기초과학연구원
Institute for Basic Science

Unitarity in Einstein-Cartan Higgs inflation with quantum correction

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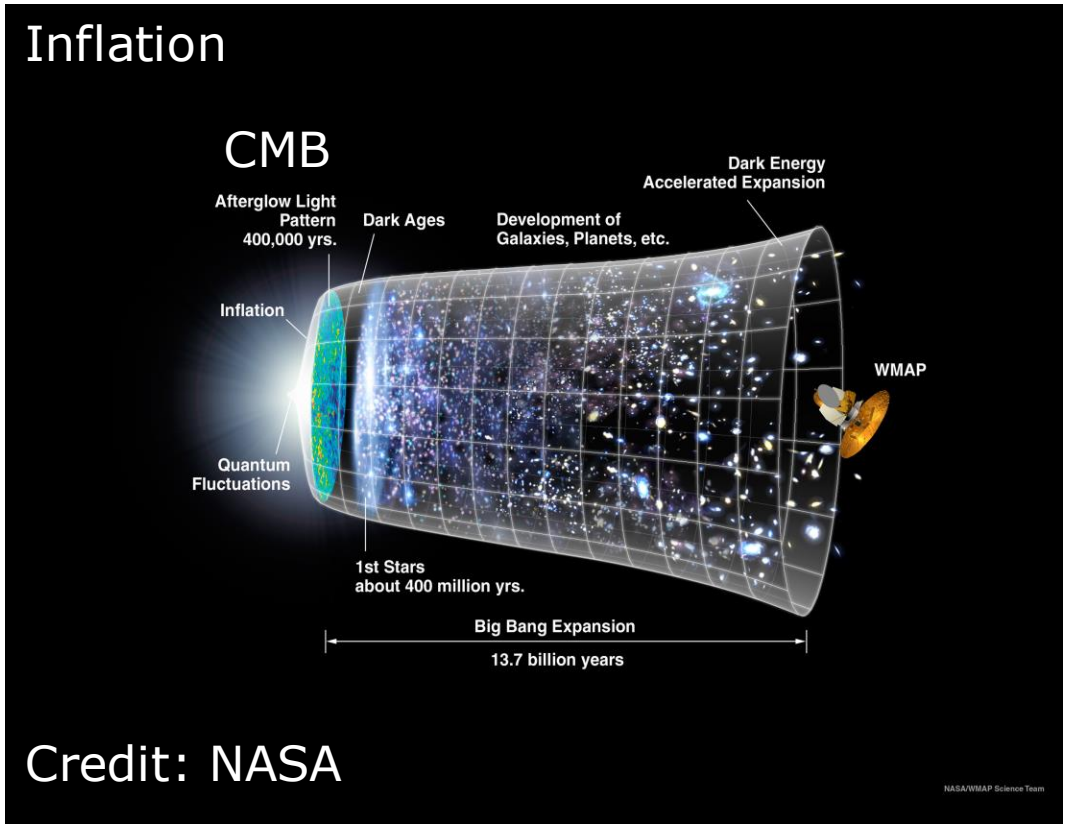
MH, Kohei Kamada, Kyohei Mukaida, 2308.14398, 2308.15420.

Higgs field in the early Universe

- Early Universe scenario
  - Cosmic inflation
  - Reheating
- Inflation model
  - Scalar field
  - Coupling with SM



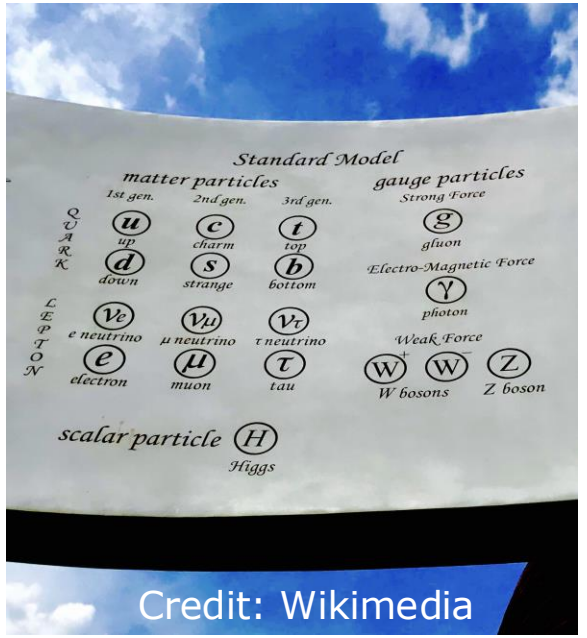
Higgs field in the early Universe

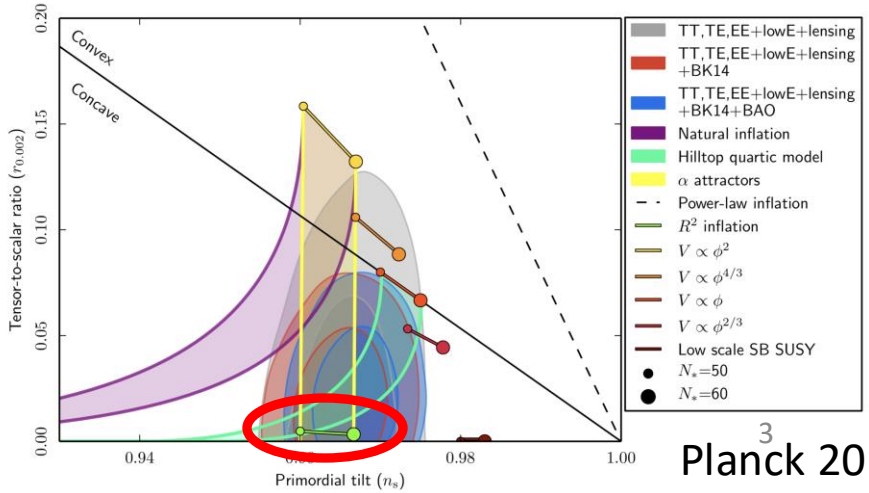
- Higgs boson discovered in LHC
  - Only scalar in SM
- Higgs inflation
  - Higgs field as inflaton (with large ξ)

$$\mathcal{L} \supset \frac{\xi}{2}\phi^i\phi_i R - \frac{1}{2}g^{\mu\nu}\partial_\mu\phi^i\partial^\nu\phi_i - \frac{\lambda}{4}(\phi^i\phi_i)^2$$

  - Known couplings with SM particles
  - Inflation predictions

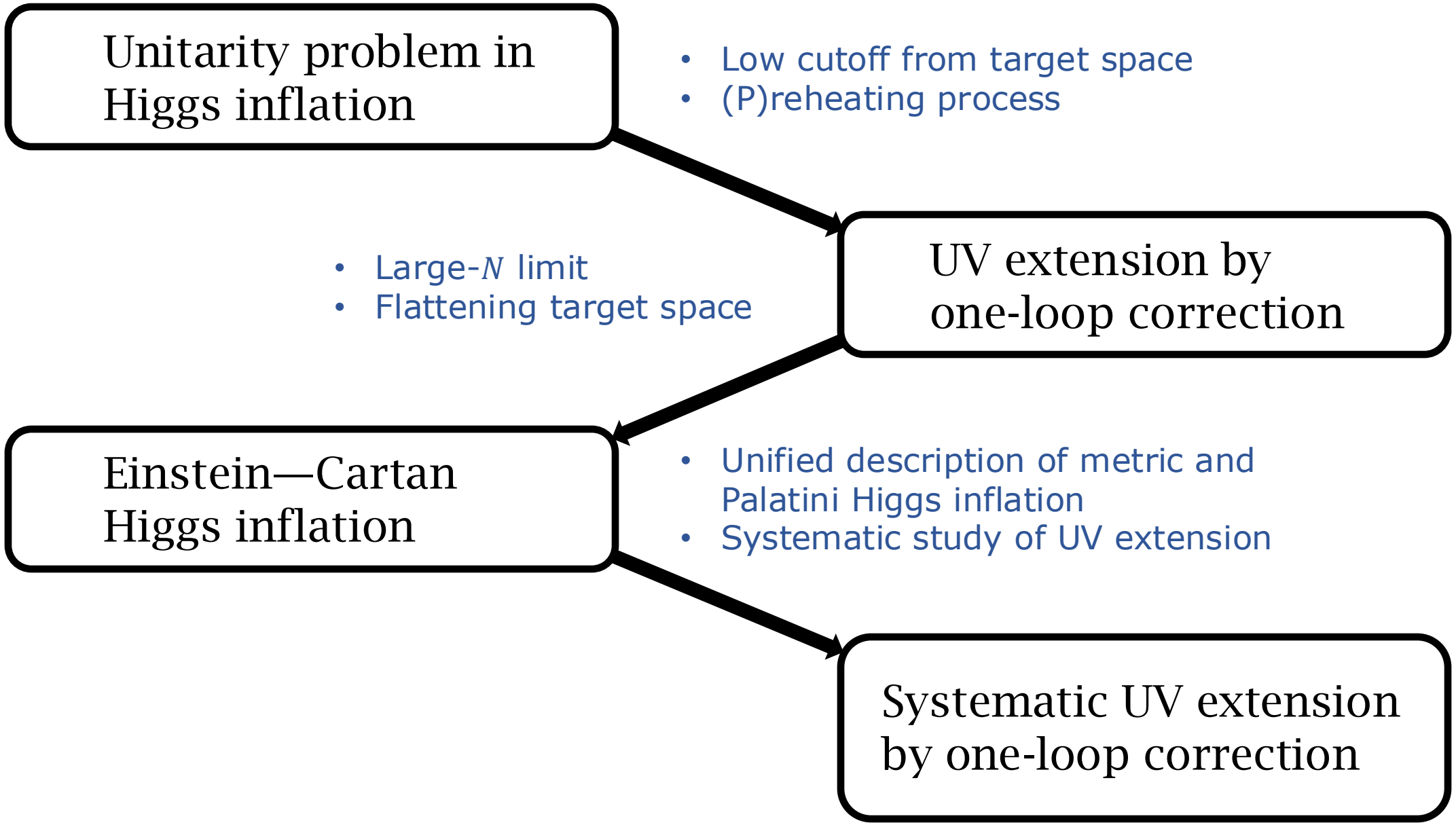
$$\frac{\xi}{\sqrt{\lambda}} \sim O(10^4)$$





Planck 2018

Cervantes-Cota, Dehnen (1995)
Bezrukov, Shaposhnikov (2007)
Barvinsky, Kamenshchik, Starobinsky (2008)


Unitarity problem in Higgs inflation
• Low cutoff from target space
• (P)reheating process
UV extension by one-loop correction
• Large-N limit
• Flattening target space
Einstein—Cartan Higgs inflation
• Unified description of metric and Palatini Higgs inflation
• Systematic study of UV extension
Systematic UV extension by one-loop correction

Unitarity problem in Higgs inflation

- Cutoff scale during Higgs inflation and reheating
  - Inflation $\Lambda/\Lambda_G \sim 1/\sqrt{\xi}$
  - Reheating $\Lambda/\Lambda_G \sim 1/\xi$

Hertzberg (2010)
Bezrukov, Magnin, Shaposhnikov (2011)
Ren, Xianyu, He (2014)
Mikura, Tada (2022)…

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- Non-linear sigma model

$$S=\int \mathrm{d}^4x\sqrt{-g_{\mathrm{J}}}\left(\frac{M_{\mathrm{Pl}}^2+\xi\phi^2}{2}R_{\mathrm{J}}-\frac{g_{\mathrm{J}}^{\mu\nu}}{2}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j-V\right)$$

$$\begin{cases} V=\dfrac{\lambda}{4}\phi^4 \\ G^{\mathrm{J}}_{ab}=\delta_{ij} \end{cases}$$

Maeda 1988, 1989

$$g_{\mathrm{E}\mu\nu}=\Omega^2 g_{\mathrm{J}\mu\nu},\qquad \Omega^2\equiv 1+\frac{\xi\phi^2}{M_{\mathrm{Pl}}^2},$$

$$S=\int \mathrm{d}^4x\sqrt{-g_{\mathrm{E}}}\left(\frac{M_{\mathrm{Pl}}^2}{2}R_{\mathrm{E}}-\frac{g_{\mathrm{E}}^{\mu\nu}}{2}G^{\mathrm{E}}_{ij}\partial_\mu\phi^i\partial_\nu\phi^j-\frac{V}{\Omega^4}\right)$$

Weyl transformation causes ambiguity!

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- Non-linear sigma model
- Extended target space---inclusion of conformal mode

$$g_{\bullet\mu\nu} = \frac{\Phi_\bullet^2}{6M_{\rm Pl}^2}\tilde{g}_{\mu\nu}, \qquad \det\left(\tilde{g}_{\mu\nu}\right) = -1$$ in any frame

$$\Phi_{\rm E}^2 = \Omega^2\Phi_{\rm J}^2 \qquad R = \frac{6M_{\rm Pl}^2}{\Phi^2}\tilde{R} - \frac{36M_{\rm Pl}^2}{\Phi^3}\tilde{\nabla}^2\Phi$$

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- Non-linear sigma model
- Extended target space---inclusion of conformal mode
  - Example: in Jordan frame

$$(\varphi^a_{\mathrm{J}}) = (\Phi_{\mathrm{J}}, \phi^i) \qquad \left(G^{\mathrm{J}}_{ab}\right) \equiv \begin{pmatrix} -\Omega^2 & -\xi\Phi_{\mathrm{J}}\phi_j/M_{\mathrm{Pl}}^2 \\ -\xi\Phi_{\mathrm{J}}\phi_i/M_{\mathrm{Pl}}^2 & \frac{\Phi_{\mathrm{J}}^2}{6M_{\mathrm{Pl}}^2}\delta_{ij} \end{pmatrix}$$

$$\text{Cutoff} \quad \frac{\Lambda}{\Lambda_{\mathrm{G}}} \propto \frac{\bar{R}^{-1/2}}{\Lambda_{\mathrm{G}}}$$

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- Non-linear sigma model
- Extended target space---inclusion of conformal mode
- Unitarity violation
  - $\Lambda_{\mathrm{inf}} < \Lambda$ No
  - $p_{\mathrm{NG}} > \Lambda$ Yes

Ema, Jinno, Mukaida, Nakayama (2017)

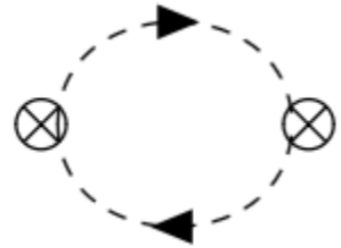
Need UV extension!

One-loop correction

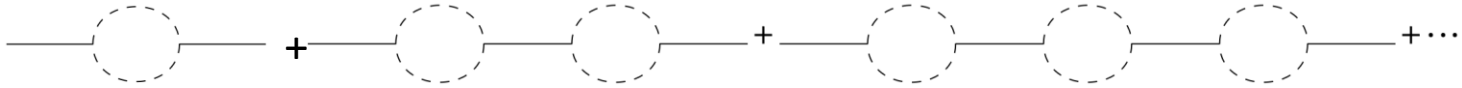
- Large-N limit

Calmet, Kuntz (2016)
Ghilencea (2018)
Ema (2019)

$$\mathcal{L} \supset \frac{\xi}{2}\phi^i\phi_i R$$



  - Closed Higgs loops enhanced by number of species N
  - Counter term αR^2
  - Resummation of one-loop diagrams



- Large-ξ suppresses other terms such as $R^{\mu\nu}R_{\mu\nu}$

$$\alpha \sim N\xi^2$$

One-loop correction

- Higgs-R^2 inflation as a linear sigma model with flat target space

$$S=\int \mathrm{d}^4x\sqrt{-g_{\mathrm{J}}}\left(\frac{M_{\mathrm{Pl}}^2+\xi\phi^2}{2}R_{\mathrm{J}}-\frac{g_{\mathrm{J}}^{\mu\nu}}{2}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j-V\right) \quad +\alpha R_{\mathrm{J}}^2$$

Weyl transformation

$$S=\int \mathrm{d}^4x\sqrt{-g_{\mathrm{E}}}\left[\frac{M_{\mathrm{Pl}}^2}{2}R_{\mathrm{E}}-\frac{1}{2}g_{\mathrm{E}}^{\mu\nu}\left(\partial_\mu\sigma_{\mathrm{E}}\partial_\nu\sigma_{\mathrm{E}}+e^{-\sqrt{\frac{2}{3}}\frac{\sigma_{\mathrm{E}}}{M_{\mathrm{Pl}}}}\partial_\mu\phi_i\partial_\nu\phi^i\right)-U(\phi,\sigma_{\mathrm{E}})\right]$$

Flat extended target space!

Ema (2017)
Gorbunov, Tokareva (2018)
Ema, Mukaida, van de Vis (2020,2021)

- Preheating is not violating unitarity

MH, Jinno, Kamada, Park, Starobinsky, Yokoyama (2019)

Einstein—Cartan Higgs inflation

- Palatini formalism
  - GR: $\Gamma^{\rho}{}_{\mu\nu}(g) \quad \Longrightarrow \quad \Gamma^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\nu\mu}$
  - Palatini: $\Gamma^{\rho}{}_{\mu\nu}$ independent of metric but $\Gamma^{\rho}{}_{\mu\nu} = \Gamma^{\rho}{}_{\nu\mu}$
  - Still low cutoff $\Lambda/\Lambda_G \sim 1/\sqrt{\xi}$ (but no violent preheating)

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- Einstein—Cartan formalism
  - $\Gamma^{\rho}_{\mu\nu}$ independent of metric
  - Torsion: $T^{\rho}{}_{\mu\nu} \equiv \bar{\Gamma}^{\rho}{}_{\mu\nu} - \bar{\Gamma}^{\rho}{}_{\nu\mu}$
  - New terms:
    - Nieh-Yan term $\partial_\mu \left(\sqrt{-g_{\rm J}} E^{\mu\nu\rho\sigma} T_{\nu\rho\sigma} \right)$

Nieh, Yan (1982)

Einstein—Cartan Higgs inflation

- EC Higgs inflation

Shaposhnikov, Shkerin, Timiryasov, Zell (2021)

$$S = \int \mathrm{d}^4x\sqrt{-g_\mathrm{J}}\left(\frac{M_\mathrm{Pl}^2+\xi\phi^2}{2}R_\mathrm{J} - \frac{g_\mathrm{J}^{\mu\nu}}{2}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j - V\right)$$

but $\Gamma^\rho_{\mu\nu}$ is torsionful

$$+ \qquad S_{\mathrm{N\text{–}Y},J} = -\frac{\xi_\eta}{4}\int \mathrm{d}^4x\, G(\phi)\partial_\mu\left(\sqrt{-g_\mathrm{J}}E^{\mu\nu\rho\sigma}T_{\nu\rho\sigma}\right)$$

$$G(\phi) = \phi^2$$

$$r = \xi_\eta/\xi$$

Einstein—Cartan Higgs inflation

- EC Higgs inflation

Shaposhnikov, Shkerin, Timiryasov, Zell (2021)

$$S=\int \mathrm{d}^4x\sqrt{-g_\mathrm{J}}\left(\frac{M_\mathrm{Pl}^2+\xi\phi^2}{2}R_\mathrm{J}-\frac{g_\mathrm{J}^{\mu\nu}}{2}\delta_{ij}\partial_\mu\phi^i\partial_\nu\phi^j-V\right)$$

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$$G(\phi)=\phi^2$$

$$r=\xi_\eta/\xi$$

- Connecting metric and Palatini versions
  - $r=1$ metric limit
  - $r=0$ Palatini limit

$$\Lambda^0_{\mathrm{E\text{–}C}}=|R_N|^{-1/2}=\frac{\sqrt{6}M_\mathrm{Pl}}{\sqrt{1+12\xi\left(1+3r^2\xi\right)}}\sim\begin{cases}\dfrac{M_\mathrm{Pl}}{r\xi} & \text{for} \quad \frac{1}{\sqrt{\xi}}\lesssim r\lesssim 1\\ \dfrac{M_\mathrm{Pl}}{\sqrt{\xi}} & \text{for} \quad 0\leqslant r\lesssim\frac{1}{\sqrt{\xi}}\end{cases}$$

at $\phi^i=(0,\ldots,0)$

Einstein—Cartan Higgs inflation

- Systematic UV extension by one-loop

$$\mathcal{L} \supset \frac{\xi}{2}\phi^i\phi_i(R(g,\Gamma) + \frac{r}{2}\nabla_\mu(E^{\mu\nu\rho\sigma}T_{\nu\rho\sigma})) \quad \Rightarrow \quad \alpha\left(R(g,\Gamma) + \frac{r}{2}\nabla_\mu(E^{\mu\nu\rho\sigma}T_{\nu\rho\sigma})\right)^2$$

Einstein—Cartan Higgs inflation

- Systematic UV extension by one-loop

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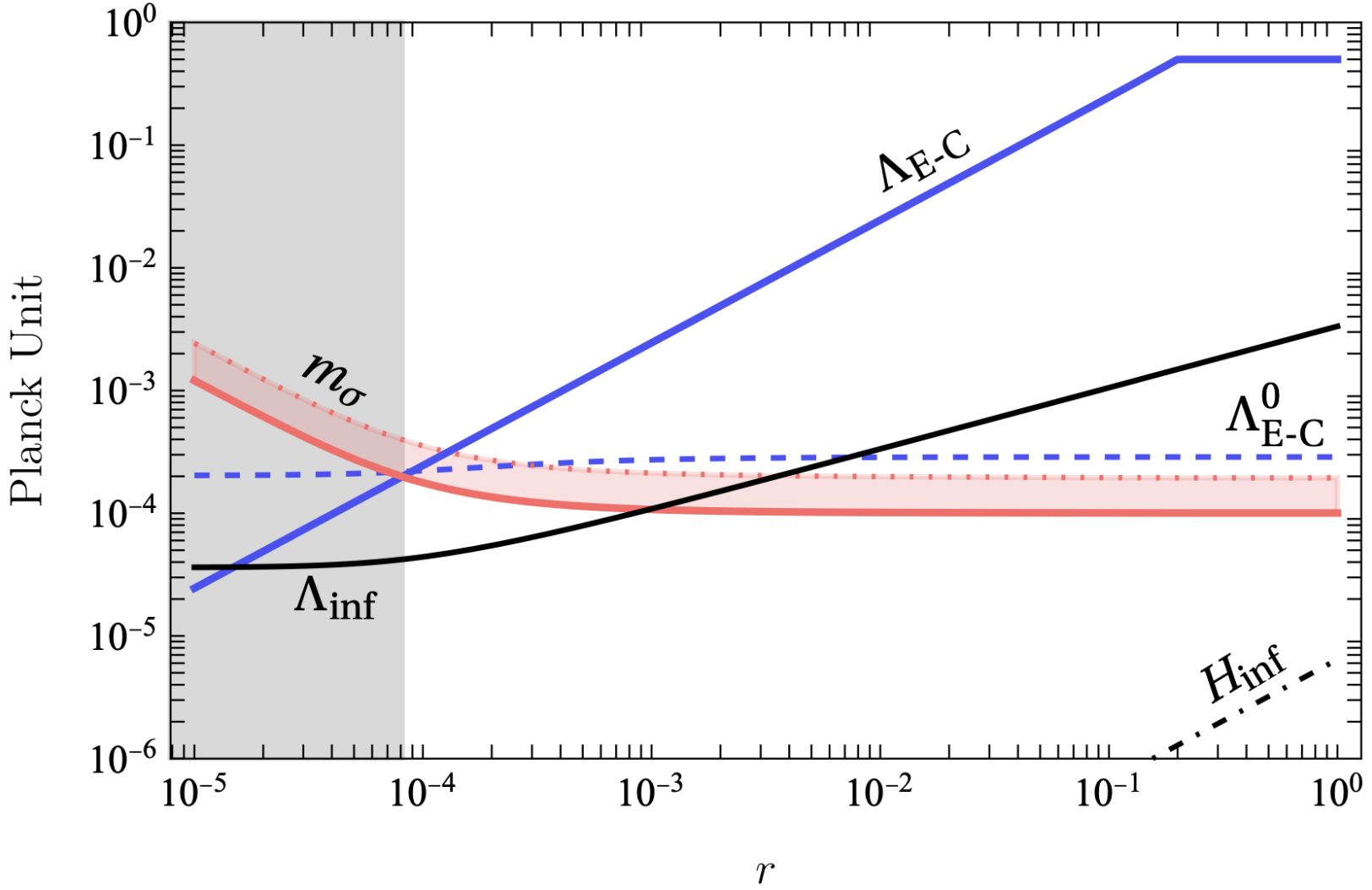
$$S = \int \mathrm{d}^4x\sqrt{-g_\mathrm{E}}\left[\frac{M_\mathrm{Pl}^2}{2}R_\mathrm{E} - \frac{1}{2}g_\mathrm{E}^{\mu\nu}\left(\partial_\mu\sigma_\mathrm{E}\partial_\nu\sigma_\mathrm{E} + e^{-\sqrt{\frac{2}{3}}\frac{\sigma_\mathrm{E}}{rM_\mathrm{Pl}}}\partial_\mu\phi_i\partial_\nu\phi^i\right) - U(\phi,\sigma_\mathrm{E})\right]$$

A mass for scalaron σ, parametrized by r: $m_\sigma^2 = \frac{M_\mathrm{Pl}^2}{12\alpha r^2}$

$$\Lambda_{\mathrm{E-C}} = |R_{N+s}|^{-1/2} = \frac{\sqrt{6}r}{\sqrt{|1-r^2|}}M_\mathrm{Pl}$$

Einstein—Cartan Higgs inflation



Summary

- Higgs as inflaton
  - Unitarity violation
- UV extension by one-loop quantum correction
  - Scalaron flattens (extended) target space
- Einstein—Cartan Higgs inflation
  - A large class of models
  - Systematic UV extension

Thank you for your attention!

Einstein—Cartan Higgs inflation

- Preheating
  - Instantaneous preheating

$$T_{\mathrm{w.b.}} \sim \Lambda_{\mathrm{inf}}$$

Ema, Jinno, Nakayama (2021)

  - Effective mass for NG mode

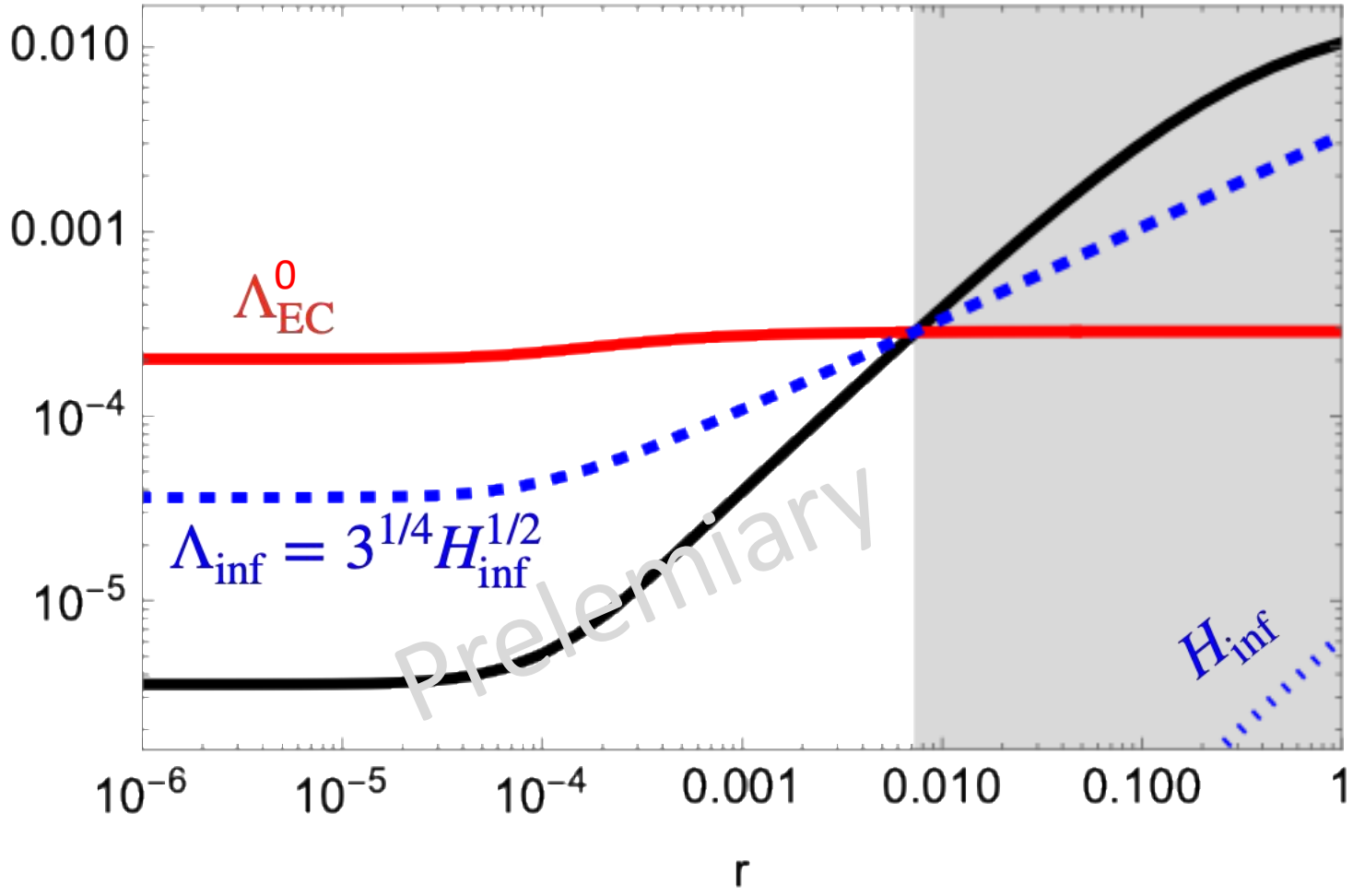
$$m_{\mathrm{eff}}^2 = \square\,\dot{\phi}^2 + \square\,\phi^2$$

$$m_{\mathrm{eff}}^2 \sim \frac{1+3r^2\xi}{M_{\mathrm{Pl}}^2}\xi \times V_{\mathrm{end}} \sim \frac{1+3r^2\xi}{\xi}\lambda M_{\mathrm{Pl}}^2$$

MH, Kohei Kamada, Yusuke Mikura, Kyohei Mukaida, in preparation.

Einstein—Cartan Higgs inflation

Typical momentum of NG modes



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