Can QCD scale be the origin of EW scale ?

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Based on ongoing project

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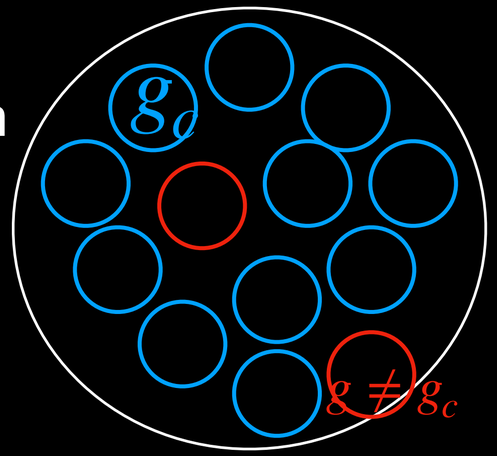
QCD scale is the only scale that is dynamically generated in the SM

$$\Lambda_{\rm QCD} \sim \Lambda_{\rm UV} e^{-\frac{c}{g^2_{\rm QCD}}}$$

So, it is quite natural to think of its connection to the EW scale.

One interesting approach is "Hubble Selection"

Inflation epoch



Hubble patches with critical couplings dominate the Universe

Hubble Selection

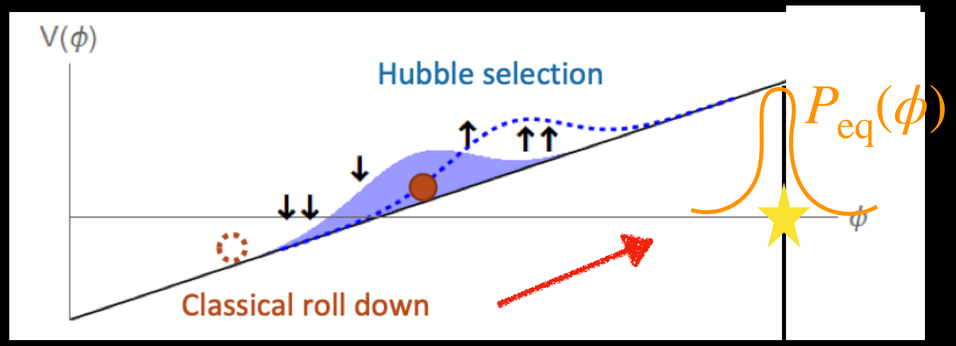
[G. F. Giudice, M. McCullough, T. You (21), S. Jung and T. Kim (21)]

- Assume a long inflation ($N \gtrsim M_{\rm Pl}^2/H_{\rm inf}^2$) and scalar field ϕ that scans the Higgs VEV (i.e. relaxion)
- Calculate the volume-weighted probability distribution $P(\phi, t)$ by solving the Fokker-Planck equation

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V'(\phi)}{3H}P\right) + \frac{1}{8\pi^2}\frac{\partial^2 (H^3 P)}{\partial \phi^2} + 3\Delta H P \ , \ \Delta H \propto \frac{V(\phi)}{M_{\rm pl}^2}$$

- Here, $V(\phi)$ is the total potential except for inflaton .

The equilibrium distribution is strongly peaked
at critical point= (local) maximum point
~ phase transition points



$\therefore$ Higgs VEV is fixed at a critical point, $v = v(\phi_c)$

ϕ_c

Where is critical point ?

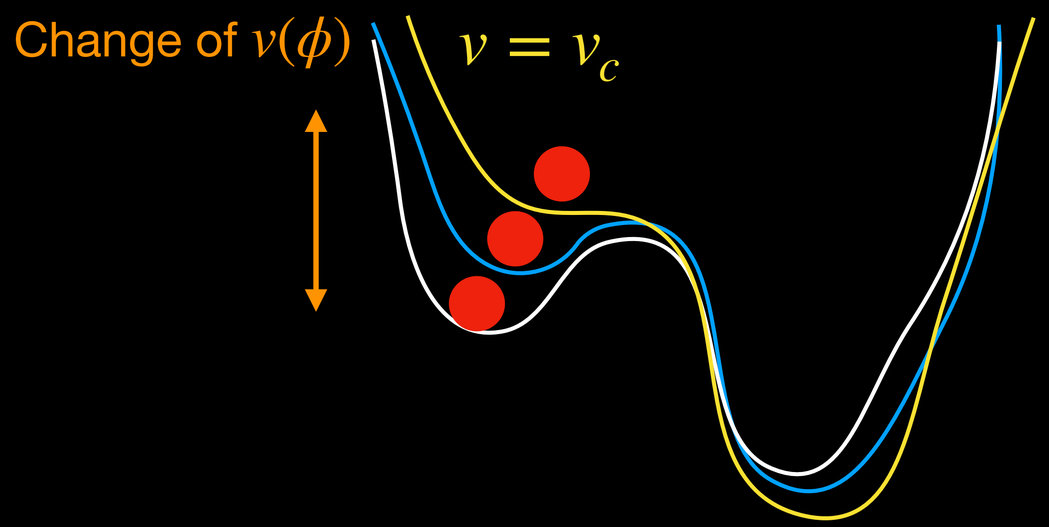
- In [S. Jung and T. Kim (2021)], they studied QCD effective potential

$$V_\Sigma = \mu^2 \mathrm{Tr}[\Sigma\Sigma^\dagger] + \lambda_1 (\mathrm{Tr}[\Sigma\Sigma^\dagger])^2 + \lambda_2 \mathrm{Tr}[(\Sigma\Sigma^\dagger)^2] - c(\det\Sigma + \det\Sigma^\dagger) - \mathrm{Tr}[\mathcal{H}(\Sigma + \Sigma^\dagger)],$$

$\propto m_q \propto v$

$\Sigma =$ meson fields

Gell-Mann, Levy (60), B.W.Lee (70)



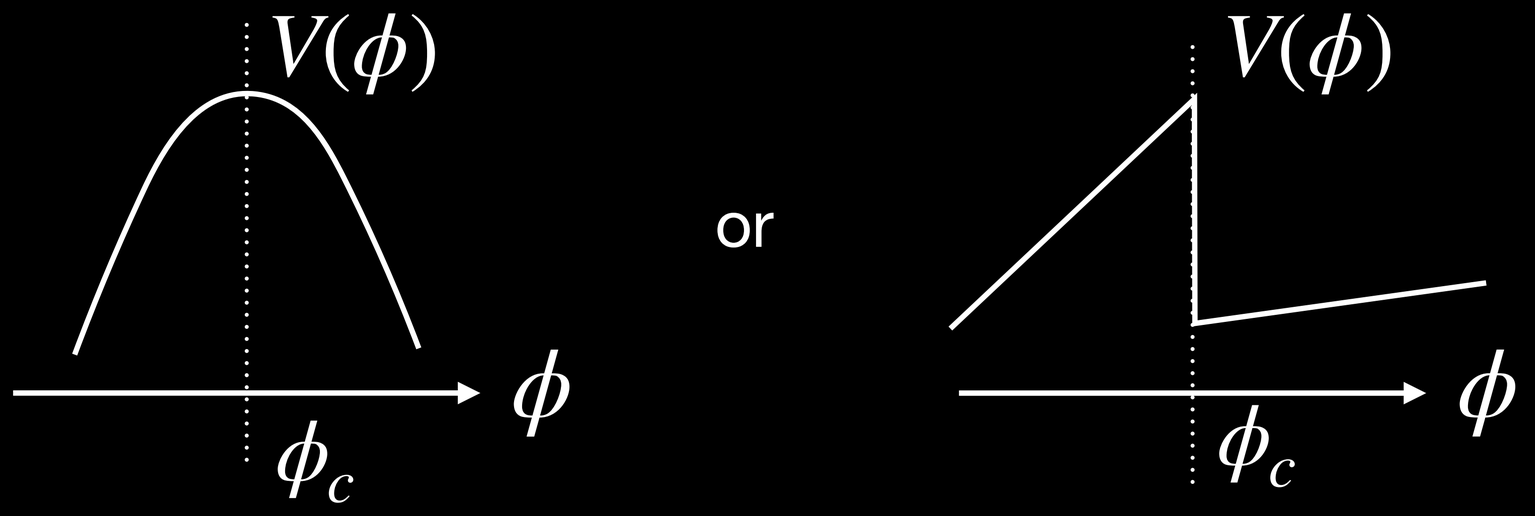
Rough sketch of QCD potential

1. QCD potential changes as ϕ scans the Higgs VEV
2. At a critical point $v = v_c$, one of vacua takes a maximum potential energy and disappears

$\therefore$ Higgs VEV is fixed at the QCD critical point !

They numerically found $v_c \simeq 20\ \mathrm{MeV}$

The essence of Hubble selection:
Assume a long inflation and scalar fields that scan parameters.
Then, they are fixed at a critical point where their potential becomes (locally) maximum



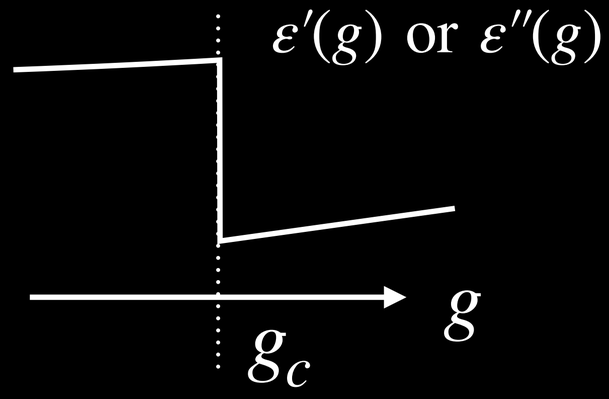
When I learned this theory for the first time, I strongly felt déjà vu (デジャブー)

Multi-critical point principle (MPP)

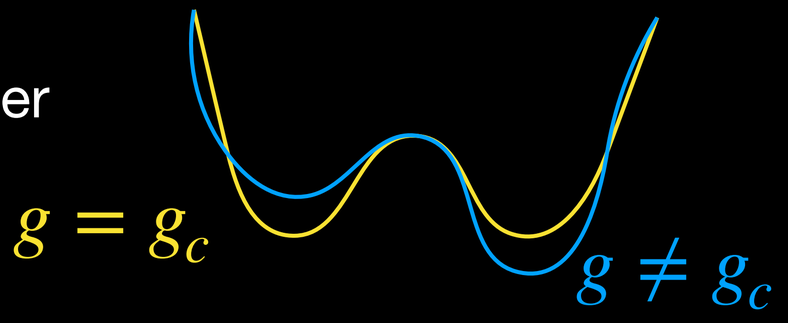
[Froggatt, Nielsen, Stominger, Kawai, Okada, KK,…]

The essence of MPP:

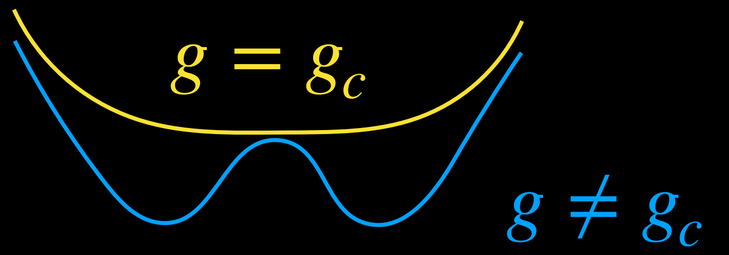
Treat coupling constant g like temperature T and consider its micro-canonical ensemble.
Then, coupling constants are fixed at a critical point where (higher-order) derivative of vacuum energy $\varepsilon(g)$ changes discontinuously



First-order



Second-order



MPP is much simpler than Hubble selection

- In MPP, everything is explained within the SM with a Micro-canonical ensemble

e.g. $$Z_{\text{Micro}} = \int \mathcal{D}\phi\ \delta\left(\int d^4x H^\dagger H - A\right) e^{iS_{\text{SM}}}$$

← Higgs mass is treated in a micro-canonical way

[Froggat, Nielsen (95)]

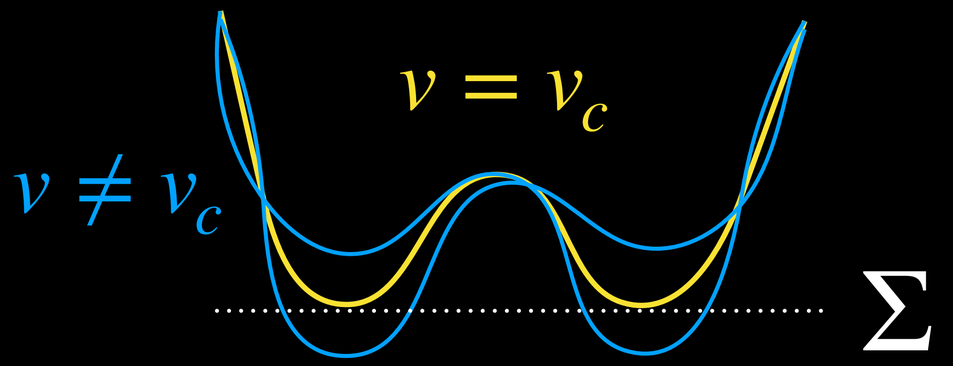
$$\sim Z_{\text{Can}}\Big|_{v=v_c} \qquad V \to \infty$$

Ensemble equivalence as in statistical mechanics

We don't need eternal inflation and relaxion fields !

See also [Kawai, Oda, Yagyu, KK (22)]

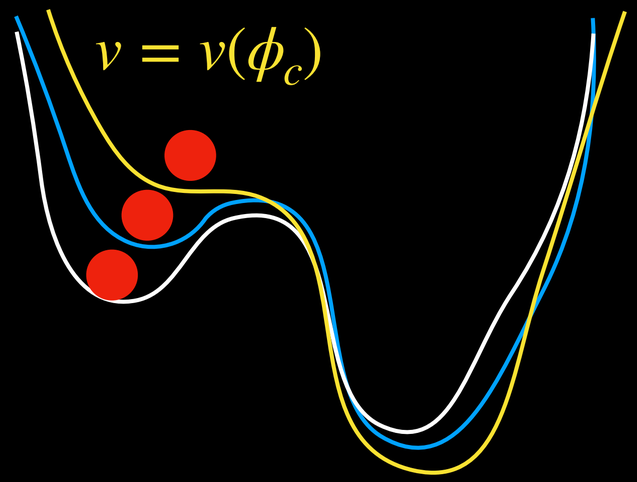
So, what we want to see is



1st order phase transition

in QCD potential

c.f. critical point in Hubble selection



We are now examining the existence of such a 1st order PT

$$V_\Sigma = \mu^2 \mathrm{Tr}[\Sigma\Sigma^\dagger] + \lambda_1 (\mathrm{Tr}[\Sigma\Sigma^\dagger])^2 + \lambda_2 \mathrm{Tr}[(\Sigma\Sigma^\dagger)^2] - c(\det\Sigma + \det\Sigma^\dagger) - \mathrm{Tr}[\mathcal{H}(\Sigma + \Sigma^\dagger)],$$

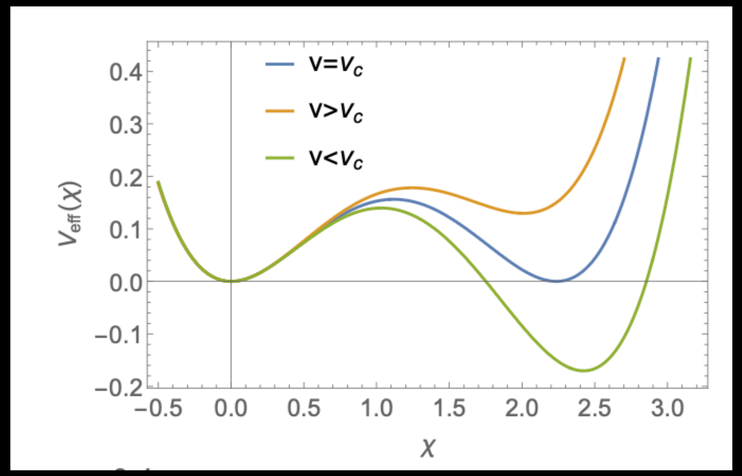
No concrete results yet.

A simple toy model before QCD

- The SM Higgs + single real scalar χ with a (dynamically generated) scale M

$$V(H,\chi) = -\mu^2 H^\dagger H + \lambda_H (H^\dagger H)^2 + \kappa H^\dagger H \chi^2 - \frac{M}{3}\chi^3 + \frac{\lambda_\chi}{4!}\chi^4 \qquad \kappa,\ \lambda_\chi > 0$$

- When varying μ^2, the vacuum structure is changing

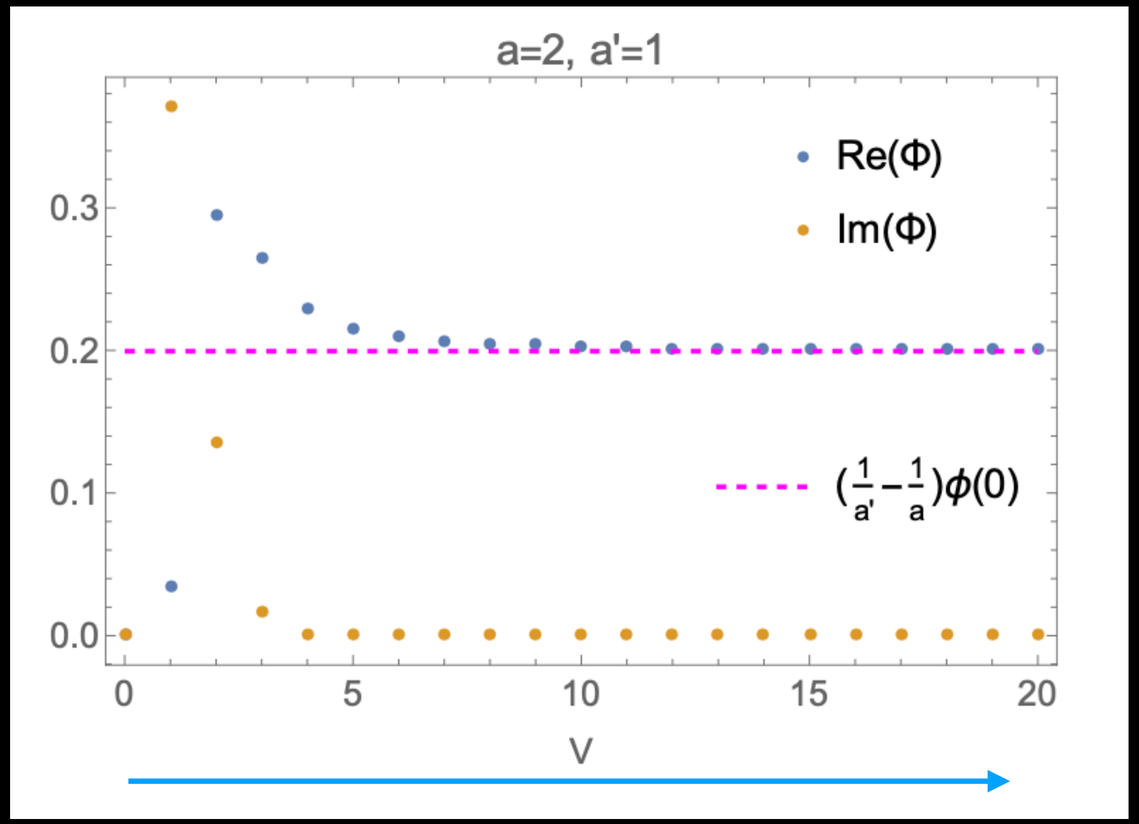


Critical point: $v_c = v(\mu_c^2) = \frac{1}{3}\sqrt{\frac{2\lambda_\chi}{\kappa}}M$

1st order phase transition !

Equivalence of ensembles (mean-field level)

$f(\mu^2)$ = Integrable test function



Spacetime volume

$$\Phi(V) := i\int_{-\infty}^{\infty} d\mu^2 e^{-iV\varepsilon(\mu^2)} f(\mu^2)$$

$$cf(\mu_c^2) = c \times e^{-iV\varepsilon(\mu_c^2)} f(\mu^2 = \mu_c^2)$$

for $V \to \infty$

Only the critical point $\mu^2 = \mu_c^2$ contributes
in the MC ensemble

Summary

- There might be relation between QCD scale and EW scale
- Hubble selection is one of approaches, but it needs a few assumptions
- MPP is another similar approach, but it only need a single assumption = Micro-canonical ensemble
- We are now analyzing the phase diagram, using the effective QCD potential
- Realization of the EW scale is just one example. Both mechanisms are applicable to any models exhibiting quantum phase transitions

Thank you so much !

Back up

Volume Weighted Fokker-Planck equation

- The usual FP equation $\frac{\partial P_{\rm FP}}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V'(\phi)}{3H}P_{\rm FP}\right) + \frac{1}{8\pi^2}\frac{\partial^2 (H^3 P_{\rm FP})}{\partial \phi^2}$ describes the probability distribution of ϕ in each Hubble patch

- However, this does not measure the true spacial distribution of ϕ because different values of ϕ correspond to different local expansion rate $H[\phi]$

  → We should multiply the expansion factor as $P(\phi, t) = \left\langle \delta(\phi - \phi(t)) e^{3\int^t dt' H[\phi(t')]} \right\rangle_{\rm Langevin}$

- After some calculations, we can obtain the volume-weighted FP equation

$$\frac{\partial P}{\partial t} = \frac{\partial}{\partial \phi}\left(\frac{V'(\phi)}{3H}P\right) + \frac{1}{8\pi^2}\frac{\partial^2 (H^3 P)}{\partial \phi^2} + 3\Delta H P \ , \ \Delta H := H[\phi] - \langle H[\phi] \rangle_P$$

Mechanism of climbing up potential

- Assuming the slow-roll of ϕ, i.e., $\dot{\phi}_{\rm cl} \sim -V'(\phi)/3H$, we can solve the FP equation analytically

$$P(\phi,t) \propto \exp\left\{-\frac{1}{2\sigma_\phi^2}\left[\phi-\phi_{\rm ini}-\dot{\phi}_c t-\frac{3}{2}(\Delta H)'\sigma_\phi^2 t\right]\right\}\ , \quad \sigma_\phi^2=\left(\frac{H}{2\pi}\right)^2 Ht$$

- Initially, the volume-effect (=yellow term) is negligible → The peak rolls down the potential
- However, after a long duration determined by $|\dot{\phi}_c| \sim \Delta H'\sigma_\phi^2 \ \leftrightarrow\ \sigma_\phi^2 \sim M_{\rm PL}^2$

  the volume-effects beats the classical force, and the peak starts to climb up the potential

- The e-folding until this time is $N \sim \frac{8\pi^2}{3}\frac{M_{\rm Pl}^2}{H^2}$, which already saturates the de-Sitter entropy bound $\sim \frac{2\pi^2}{3}\frac{M_{\rm Pl}^2}{H^2}$

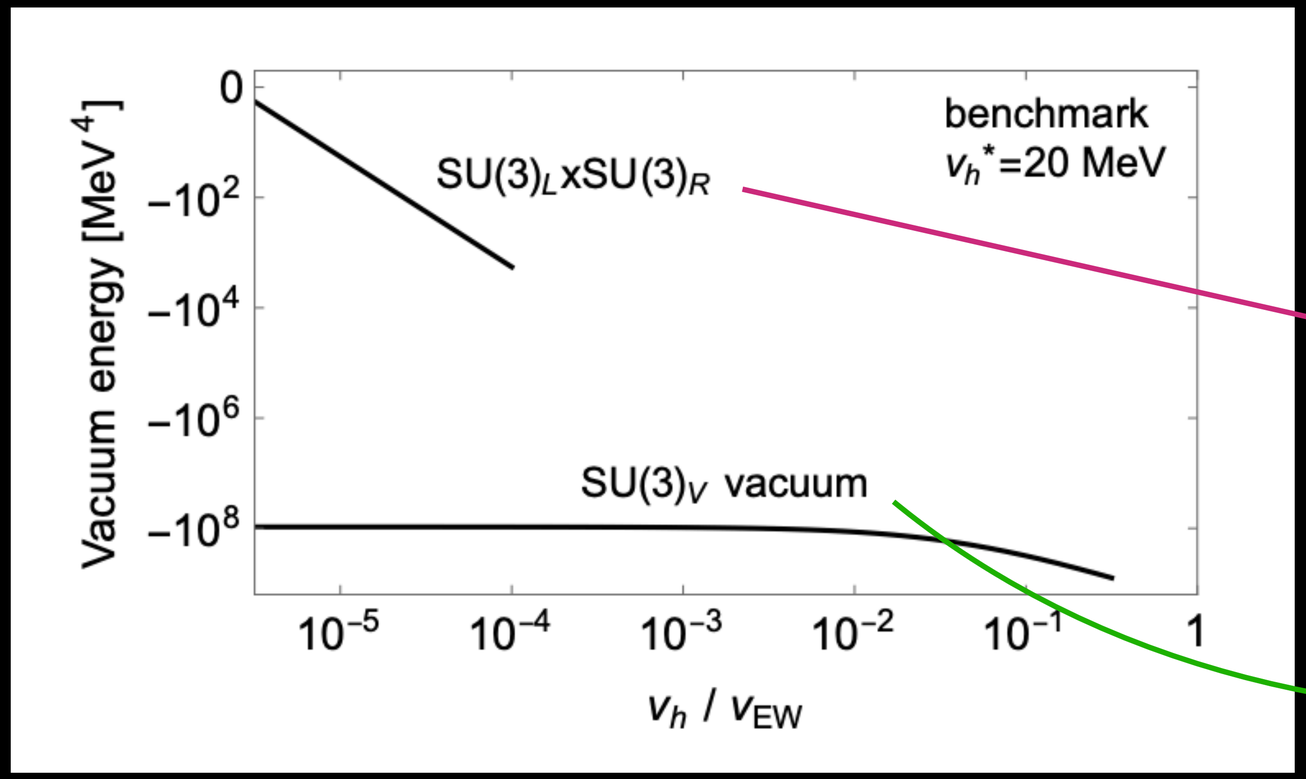
  → We need a quite long inflation ~ eternal inflation

Benchmark parameters

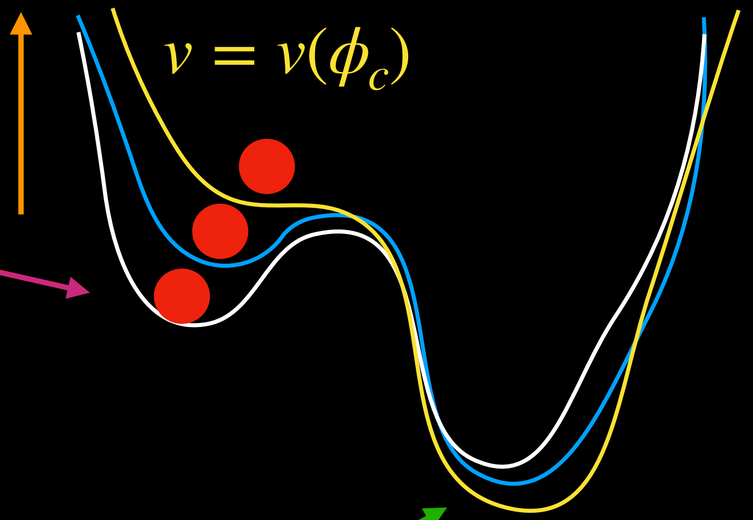
parameter	f_π	f_K	m_π	m_K	m_η	$m_{\eta'}$	m_{a_0}	$m_{f_0(500)}$	$m_{f_0(1370)}$	$m_{K^*(1430)}$
measured	92.4	113	$139.57^{\pm 0.005}$	$497.61^{\pm 0.013}$	$547.86^{\pm 0.017}$	$957.78^{\pm 0.06}$	$980^{\pm 20}$	$500^{\pm 150}$	$1370^{\pm 150}$	$1425^{\pm 50}$
benchmark	92.4	113	137	491	534	973	1050	731	1260	1140

TABLE I. Predictions of the benchmark SM point [Eq. (6) and (7)], compared with data from the Particle Data Group [55]. In units of MeV.

QCD vacuum energy as a function of Higgs VEV



Change of $v(\phi)$

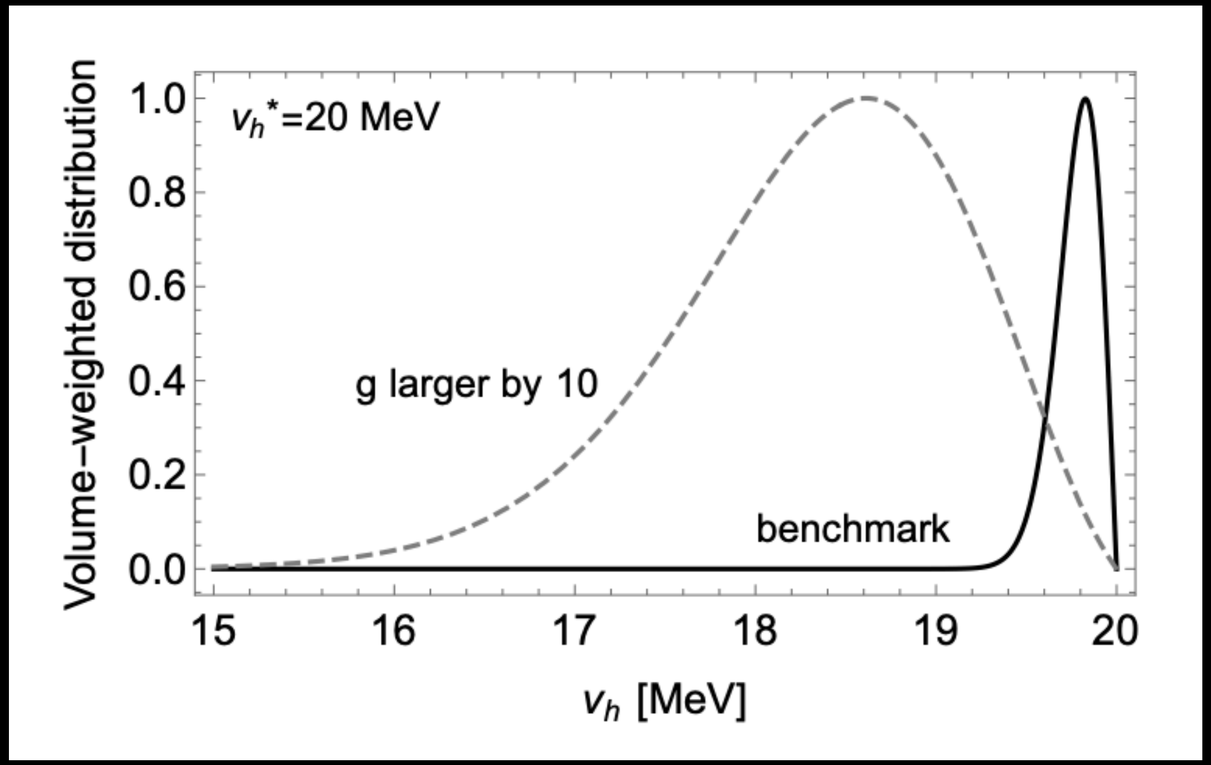


Other parameters are fixed so that they are consistent with meson data

[S. Jung and T. Kim (2021)]

Volume-Weighted probability distribution

$P_{\mathrm{eq}}(\phi)$



[S. Jung and T. Kim (2021)]