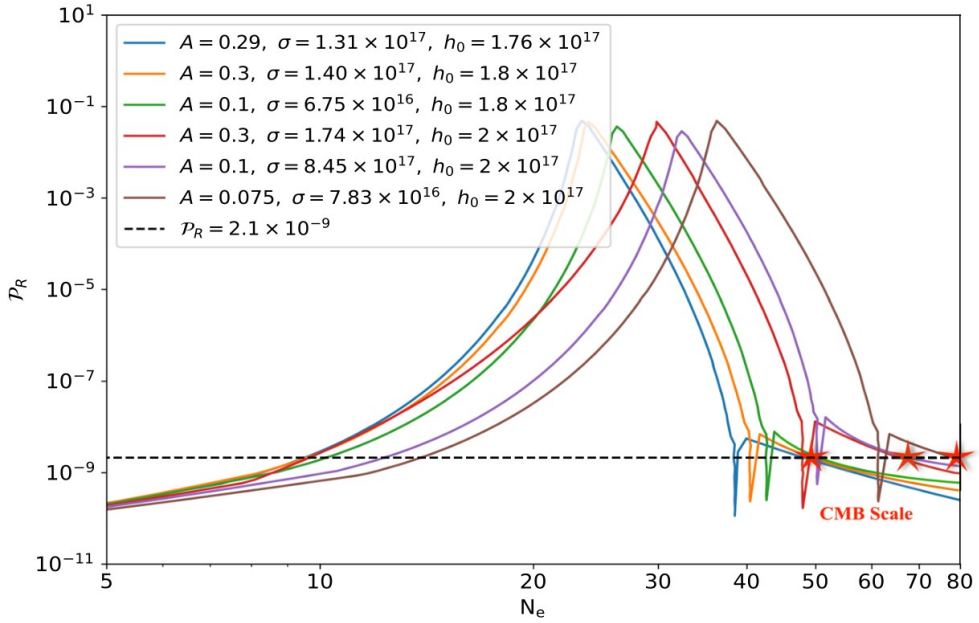

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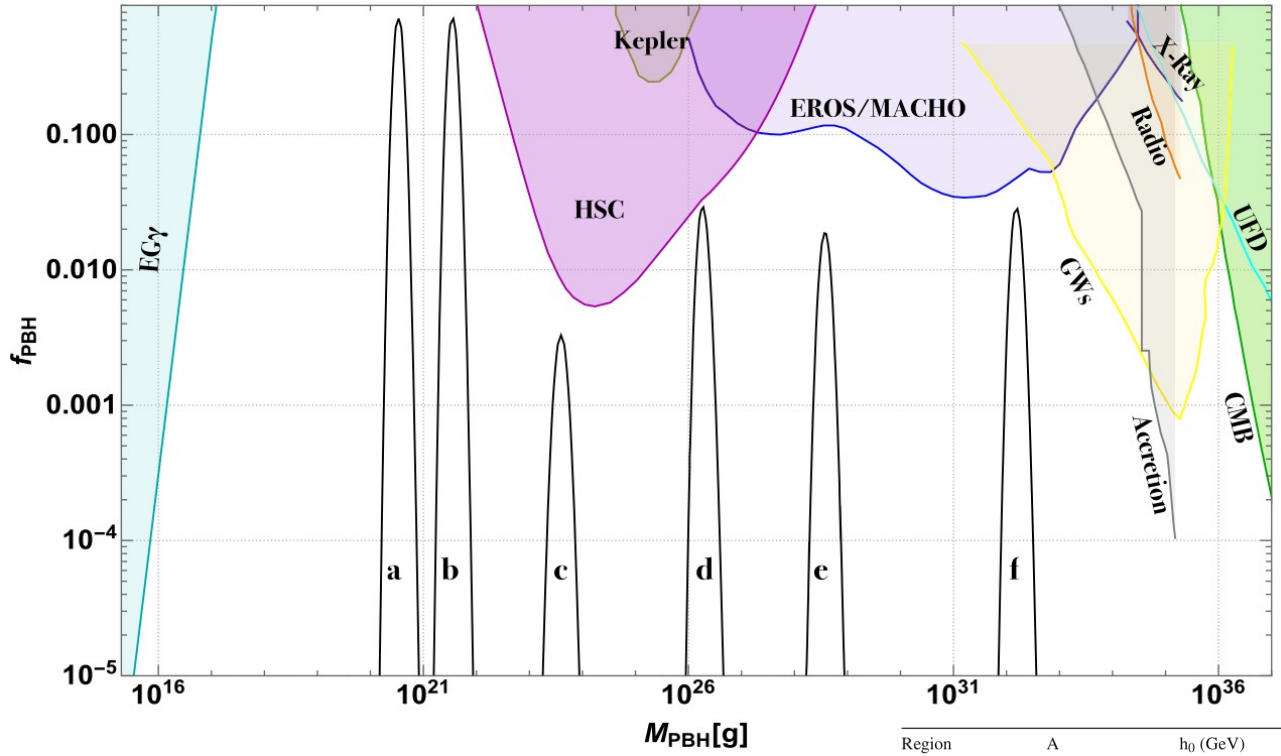
$$\beta(M_{PBH}) = 3.7\times10^{-9}\left(\frac{\gamma}{0.2}\right)^{-1/2} \times\left(\frac{g_{*form}}{10.75}\right)^{1/4}\left(\frac{M_{PBH}}{M_\odot}\right)^{1/2} f_{PBH}$$

$$M_{PBH} = 1.36\times10^{18}\left(\frac{\gamma}{0.2}\right)\left(\frac{g*}{106.75}\right)^{-1/6} \times\left(\frac{k}{7\times10^{13}\ Mpc^{-1}}\right)^{-2}$$

PBH formation

- PBH fractions for benchmark points:



Region	A	h_0 (GeV)	σ (GeV)	N	n_s	r
a	0.29	1.76×10^{17}	1.31×10^{17}	N = 47	0.958287	0.00960803
b	0.3	1.8×10^{17}	1.40×10^{17}	N = 50	0.964321	0.00951552
c	0.1	1.8×10^{17}	6.75×10^{16}	N = 51	0.960781	0.00347376
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e	0.1	2×10^{17}	8.45×10^{16}	N = 68	0.952763	0.00396931
f	0.075	2.1×10^{17}	7.83×10^{16}	N = 78	0.952688	0.00290832

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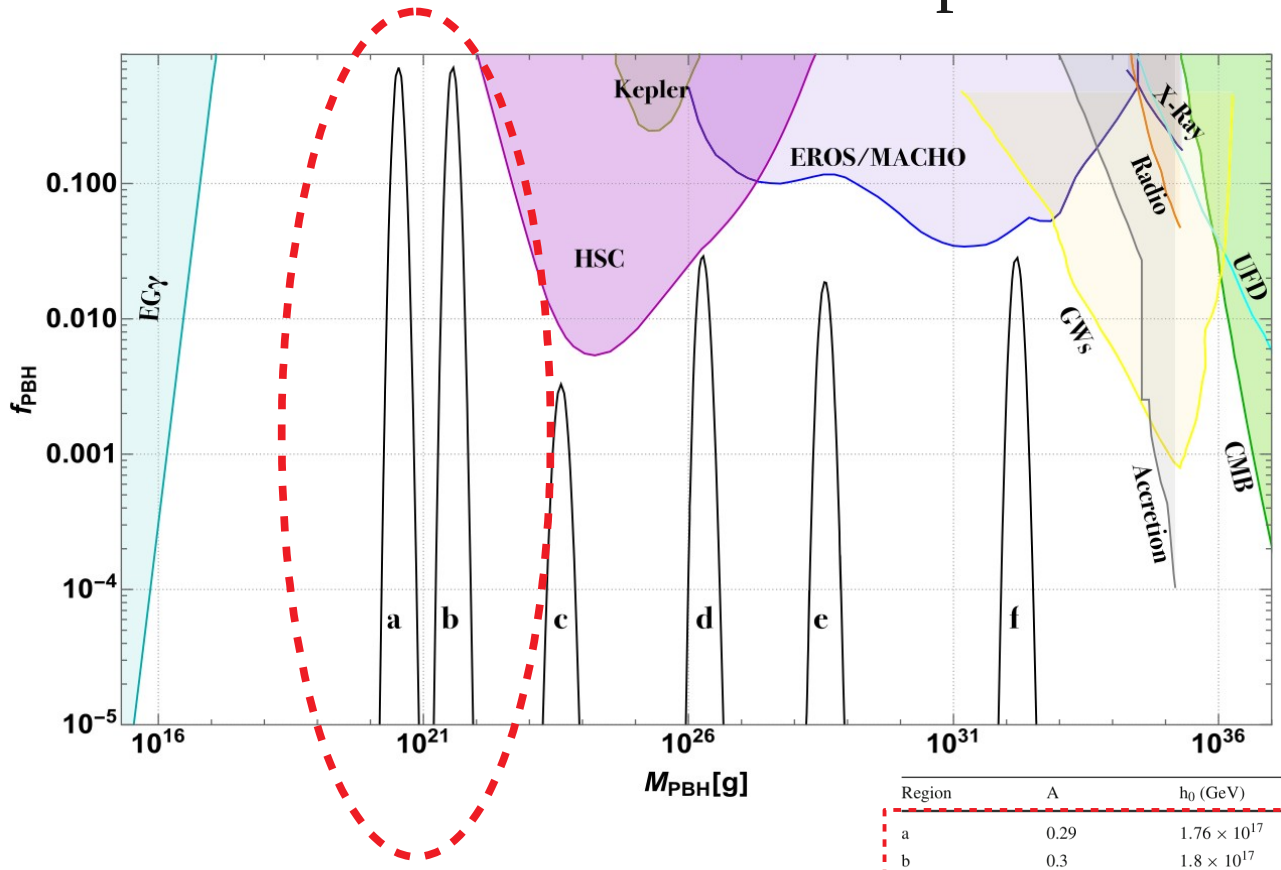

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Stochastic GWs

- Production of GWs during formation of PBHs when the modes re-enter the Hubble radius. It gives the relationship of PBH mass and GW frequency at present:

$$f_{GW} \simeq 10^{-9}\left(\frac{M_{PBH}}{30M_{\odot}}\right)^{-\frac{1}{2}} \text{Hz}$$

M.Sasaki, T.Suyama, T.Tanaka, S.Yokoyama: Class Quantum Gravity 35(6), 063001 (2018).

- We obtain the second-order perturbation via solving the Bardeen equation in $\Phi = \Psi$, $\Phi = \frac{2\mathcal{R}}{3}$:

$$ds^2 = a^2(\tau)\Big[-(1+\Phi)d\tau^2 + \Big[(1-2\Psi)\delta_{ij} + \frac{1}{2}h_{ij}\Big]dx^i dx^j\Big]$$

$$\frac{d^2h(\mathbf{k},\tau)}{d\tau^2} + \frac{2}{\tau}\frac{dh(\mathbf{k},\tau)}{d\tau} + k^2h((k),\tau) = \mathcal{S}((k),\tau)$$

$$\frac{d^2\Phi}{d\tau^2} + \frac{4}{\tau}\frac{d\Phi}{d\tau} + \frac{1}{3}k^2\Phi = 0$$

K.N.Ananda, C.Clarkson, D.Wands,: PRD75, 123518 (2007).
H.Di, Y.Gong: JCAP07,007(2018).
D.Baumann, P.J.Steinhardt, K.Takahashi, K.Ochiki: PRD76,084019 (2007)

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- The present energy density attributed to GWs:

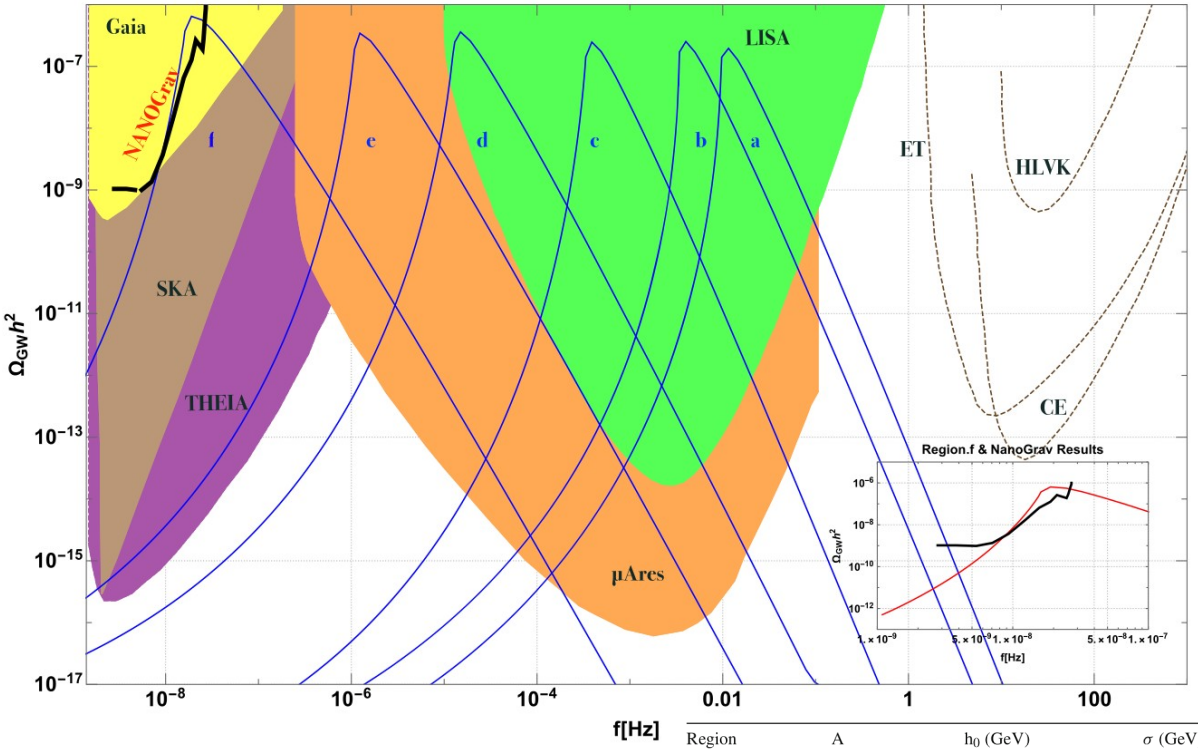
$$\Omega_{GW} = c_g \frac{\Omega_{r,0}}{6} \int_0^{\infty} dv \int_{|1-v|}^{1+v} du \left(\frac{4v^2-(1+v^2-u^2)^2}{4uv}\right)^2 \overline{\mathcal{I}^2(u,v)} \mathcal{P}_{\mathcal{R}}(kv)\mathcal{P}_{\mathcal{R}}(ku)$$

$$\Omega_{r,0} \sim 5.38 \times 10^{-5}$$

$$\overline{\mathcal{I}^2(u,v)} = \frac{1}{2}\left[\frac{3(u^2+v^2-3)}{4u^3v^3}\right]^2 \times \left[\left(-4uv+(u^2+v^2-3)\ln\left|\frac{3-(u+v)^2}{3-(u-v)^2}\right|\right)^2 + \pi^2(u^2+v^2-3)^2\Theta(v+u-\sqrt{3})\right]$$

Stochastic GWs

- Stochastic GWs from benchmark points:



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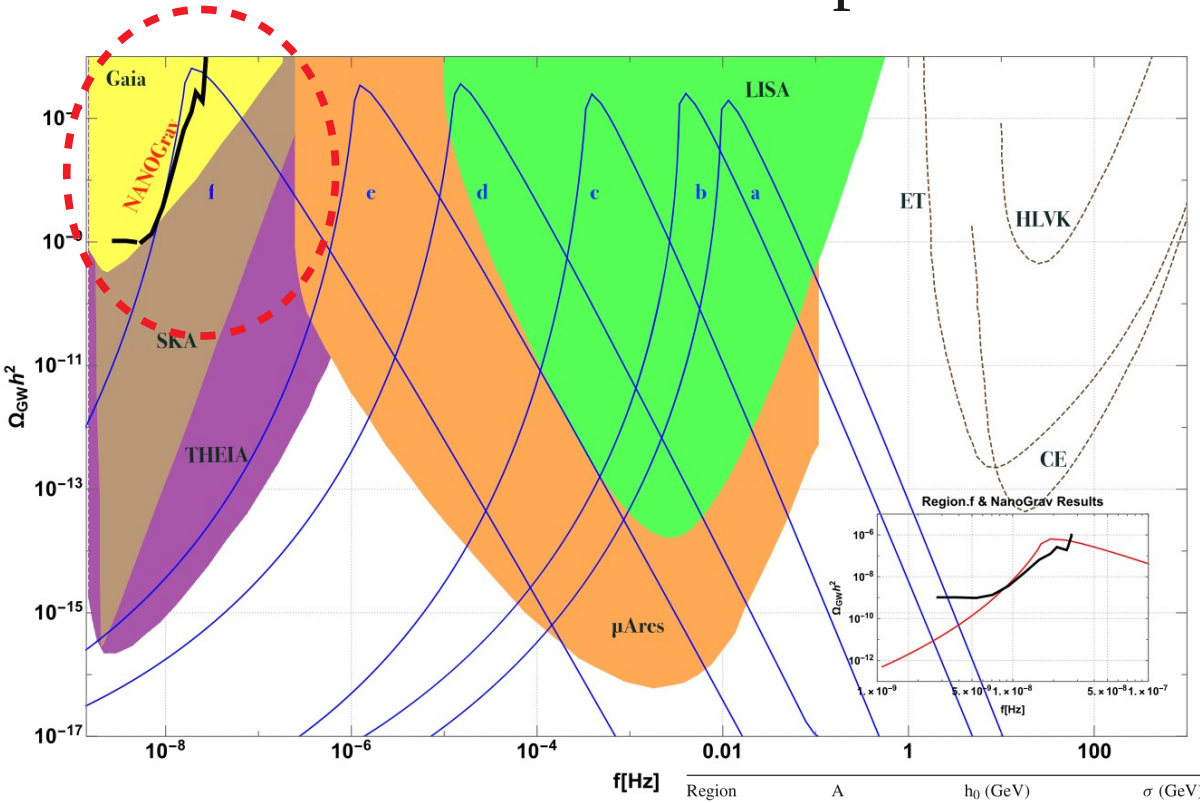

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K.Cheung, C.J.Ouseph, P.Y.Tseng: EPJC 84 (2024) 9,906.

Summary

- We investigated the modifications to the Higgs inflation model, adding a **dip/dump** to the slow-roll potential.
- The introduction of the dip in the potential enhance the power spectrum of **curvature perturbation**, which allows for **PBHs formation** in masses range 1.5×10^{20} g to 9.72×10^{32} g
- This also results in the amplification of second-order stochastic **GWs productions**.
- Parameter (a,b): substantial portion of DM in the form of PBHs.
- Parameter (f): GW elucidates the NANOGrav signal.

Thank you for your attention!

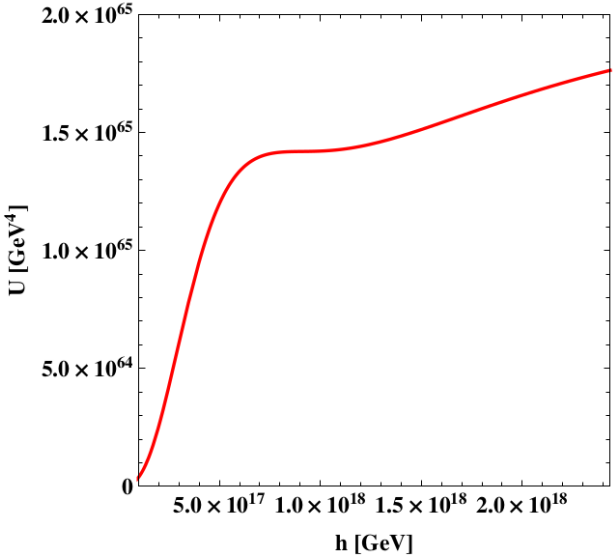
Back up

UV models

- A bump-like feature can be generated by a small local radiative correction to the base potential.

S.S.Mishra, V.Sahni: JCAP 04, 007 (2020).

- The closest UV semi-realistic model to realize the effective potential with a dip structure is to supplement the Higgs inflation with a TeV scale DM and a heavy right-handed neutrino.



N.Haba, H.Ishida, R.Takahashi: PTEP 2015(5),053B01 (2015).
N.Okada, Q.Shafi, PRB 747, 223-228 (2015).

Mukhanov-Sasaki equation

- Mukhanov-Sasaki equation describes the evolution of curvature perturbation:

M.Sasaki: Prog.Theor.Phys.76,1036(1986).
V.F.Mukhanovh: Sov.Phys.JETP 67, 1297-1302 (1988).

$$v_k'' + \left(k^2 - \frac{z''}{z}\right) v_k = 0$$

$v = z\mathcal{R}$ with $z = aM_{PL}\sqrt{2\epsilon_H} = a\frac{\dot{\phi}}{H}$

- In the early times, it is sub-Hubble $k \gg aH$, we can assume v, representing a scalar quantum field, to be in the Bunch-Davies vacuum $v_k \to \frac{1}{\sqrt{2k}} e^{-ik\tau}$
- During inflation, modes begin to enter the super-Hubble region, indicates $k \ll aH$ and $\mathcal{R}_k$ approaches a constant value.

$$\mathcal{P}_{\mathcal{R}}(k) = \frac{k^3}{2\pi^2}|\mathcal{R}_k^2|\Big|_{k\ll aH} = \frac{k^3}{2\pi^2}\frac{|v_k|^2}{z^2}\Big|_{k\ll aH}$$

PBH formation

- The curvature perturbation is related to the density contrast by

$$\delta(t,k) = \frac{2(1+\omega)}{5+3\omega}\left(\frac{k}{aH}\right)^2 \mathcal{R}_k$$

- In the Press-Schechter formalism, the Gaussian density contrast, the Gaussian curvature perturbations coarse-grained over the comoving Hubble scale by a window function, is greater than the critical density for PBH formation:

$$\beta(M_{PBH}) = 2\gamma \int_{\delta_c}^{\infty} \frac{d\delta}{\sqrt{2\pi}\sigma_{MPH}} exp\left(-\frac{\delta^2}{2\sigma_{PBH}^2}\right) \simeq \sqrt{\frac{2}{\pi}}\frac{\gamma}{\nu_c} exp\left(-\frac{\nu_c^2}{2}\right),$$

$$W(kR) = \exp\left(-\frac{k^2R^2}{2}\right)$$

$$\sigma^2_{M_{PBH}} = \int_0^{\infty} \frac{dk}{k}\frac{16}{81}(kR)^4 W^2(kR)\mathcal{P}_R(k)$$

$$\nu_c = \delta_c/\sigma_{M_{PBH}}$$

$$\gamma = 0.4$$

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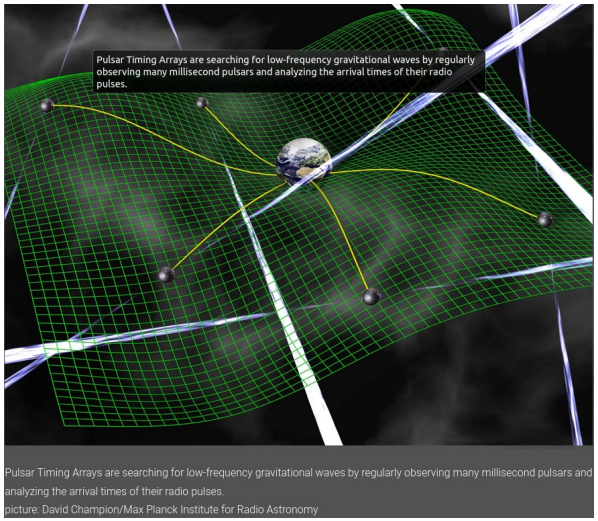
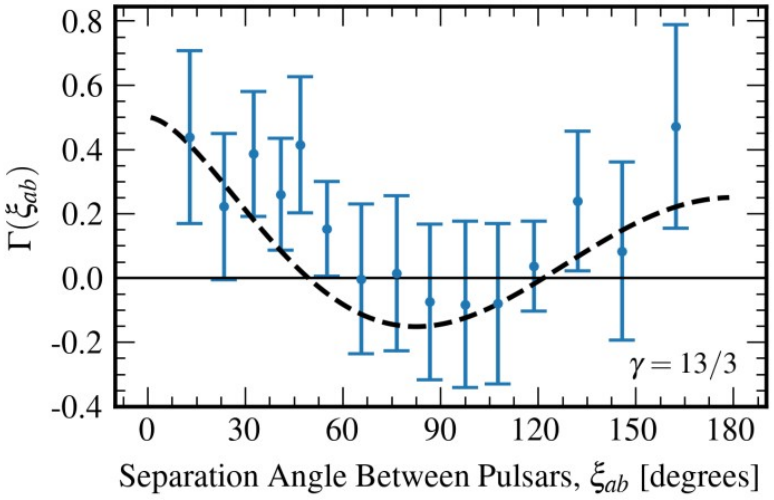


Pulsar Timing Arrays | Max Planck Institute for Gravitational Physics (Albert Einstein Institute)



Helling-Downs pattern from NANOGrav 15 yr, arXiv:2306.16213

PBH: Pulsar Timing Array

- Pulsar timing as probe of PBHs. J.T.Acuna,P.Y.Tseng, arXiv:2306.16213
- Pulsar signals arriving on Earth can be characterized by its phase

$$\phi(t) = \phi_0 + \nu t + \frac{\dot{\nu}}{2} t^2$$

frequency

Spin-down

- Third-order from transiting PBHs: i) in the vicinity of the pulsar induce-->*Doppler phase shift*, ii) close to the Earth-pulsar line of sight-->*Shapiro time delay*.

Doppler:

$$\left(\frac{\delta\nu}{\nu}\right)_D = \frac{1}{c}\int dt\ \hat{d}\cdot\vec{\nabla}\Phi,$$

Shapiro:

$$\left(\frac{\delta\nu}{\nu}\right)_S = -\frac{2}{c^3}\int_{LOS} dz\ \vec{v}\cdot\vec{\nabla}\Phi,$$

PBH gravitational potential

PBH: Pulsar Timing Array

- The corresponding phase shift is

$$\delta\phi(t) = \int^{t} dt' \; \delta\nu(t')$$

- The residual signal amplitude after subtracting the phase shift from the 1st and the 2nd order

$$h_I(t) = \sum_{i=1}^{N} \delta\phi_{I,i}(t) - \delta\phi_{0,I}(t), \quad \delta\phi_{0,I} \equiv \sum_{n=0}^{2} f_n(t) \left[\frac{1}{T_{obs}} \int_0^{T_{obs}} dt' \; \sum_{i=1}^{N} \delta\phi_{I,i}(t') f_n(t') \right]$$

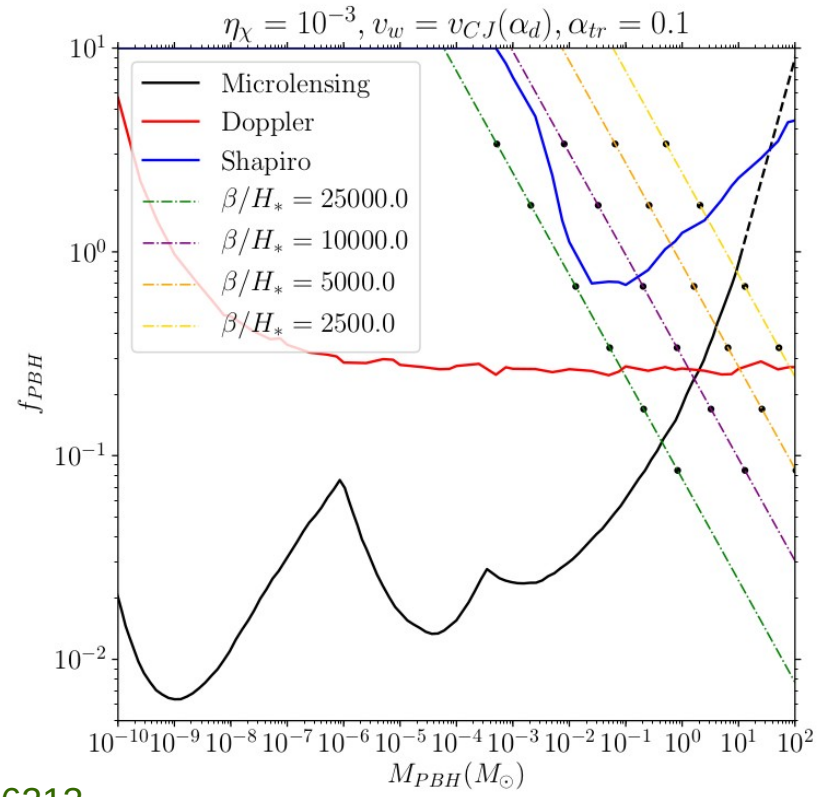
- The signal-to-noise ratio is given as

$$\mathrm{SNR}_I^2 = \frac{1}{\nu_I^2 t_{rms}^2 \Delta t} \int_0^{T_{obs}} dt \; h_I^2(t)$$

J.T.Acuna,P.Y.Tseng, arXiv:2306.16213

PBH: Pulsar Timing Array

- The projecting sensitivity of SKA, assuming 20 yr observation of 200 pulsars with 50 ns uncertainty in the pulsar timing data and 2 weeks cadence:



J.T.Acuna,P.Y.Tseng, arXiv:2306.16213

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